Assignment
Statistics and Hypothesis Testing

By:

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Question 1:

The quality assurance checks on the previous batches of drugs found that — it is 4 times more likely that a drug is able to produce a satisfactory result than not.

Given a small sample of 10 drugs, you are required to find the theoretical probability that at most, 3 drugs are not able to do a satisfactory job.

a.) Propose the type of probability distribution that would accurately portray the above scenario, and list out the three conditions that this distribution follows.

b.) Calculate the required probability.

Solution:

a. Based on the above problem statement, Binomial distribution will be the best way to find out the probability.

b. Data given in the problem statement is:

No. Of samples , n = 10
No .of un successful drug trails (utmost) , r = 3
Probability of being unsuccessful :
P(not producing satisfactory results) = x
4x+x = 1
X = 1/5

Binomial distribution formula:

$P(X=r) = {}^{n}C_{r}\,(p)^{r}\,(1-p)^{(n-r)}$

We will calculate X =0, 1, 2, 3(because it is mentioned that utmost 3)

$P(X\leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$

$= {}^{10}C_{0}\,(1/5)^{0}\,(1-(1/5))^{(10-0)} + {}^{10}C_{1}\,(1/5)^{1}\,(1-(1/5))^{(10-1)} + {}^{10}C_{2}\,(1/5)^{2}\,(1-(1/5))^{(10-2)}$

$+ {}^{10}C_{3}\,(1/5)^{3}\,(1-(1/5))^{(10-3)}$

$= 1 \,.\, 1 \,.\, 0.11 + 10*0.2*0.134 + 45*0.04*0.1677+120*0.008*0.2097$

$= 0.11+0.268+0.30186+0.20312$

$P(X\leq 3) = 0.8829$

Question 2:

For the effectiveness test, a sample of 100 drugs was taken. The mean time of effect was 207 seconds, with the standard deviation coming to 65 seconds. Using this information, you are required to estimate the range in which the population mean might lie — with a 95% confidence level.

a.) Discuss the main methodology using which you will approach this problem. State all the properties of the required method. Limit your answer to 150 words.
b.) Find the required range.

Solution:

a) I am using Central Limit theorem in this case as it states that sampling distribution of the sample mean approaches a normal distribution as the sample size gets larger. It does not depend on the shape of hte population distribution. This is the fact in case of the sample size is more than 30.
Sampling distibution follows the following properties:
Sampling distribution mean = poppulation distribution mean
Sampling Distribution Standard deviation (Standard Error)= $\sigma/\sqrt{n}$
Given Data :
$\overline{X}$ = Sample Mean
Z* is the Z-score associated with the confidence level (95%)
S = Standard error, sample standard deviation
n = sample size

b) Given Data:
Sample size n =100
Standard error SE = 65 sec
Sample mean μ($\overline{x}$) = 207 sec
Confidence level = 95%
Z* score = 1.96

Since the given sample size is 100 which is greater than 30, the sampling distribution is a normal distribution

Population range = [$\overline{X}$ − {(Z*SE)/$\sqrt{n}$}], [$\overline{X}$ + {($Z*SE$)/$\sqrt{n}$}]

= [207-1.96 ∗ 65/ √100, 207+1.96 ∗ 65/ √100]

= [194.26, 219.74]

Range of μ = (194.26, 219.74)

Question 3:

a) The painkiller drug needs to have a time of effect of at most 200 seconds to be considered as having done a satisfactory job. Given the same sample data (size, mean, and standard deviation) of the previous question, test the claim that the newer batch produces a satisfactory result and passes the quality assurance test. Utilize 2 hypothesis testing methods to make your decision. Take the significance level at 5 %. Clearly specify the hypotheses, the calculated test statistics, and the final decision that should be made for each method.

b) You know that two types of errors can occur during hypothesis testing — namely Type-I and Type-II errors — whose probabilities are denoted by α and β respectively. For the current sample conditions (sample size, mean, and standard deviation), the value of α and β come out to be 0.05 and 0.45 respectively.

Now, a different sampling procedure (with different sample size, mean, and standard deviation) is proposed so that when the same hypothesis test is conducted, the values of α and β are controlled at 0.15 each. Explain under what conditions would either method be more preferred than the other, i.e. give an example of a situation where conducting a hypothesis test having α and β as 0.05 and 0.45 respectively would be preferred over having them both at 0.15. Similarly, give an example for the reverse scenario - a situation where conducting the hypothesis test with both α and β values fixed at 0.15 would be preferred over having them at 0.05 and 0.45 respectively. Also, provide suitable reasons for your choice(Assume that only the values of α and β as mentioned above are provided to you and no other information is available).

Solution:

a) Given Data:
   Sample size n =100
   Standard deviation = 65 sec
   Sample mean $\mu(\overline{x})$ = 207 sec
   Significance level = 95%

Hypothesis: H_0 : μ <= 200 seconds, H_1 : μ > 200 seconds (upper tailed test)

Calculation by Critical Value method:

Standard Deviation :

$\sigma\overline{x}$ = σ /√n

= 65/ √100 = 6.5

Zprobability = 1 – 0.05 = 0.950 Zc = 1.645

UCV = μ + (Zc * $\sigma\bar{x}$) = 200 + (1.645 * 6.5) = 210.69

LCV = μ - (Zc * $\sigma\bar{x}$) = 200 - (1.645 * 6.5) = 189.31

The given sample mean $\bar{x}$ = 207 seconds is lies between the UCV and LCV we cannot reject the Null hypothesis.

Calculation by p-value method:

Given Data:
Sample mean μ($\bar{x}$) = 207 sec
Standard deviation σ = 65 sec
μ = 200 seconds
α = 5% (significance level)

Z score Z = $\bar{X} - \mu$ / (σ /√n) = 207−200/ 6.5 = 1.077 = 1.08

Zprob value can be obtained from the z score tables with correspoding 1.08 and the value is **Zprob = 0.8599**

As the calculated Z score is positive and it is one tailed test

P-Value = (1 - Zprob) * 1

= (1 – 0.8599)

= 0.1401 = 14%

As the calculated P-Value from the above , is 14% which is greater than significance level (α = 5%), So, we cannot reject the NULL hypothesis.

b)

In hypothesis testing, Type-1 (α) error refers to the scenario where we decided to reject the NULL hypothesis even though it was TRUE, and b) Type-2 (β) error refers to the scenario where we failed to reject the NULL hypothesis when it was FALSE

The data given the problem statement can be drawn in to two scenarios with the type-1 and type-2 errors as follows:

	Scenario - 1	Scenario - 2
α	0.05	0.45
β	0.15	0.15

From the above we can conclude that , generally we will keep the type – 1 error to a very low when there is a chance of accepting Alternate hypothesis(H_1).

Type-1(α) error we need to keep to a minimum as if there is any side affects related to the overdose of pain killer drug which means we are tend to reject the null hypothesis. Hene we would like to keep the value of type-1 and type-2 will be 0.05 and 0.45 respectevely.

We can be relaxed as type – 1 error does not lead to the major side effects in the pain killer drug as mentioned. This would mean that we can be more actively changing the current postion ans tend to establish alternate hypothesis , in such cases we will be restricting type-2 error.

Question 4:

Now, once the batch has passed all the quality tests and is ready to be launched in the market, the marketing team needs to plan an effective online ad campaign to attract new customers. Two taglines were proposed for the campaign, and the team is currently divided on which option to use.

Explain why and how A/B testing can be used to decide which option is more effective. Give a stepwise procedure for the test that needs to be conducted.

Answer:

A/B testing is also known as split test where users will be splitted and to test a number of variations of compaign. Simply, A version will be shown to 50% of population and B version shown to remaining 50% of population to get which is performing better among A&B .

Procedure for the A/B test:

Collect Data: First step of the A/B testing is to collect the Data which is performing the poor or great and etc.,

Identify Goals: We need to identify the Goals of the testing like changing the button to selling the product in various apspects. Add the significance value to the Goals and list out the goals and metrics which we are targeting from the A/B testing.

Generate Hypothesis: Once we have identified the Goals and current data which was collected in the step 1 we need to create the hypothesis to get better metrics compared to the current one's.

Create the Variations: Once we completed the priorities and hypothesis , we need to create variations of the same page / product / which ever we would like to see the difference based on the campaigns.

Run Experiment: Kickoff the campaigns for the users with two variations of two categories. Their interation with each page / product or any other should be measured counted and compared with the versions.

Analyze Results: Once the experiment is completed, we need to analyse the data from the experiment / campaign whcih will show the difference with the current statistics either with positive results, negative results or neutral. Such in any case you are free to start from the creating new hypothesis and continue the experiment if required.