Problem Chosen	2024 MCM/ICM Summary Sheet	Team Control Number
A		2425634

Mysterious Inhabitants of the Deep: Lamprey Impacts on Ecosystems and Their Ecological Adaptations

Summary

The lamprey is a wonderful creature. Its sex is not innate, but differentiates between sexes during the larval period based on food abundance. This article studies the advantages and disadvantages brought by the variable sexual differentiation mechanism to the lamprey population itself, as well as its impact on the entire ecosystem.

For question one, we proposed **Model I: ecological interaction dynamics mode**l. In the first step, we were inspired by the **Lotka-Volterra equation**. We used its similar structure to establish a system of differential equations for the small ecosystem, that is, a **stable model**. In the second step, we set the sex ratio as a function of plankton, resulting in a new system of equations called a **dynamic model**. In the third step, we compared the population curves solved by the two sub-models and summarized the impact on the wider ecosystem when the sex (proportion) of lampreys is variable.

For question two, we proposed **Model II: environmental dynamic adaptation analysis model**, which includes two parts: **environmental adaptability index** and **environmental factor impact analysis**. First, we proposed two indicators: reproductive success rate and resource utilization efficiency, and drew the curves of these two indicators on sex ratio; Second, we **simulated** environmental factors by changing the parameters, and analyze the advantages and disadvantages of the lamprey population in adapting to environmental changes.

For question three, we proposed **Model III: ecological diversity and dynamic stability model**, which contains two sub-models: The Shannon-Wiener index and ecological dynamics stability analysis. We start with changing the relative abundance of the population into the proportion of biomass, and then calculate the **Shannon-Wiener Index** to measure species diversity, further we use the **Lyapunov stability theories** about dynamic system to find the equilibrium point of the ecological dynamic system and analyze its stability. Finally, we compared the analysis results of two parts above and came to a consistent conclusion: **the sex ratio variable mechanism ultimately reduces the stability of the overall ecosystem.**

For question four, we **modify the differential equations** based on model one, and redraw the population curves of adult lampreys and parasites to compare the curves under the constant model and the dynamic model—changes, concluding that sex-variable mechanisms can **have a positive impact on parasite populations.**

Keywords: **Lotka-Volterra equation; Dynamic model; Stable model; Dynamic system**

Contents

1 Introduction

1.1 Background

Most species are known to be male and female. Studies have found that although the sex ratio of many species is close to 1:1 at birth, it does not mean that the sex ratio of all species is close to 1:1. Some species will undergo sex differentiation or sex reversal under certain conditions, such as the sex of lampreys changes under the influence of external environment. The U.S. Geological Survey (USGS) and Michigan State University have found that the sex of lampreys depends on their growth rate. If the larva grows slowly, the possibility of becoming male increases.The study also showed that about 78% of lampreys grow into males after three years if their environment is poor for food. By contrast, the number of males rises to 56% if the growing environment is beneficial[1]. This study provides a solution to the troublesome problem of “lamprey invasion”.

Environmentally triggered sex determination and reversals (herein termed sex determination) can lead to highly variable population sex ratios and are important when considering management tactics for valued, invasive, andhatchery-reared fishes[2][3].According to the Great Lakes Fisheries Commission (GLFC), lampreys, which originally lived in Europe and the Pacific Northwest, have invaded the Great Lakes and are regarded as a parasitic pest. Lampreys attach to fish through their mouths and teeth, piercing their sharp tongues into scales to suck blood and body fluids. The lampreys reduce the number of fish in Wumo Lake, which brings billions of yuan in fishing income every year. David Ullrich, a research funder and chairman of the Great Lakes Fisheries Commission (GLFC), said, “Lampreys’ impact on Great Lakes fish bandsIf we can control the number of hazards, we will take a key step in the development of advanced technology.”

The position of lampreys in the ecosystem is complex. In some lake habitats, they can parasitize a variety of fish, resulting in a decline in the number of fish, and are regarded as invasive species with a significant impact on the ecosystem. Lampreys, on the other hand, are very important native food sources in their native areas, such as Scandinavia, the Baltic Sea region, and some indigenous peoples in the Pacific Northwest of North America.Therefore, it is important to study the effect of lampreys sex ratio change on the ecosystem for understanding and controlling lampreys population in different environments.

1.2 Restatement of the Problem

Our task is to focus on variable sex ratios in the lamprey population and its relationship with the local environment, analyzing the advantages and disadvantages of the lamprey's ability to alter its sex ratio depending on resource availability, and revealing the impact of the change of lamprey's sex ratio on an ecosystem.

To investigate this issue, our team has been tasked to develop and examine a model to analyze the interactions that lampreys produce in an ecosystem. The model and its analysis must:

- Account for the interaction of lampreys and other species in an ecosystem, including predation, competition, etc;
- Evaluate the impact on an ecosystem when the sex ratio of lampreys changes;
- Analyze the advantages and disadvantages of the life characteristic that lampreys' variable sex ratios according to the availability of resources for the population of lampreys;
- Assess the effects of changes in the sex ratio of lampreys on ecosystem's stability;
- Consider the benefit of an ecosystem with variable sex ratios in the lamprey population on other species like parasites.

1.3 Literature Overview

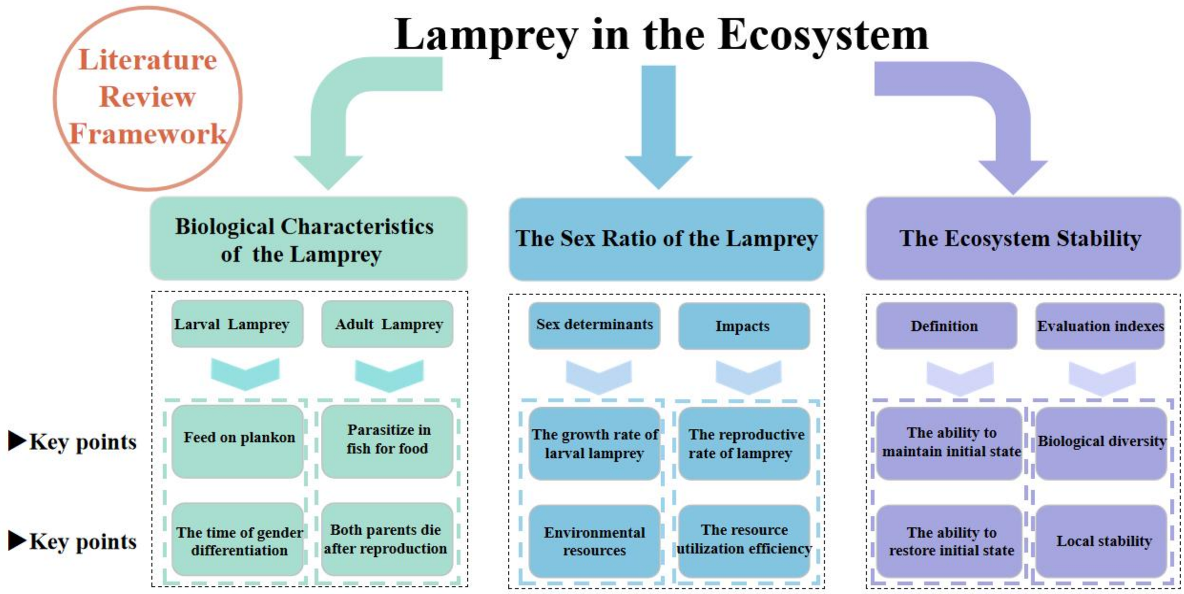


Figure 1: Literature Review Framework

We mainly searched the biological characteristics of the sea lamprey, the sex ratio of the sea lamprey, and the ecosystem stability, and obtained the following effective information:

- **Biological Characteristics of the Lamprey**

Lampreys (Petromyzontiformes) are a key component of freshwater ecosystems throughout temperate parts of the world, It is distributed in northeast China, Europe, northern Asia, North Korea, Japan and North America. To achieve our goals, we need to understand the basic biological information of sea lamprey, with the core contents including survival, foraging, and reproduction[4].Some lampreys live in fresh water for life; some are migratory, live in the sea, grow to freshwater rivers to lay eggs.larval lamprey feed mainly on plankton, and adult lamprey parasitize in almost all variety of fish for food. During the breeding season, the lamprey flock from their habitat to the middle and upper reaches of the river, where the male chooses shallow, fast and gravel waters to build their nest, and the female then arrives and nests together. Figure 1 has showed the life stages of lamprey. Adults spawn in freshwater streams, then the larvae live in sediments, ultimately migrate to the ocean or freshwater lakes. After a life in the ocean they return to streams to spawn [5]. Soon after breeding, all the parents died.

Figure2: A diagram of the life stages of Pacific lamprey

- **The Sex Ratio of the Lamprey**

Sex is determined by chromosomes in mammals and by temperature in many reptiles. But for sea lampreys — eel-like creatures that dine on blood — the growth rate of their larvae seems to control whether they are male or female. They are the first creatures knownto undergo sex determination in this way[6].The U.S. Geological Survey (USGS) and Michigan State University have found that the sex of lampreys depends on their growth rate. If the larva grows slowly, the possibility of becoming male increases.The few studies on Petromyzontiformes suggest that environmentally triggered sex determination occurs and may be influenced by density[7].They found that lamprey in productive streams with lots of food were larger, reached maturity earlier and were more likely to be female. But in unproductive sites, smaller, male lamprey dominated, Johnson’s team reports in a paper published on 29 March inProceedings of the Royal Society B.

- **The Ecosystem Stability**

Ecosystem stability refers to the ability of the system to maintain and restore its initial state after being subjected to external perturbations[8].Biodiversity can improve the system resistance to perturbations[9] or increase the ecosystem reliability[10].The greater the connection in the system, the more unstable the system is, while the high diversity is always associated with the weakened dipartition structure and interaction strength[11].Stability can be divided into local stability, global stability, relative stability, and structural stability. In the equation of state of the system, the change of parameters (perturbation), through the transfer of the transfer matrix, makes this change is reflected in the solution space[12].

1.4 Our Work

Based on the problem analysis and literature overview, we construct three distinctive models—Eco-Interaction Dynamics Model, Environmental Dynamics Adaptation Analysis and Ecosystem Stability Model. Then we study the impact from the variable sex ratios of lamprey on our models. Finally, we conduct sensitivity to test the stablitity of our models. The work we have done is mainly shown in Figure 3.

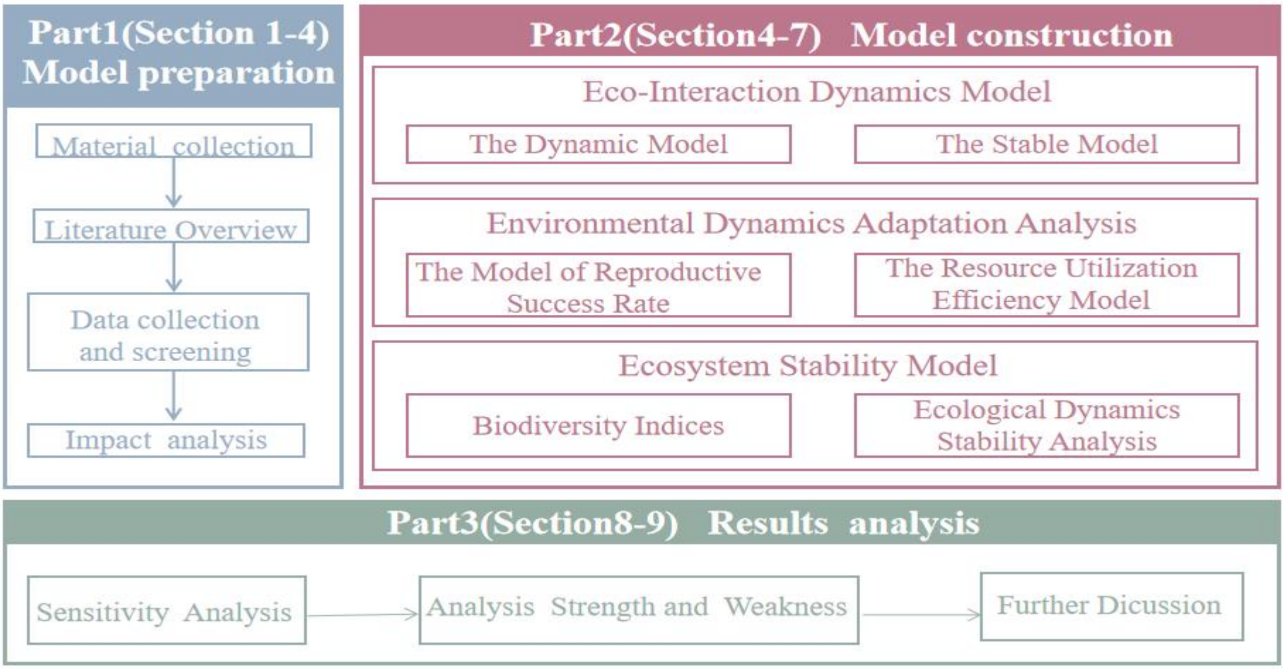


Figure 3: Flow Chart of Our Work

2 Assumptions and Justifications

▼**Assumption 1:** Lampreys have no natural enemies (disregarding humans)

▲ **Justifications 1:** Lampreys are at the top of the Marine ecosystem food chain, have few predators, are very little fed on by humans, and can be considered in the normal mortality of lamprey.

▼**Assumption 2:** The parasite only parasitizes on adult lampreys.

▲**Justifications 2:** The larval lampreys are too small to parasitize on.

▼**Assumption 3：** There are no drastic changes in environmental factors during the forecast period.

▲**Justifications 3:** Keep parameters fixed to reduce complexity of the model.

▼**Assumption 4：** The data obtained in this article are all true and valid, such as population size, natural growth rate.

▲**Justifications 4:** The data is obtained from the sampling survey of the scientific research team obtained from the official website and is guaranteed to be authentic and reliable.

3 Notations

The primary notations used in this paper are listed in Table 1.

Table 1: Notations

Symbol	Description
x_1	Population density of the plankton
x_2	Population density of salmon
y_1	Population density of larval lamprey
y_2	Population density of adult lamprey
y_3	The number of parasite population
r	Sex ratio of sea lamprey adults，that is defined as the probability of female/（male + female）
ρ	The reproductive rate of lamprey
Q	The breeding success rate of lamprey
R	the probability of male/（male + female）
A	Environmental resources
E	the resource utilization efficiency
H	Shannon Diversity Index
s	The Number of species
p_i	Relative abundance of the species i

4 Problem 1: Model establishment and solution

4.1 The Location of Lamprey in an Ecosystem

To account for the interaction of lampreys and other species in an ecosystem, we should firstly know the location of lamprey in an ecosystem. Lamprey is a key component of freshwater ecosystems throughout temperate parts of the world. In an ecosystem, larval lamprey feed mainly on plankton, and adult lampreys both parasitize fish and filter-feed plankton.

4.2 Data Overview

The problem does not provide us with data directly, so we need to consider what data to collect when building the model and what data to collect during the process of building the model. Through the analysis of the problem, we collected the main data in Table 2. Since the amount of data is too large to list them all, it is a good way to visualize the data.

Table 2: Main Data Description and Sources

Data Description	Data Sources
	https://www.gbif.org/dataset/
larval lamprey	Abundances and biological traits of the larval lamprey sampled in the survey of lamprey abundance Indice on the Scorff river (France)
adult lamprey	Phenology and biological traits of migrating river lamprey (Lampetra fluviatilis) sampled by trapping in the Bresle river (France)
salmon	Phenology and biological traits of migrating salmon (Salmo salar) sampled by trapping in the Scorff river (France)
plankton	Inner Oslofjorden Phytoplankton Database

To avoid the interference of more unknown factors in the model and to simplify the model complexity, based on the ecosystem of the Scorff River, we collected data on larval lamprey, adult lamprey, plankton and salmon obtained from sampling surveys of the Scorff River and around it. Among them, larval lamprey feed mainly on plankton. Since the lamprey adults live on almost all kinds of fish, but the data of all kinds of fish is not easy to obtain, here we use the data of salmon as the food for the lamprey for two main reasons: first, the lamprey prefers salmon; second, we choose the data from the sampling data of the Scorff River, which is the site of salmon migration, and the number of salmon is large. Moreover, lamprey are at the top of the ecosystem food chain and have few predators, so species which prey lamprey are not considered here.

4.3 Lotka-Volterra Model

4.3.1 Model Construction

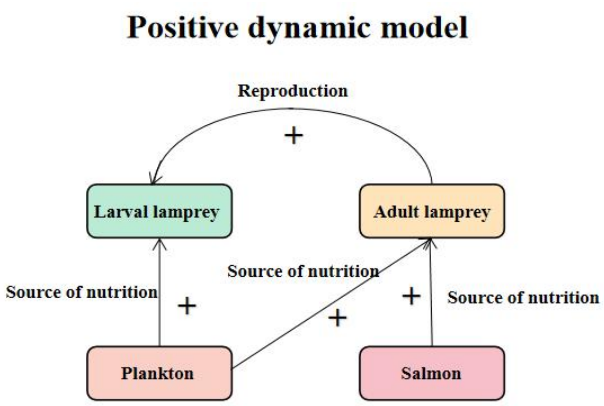


Figure 4: Positive dynamic model and negative dynamic model

To investigate the self-interactions and interactions with other species, we examined the classical Lotka-Volterra model as a basis for considering the predator–prey relationship between sea lamprey and its food such as plankton, salmon. An abstract version of the predator–prey equation is:

$$\frac{dx}{dt} = \alpha x - \beta xy$$

$$\frac{dy}{dt} = \delta xy - \gamma y$$

where α and δ are the growth rates of each of the prey predator species respectively, while β and δ represent the competitive interaction between the two species.

Since the sex of adult lamprey is related to the environmental conditions of larval growth, to study the effect of lamprey sex ratio change on the ecosystem, we added the influencing factor of lamprey to the Lotka-Volterra model and established a variant of the Lotka-Volterra model as follows.

First, we need to understand the interaction between species in the ecosystem when the sex ratio is relatively stable, so we established a Lotka-Volterra model with a stable sex ratio:

$$\frac{dx_1}{dt} = a_1 x_1 - b_{11} x_1 y_1 - b_{12} x_1 y_2 \tag{1}$$

$$\frac{dx_2}{dt} = a_2 x_2 - b_{22} x_2 y_2 \tag{2}$$

$$\frac{dy_1}{dt} = d_{11} x_1 y_1 - c_1 y_1 + k\rho y_2 \tag{3}$$

$$\frac{dy_2}{dt} = d_{21} x_1 y_2 + d_{22} x_2 y_2 - c_2 y_2 - 2\rho y_2 \tag{4}$$

$$\rho = \frac{1}{2}\ \mathrm{r} \tag{5}$$

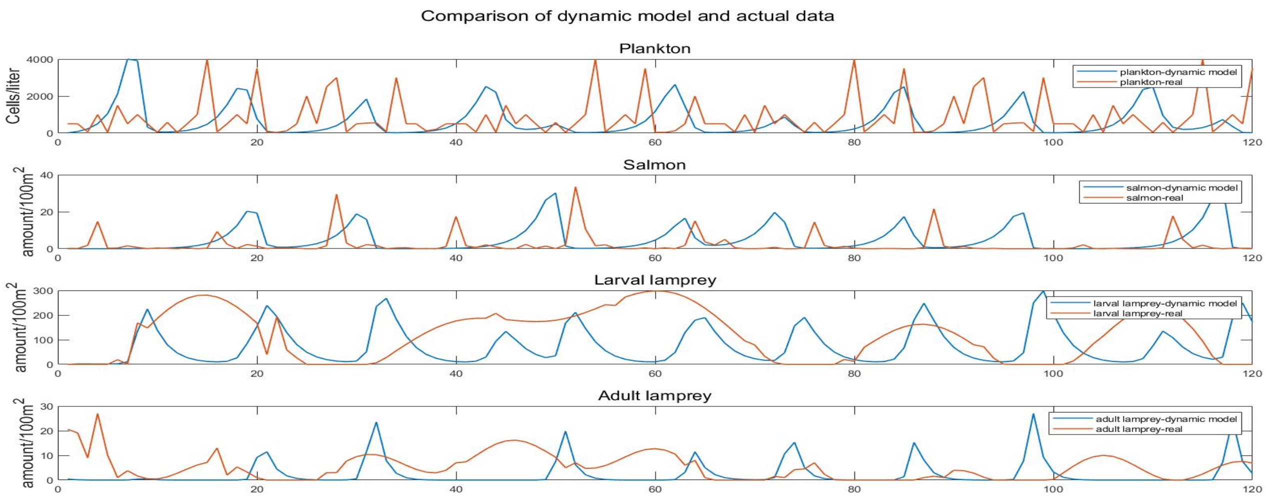
Specifically, a_1, a_2, c_1, c_2 respectively represent the growth rate of the plankton, the growth rate of salmon, the natural mortality rate of larval lamprey and the natural mortality rate of adult lamprey. These parameters are determined by reference to the relevant data, b_{11},b_{12},b_{22} represent the predation rate of the plankton and salmon by larval lamprey and adult lamprey.d_{11},d_{12}, d_{22} represent the hunting rates of larval lamprey and adult lamprey. These parameters were determined through continuous iteration and optimization. In addition, we also reviewed the data and learned that the female individual ratio r of lampreys in the general environment is approximately between 0.2 and 0.6, and set this value to 0.4 to apply to the stable model. Then we modified the model on the basis of the variable sex ratios, as follows:

$$r = \frac{ln(x_1 + 2)}{2} \tag{6}$$

By reading the literature, the sex ratio is determined by the larval lamprey food conditions, the higher the plankton population density, the more environmental resources, the greater the probability of lamprey differentiation into female. What's more, through consulting data that the lamprey sex ratio is generally not higher than 0.5. Inspired by the Logistic model, we established the sex ratio determination model.

4.3.2 Model Solution

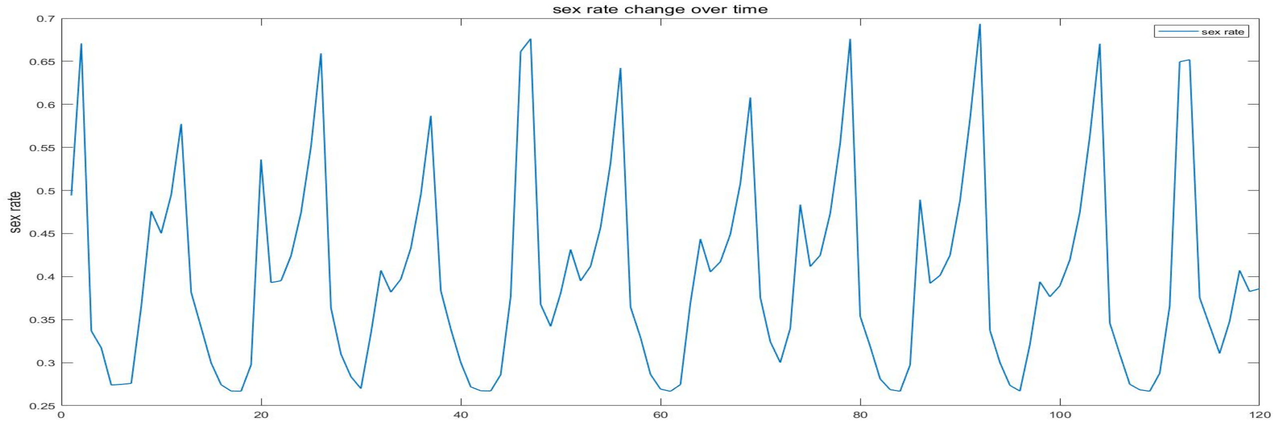
After establishing the model, to verify the accuracy and reliability of the model, we used actually obtained data from the sampling survey of plankton, salmon, larval lamprey, and adult lamprey populations in the Scorff River from January 2012 to December 2021. As shown in Figure 5, both the actual data and the dynamic model show obvious periodicity, and the curve of the model is roughly the same as the curve of the actual data, indicating that the established model can better simulate the interaction relationship between lamprey and other species in the real ecosystem. In addition, the dynamic model is consistent with the performance of the conventional Lotka-Volterra equation, while the actual data fluctuates more sharply than the model, because the reality is also affected by many other environmental variables and stochastic factors.



Remark：The starting month of the x-axis is 2012-01 and the termination month is 2021-12

Figure 5：Comparison of dynamic model and actual data

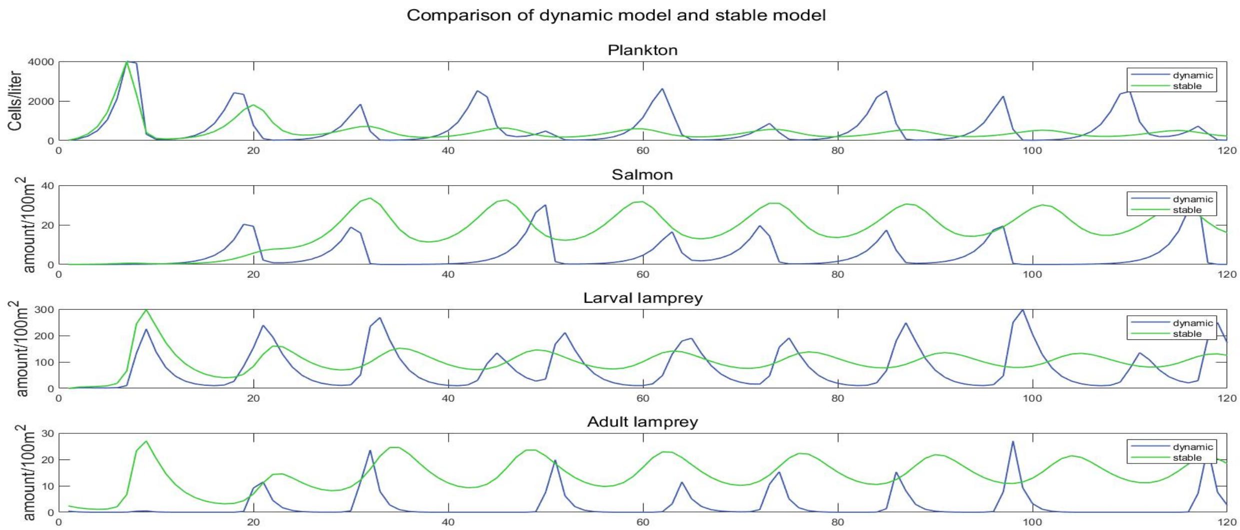
As shown in Figure 6, in the dynamic model, the sex ratio also shows an obvious periodicity because the sex ratio of adult lamprey is correlated with the population of plankton.



Remark：The starting month of the x-axis is 2012-01 and the termination month is 2021-12

Figure 6: Dynamic trend chart of sex ratio

After verifying the reliability and accuracy of the model, we compared this sex ratio dynamic model with the sex ratio stable model, and the results are shown in Figure 7, and from the figure, we have summarized the effects of the variable sex ratios of lamprey on the ecosystem.



Remark：The starting month of the x-axis is 2012-01 and the termination month is 2021-12

Figure 7：Comparison of dynamic model and stable model

4.3.3 Model Analysis

By developing a sex ratio stable model and a sex ratio dynamic model, we summarize the following conclusions:

- **The periodicity of sex ratio:** When the sex of lamprey is variable and determined by resource richness of the larval period, the sex ratio of lamprey is no longer fixed but periodically variable, and the probability of an individual evolving into a female fluctuates roughly between 0.3 and 0.65.

- **Hysteresis:** The quantity curve of adult lamprey fluctuations always slightly lagged behind the salmon quantity curve, and larval lamprey quantity curve fluctuations always lagged slightly behind the plankton quantity curve, which verify that adult lamprey feeds mainly on salmon and larval lamprey mainly on plankton.
- **The periodicity of fluctuations:** Whether in the stable model or in the dynamic model, the quantity curve fluctuations of each species (Here larval lamprey and adult lamprey are regarded as two different species, because they occupy different ecological niches) have obvious periodicity. In the dynamic model, due to the addition of variable sex ratios, different from the stable periodicity of the stable model, the periodicity of the dynamic model is weakened, which turns to be less regular, and the fluctuation period is shortened.
- **Statistical characteristics:** In the dynamic model, the variance of the population of each species is significantly higher than that of the stable model, that is, the fluctuation amplitude of the population in the dynamic model is larger than that of the stable model. On the other hand, except that the average number of plankton increases, the average number of other species is slightly or significantly decreased.

5 Problem 2: Model Establishment and Solution

5.1 The Determination of Lamprey' Sex

Professor Nick Johnson estimated the lamprey sex data from different stream and lentic areas in which sea lamprey were stocked by building a Bayesian hierarchical logistic regression model. The results were found that, as shown in Figure 8,whereas in lentic environments the probability of being male increased slightly over time, in stream environments the probability of being male decreased over time. From these results，and a study evaluating effects of coded wire tagging on larval sea lamprey, they have reached the following conclusions: In environments where environmental traits favor growth and conditions, larval lamprey are more likely to differentiate into females, with sex ratios less skewed toward males. Conversely, the greater the proportion of males in environmentally poor environments.

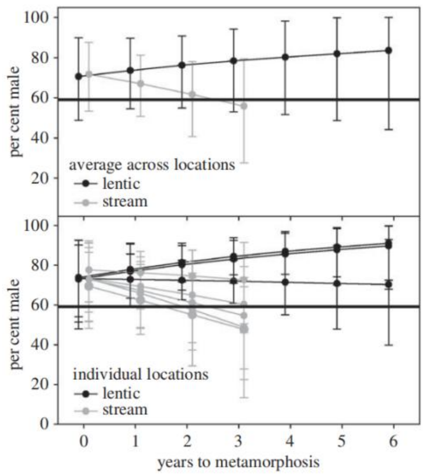


Figure 8: Predictions from the Bayesian hierarchical logistic model
Source:http://dx.doi.org/10.1098/rspb.2017.0262

5.2 Environmental Adaptability Index

5.2.1Model Construction

Since sex ratio is related to population reproduction rate, the relationship between reproduction rate ‘ρ’ and the sex ratio ‘r’ can be derived.

If $r < 1/2$,

Then $\rho = 2r$;

If $r > 1/2$,

Then $\rho=2(1-r)$.

Inspired by Professor Nick Johnson's research, we learned that the sex ratio of lamprey is determined by the amount of environmental resources, and on this basis, we tried to build the model of reproductive success rate and the resource utilization efficiency model:

$$Q = \alpha R(1 - R)A \tag{7}$$

$$E(R) = \beta \frac{R}{1 - R} \tag{8}$$

In the model, α and β are constants, the reproductive success rate of the population is determined by the sex ratio and the amount of environmental resources. The model assumes that reproductive success is proportional to the product of resource amount and sex ratio rate, the rate of resource utilization efficiency is directly proportional to the sex ratio rate. As shown in Figure 9, the curve of reproductive success with sex ratio shows a maximum of reproductive success at some sex ratio rates. This implies that there is an optimal sex ratio that maximizes reproductive success. The curve of the resource utilization efficiency changing with the sex ratio reveals that the sex ratio has a significant impact on the efficiency of resource utilization, and there may be a sex ratio rate that maximizes the efficiency of resource utilization.

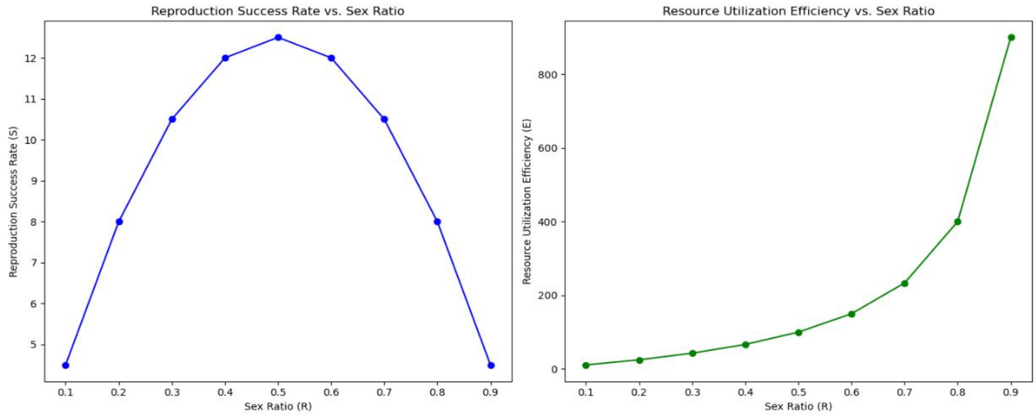


Figure 9: The rate of reproductive success and the rate of resource utilization efficiency

5.2.2 Model Analysis

By establishing and solving the reproductive success model and the resource utilization efficiency model, we conclude the advantages and disadvantages of the sex ratio change on the survival and reproduction of the lamprey population:

Advantages: Different from the total resources, the sea lamprey can adjust the sex ratio of its population according to the growth rate of the larvae, and in the process of adjusting the sex ratio, an optimal sex ratio can be achieved, which can maximize the reproductive success. When insufficient resources or survival difficult, seven lamprey population sex at this time more to male, so male lamprey for natural resources utilization efficiency, resource utilization efficiency with the change of sex ratio curve reveals, sex ratio also has a significant influence on resource utilization efficiency, can adjust the sex ratio make resource utilization efficiency increase, which is conducive to the population growth and survival.

Disadvantages: The sex ratio regulation mechanism does not guarantee the reproductive success and resource utilization efficiency reached the highest value at the same time, when lamprey sex ratio makes his resource utilization efficiency is higher, perhaps his reproductive success is relatively low, to hinder its population reproduction rate, the same, if the reproductive rate reaches the highest, its resource utilization efficiency may be reduced. Therefore, the regulatory mechanisms of the lamprey own population regarding the sex ratio place it in a dynamic equilibrium, maintaining different survival states in different environments.

5.3 Environmental Factor Impact Analysis

5.3.1 Model Solution

In addition to examining the strengths and weaknesses of sex ratio on lamprey populations through the model of reproductive success rate and the resource utilization efficiency model, we have examined the effects of changes in sex ratio on the ecosystem. We simulated the ecosystem changes by adjusting the parameters of the Lotka-Volterra model with dynamic sex ratio established in question 1, and the specific simulation results are shown below.

Case 1: Adjust the growth rate of the plankton('a_1')

Unsensitive to the number of plankton: As shown in Figure 10(a) and Figure 10(b), a_1 decreases slightly (from 0.7 to 0.5), the results are not significantly changed; the succession cycle of the entire ecosystem is significantly longer when the natural growth rate of plankton decreases significantly (decline from 0.7 to 0.2)

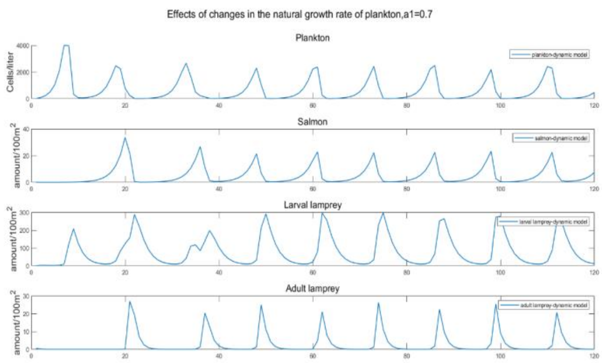


Figure 10(a): Effects of changes in the natural rate of plankton ,a_1=0.7

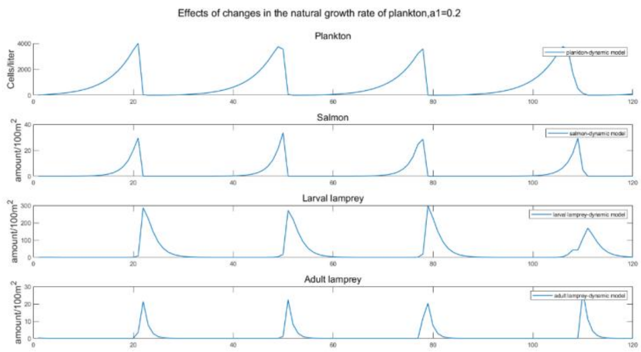
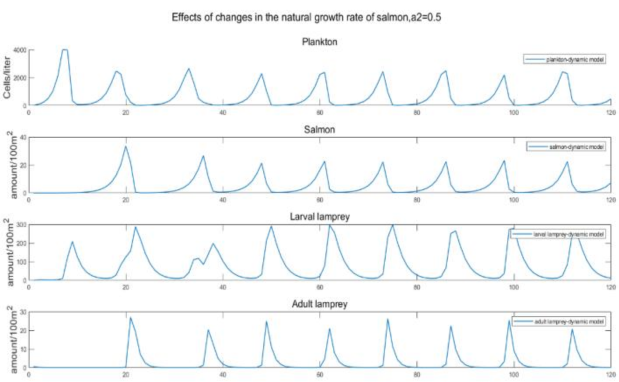


Figure 10(b): Effects of changes in the natural rate of plankton ,a_1=0.2

Case 2: Adjust the growth rate of the salmon('a_2')

Sensitive to fish abundance: As shown in Figure 11(a) and Figure 11(b),when the natural growth rate of salmon decreases slightly (from 0.5 to 0.3), the population succession cycle of adult lamprey became significantly longer, and the average and peak populations of larval lamprey decrease.



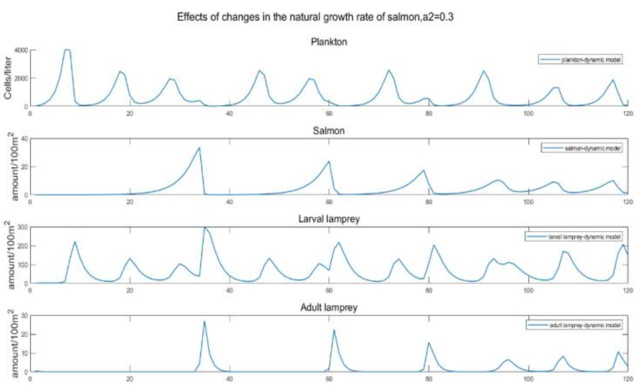


Figure 11(a): Effects of changes in the natural growth rate of salmon ,a_2=0.5

Figure 11(b): Effects of changes in the natural growth rate of salmon ,a_2=0.3

Case 3: Adjust the natural death rate of the larval lamprey('c_1')

Sensitive to the natural death rate of the larval lamprey: As shown in Figure 12(a) and Figure 12(b), the natural death rate of the larval lamprey changes from 0.5 to 0.3, the plankton succession cycle becomes longer and the average number decreases, which leads to the lengthening of the larval and adult lamprey succession cycle.

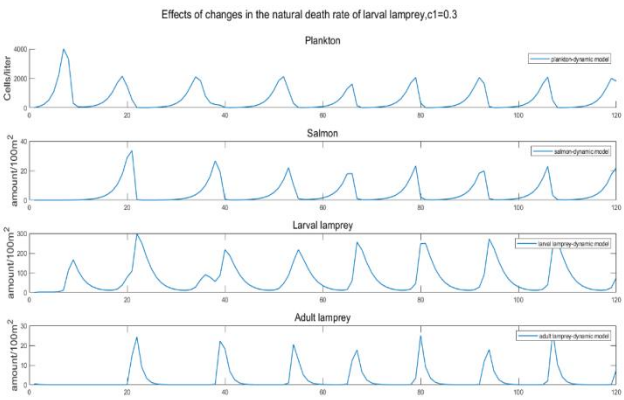


Figure 12(a): Effects of changes in the natural death rate of larval lamprey ,c_1=0.3

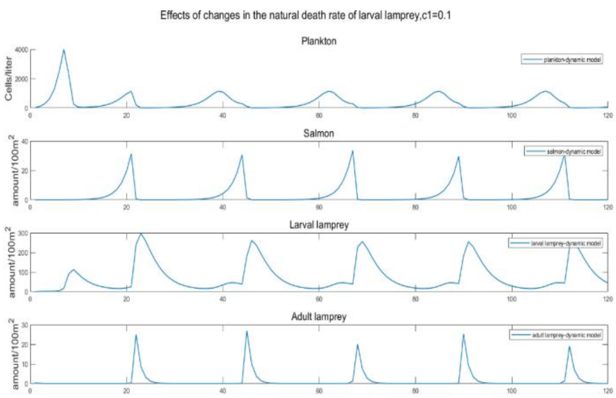


Figure 12(b): Effects of changes in the natural death rate of larval lamprey ,c_1=0.1

Case 4: Adjust the natural death rate of the adult lamprey('c_2')

Unsensitive to natural mortality of adult lamprey: As shown in Figure 13(a) and Figure 13(b), c_2 changes from 0.5 to 0.3 and then to 0.1,there are no significant changes in the four succession curves ,which indicate that the main source of death for adult lamprey is mating.

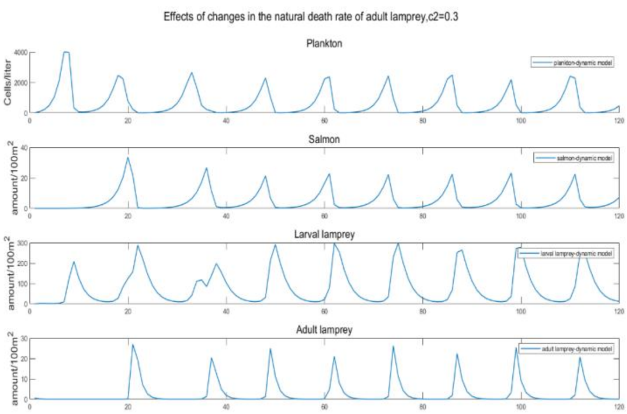


Figure 13(a): Effects of changes in the natural death rate of adult lamprey ,c_2=0.3

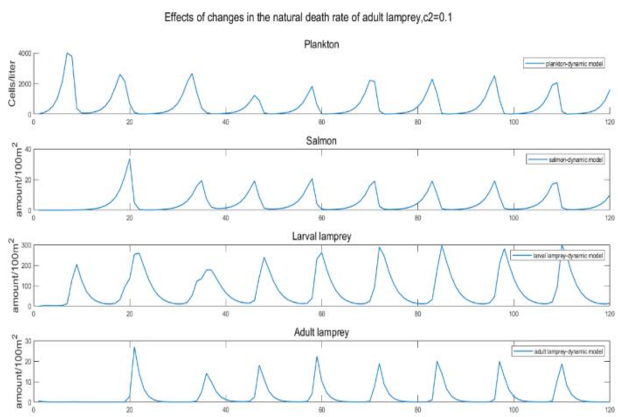


Figure 13(b): Effects of changes in the natural death rate of adult lamprey ,c_2=0.1

Case 5: Adjust The reproductive rate of lamprey('ρ')

Sensitive to reproduction coefficient: As shown in Figure 14(a) and Figure 14(b), reproduction coefficient ρ is set as a multiple of sex ratio ‘r’， which is approximately understood as the reproductive rate of lamprey (the proportion of individuals involved in reproduction in adult lamprey). When $\rho = 2r, r\ 0.5r$ is set respectively, the succession curve of the whole ecosystem has changed significantly.

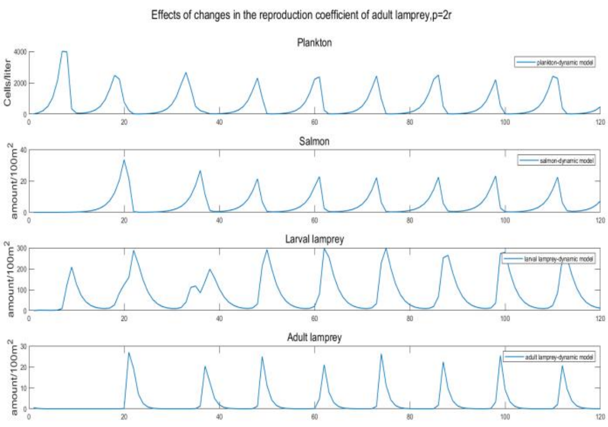


Figure 14(a): Effects of changes in the reproductive rate of adult lamprey , $\rho = 2r$

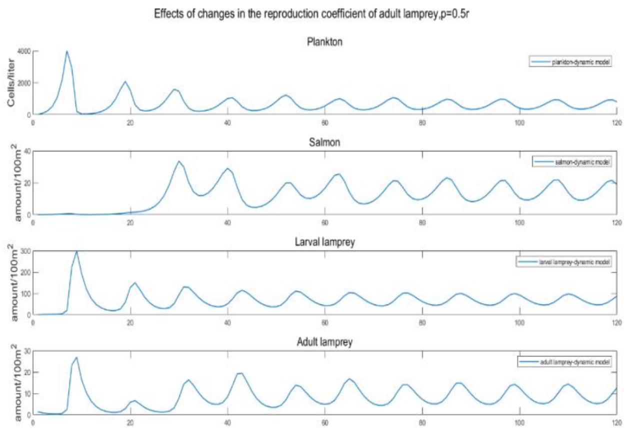


Figure 14(b): Effects of changes in the reproductive rate of adult lamprey , $\rho = 0.5r$

5.3.2 Conclusions

In general, the advantage of to the population of lampreys is that it is not sensitive to the number of plankton and the adult lamprey, and can better maintain the breeding trend and stability of the population. However ,the disadvantage to the population of lampreys is that it is sensitive to the death rate of larvae, the number of fish, if the hatching and surviving fertilized eggs decrease, or the number of fish decreased, the reproduction curve of the population is easy to decline rapidly.

6 Problem 3: Model Establishment and Solution

6.1Assessment and measurement indicators of ecosystem stability

This issue has been hotly debated ever since the diversity-stability theory was proposed in the fifties. The initial stability was simply defined as the change in population density in a community or ecosystem, and later it relates to resistance, local stability and global stability. From the extension of stability, stability can be divided into local stability (also known as adjacent stability), the system can return to the original equilibrium point, but after large disturbance, the system can not return to the original equilibrium point, the equilibrium point is called local stability. After the system receives large perturbations, the system is far away from the equilibrium point, but can still return to the original equilibrium point, the stability of the balance point is called global stability.

The first way to analyze the stability is to measure the biodiversity, inspired by reading the literature, we use species richness to measure it. More specifically, through the improved Shannon ecological diversity index. The second method is to judge Lyapunov stability. It is necessary to find the equilibrium points of the dynamic system and the Jacobian determinants at these points, and judge the local stability of the point through the positive and negative nature of its eigenvalues.

6.2 Shannon Diversity Index

We use Shannon Diversity Index to measure the indicator of biodiversity.

6.2.1 Background

Introduced by the concept of information entropy in information theory, Shannon Diversity Index is an index that considers the number and relative abundance of species. When Shannon Diversity Index increases, biodiversity is richer, because it considers the number of species and their relative proportions in the ecosystem, making the index more sensitive to uniformity in the ecosystem.

$$\mathrm{H} = -\sum_{i=1}^{s} \left(p_i \cdot \ln\left(p_i\right)\right) \tag{9}$$

The larger H is, the higher the species diversity in the community, that is, the species are more evenly distributed in the community; when H is smaller, the relative abundance of species is less evenly distributed, and some species may dominate.

6.2.2 Model construction and solution

Since several organisms we studied are about organisms of different levels in the lamprey biological chain that cannot be measured by the number of individuals, we should apply biomass to calculate its relative abundance.

$$\text{Relative abundance } = \left(\frac{\text{Biomass of a specific species}}{\text{The total biomass of the entire community}}\right) \times 100\% \tag{10}$$

In the process of calculating the biomass, we converted the units. According to the collected data, according to the reference, the average depth of Scorff River is 2.5 m. We convert the plankton's unit “cells / lines” into “individuals/100 square meters of water area”, from this, the relative abundance is calculated, and the following Figure 15 is drawn according to the calculation formula of Shannon Diversity Index.

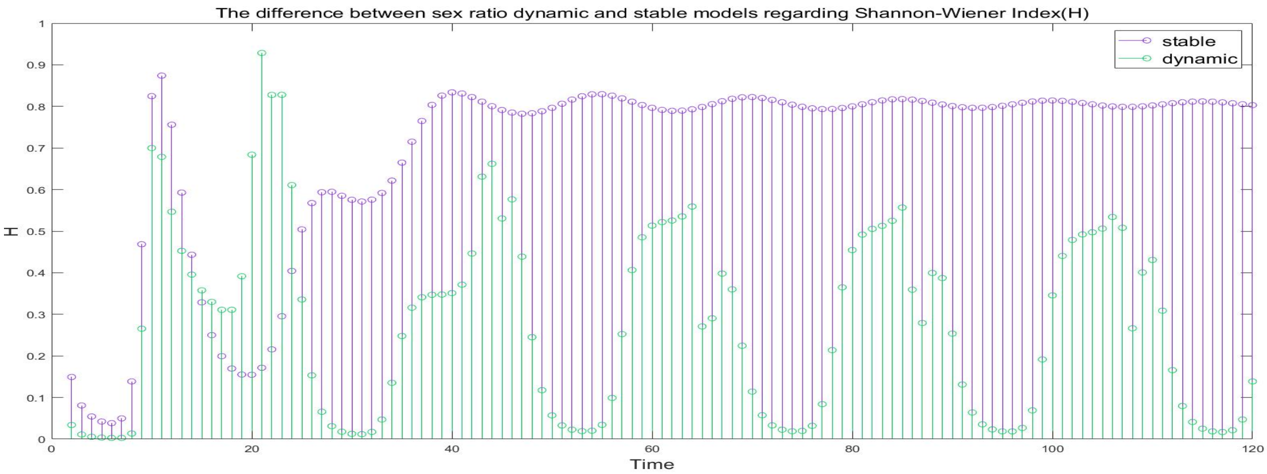


Figure 15: The difference between sex ratio and stable models regarding Shannon-Wiener Index(H)

6.3 Equilibrium points and Lyapunov stability of dynamical systems

6.3.1 Lyapunov stability

For the local stability of the ecosystem, we first analyze the model equilibrium point,

$$\begin{gathered}\frac{d\mathbf{x}}{dt} = \mathbf{f}(t, \mathbf{x}), \mathbf{x} \in \mathbb{R}^n \\ \mathbf{f}(t, \tilde{\mathbf{x}}) = 0\text{"is true for any "t"",} \\ \text{"Then "}\tilde{\mathbf{x}}\text{" is called the equilibrium point of this differential equation".}\end{gathered} \tag{11}$$

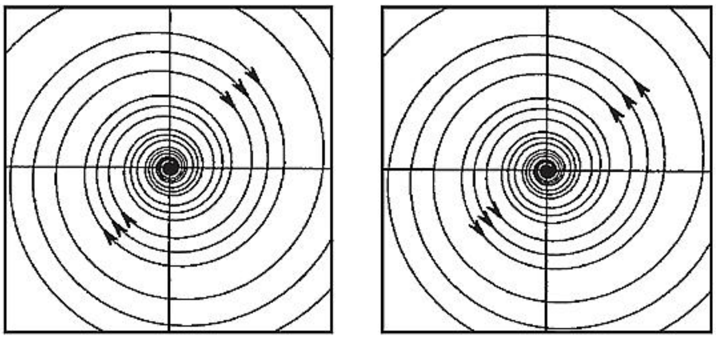


Figure 16: Phase diagrams of a dynamic system

Left picture: Stable point. After the power system reaches this point, it will fluctuate or vibrate slightly near this point for a long time; Right picture: Unstable point. After the power system passes this point, it will oscillate violently and be in an unstable state.

In this study system, the four differential equations representing Lotka-Volterra (1),(2),(3),(4), We can get four balance points based on each different sex ratio. Next, we construct the Jacobian matrix of the equilibrium point and use the four organisms as the state variables of the model $\mathbf{x}= [x_1, x_2, y_1, y_2]$. Calculate the partial derivatives of this set of equations with respect to each state variable to form a matrix which is the Jacobian matrix. Computing the eigenvalues of the Jacobian matrix, which provide information about the stability of the system near the equilibrium point. An equilibrium point is stable if the real parts of all eigenvalues are negative.

6.3.2 Result Analysis

Through calculation, we found that the eigenvalues of most of the Jacobian matrices in the equilibrium point are not all positive or not all negative, so we think it is a meaningless equilibrium point. When r>0.23, the stable point always is (396, 28.9, 104.9, 13). The system can tend to be stable and fluctuate up and down at the stable point. When r<0.23, there is an unstable equilibrium point (49.3, 16.9, 1.88, 0). After this point, the system is no longer stable, tends to collapse, and eventually the number of each species drops sharply.

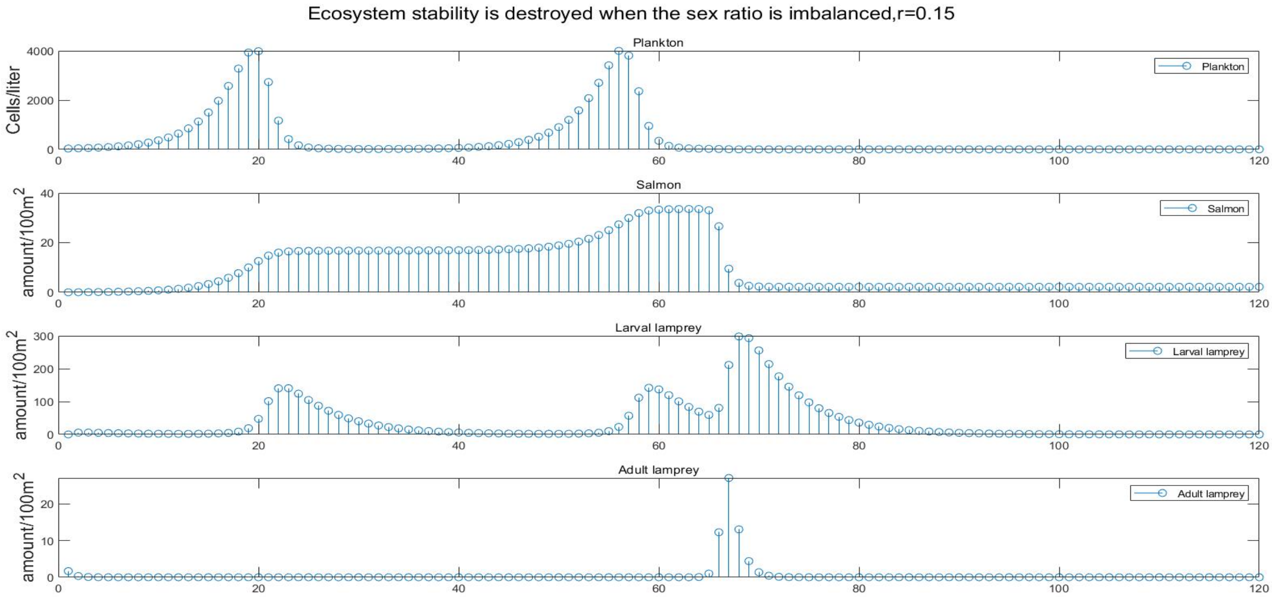


Figure 17: Ecosystem stability is destroyed when the sex ratio is imbalanced, r=0.15

Figure 17 shows that at the fortieth month, when the sex ratio r is 0.15, each species in the ecosystem reaches a balance point. The number of species in the system first increases and then drops sharply, approaching zero, and the ecosystem is on the verge of collapse.

6.4 Conclusions

Based on the results of the Shannon Diversity Index, we know that in ecosystems where the sex ratio of lamprey populations changes, their biodiversity fluctuates, and species richness is lower than in ecosystems where the sex ratio remains unchanged. When unchanged, the biodiversity index is more stable.

For local stability, we look for the equilibrium points of the four differential equations of Lotka-Volterra. According to the eigenvalue analysis of the Jacobian matrix of each equilibrium point, when the sex ratio changes, except when there is a stable point, There may be a certain sex ratio that throws the ecosystem out of balance and collapses.

7 Problem 4: Model Establishment and Solution

7.1 Model Construction

In order to analysis if an ecosystem with variable sex ratios in the lamprey population can offer advantages to others in the ecosystem, such as parasites,we add the parasite population to the Lotka-Volterra model with a stable sex ratio and the Lotka-Volterra model with a dynamic sex ratio established in problem 1.Assume the parasite lives exclusively in adult lamprey, we modify the models established in problem 1, formula(1),(2),(3),(5) remain unchanged, modify formula (4) to formula (12), and add a new formula (13). d_5 represents parasitism rate in adult lamprey, c_5 represents the natural death rate of parasite.

$$\frac{dy_2}{dt} = d_{21}x_1y_2 + d_{22}x_2y_2 - c_2y_2 - b_5y_2xy_3 \tag{12}$$

$$\frac{dy_3}{dt} = d_5y_2y_3 - c_5y_3 \tag{13}$$

7.2 Model Solution

Separately solve the modified dynamic model and the modified stable model, and plot the population curves, we can contrast them and get the following results:

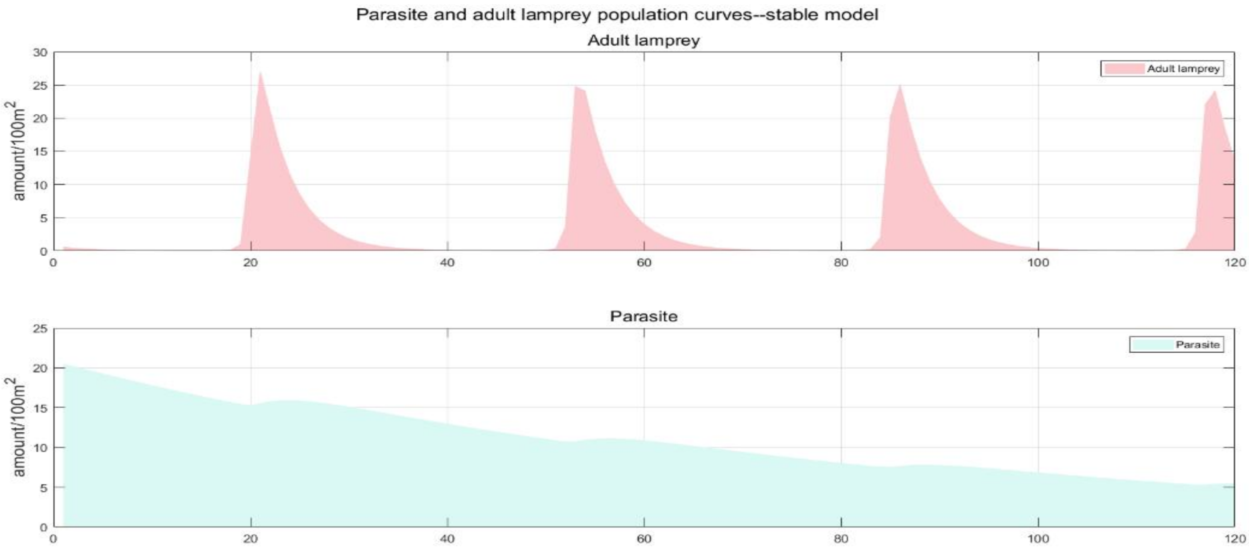


Figure 18: Parasite and adult lamprey population curves-stable model

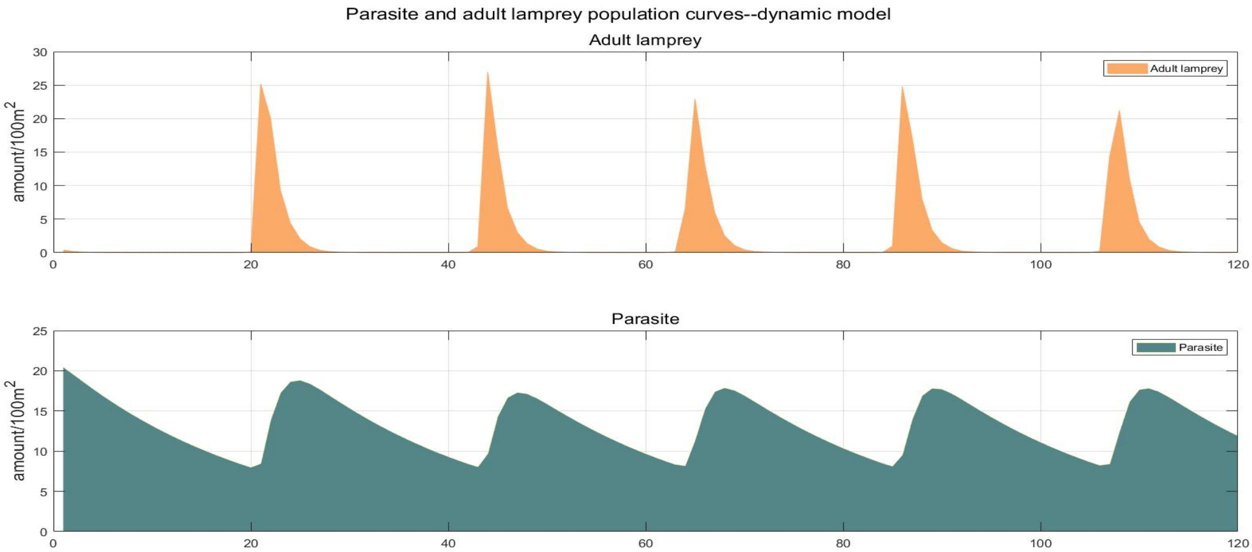


Figure 19: Parasite and adult lamprey population curves-dynamic model

From Figure 18 and 19, it can be seen that the average number of parasite population in the dynamic model is higher than that in the stable model. In the stable model, the number of parasite population generally shows a downward trend. Therefore, the variable sex ratios of lamprey will have a beneficial impact on the parasite population.

8 Sensitivity Analysis

In our model, the natural growth rate a_1,a_2,the natural death rate c_1,c_2 and the reproductive rate ρ are all extremely important coefficients that are affected by the environment. These coefficients in our model are mainly obtained by checking the data, so to ensure the accuracy of the model, it is necessary to conduct sensitivity tests on the above coefficients.

Take a_1 as an example, the value of 0.6 (that is, the natural growth rate of plankton is 60%) is applied in the model. We adjust a_1 to 0.6-0.05 and 0.6+0.05 respectively, calculate the new adult lamprey population curve, and plot the difference between the new curve and the previous curve of a_1=0.6. The results in Figure 20 show that in both cases the absolute value of the deviation function is limited within 3, that is, the relative deviation is limited to within 10%, which is in the acceptable deviation range. The analysis methods and results for a_2, c_1,c_2, ρ are similar, which prove that our model has good stability under interference.

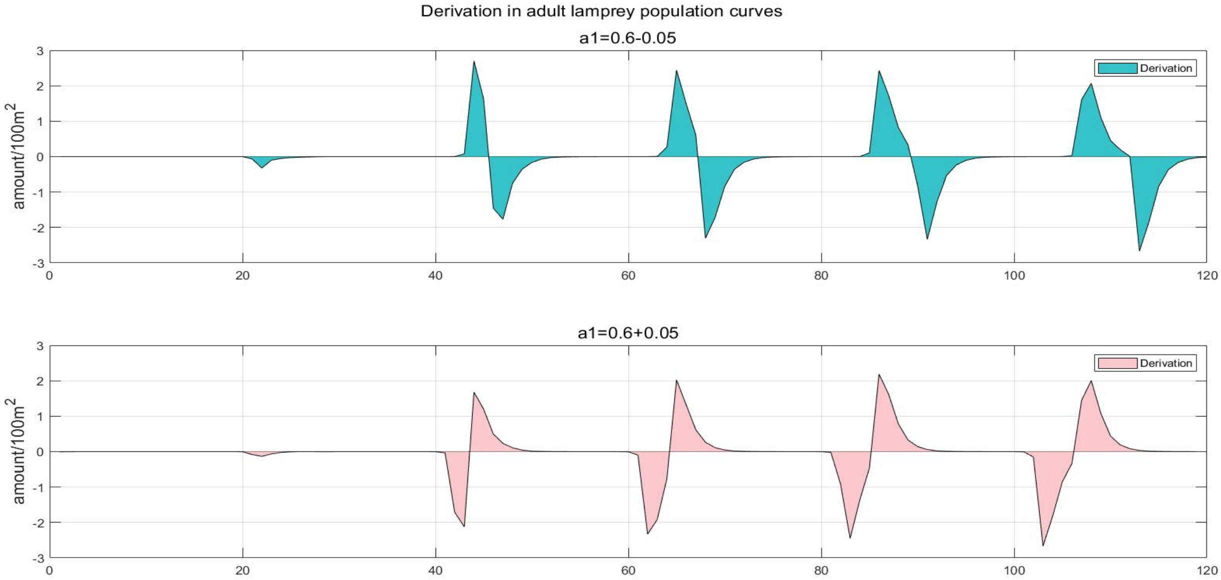


Figure 20: Derivation in adult lamprey population curves, a_1 =0.6±0.05

9 Model Evaluation and Further Discussion

9.1 Strengths of the Model

- **The Lotka-Volterra model initially expresses ecological interactions and explores cyclic fluctuations.**

It predicts changes in the relative sizes of predator and prey populations over time, and explores possible cyclic fluctuations between predators and prey. The predator has a lag compared to the prey's fluctuations, and the population curve shows good periodicity. Consist with the results of the L-V equation, and is a reasonable improvement and extension of the L-V equation.

- **The Shannon index has mathematical clarity and sensitivity to species richness.**

Its calculation is based on the mathematical principles of information theory, which makes its definition very clear and operational. Not only the abundance of species is taken into account, but also the evenness of those species. This makes it more sensitive relative to the distribution of various species in the community, providing a more comprehensive assessment of diversity.

- **The equilibrium point stability analysis model reflects the determination of system stability and the understanding of local dynamics.**

The behavior of the system near the equilibrium point can be predicted, which has important implications for understanding the long-term evolution of ecosystems and responses to external perturbations.

9.2 Weaknesses of the Model

- **In our model, adult lampreys are driven by eating plankton and salmon, not from larval lampreys.**
- **The model does not specifically discuss the impact of changes in environmental factors, and it does not consider competition and substitution, the food chain structure is too simplistic.**

9.3 Further Discussion

- For weakness 1, the better model is to set the intake of plankton and salmon as penalty terms, the greater the intake, the smaller the value is, which is always negative. And set that the growth momentum of adult lampreys comes from the number of juveniles, which is written in the form of the delay differential equation.
- For weakness 2, factors such as competition, interaction and human influence are considered based on the original model.

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