problems

Solutions :-

1) Given

$$x(n) = \begin{cases} 1+\frac{n}{3}, & -3 \le n \le -1 \\ 1, & 0 \le n \le 3 \\ 0, & \text{elsewhere} \end{cases}$$

a)

$1+\frac{n}{3}$, $-3 \le n \le -1$ $\Rightarrow$ at $n = -3 \Rightarrow 0$

$n = -2 \Rightarrow \frac{1}{3}$

$n = -1 \Rightarrow \frac{2}{3}$

from $0 \le n \le 3$; 1

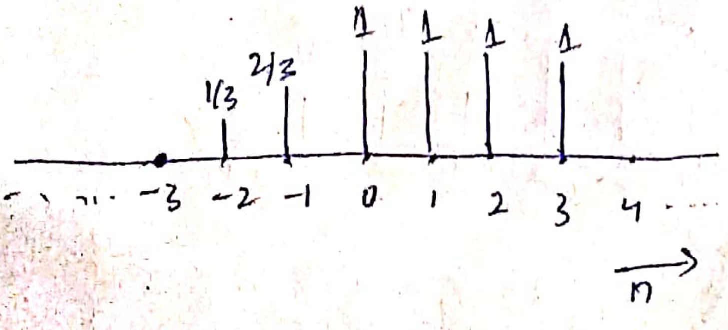
elsewhere, 0

So $x(n) = \{ \ldots\ldots 0, 1/3, \frac{2}{3}, \underset{\uparrow 0}{1}, 1, 1, 1, 0, \ldots \}$

b)

1) First fold x(n) and then delay the resulting by four samples.

$x(n)$



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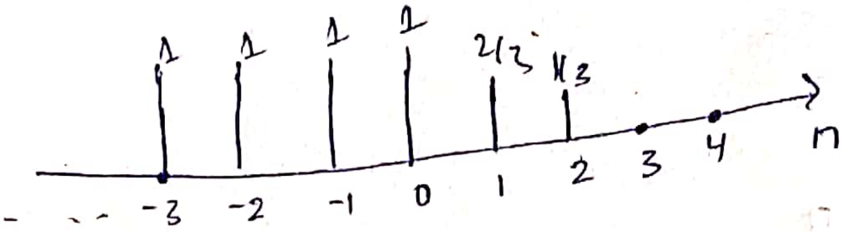
Folding $x(-n)$

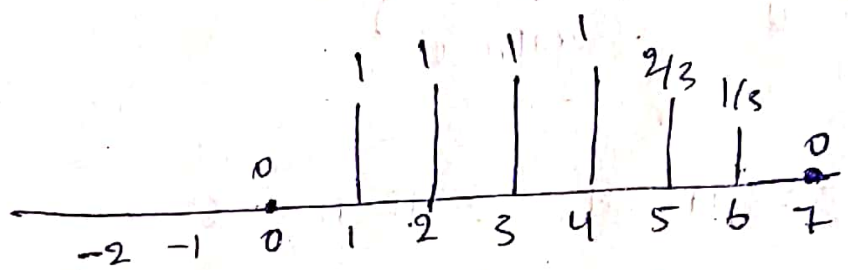


Delaying by four samples $x(-n+4)$

$-3 \le -n+4 \le 2$

$-7 \le -n \le -2$

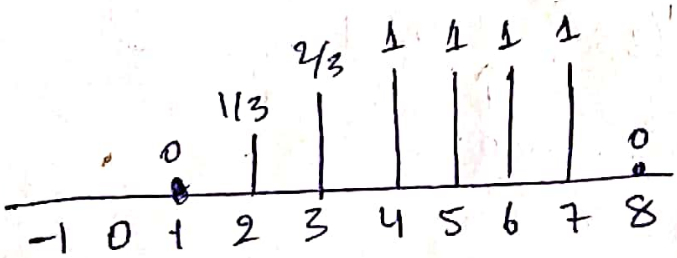
$7 \ge n \ge 2$

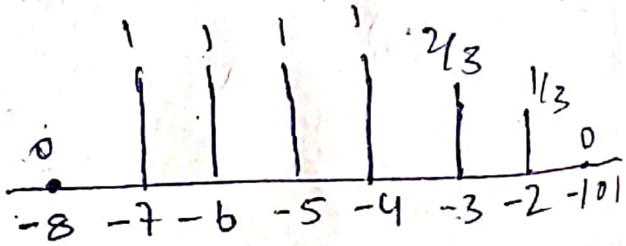


2) First delay $x(+n)$ by four samples and then fold the resulting signal

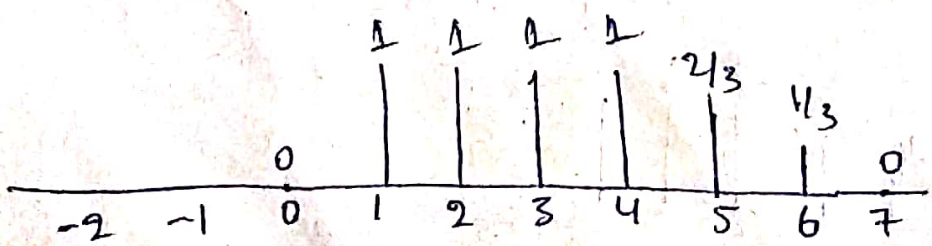
$x(-n-4)$

$x(n-4)$





c) $x(-n+4)$



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d) By comparing results in parts (b) and (c) to get $x(-n+k)$ from $x(n)$ first we need to fold $x(n)$ which is $x(-n)$ and shift by k samples to right if $k>0$ or to left if $k<0$ results in $x(-n+k)$

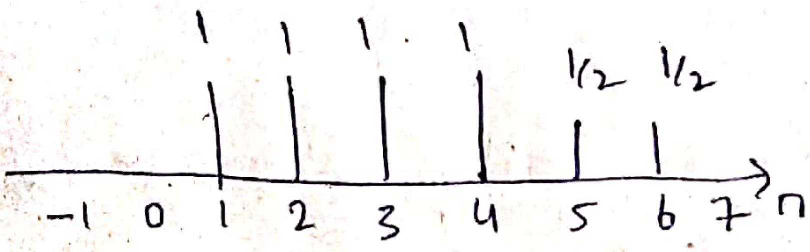
e) Expression

Yes; $x(n) = \frac{1}{3}\delta(n-2) + \frac{2}{3}\delta(n-1) + u(n) - u(n-4)$

2) Given

$x(n)$

a) $x(n-2)$

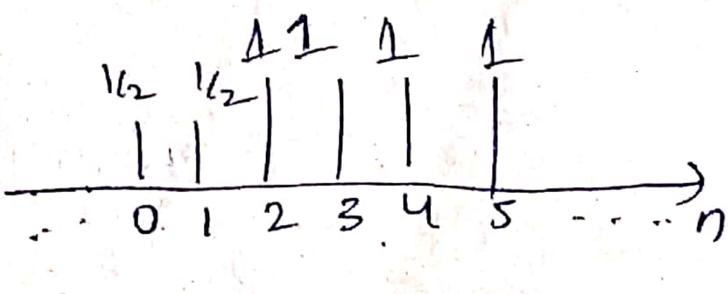


b) $x(4-n)$

$-1 \le 4-n \le 4$

$-5 \le -n \le 0$

$5 \ge n \ge 0$



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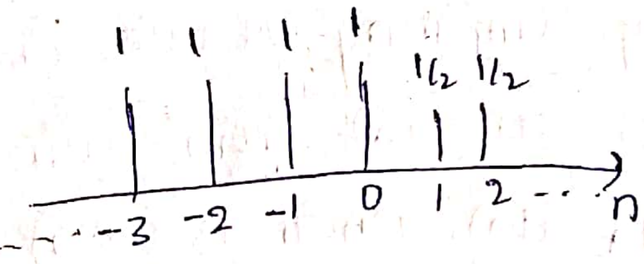
c) $x(n+2)$

$-1 \leq n+2 \leq 4$

$-3 \leq n \leq 2$

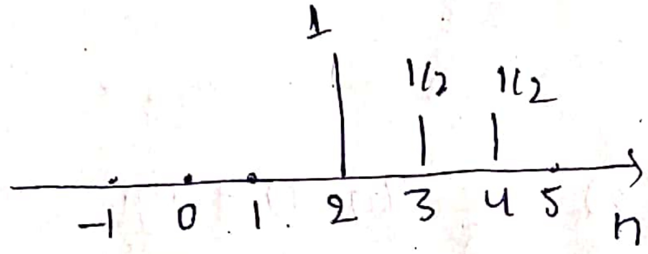


d) $x(n)\, u(2-n)$ $\quad u(2-n) \Rightarrow 1$ $\quad 2-n \geq 0$

$-n \geq -2$

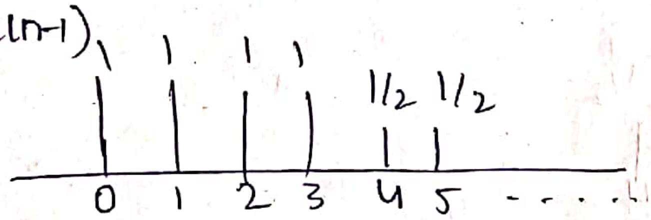
$n \geq 2$

$\therefore$

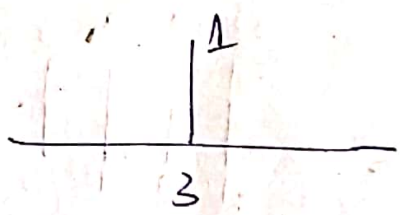


e) $x(n-1)\, \delta(n-3)$

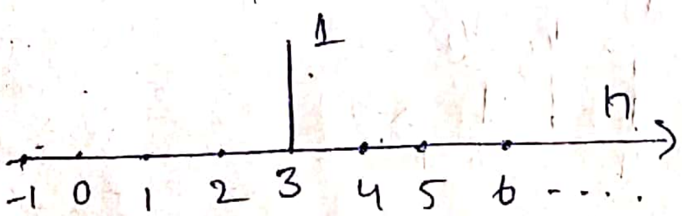
$\therefore x(n-1)$



$\delta(n-3)$



$x(n-1)\; \delta(n-3)$



f) $x(n^2) \Rightarrow x(n) = \{x(-2), x(-1), x(0), x(1), x(2), x(3), x(4), \ldots\}$

$x(n^2) = \{x(4), x(1), x(0), x(1), x(4), x(9), x(16), \ldots\}$

$= \{\ldots \frac{1}{2}, 1, 1, 1, \frac{1}{2}, 0, 0 \ldots\}$

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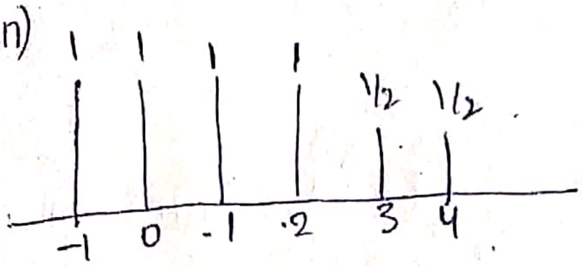
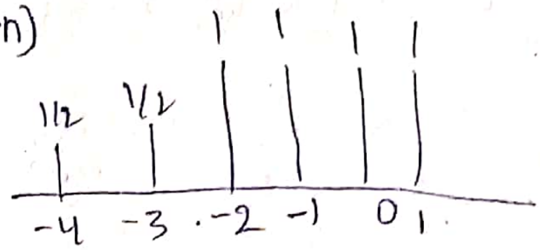
g) even part of $x_e(n) = \frac{x(n) + x(-n)}{2}$

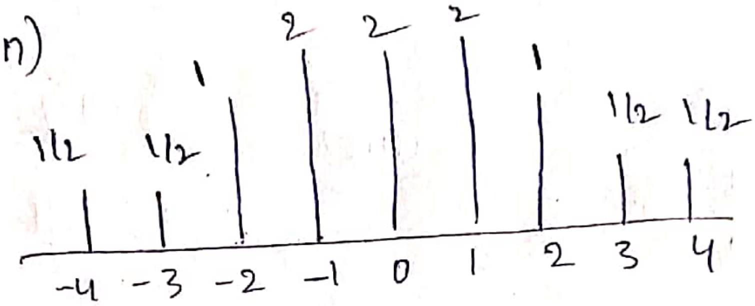
$x(n)$



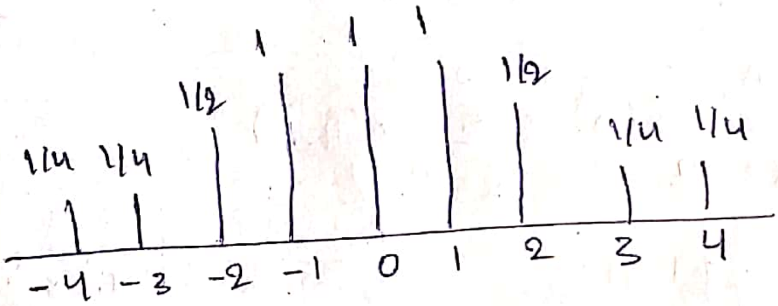
$x(-n)$



$x(n) + x(-n)$

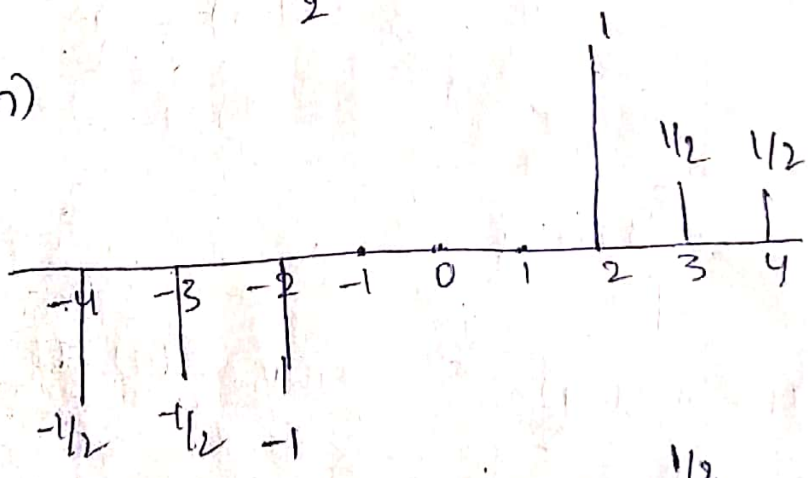


$\frac{x(n) + x(-n)}{2}$

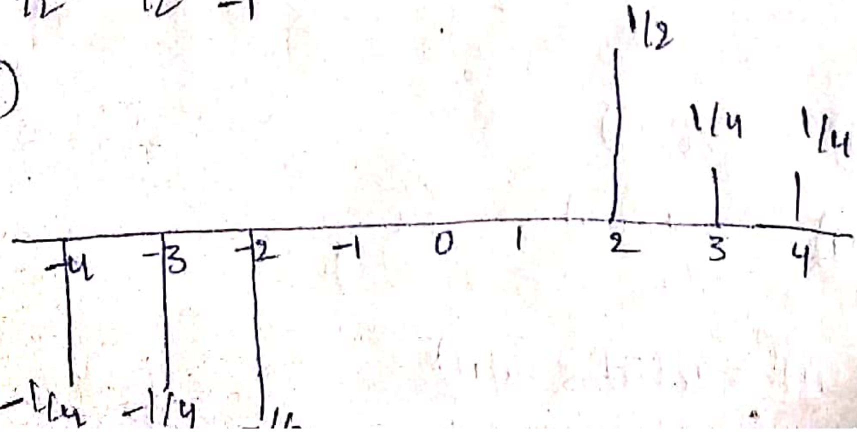


odd part $x_o(n) = \frac{x(n) - x(-n)}{2}$

$x(n) - x(-n)$



$\frac{x(n) - x(-n)}{2}$



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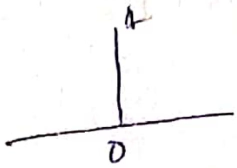
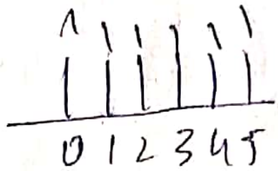
3) $\delta(n) = u(n) - u(n-1)$

a) we know that $\delta(n) =$



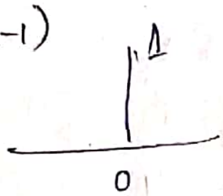
$u(n) =$



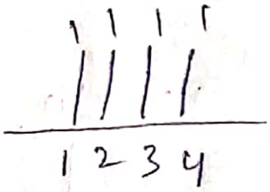
$\left\{ u(n) = \{1, n \geq 0 \right.$

$u(n) - u(n-1)$

$\Rightarrow$



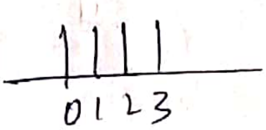
$u(n-1) =$



$\therefore \delta(n) = u(n) - u(n-1)$

b) $u(n) = \sum_{k=-\infty}^{n} \delta(k) = \sum_{k=0}^{\infty} \delta(n-k)$

$u(n) =$



$\Rightarrow \sum_{k=-\infty}^{n} \delta(k) = u(n) = \begin{cases} 0 ; n < 0 \\ 1 ; n \geq 0 \end{cases}$

$\sum_{k=0}^{\infty} \delta(n-k) = \begin{cases} 0, n < 0 \\ 1, n \geq 0 \end{cases}$

4) $x(n) = \{2, 3, \underset{\uparrow}{4}, 5, 6\}$

$x_e(n) = \dfrac{x(n) + x(-n)}{2}$ $\qquad x_o(n) = \dfrac{x(n) - x(-n)}{2}$

$x_e(n) = x_e(-n)$

$x_o(n) = -x_o(n)$

$\Rightarrow x(n) = x_e(n) + x_o(n)$

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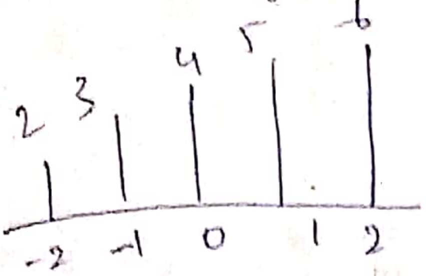
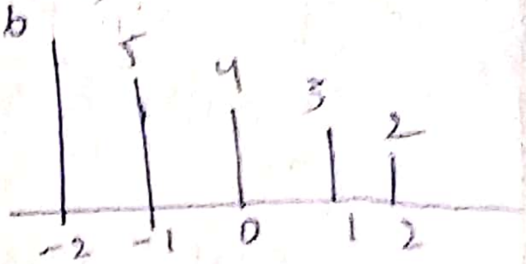
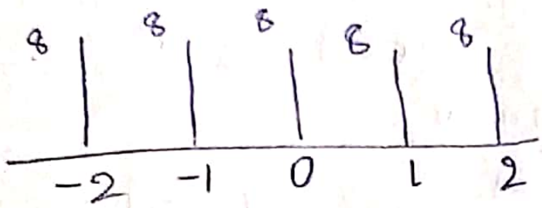
Given $x(n) = \{2, 3, 4, 5, 6\}$
$\uparrow$



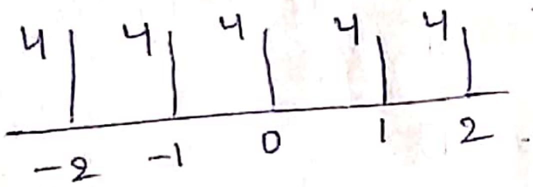
$x(-n)$



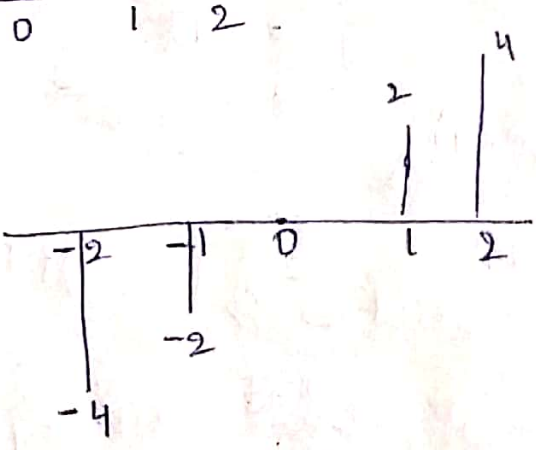
$x(n) + x(-n)$



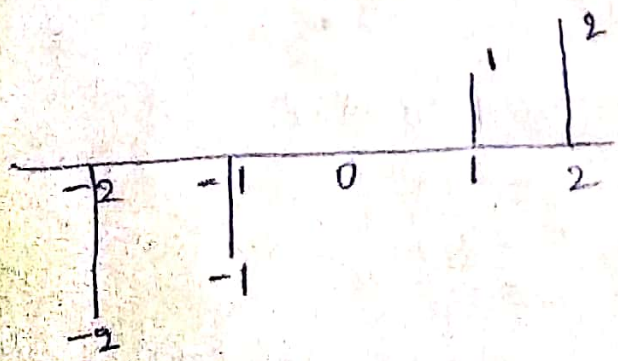
$$x_e(n) = \frac{x(n) + x(-n)}{2}$$



$x(n) - x(-n) \Rightarrow$



$$x_o(n) = \frac{x(n) - x(-n)}{2}$$



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5) Power (energy)

Sol First proving that

$$\sum_{n=-\infty}^{\infty} x_e(n) + x_o(n) = 0$$

$$\sum_{n=-\infty}^{\infty} x_e(n) + x_o(n) = \sum_{m=-\infty}^{\infty} x_e(-m)\, x_o(-m)$$

$$= -\sum_{m=-\infty}^{\infty} x_e(m)\, x_o(m)$$

$$= -\sum_{n=-\infty}^{\infty} x_e(n)\, x_o(n)$$

$$= \sum_{n=-\infty}^{\infty} x_e(n)\, x_o(n)$$

$$= 0$$

Energy (power)

$$\Rightarrow \sum_{n=-\infty}^{\infty} x^2(n) = \sum_{n=-\infty}^{\infty} [x_e(n) + x_o(n)]^2$$

$$= \sum_{n=-\infty}^{\infty} x_e^2(n) + x_o^2(n) + 2x_e(n)\, x_o(n)$$

$$= \sum_{n=-\infty}^{\infty} x_e^2(n) + \sum_{n=-\infty}^{\infty} x_o^2(n) + 2\sum_{n=-\infty}^{\infty} x_e(n)$$

$$= E_e + E_o + 0$$

$$\boxed{E = E_e + E_o}$$

6) $y(n) = T[x(n)] = x(n^2)$

a) Given $y(n) = \gamma[x(n)] = x(n^2)$

$x(n-K) \rightarrow y_1(n) = x[(n-K)^2]$

$= x(n^2 + K^2 - 2nK)$

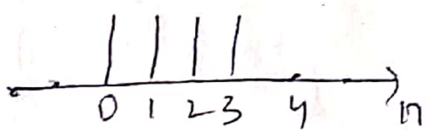
$x(n-K) \neq y(n-K)$

so, given system is time varient.

b) Given -

$$x(n) = \begin{cases} 1 ; 0 \le n \le 3 \\ 0 ; \text{elsewhere} \end{cases}$$

1) $x(n) = \{ \ldots\ldots 0, \underset{\uparrow}{1}, 1, 1, 1, 0, 0, \ldots \}$



2) $y(n) = \gamma[x(n)]$

$= x(n^2) = \{x(0), x(1), x(2^2), x(3^2), x(4^2) \ldots\}$

$= \{x(0), x(1), x(4), x(9), x(16) \ldots\}$

$y(n) = x(n^2) = \{ \ldots \underset{\uparrow}{1}, 1, 0, 0, 0 \ldots \}$

3) $y_2(n) = y(n-2)$

$y(n-2) = \{ \ldots\ldots, \underset{\underset{0}{\uparrow}}{0}, 0, 1, 1, 0, 0, 0, \ldots \}$

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7)

a) $y(n) = \cos[x(n)]$

(i) Static (only present i/p)

(ii) $y_1(n) = \cos[x_1(n)]$

$y_2(n) = \cos[x_2(n)]$

$y(n) = \cos[x_1(n)] + \cos[x_2(n)]$

$y'(n) = \cos[x_1(n) + x_2(n)]$

$\Rightarrow$ non linear

(iii) $y(n) = \cos[x(n-n_0)]$

$y'(n) = \cos[x(n-n_0)]$

$\Rightarrow$ Time invariant

(iv) only present i/p $\rightarrow$ casual

(v) stable

b) $y(n) = \sum_{k=-\infty}^{n+1} x(k)$

Dynamic (depends on Future values)

linear, Time invariant, non casual (also depends o Future value), unstable

c) $y(n) = x(n) \cos(\omega_0(n))$

static, linear, Time varient, casual, stable

$y(n) = x(n-n_0) \cos(\omega_0(n-n_0))$

$y'(n) = x(n-n_0) \cos(\omega_0 n)$

d) $y(n) = x(-n+2)$

Dynamic

$\Downarrow$

at $n=0 \Rightarrow y(0) = x(2)$

$y_1(n) = x_1(-n+2) + x_2(-n+2)$

$y_2(n) = x_1 + x_2(-n+2)$

$\Rightarrow x_1(-n+2) + x_2(-n+2)$

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⇒ linear, Non-causual, Stable, Time invariant.

e) $y(n) = \text{Trun}[x(n)]$

static, non linear, time invarient, causal, stable.

f) $y(n) = \text{Round}[x(n)]$

static, non-linear, time invarient, causal, stable.

g) $y(n) = |x(n)|$

static, non-linear, time invarient, causal, stable.

h) $y(n) = x(n)\,u(n)$

static, linear, time invarient, causal, stable.

i) $y(n) = x(n) + n\,x(n+1)$

Dynamic, linear, time varient, non causal, stable

k) $y(n) = \begin{cases} x(n) \ ; \text{if } x(n) \geq 0 \\ 0 \ ; \text{if } x(n) < 0 \end{cases}$

Static, linear, time invarient, non causal, stable.

1)

$x(n) = x(n+N) \ \forall n \geq 0$

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$$

$$y(n+N) = \sum_{k=-\infty}^{n+N} h(k)\, x(n+N-k)$$

$$y(n+N) = \sum_{k=n+1}^{n+N} h(k)\, x(n-k) + \sum_{k=-\infty}^{n} h(k)\, x(n-k)$$

$$y(n+N) = y(n) + \sum_{k=n+1}^{n+N} h(k)\, x(n-k)$$

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for BIBO system $\lim_{n\to\infty} |h(n)| = 0$

$$\lim_{n\to\infty} \sum_{K=n+1}^{n+N} h(K)\, x(n-K) = 0$$

$$\lim_{n\to\infty} y(n+K) = y(N)$$

$$\therefore y(N) = y(n+N)$$

b) $x(n) = x_0(n) + a\,u(n)$. $x_0(n)$ ∃ bounded with $\lim_{n\to\infty} x_0(n) = 0$

$$\Rightarrow y(n) = a\sum_{K=0}^{\infty} h(K)\, u(n-K) + \sum_{K=0}^{\infty} h(K)\, x_0(n-K) = a\sum_{K=0}^{n} h(K) + y_0(n)$$

$$\Rightarrow \sum_n x_0^2(n) < \infty \Rightarrow \sum_n y_0^2(n) < \infty$$

Hence $\lim_{n\to\infty} |y_0(n)| = 0 = a\sum_{K=0}^{n} h(K) = \text{constant}$

c) $y(n) = \sum_K h(K)\, x(n-K)$

$$\sum_{-\infty}^{\infty} y^2(n) = \sum_{-\infty}^{\infty}\Big(\sum_K h(K)\, x(n-K)\Big)^2 = \sum_K \sum_i h(K)\, h(i) \sum_n x(n-K)\, x(n-i)$$

$$\sum_n x(n-K)\, x(n-i) \le \sum_n x^2(n)\, |h(i)|$$

BIBO stable system $\sum_K |h(K)| < M$

Hence $E_y \le E_x$, So that $E_y < 0$ if $E_\infty < 0$.

10. The following i/p - o/p pairs have

As this is a time-invarient system

$y_2(n)$ should have only 3 elements and

$y_3(n)$ should have 4 elements.

so its linear.

11 since $x_1(n) + x_2(n) = \delta(n)$

system is linear, the impulse response of system

is $y_1(n) + y_2(n) = \{0, \underset{\uparrow}{3}, -1, 2, 1\}$

If system we time invarient the response of $x_3(n)$

would $\{\underset{\uparrow}{3}, 2, 1, 3, 1\}$

12 The only available information

(a) Ay linear combination of signal in the form of

$x_i(n)$; $i = 1, 2, \cdots N$

because if we take $i = 1, 3$

$y_1(n) = x_1(n)$

$y_3(n) = x_3(n) \Rightarrow y(n) = y_1(n) + y_3(n) = x_1(n) + x_3(n)$

$y(n) = x(n) + x_3(n)$

linear

(b) Same

Any $a_i(n-k)$ where k is anyinteger, $i = 1, 2, \cdots N$

1st replace $n = n - n_0 \Rightarrow x_i(n - n_0 - k)$

$x(n)$ by $x(n - n_0) \Rightarrow x_i(n - k - n_0)$ [Time invarient]

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13 Show that

A system to be BIBO stable only when bounded o/p produce bounded i/p.

$$y(n) = \sum_{K} h(K)\,x(n-K)$$

$$|y(n)| = \sum_{K} |h(K)|\,|x(n-K)|$$

$$= \sum_{K} |x(n-K)| \le m_n \text{ [some constant]}$$

So $|y(n)| = m_n \sum_{K} |h(K)|$

$|y(n)| < \infty$ for all n, if and only if $\sum_{K} |h(K)| < \infty$

So $\sum_{n=-\infty}^{\infty} |y(n)|$

A system to be BIBO stable only when bounded i/p produces bounded o/p.

$$y(n) = \sum_{K=-\infty}^{\infty} h(K)\,x(n-K) \; ; \; n \le n-K$$

$$K \ge 0$$

$$K \ge 0$$

$$|y(n)| = \sum_{K=-\infty}^{\infty} |h(K)|\,|x(n-K)|$$

as $\sum_{K=-\infty}^{\infty} |x(n-K)| \le m_n$ for some constant

$$|y(n)| = m_n \sum_{K=-\infty}^{\infty} |h(K)| \; ; \; n \le n-K \; ; K \ge 0$$

$|y(n)|$ is $< \infty$ if and only if $\sum_{K=-\infty}^{\infty} |h(K)| < \infty$

So $\sum_{n=-\infty}^{\infty} |h(n)| < \infty$

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14) Show that

(a) If a system is causal output depends only on the present and past i/p's as $x(n) = 0$ for $x(n) = 0$, for $n < n_0$ then $y(n)$ also zero for $n < n_0$.

(b) $y(n) = \sum_{K=-\infty}^{\infty} h(K)\, x(n-K)$

for finite impulse response.

$h(n) = 0,\ n < 0$ and $n \geq m$.

so $y(n)$ reduces to $y(n) = \sum_{K=0}^{n-1} h(K) x(n-K)$

15)

(a) for $a = 1$, $\sum_{n=m}^{N} a^n = N - m + 1$

for $a \neq 1$, $\sum_{n=m}^{N} a^n = \frac{a^m - a^{N+1}}{1-a}$

$(1-a)^N \sum_{n=m}^{N} a^n = a^m + a^{m+1} - a^{m+1} + \ldots a^n a^N - a^{N+1}$

$= a^m - a^{N+1}$

(b) for $m = 0$, $|a| < 1$ and $N \to \infty$

$\Rightarrow \sum_{n=-\infty}^{\infty} a^n = \frac{1}{1-a},\ |a| < 1.$

16) (a) if $y(n) = x(n) * h(n) \ldots$

Ans. $y(n) = \sum_{K} h(K)\, x(n-K)$

$\sum_{n} y(n) = \sum_{n} \sum_{K} h(K)\, x(n-K)$

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$$\sum_n y(n) = \sum_k h(k) \sum_{n=-\infty}^{\infty} x(n-k)$$

$$\sum_n y(n) = \left(\sum_k h(k)\right)\left(\sum_n x(n)\right)$$

(b) Compute

(1) $x(n) = \{1,2,4\}$ $h(n) = \{1,1,1,1,1\}$

<u>Sol</u> $y(n) = \{1,3,7,7,7,6,4\}$

$h(n)$ \ $x(n)$	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4

$\sum_n y(n) = 35$ $\sum_n x(n) = 7$ $\sum_n h(n) = 5$

$\sum_n y(n) = \sum_n x(n) \sum_n h(n)$

$35 = 7 \times 5$

$35 = 35$

(2) $x(n) = \{1,2,-1\}$, $h(n) = x(n)$

<u>Sol</u> $y(n) = x(n) * h(n)$

$y(n) = \{1,4,2,-4,-1\}$

$x(n)$ \ $h(n)$	1	2	-1
1	1	2	-1
2	2	4	-2
-1	-1	-2	1

$\sum_n y(n) = 4$; $\sum_n x(n) = 2$, $\sum_n h(n) = 2$

$\sum_n y(n) = \sum_n x(n) \cdot \sum_n h(n)$

$4 = 2 \times 2$

$4 = 4$

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(3) $x(n) = \{0, 1, -2, 3, -4\}$ $h(n) = \{\frac{1}{2}, \frac{1}{2}, 1, \frac{1}{2}\}$

Sol $y(n) = \{0, \frac{1}{2}, -\frac{3}{2}, \frac{3}{2}, -2, 0, -\frac{5}{2}, 2\}$

$\sum_n y(n) = -5$, $\sum_n x(n) = -2$, $\sum_n h(n) = \frac{5}{2}$

$\sum_n y(n) = \sum_n x(n) \, h(n)$

$-5 = -5$

$h(n) \backslash x(n)$	0	1	-2	3	-4
1/2	0	1/2	-1	3/2	-2
1/2	0	1/2	-1	3/2	-2
1	0	1	-2	3	-4
1/2	0	1/2	-1	3/2	-2

(4) $x(n) = \{1, 2, 3\}$, $h(n) = \{0, 0, 1, 1, 1, 1\}$

Sol $y(n) = \{0, 0, 1, -1, 2, 2, 1, 3\}$

$\sum_n y(n) = 8$; $\sum_n x(n) = 2$; $\sum_n h(n) = 4$

$\sum_n y(n) = \sum_n x(n) \sum_n h(n)$

$8 = 8$

$h(n) \backslash x(n)$	1	-2	3
0	0	0	0
0	0	0	0
1	1	-2	3
1	1	-2	3
1	1	-2	3
1	1	-2	3

(5) $x(n) = \{0, 1, 4, -3\}$, $h(n) = \{\underset{\uparrow}{1}, 0, -1, -1\}$

$y(n) = \{0, 1, 4, -4, -5, -1, 3\}$

$\sum_n y(n) = 2$; $\sum_n x(n) = -2$; $\sum_n h(n) = 1$

$\sum_n y(n) = \sum_n x(n) \sum_n h(n)$

$-2 = -2$

$h(n) \backslash x(n)$	0	1	4	-3
1	0	1	4	-3
0	0	0	0	0
-1	0	-1	-4	3
-1	0	-1	-4	3

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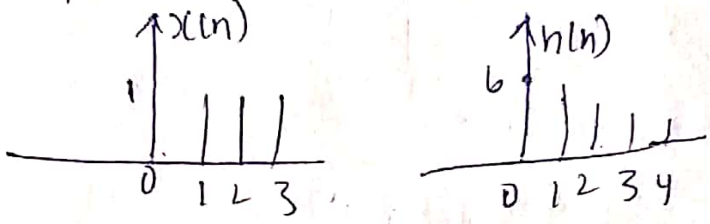
1) $x(n) = \left(\frac{1}{2}\right)^n u(n)$, $h(n) = \left(\frac{1}{4}\right)^n u(n)$

Sol $y(n) = \left[2\left(\frac{1}{2}\right)^n - \left(\frac{1}{4}\right)^n u(n)\right]$

$\sum_n y(n) = \frac{8}{3}$, $\sum_n h(n) = \frac{4}{3}$, $\sum_n x(n) = 2$

17 compute and plot convolutions

a)

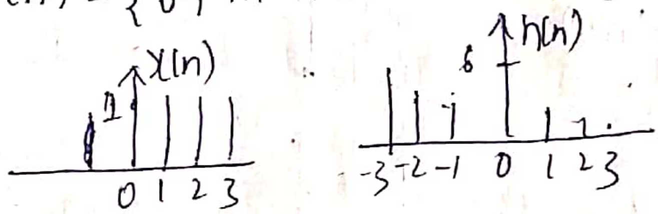


Sol $x(n) = \{1, 1, 1, 1\}$ $h(n) = \{6, 5, 4, 3, 2, 1, 0\}$

$y(n) = x(n) * h(n)$

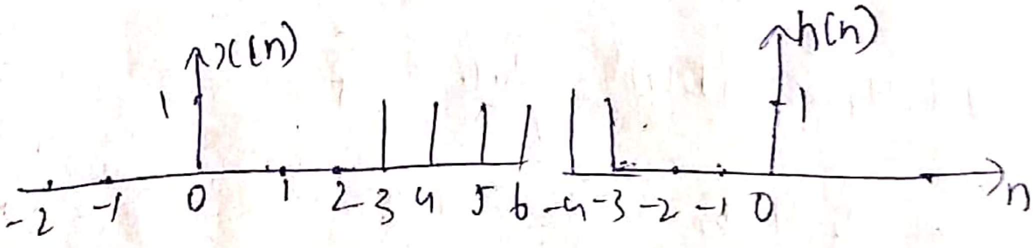
$y(n) = \{6, 11, 15, 18, 14, 10, 6, 3, 1\}$

b)



$y(n) = \{6, 11, 15, \underset{\uparrow}{18}, 14, 10, 6, 3, 1\}$

(C)



Sol $x(n) = \{0, 0, 0, 1, 1, 1, 1\}$

$h(n) = \{1, 1, 0, 0, \underset{\uparrow}{0}\}$

$y(n) = \{0, 0, 0, 1, 2, 2, 2, 1, 0, 0, 0\}$

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18 Determine and Stable

a) graphically

$x(n) = \{0, \frac{1}{3}, \frac{2}{3}, 1, \frac{4}{3}, \frac{5}{3}, 2\}$

$h(n) = \{1,1,1,1,1\}$

$y(n) = \{0, 1/3, 1, 2, \frac{10}{3}, 5, \frac{20}{3}, 6, 5, \frac{11}{3}, 2\}$

b) analytically

$x(n) = \frac{1}{3} n[u(n) - u(n-7)]$

$h(n) = u(n+2) - u(n-3)$

$y[n] = \frac{1}{3} n[u(n) - u(n-7)] * u(n+2) - u(n-3)$

$= \frac{1}{3} n u(n) * u(n+2) - \frac{1}{3} n(u(n) * u(n-3)) - \frac{1}{3} n(u(n-7)$

$* u(n+2) + \frac{1}{3} n(u(n-7) * u(n-3)).$

19)

Sol

$$y(n) = \sum_{k=0}^{4} h(k) x(n-k)$$

$$x(n) = \{\alpha^{-3}, \alpha^{-2}, \alpha^{-1}, \underset{\uparrow}{1}, \alpha, \ldots \alpha^{5}\}$$

$$h(n) = \{\underset{\uparrow}{1}, 1, 1, 1, 1\}$$

$$y(n) = \sum_{k=0}^{4} x(n-k), \; -3 \le n \le 9$$

$= 0$, otherwise

$\therefore\ y(-3) = \alpha^{-3}$

$y(-2) = x(-3) + x(-2) = \alpha^{-3} + \alpha^{-2}$

$y(-1) = \alpha^{-3} + \alpha^{-2} + \alpha^{-1}$

$y(0) = \alpha^{-3} + \alpha^{-2} + \alpha^{-1} + 1$

$y(1) = \alpha^{-3} + \alpha^{-2} + \alpha^{-1} + 1 + \alpha$

$y(2) = \alpha^{-3} + \alpha^{-2} + \alpha^{-1} + 1 + \alpha + \alpha^{2}$

$y(3) = \alpha^{-1} + 1 + \alpha + \alpha^{2} + \alpha^{3}$

$y(4) = \alpha^{4} + \alpha^{3} + \alpha^{2} + \alpha + 1$

$y(5) = \alpha + \alpha^{2} + \alpha^{3} + \alpha^{4} + \alpha^{5}$

$y(6) = \alpha^{2} + \alpha^{3} + \alpha^{4} + \alpha^{5}$

$y(7) = \alpha^{3} + \alpha^{4} + \alpha^{5}$

$y(8) = \alpha^{4} + \alpha^{5}$

$y(9) = \alpha^{5}$.

20)

(a) $131 \times 122 = 15982$

(b) $y(n) = \{1, 5, 9, 8, 2\}$

(c) $(z^2 + 3z + 1) * (2z^2 + 2z + 1)$

$= 2z^4 + 5z^3 + 9z^2 + 5z + 1$

(d) $= 1.31 \times 12.2 = 15.982$

(e) These are different ways to perform convolution

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2)

a) $y[n] = x(n) * h(n)$

$= a^n u(n) * b^n u(n)$

$= [a^n * b^n] u(n)$

$$y[n] = \sum_{k=0}^{n} a^k u(k) b^{n-k} u(n-k)$$

$$= b^n \sum_{k=0}^{n} a^k u(k) b^{-k}$$

$$= b^n \sum_{k=0}^{n} (ab)^{-k}$$

if $a \neq b$ then $y(n) = \frac{b^{n+1} - a^{n+1}}{b-a} u(n)$

if $a = b \Rightarrow b^n (n+1) u(n)$

b) $x(n) = \{1,2,1\}$ $h(n) = \{1,-1,0,0,1,1\}$

$y(n) = \{1,1,-1,0,0,3,3,2,1\}$

c) $x(n) = \{1,\underset{\uparrow}{1},1,1,1,0,-1\}$

$h(n) = \{1,2,\underset{\uparrow}{3},2,1\}$

$y[n] = \{1,3,6,8,9,8,5,1,-2,-2,-1\}$

2)

We can express $\delta(n) = u(n) - u(n-1)$

$h(n) = h(n) * \delta(n)$

$= h(n) * [u(n) - u(n-1)]$

then $y(n) = h(n) * x(n)$

$= (\delta(n) - \delta(n-1)) * x(n)$

$= \delta(n) * x(n) - \delta(n-1) * x(n)$

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23 $y(n) = ny(n-1) + x(n)$, $n \geq 0$

$y_1(n) = ny_1(n-1) + x_1(n)$

$y_2(n) = ny_2(n-1) + x_2(n)$

Add both

$y(n) = ny_1(n-1) + x_1(n) + ny_2(n-1) + x_2(n)$

$y(n) = ny(n-1) + x(n)$

$x(n) = ax_1(n) + bx_2(n)$

$y(n) = ay_1(n) + by_2(n)$

Hence the system is linear

$\Rightarrow y(n-1) = (n-1)y(n-2) + x(n-1)$

delayed $\Rightarrow y(n-1) = ny(n-2) + x(n-1)$

So, system is time invarient.

24 signal $x(n) = a^n u(n)$, $0 < a < 1$

Sol (a) $\delta(n) = \delta(n) - ar(n-1)$

$\delta(n-k) = r(n-k) - ar(n-k-1)$

$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k)$$

$$= \sum_{k=-\infty}^{\infty} x(k)\,[r(n-k) - ar(n-k-1)]$$

$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\,[r(n-k) - a\sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k-1)$$

$$x(n) = \sum_{k=-\infty}^{\infty} [x(k) - ax(k-1)\;\delta(n-k)]$$

Thus $c_k = x(k) - ax(k-1)$

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(b) $y(n) = T[x(n)]$

$$= T\left[\sum_{k=-\infty}^{\infty} c_k \delta(n-k)\right]$$

$$= \sum_{k=-\infty}^{\infty} c_k T(\delta(n-k))$$

$$= \sum_{k=-\infty}^{\infty} c_k g(n-k)$$

(c) $h(n) = T[\delta(n)]$

$h(n) = T[\delta(n) - a\delta(n-1)]$

$$= g(n) - a g(n-1)$$

Q5 Determine the zero input response

Sol Given $x(n) - 3y(n-1) - 4y(n-2) = 0$

with $x(n) = 0$

$-3y(n-1) - 4y(n-2) = 0 \quad (\div(-3))$

$y(n-1) + \frac{4}{3} y(n-2) = 0$

at $n = 0$

$y(-1) = -\frac{4}{3} y(-2)$

at $n = 1$

$y(0) = -\frac{4}{3} y(-1) = \left(-\frac{4}{3}\right)^2 y(-2)$

$y(1) = \left(-\frac{4}{3}\right)^3 y(-2)$

$y[k] = \left(\frac{-4}{3}\right)^{k+2} y(-2)$

→ zero - input response

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26) $y[n] = \frac{5}{6}y(n-1) - \frac{1}{6}y(n-2) + x(n)$

$x(n) = y(n) - \frac{5}{6}y(n-1) + \frac{1}{6}y(n-2)$

Characteristic equation is

$\lambda^2 - \frac{5}{6}\lambda + \frac{1}{6} = 0$; $\lambda = \frac{1}{2}, \frac{1}{3}$

So $y_h(n) = C_1\left(\frac{1}{2}\right)^n + C_2\left(\frac{1}{3}\right)^n$

$x(n) = 2^n u(n)$

$y_p(n) = K(2^n)u(n)$

Here $K = \frac{8}{5}$ when $n=2$

Total solution is

$y_p(n) + y_h(n) = y(n)$

$y(n) = \frac{8}{5}(2^n)u(n) + C_1\left(\frac{1}{2}\right)^n u(n) + C_2\left(\frac{1}{3}\right)^n u(n)$

Assume $y(-2) = y(-1) = 0$ so $y(0) = 1$

then $y(1) = \frac{5}{6}y(0) + 2 = \frac{17}{6}$

So $\frac{8}{5} + C_1 + C_2 = 1$ - (1) $3C_1 + 2C_2 = \frac{-11}{5}$ (2)

therefore By solving

$C_1 = -1$

$C_2 = 2/5$

So, $y[n] = \left[\frac{8}{5}(2)^n - \left(\frac{1}{2}\right)^n + \frac{2}{5}\left(\frac{1}{3}\right)^n\right]u(n)$

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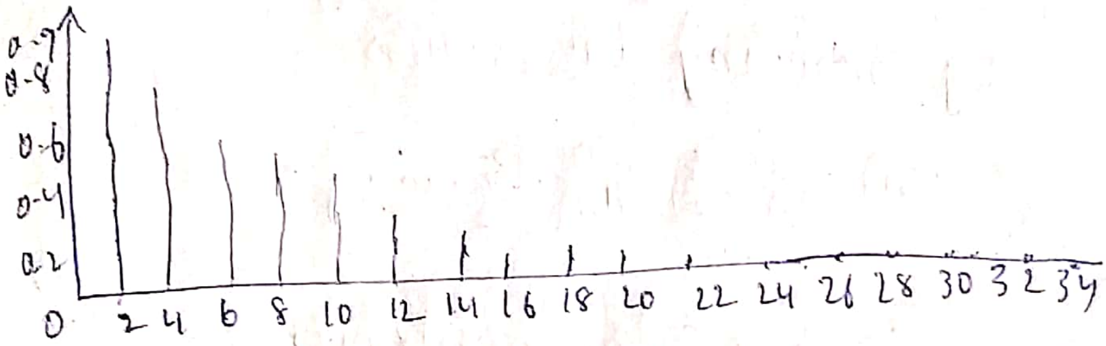
28. At $y(-1) = 1$

The given equation $y(n) = (-a)^{n+1} + \frac{(1-(-a)^{n+1})}{1+a}$

$y(n) = y_{zi}(n) + y_{zs}(n)$

= Transient State



29 (a) $L_1 = N_1 + m_1$ and $L_2 = N_2 + m_2$

(b) Partial overlap from left

low $N_1 + m_1$ high $N_1 + M_2 - 1$

Full over lap; low $N_1 + m_2$ hig $N_2 + m_1$

partial overlap from right

low $N_2 + M_1 + 1$ high $N_2 + m_2$.

(c) $x(n) = \{1, 1, 1, 1, 1, 1, 1\}$ $h(n) = \{2, 2, 2, 2\}$
↑

$N_1 = -2$ $m_1 = -1$

$N_2 = 4$ $m_2 = 2$

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31

$h(n) = \{1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}\}$

$y(n) = \{\underset{\uparrow}{1}, 2, 2, 2, 5, 3, 3, 3, 2, 1, 0\}$

$x(0)\, h(0) = y(0) \Rightarrow x(0) = 1$

$\frac{1}{2} x(0) + x(1) = y(1) \Rightarrow x(1) = \frac{3}{2}$

By continuing this process

$x(n) = \{1, \frac{3}{2}, \frac{3}{2}, \frac{7}{4}, \frac{3}{2}, \dots\}$

32 (a) $h(n) = h_1(n) * [h_2(n) - h_3(n) * h_4(n)]$

(b) $h_3(n) * h_4(n) = (n-1)\, u(n-2)$

$h_2(n) - h_3(n) * h_4(n) = 2u(n) - \delta(n)$

$h_1(n) = \frac{1}{2}\delta(n) + \frac{1}{4}\delta(n-1) + \frac{1}{2}\delta(n-2)$

(c) $x(n) = \{1, 0, \underset{\uparrow}{0}, 3, 0, -4\}$

$y(n) = \{\frac{1}{2}, \frac{5}{4}, 2, \frac{25}{4}, \frac{13}{2}, 5, 2, 0, 0, \dots\}$

$s(n) = u(n) * h(n)$

$$s(n) = \sum_{k=0}^{\infty} u(k)\, h(n-k)$$

$$= \sum_{k=0}^{n} h(n-k)$$

$$= \sum_{k=0}^{\infty} a^{n-k}$$

$$= \frac{a^{n+1}-1}{a-1}, \quad n \geq 0.$$

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33 $x(n) = u(n+5) - u(n-10)$

$$s(n+5) - s(n-10) = \frac{a^{n+6}-1}{a-1} u(n+5) - \frac{a^{n-9}-1}{a-1} u(n-10)$$

$$y(n) = x(n) * h(n) - x(n) * h(n-2)$$

$$y(n) = \frac{a^{n+6}-1}{a-1} u(n+5) - \frac{a^{n-9}-1}{a-1} u(n-10)$$

$$- \frac{a^{n+4}-1}{a-1} u(n+3) + \frac{a^{n+1}-1}{a-1} u(n-12)$$

34 $h(n) = [u(n) - u(n-m)]/m$

$$s(n) = \sum_{k=-\infty}^{\infty} u(k)\, h(n-k)$$

$$s(n) = \sum_{k=0}^{n} h(n-k)$$

$$s(n) = \begin{cases} \frac{n+1}{m}, & n < m \\ 1, & n \geq m \end{cases}$$

35 $$\sum_{n=-\infty}^{\infty} |h(n)| = \sum_{n=0,\, n=\text{even}}^{\infty} |a|^n$$

$$= \sum_{n=0}^{\infty} |a|^{2n}$$

$$= \frac{1}{1-|a|^2}$$

stable $*$ if $|a|<1$

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36 $x(n) = u(n) - u(n-10)$

$h(n) = a^n u(n)$, the response $u(n)$ is

$$y_1(n) = \sum_{k=0}^{\infty} u(k)\, h(n-k)$$

$$= \sum_{k=0}^{n} a^{n-k}$$

$$= a^n \sum_{k=0}^{n} a^{-k} = \frac{1-a^{n+1}}{1-a} u(n)$$

Then, $y(n) = y_1(n) - y_1(n-10)$

$$= \frac{1}{1-a}\left[(1-a^{n+1}) u(n) - (1-a^{n-1})\, u(n-10) \right]$$

38 $h(n) = \left(\frac{1}{2}\right)^n u(n)$ $y_1(n) = \sum_{k=0}^{\infty} u(k)\, h(n-k)$

$$= \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^{n-k}$$

$$= \left(\frac{1}{2}\right)^n \sum_{k=0}^{\infty} \left(\frac{1}{a}\right)^{k}$$

$$= \frac{1-\left(\frac{1}{2}\right)^{n+1}}{1-\frac{1}{a}} u(n)$$

(a) $a = 2$

$$y(n) = \frac{1}{1-2}\left[1-(2)^{n+1} \right] u(n) + \left[(1-(2)^{n-9})\, u(n-10) \right]$$

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39 $h(n) = h_1(n) * h_2(n) * h_3(n)$

$= [\delta(n) - \delta(n-1)] * u(n) * h(n)$

$= [u(n) - u(n-1)] * h(n)$

$= \delta(n) * h(n) = h(n)$

(b) not affected

40

(a) $x(n)\,\delta(n-n_0) = x(n_0)$

$n = n_0$ is of interest

$x(n) * \delta(n-n_0) = x(n-n_0)$

Thus, we obtain the shifted version of the sequence $x(n)$

(b) $y(n) = \sum_{k=-\infty}^{\infty} h(k)\,x(n-k)$

$y(n) = h(n) * x(n)$

Linearity

$$\boxed{y(n) = \alpha y_1(n) + \beta y_2(n)}$$

Time invarience

$x(n) \to y(n) = h(n) * x(n)$

$x(n-n_0) \to y_1(n) = h(n) * x(n-n_0)$

$= \sum_n h(k)\,x(n-n_0-k)$

$= y(n-n_0)$

(c) $h(n) = \delta(n-n_0)$

45

(a) $y(n) = ay(n-1) + bx(n)$

$\rightarrow h(n) = ba^n u(n)$

$\Rightarrow \sum_{n=0}^{\infty} h(n) = \frac{b}{1-b} = 1 \quad (\because b = 1-a)$

(b) $s(n) = \sum_{k=0}^{n} h(n-k) = b\left[\frac{1-a^{n+1}}{1-a}\right] u(n)$

$s(\infty) = \frac{b}{1-a} = 1$

(c) $b = 1-a$ in both case.

46 $y(n) = 0.8\,y(n-1) + 2x(n) + 3x(n-1)$

$y(n) = 0.8\,y(n-1) = 2x(n) + 3x(n-1)$

The characteristic eauation is

$\lambda - 0.8 = 0$

$\lambda = 0.8$

$y_n(n) = C(0.8)^n$

Here $x(n) = \delta(n)$

$y(0) = 1$, it flows that $C = 1$

Then, impulse response

$h(n) = 2(0.8)^n u(n) + 3(0.8)^{n-1} u(n-1)$

$h(n) = 2\delta(n) + 4.6\,(0.8)^{n-1} u(n-1)$

(b) $x(n) = -1.5x(n-1) + \frac{1}{2}y(n) - 0.4\,y(n-1)$

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(a) $y(n) = 0.9y(n-1) + x(n) + 2x(n-1) + 3x(n-2)$

for $x(n) = \delta(n) = \{0, 1, 0, 0, 0\}$

$y(0) = 0.9(0) + 1 + 0 + 0 = 1$

$y(1) = 0.9(1) + 0 + 2 + 0 = 2.9$

$y(3) = 0.9(2.9) + 0 + 0 + 3 = 5.61$

$y(4) = 0.9(5.049) + 0 + 0 + 0 = 5.049$

$y(5) = 0.9(4.544) + 0 + 0 + 0 = 4.08969$

$y(6) = 0.9(y(3) + x(6) + 2x(5)) + 3x(4)) = 3.686$

(b) $s(0) = y(0) = 1$

$s(1) = y(0) + y(1) = 3.91$

$s(2) = y(0) + y(1) + y(2) = 9.51$

$s(3) = y(0) + y(1) + y(2) + y(3) + y(4) = 19.16$

$s(5) = y(0) + y(1) + y(2) + y(3) + y(4) + y(5) = 23.19$

$s(6) = y(0) + \ldots\ldots + y(6) = 26.87$

(c) $h(n) = (0.9)^n u(n) + 2(0.9)^{n-1} u(n-1) + 3(0.9)^{n-2}$

$u(n-2)$.

$h(n) = \delta(n) + 2.9\delta(n-1) + 5.6(0.9)^{n-2} u(n-2)$

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49

$h_1(n) = c_0\delta(n) + c_1\delta(n-1) + c_2\delta(n-2)$

$h_2(n) = b_2\delta(n) + b_1\delta(n-1) + b_0\delta(n-2)$

$h_3(n) = a_0\delta(n) + (a_1 + a_0a_2)\delta(n-1) + a_1a_2\delta(n-2)$

(b) Let $a_0 = c_0$

$a_1 + a_2c_0 = c_1$

$a_0 = a_1 = c_2$

Hence

$\frac{c_2}{a_2} + a_2c_0 - c_1 = 0$

$\Rightarrow c_0a_2^2 - c_1a_2 + c_2 = 0$

For $c_0 \neq 0$, the quadratic has a real solutions if and only if $c_1^2 - 4c_0c_2 \geq 0$

50

(a) $y(n) = \frac{1}{2}y(n-1) + x(n) + x(n-1)$

$y(n) - \frac{1}{2}y(n-1) = \delta(n)$ the solution of s/m is

$h(n) = \left(\frac{1}{2}\right)^n u(n) - \left(\frac{1}{2}\right)^{n-1} u(n-1)$

(b) $h(n) * [\delta(n) + \delta(n-1)] = \left(\frac{1}{2}\right)^n u(n) + \left(\frac{1}{2}\right)^{n-1} u(n-1)$

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(a) convolution $y_1(n) = x_1(n) * h_1(n)$

$y_1(n) = \{1, 3, 7, 7, 7, 6, 4\}$

	1	1	1	1	1
1	1	1	1	1	1
2	2	2	2	2	2
4	4	4	4	4	4

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correlation:-

$r_1(n) = x(n) * h(-n)$

$h(-n) = \{1,1,1,1,1\}$

$x(n) = \{1,2,4\}$

$r_1(n) = \{1,3,7,7,7,6,4\}$

b) $x_2(n) = \{0,1,-2,3,4\}$ $h_2(n) = \{\frac{1}{2},1,2,1,\frac{1}{2}\}$

convolution : $y_2(n) = x_2(n) * h_2(n)$

$y_2(n) = \{0,\frac{1}{2},0,-2,\frac{1}{2},-6,-5/2,-2\}$

correlation

$r_1(n) = x(n) * h(n)$

$h(-n) = \{\frac{1}{2},1,2,1,\frac{1}{2}\}$

$x(n) = \{0,1,2,3,-4\}$

$r_2(n) = \{0,\frac{1}{2},0,\frac{3}{2},-2,-\frac{1}{2},-6,-\frac{5}{2},2\}$

c) convolution $\Rightarrow y_3(n) = x_3(n) * h_3(n)$

$y_3(n) = \{4,11,20,30,20,11,4\}$

	4	3	2	1
1	4	3	2	1
2	8	6	4	2
3	12	9	6	3
4	16	12	8	4

correlation

$h_3(-n) = \{1,2,3,4\}$

$x_3(n) = \{1,2,3,4\}$

$r_3(n) = \{1,4,10,20,25,24,16\}$

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52 $x(n) = \{1,3,3,1\}$

$y(n) = \{1,4,6,4,1\}$

The length of i/p, $L_1 = 4$

The length of o/p, $N = 5$

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$$

Length of $h(n)$ is L_2

$$L_1 + L_2 - 1 = N$$

$$4 + L_2 - 1 = 5$$

$$3 + L_2 = 5$$

$$L_2 = 5 - 3 = 2$$

$\therefore h(0) = y(0) = x(0) = 1$ $\quad h(n) = \{h(0), h(1)\}$

$$3h(0) + h(1) = y(1) = 4$$

$$\Rightarrow 3(1) + h(1) = 4$$

$$3 + h(1) = 4$$

$$h(1) = 1$$

$$\therefore h(n) = \{1,1\}$$

54 $y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$

$$\lambda^2 - 4\lambda + 4 = 0$$

$$\lambda^2 - 2\lambda - 2\lambda + 2 \times 2 = 0$$

$$\lambda(\lambda - 2) + (-2)(\lambda - 2) = 0$$

$$(\lambda - 2)(\lambda - 2) = 0$$

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$\lambda = 2,2$ Hence $y_n(n) = 2C_1 2^n + C_2 n_2^n$

The particular solution is $y_p(n) = K(-1)^n u(n)$.

$K(-1)^n u(n) - 4K(-1)^{n-1} u(n-1) + 2u(n-2) = (-1)^n u(n) - (-1)^n u(n)$

5) $y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$

The characteristic equation is

$$\lambda^2 - 4\lambda + 4 = 0$$

$$\lambda^2 - 2\lambda - 2\lambda + 2\times 2 = 0$$

$$(\lambda-2)(\lambda-2) = 0$$

$$\lambda = 2,2$$

$$\therefore y_h(n) = 2^n c_1 + n_2^n c_2$$

when $x(n) = \delta(n)$

with $y(0) = 1$ & $y(1) = 3$

$\Rightarrow C_1 = 1$

$\Rightarrow 2C_1 + 2C_2 = 3$

$\Rightarrow 2(C_1 + C_2) = 3$

$$1 + C_2 = \frac{3}{2}$$

$$C_2 = \frac{3}{2} - 1 = \frac{1}{2}$$

$$\therefore h(n) = \left[2^n + \frac{1}{2} n_2^n\right] u(n)$$

6.

$x(n) = x(n) * \delta(n)$

$\Rightarrow x(n) = x(n) * [u(n) - u(n-1)]$

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$\Rightarrow x(n) = x(n) * [u(n) - u(n-1)]$

$\Rightarrow x(n) = u(n) * [x(n) - x(n-1)]$

$$= \sum_{k=-\infty}^{\infty} [x(k) - x(n-k)]\, u(n-k)$$

57

Let $h(n)$ be the Impulse response

$$S(k) = \sum_{m=-\infty}^{\infty} h(m)$$

$\Rightarrow h(k) = S(k) - S(k-1)$

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$$

$$\Rightarrow y(n) = \sum_{k=-\infty}^{\infty} [S(k) - S(k-1)]\, x(n-k)$$

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$x(n) = \{1, 2, 1, 1\}$ $\quad x(0) = 1,\ x(1) = 2$

$x(2) = 1,\ x(3) = 1$

$$r_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n)\, x(n-l) = \sum_{n=0}^{3} x(n)\, x(n-l)$$

$r_{xx}(-3) = x(0)\, x(0-(-3)) = x(0)\, x(3) = (1)(1) = 1$

$r_{xx}(-2) = x(0)\, x(0-(-2)) = x(0) + x(2) + x(1) + x(3) = 1+2 = 3$

$r_{xx}(-1) = x(0)\, x(1) + x(1)\, x(2) + x(2)\, x(3)$

$= 2+2+1 = 5$

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$r_{xx}(0) = \sum_{n=0}^{3} x^2(n) = 7$ also $r_{xx}(-l) = r_{xx}(l)$

also $r_{xx}(2) = \{1, 3, 5, 7, 5, 3, 1\}$

(b) $y(n) = \{\underset{\uparrow}{1}, 1, 2, 1\}$

$$r_{yy}(l) = \sum_{n=-\infty}^{\infty} y(n)\, y(n-l)$$

$$r_{yy}(l) = \{1, 3, 5, 7, 5, 3, 1\}$$

we observe that $y(n) = x(-n+3)$, which is evalualent to reversing the sequence of $x(n)$

60 what is the normalited auto correlation

Sol $r_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n)\, x(n-l)$

$$r_{xx}(l) = \begin{cases} 2N+1-|l| & ; -2N < l < 2N \\ 0 & ; \text{otherwise} \end{cases}$$

$r_{xx}(0) = r_{xx}(0) = 2N+1-|0| = 2N+1$

$\therefore$ The normalised auto correlation is

$$r_{xx}(l) = \begin{cases} \frac{1}{2N+1}(2N+1-|l|) & ; -2N \le l < 2N \\ 0 & , \text{else} \end{cases}$$

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61 An Audio signal

(a)

$$r_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n)\, x(n-l)$$

$$= \sum_{n=-\infty}^{\infty} (s(n) - r_1 s(n-k) + r_2 s(n-k_2)] * [s(n-l)$$

$$+ n(n-l-k_1) + r_2(s(n-l)$$

$$= [1 + r_1^2 + r_2^2] r_{33}(l) + r_1 [r_{22}(1+k_1) + r_{23}(k+3)]$$

$$+ r_2 (r_{33}(l+10) + r_{33}(l-k_2)] f$$

$$r_1 r_2 [r_{33}(l + k_1 - x_2) + r_{33}(l + r_3 - k_1)$$

(b) $r_{xx}(l)$ has peaks at $l = 0$, $\pm 1k$, $\pm k_2$ & $\pm(k_1+1$
suppose that $k_1 < k_2$. Then we can determine n & k_1. The problem is determine r_2 & k_3 from the other peaks.

(c) if $r_2 = 0$, the peaks occur at $l = 0$ & $l = \pm k_1$
Then, it is easy obtain r_1 & k_1.

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