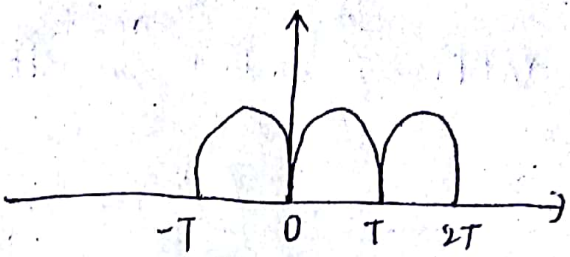
DTFT problems (Assignment -2)

1) consider the full wave rectified sinusoidal

(a) determine the spectrum $x_a(F)$



Sol

$$x_a(t) = \sum_{k=-\infty}^{\infty} c_k e^{j2\pi k f_0 t}$$

let $f_0 = \frac{1}{\tau}$

$$= \sum_{k=-\infty}^{\infty} c_k e^{j2\pi \frac{k}{\tau} t}$$

$$c_k = \frac{1}{T}\int_0^T A \sin\left(\frac{\pi t}{\tau}\right) e^{-j2\pi f k t/\tau} dt$$

$$= \frac{A}{2j.\tau}\int_0^{\tau}\left(e^{\frac{j\pi t}{\tau}} - e^{\frac{-j\pi t}{\tau}}\right) e^{-j2\pi k\frac{t}{\tau}} dt$$

$$= \frac{A}{2j.\tau}\int_0^{T}\left(e^{\frac{j\pi t}{\tau}} . e^{\frac{-j2\pi kt}{\tau}} - e^{\frac{-j\pi t}{\tau}} . e^{-j2\pi\frac{kt}{\tau}}\right) dt$$

$$= \frac{A}{2j.\tau}\left[\int_0^T e^{j\pi(1-2k)\frac{t}{\tau}} - e^{-j\pi(1-2k)\frac{t}{\tau}}\right] dt$$

$$= \frac{A}{2j\tau}\int_0^T e^{j\pi(1-2k)\frac{t}{\tau}} dt - e^{-j\pi(1-2k)\frac{t}{\tau}} dt$$

$$= \frac{A}{2j\tau}\left[\frac{e^{j\pi(1-2k)\frac{t}{\tau}}}{j\pi(1-2k)\frac{1}{\tau}}\right]_0^{\tau} - \left[\frac{e^{-j\pi(1-2k)\frac{t}{\tau}}}{-j\pi(1-2k).\frac{1}{\tau}}\right]_0^{T}$$

$$= \frac{A}{2j\tau}\left[\frac{e^{j\pi(1-2k)} - e^0}{j\pi(1-2k).\frac{1}{\tau}} - \frac{e^{-j\pi(1-2k)} - e^0}{-j\pi(1-2k).\frac{1}{\tau}}\right]$$

$$= \frac{A}{2j\tau} . \frac{\tau}{j\pi}\left(\frac{-1-1}{(1-2k)} + \frac{(-1-1)}{(1+2k)}\right)$$

$$= \frac{-A}{2\pi}\left[-2\left[\frac{1}{1-2k}+\frac{1}{1+2k}\right]\right]$$

$$= \frac{A}{\pi}\left[\frac{1+2k+1-2k}{1+2k-2k-4k^2}\right] = \frac{2A}{\pi(1-4k^2)}$$

$$\therefore x_a(f) = \int_{-\infty}^{\infty} x_a(t)\, e^{-j2\pi f t}\, dt$$

$$= \int_{-\infty}^{\infty} \sum_{k=-\infty}^{\infty} c_k\, e^{j2\pi k f_0 t} \cdot e^{-j2\pi f t}\, dt$$

$$= \sum_{k=-\infty}^{\infty} \int_{-\infty}^{\infty} c_k\, e^{-j2\pi (f-kf_0)t}\, dt$$

$$= \sum_{k=-\infty}^{\infty} \int_{-\infty}^{\infty} c_k\, e^{-j2\pi \left(F-\frac{k}{\gamma}\right)t}\, dt$$

$$\boxed{x_a(f) = \sum_{k=-\infty}^{\infty} c_k\, \delta\left(F-\frac{k}{\gamma}\right)}$$

(b) compute the power of the signal.

$$P_x = \frac{1}{T}\int_0^T x_a^2(t)\, dt$$

$$= \frac{1}{T}\int_0^T \left(A \sin\frac{\pi t}{\gamma}\right)^2 dt$$

$$= \frac{A^2}{T}\int_0^T \sin^2\frac{\pi t}{\gamma}\, dt$$

$$= \frac{A^2}{T}\int_0^T \frac{1-\cos 2\left(\frac{\pi}{\gamma}\right)t}{2}\, dt$$

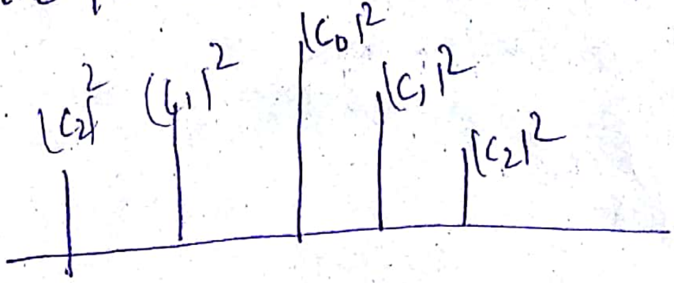
$$= \frac{A^2}{T}\int_0^T \frac{1}{2} - \cos 2\left(\frac{\pi}{\gamma}\right)t\, dt$$

$$= \frac{A^2}{T}\left[\frac{t}{2} - 0\right]$$

$$= \frac{A^2}{T} \cdot \frac{T}{2} = \frac{A^2}{2}$$

$$\therefore \boxed{P_x = \frac{A^2}{2}}$$

(c) Plot the power spectral density



(d) check the validity of parsevals relation for given

sol $$P_x = \sum_{K=-\infty}^{\infty} |c_K|^2$$

$$= \sum_{K=-\infty}^{\infty} \left| \frac{2A}{\pi(1-4K^2)} \right|^2$$

$$= \frac{4A^2}{\pi^2} \sum_{K=-\infty}^{\infty} \frac{1}{(4K^2-1)^2}$$

$$= \frac{4A^2}{\pi^2} \sum_{K=0}^{\infty} \left| \frac{1}{(4K^2-1)^2} \right|_{K=0} + 2 \sum_{K\geq 1}^{\infty} \frac{1}{(4K^2-1)^2}$$

$$= \frac{4A^2}{\pi^2} \left[1 + \frac{2}{3^2} + \frac{2}{15^2} + \dots \right]$$

$$= \frac{4A^2}{\pi^2} [1.231] = 0.498 A^2 = 0.5 A^2 = \frac{A^2}{2}.$$

4.2 Compute and sketch the magnitude and phase spectrum for the following signals.

Sol

1(a) $x_a(t) = \begin{cases} Ae^{-at} & t \geq 0 \\ 0, & t < 0 \end{cases}$

$$x_a(f) = \int_0^\infty Ae^{-at} e^{-j2\pi ft} dt$$

$$= A \int_0^\infty e^{-(a+2\pi fj)t} dt$$

$$= A \left[\frac{e^{-(a+2\pi fj)t}}{-(a+2\pi fj)} \right]_0^\infty$$

$$= A \cdot \frac{1}{a+j2\pi f}$$

$$|x_a(f)| = \frac{A}{\sqrt{a^2+(2\pi f)^2}}$$

$$\angle |x_a(f)| = -\tan^{-1}\left[\frac{2\pi f}{a}\right]$$

(b) $x_a(t) = Ae^{-a|t|}$

$$x_a(f) = \int_{-\infty}^{\infty} Ae^{-a|t|} e^{-j2\pi ft} dt$$

$$= \int_{-\infty}^{0} Ae^{at} e^{-j2\pi ft} + \int_0^\infty A e^{-at} e^{-j2\pi ft}$$

$$= \int_0^\infty Ae^{-at} e^{j2\pi ft} + \int_0^\infty Ae^{-at} e^{-j2\pi ft}$$

$$= A \int_0^\infty e^{-(-j2\pi ft+a)t} dt + A \int_0^\infty e^{-(a+j2\pi f)t} dt$$

$$= A \left[\frac{e^{-(a-j2\pi ft)}}{-(a-j2\pi f)} \right]_0^\infty + A \left[\frac{e^{-(a+j2\pi f)t}}{-(a+j2\pi f)} \right]_0^\infty$$

$$= A \left[\frac{1}{a-j2\pi f} \right] + A \left[\frac{1}{a+j2\pi f} \right]$$

$$= \frac{Aa + Aj2\pi f + Aa - Aj2\pi f}{a^2 - (j2\pi f)^2}$$

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$$= \frac{2aA}{a^2+(2\pi f)^2}$$

$$|Xa(f)| = Xa(f)$$

$$\angle Xg(F) = \tan^{-1}\left(\frac{0}{\text{some}}\right)$$

$$= 0$$

4.3 consider the signal

$$x(t) = \begin{cases} 1-\frac{|t|}{\tau}, & |t| = \tau \\ 0, & \text{elsewhere} \end{cases}$$

(a) Determine and sketch its magnitude and phase spectrum $|Xa(f)|$ and $Xa(f)$

Sol: $$Xa(f) = \int_{T}^{0}\left(1+\frac{t}{\tau}\right)e^{-j2\pi ft}\,dt + \int_{0}^{T} -\frac{1}{\tau}\, e^{-j2\pi ft}\,dt.$$

let $y(t) = \left\{\frac{t}{T} ; -\tau \le t \le \tau\right\}$

$$X(F) = \int_{-T}^{0}\frac{1}{\tau}\, e^{-j2\pi ft}\,dt + \int_{0}^{T} -\frac{1}{T}\, e^{-j2\pi ft}\,dt$$

$$= \frac{-2\sin^2\pi ft}{j\pi ft}$$

$$X(F) = \frac{1}{j2\pi f}\, y(F)$$

$$= \tau\left(\frac{\sin\pi f\tau}{\pi ft}\right)^2$$

$$|X(f)| = T\left(\frac{\sin\pi FT}{\pi FT}\right)^2$$

$$\angle Xa(F) = 0$$

$$\tau \cdot \left(\frac{\sin \pi F \tau}{\pi F \tau}\right)^2$$

let $F = t/\tau$

$$\left(\frac{\sin \pi t}{\pi t}\right)^2 = \text{sinc}^2(t).$$

b

Sol
$$c_k = \frac{1}{T_p} \int_{-T_p/2}^{T_p/2} x_1(t)\, e^{-j2\pi k t/T_p}\, dt$$

$$= \frac{1}{T_p} \int_{-\tau}^{0} \left(1+\frac{t}{\tau}\right) e^{-j2\pi k t/T_p} dt + \int_{0}^{\tau} \left(1-\frac{t}{\tau}\right) e^{-j2\pi k t/T_p} dt$$

$$= \frac{\tau}{T_p} \left[\frac{\sin \pi k \tau / T_p}{\pi k \tau / T_p}\right]^2$$

c $c_k = \left(\frac{1}{T_b}\right) x_a\left[\frac{k}{T_b}\right]$

Sol
$$\frac{1}{T_b}\, x_a\left(\frac{k}{T_b}\right)$$

$$\frac{1}{T_b} \cdot \tau \cdot \left(\frac{\sin \pi \frac{k\tau}{T_b}}{\pi \frac{k \cdot \tau}{T_b}}\right)^2$$

$\therefore c_k = \frac{1}{T_b}\, x_a\left(\frac{k}{T_b}\right)$ Hence proved

4.4 consider the following

$$x(n) = \{\ldots, 1, 0, 1, 2, 3, 2, 1, 0, 1, \ldots\}$$

(a) Sketch the signal $x(n)$ and its magnitude and phase spectrum

1 0 1 2 3 2 1 0 1

Sol

$N=6$

$$C_k = \frac{1}{N}\sum_{n=0}^{N-1} x(n)\, e^{-j2\pi \frac{kn}{N}}$$

$$= \frac{1}{6}\sum_{n=0}^{5} x(n)\, e^{-j2\pi \frac{kn}{6}}$$

For $n=0 \rightarrow x(0) \cdot e^{-\frac{j2\pi k}{6}(0)} = 3\times 1 = 3$

$n=1 \rightarrow x(1) \cdot e^{-\frac{j2\pi k}{6}(1)} = 2e^{-j\frac{\pi 2k}{6}}$

$n=2 \rightarrow e^{-\frac{j2\pi k}{3}}$

$n=3 \rightarrow 0$

$n=4 \rightarrow e^{-\frac{j4\pi k}{3}}$

$n=5 \rightarrow 2 \cdot e^{-\frac{j10\pi k}{6}}$

$$\therefore = \frac{1}{6}\left[3 + 2e^{-\frac{\pi 2k}{6}} + e^{-\frac{j2\pi k}{3}} + e^{-\frac{j4\pi k}{3}} + 2e^{-\frac{j10\pi k}{6}}\right]$$

For $k=0$

$$= \frac{1}{6}\left[3+2+1+1+2\right]$$

$$= \frac{1}{6}\left[9\right] = \frac{9}{6}$$

For $k=1$

$$= \frac{1}{6}\left[3 + e^{-\frac{\pi \cancel{2}}{\cancel{6}3}} + e^{-\frac{j2\pi}{3}} + e^{-\frac{j4\pi}{3}} + 2e^{-\frac{j\cancel{10}^{5}\pi}{\cancel{6}_{3}}}\right]$$

$$= \frac{1}{6}\left[3 + 2\left(\cos\left(\frac{\pi}{3}\right) - j\sin\left(\frac{\pi}{3}\right)\right) + \cos\left(\frac{2\pi}{3}\right) - j\sin\left(\frac{2\pi}{3}\right) + \dots\right]$$

$$= \frac{4}{6}$$

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Similarly;

For $k=2;\ c_2 = 6$

$k=3;\ c_3 = \frac{1}{6}$

$k=4;\ c_4 = 0$

$k=5;\ c_5 = \frac{4}{6}$

(b) parsevals By computing the power in time and frequency domains.

Sol

$$P_t = \frac{1}{6}\sum_{n=0}^{5} |x(n)|^2$$

$$= \frac{1}{6}\left[1^2 + 0^2 + 1^2 + 2^2 + 3^2 + 2^2\right]$$

$$= \frac{19}{6}$$

$$P_F = \sum_{n=0}^{5} |c(n)|^2$$

$$= \left(\frac{9}{6}\right)^2 + \left(\frac{4}{6}\right)^2 + 0^2 + \left(\frac{1}{6}\right)^2 + 0^2 + \left(\frac{4}{6}\right)^2$$

$$= \frac{114}{36} = \frac{19}{6}$$

4.5

$$x(n) = 2 + 2\cos\left(\frac{\pi n}{4}\right) + \cos\left(\frac{\pi n}{2}\right) + \frac{1}{2}\cos\left(\frac{3\pi n}{4}\right)$$

(a)
$$x(n) = 2 + 2\left[\frac{e^{j\frac{\pi n}{4}} + e^{-j\frac{\pi n}{4}}}{2}\right] + \frac{e^{j\frac{\pi n}{2}} + e^{-j\frac{\pi n}{2}}}{2}$$

$$+ \frac{1}{2}\left[\frac{e^{j\frac{3\pi n}{4}} + e^{-j\frac{3\pi n}{4}}}{2}\right]$$

$$= 2 + e^{-j\frac{\pi n}{4}} + e^{j\frac{\pi n}{4}} + \frac{1}{2}e^{j\frac{\pi n}{2}} + \frac{1}{2}e^{-j\frac{\pi n}{2}} +$$

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$\frac{1}{4} e^{\frac{j3\pi n}{4}} + \frac{1}{4} e^{\frac{-j3\pi n}{4}}$

$\therefore N = 8$

$c_k = \frac{1}{8} \sum_{n=0}^{7} x(n)\, e^{-j\frac{\pi k n}{4}}$

$x(n) = \left\{ \frac{11}{2}, 2+\frac{3}{4}\sqrt{2}, 1, 2-\frac{3}{4}\sqrt{2}, \frac{1}{2}, 2-\frac{3}{4}\sqrt{2}, 1, 2+\frac{3}{4}\sqrt{2} \right\}$

$c_0 = 2,\ c_1 = c_7 = 1,\ c_2 = c_6 = \frac{1}{2},\ c_3 = c_5 = \frac{1}{4},\ c_4 = 0$

(e) Power

Sol: $P(t) = \sum_{n=0}^{7} |c_k|^2$

$= \left[2^2 + 1^2 + 1^2 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)^2 \right]$

$= \left[4 + 2 + \frac{1}{2} + \frac{1}{8} \right]$

$\boxed{P(x) = \frac{53}{8}}$

Q: Determine and sketch magnitude and phase of periodic signals.

(a) $x(n) = 4 \sin \frac{\pi(n-2)}{3}$

Sol: $= 4 \left[\frac{e^{\frac{j\pi(n-2)}{3}} - e^{\frac{-j\pi(n-2)}{3}}}{2j} \right]$

$= 4 \left[e^{3j\pi(n-2)} - e^{-3j\pi(n-2)} \right]$

$N = 6$

$$C_k = \frac{1}{6}\sum_{n=0}^{5} x(n)\, e^{-\frac{j2\pi kn}{6}}$$

$$= \frac{4}{6}\sum_{n=0}^{5} \frac{\left[e^{\frac{j\pi(n-2)}{3}} - e^{\frac{-j\pi(n-2)}{3}} \right]}{2j} e^{-j\frac{4\pi kn}{6}\!/3}$$

$$= \frac{1}{\sqrt{3}}\left[-e^{-j2\pi k/3} - e^{-j\pi k/3} + e^{-j\pi k/3} + e^{-j2\pi k/3} \right]$$

$$= \frac{1}{\sqrt{3}}(-2j)\left[\sin\frac{2\pi k}{3} + \sin\frac{\pi k}{3} \right] e^{-j2\pi k/3}$$

$c_0 = 0$; $c_1 = -2j\, e^{-j2\pi/3}$, $c_2 = c_3 = c_4 = 0$, $c_5 = c_1$

$\angle c_1 = \frac{5\pi}{6}$, $\angle c_5 = \frac{-5\pi}{6}$, $\angle c_1 = \angle c_2 = \angle c_3 = \angle c_4 = 0°$

(b) $x(n) = \cos\left(\frac{2\pi}{3}\right)n + \sin\left(\frac{2\pi}{5}\right)n$.

<u>Sol</u> $N = 15$

$\cos\left(\frac{2\pi}{3}\right)n$

$\frac{1}{2}\left[e^{j\frac{2\pi}{3}n} + e^{-j\frac{2\pi}{3}n} \right]$

$e^{-j\frac{2\pi}{3}n} = e^{\frac{-2j\pi kn}{N}}$

$k = \frac{N}{3} = 5$

$15 - 5 = 10$

$c_k = \begin{cases} \frac{1}{2} ; & k = 5, 10 \\ 0 ; & \text{otherwise} \end{cases}$

$\sin\frac{2\pi}{5}n$

$\frac{1}{2j}\left[e^{\frac{j2\pi n}{5}} - e^{\frac{-j2\pi n}{5}} \right]$

$e^{\frac{j2\pi n}{5}} = e^{\frac{-25\pi kn}{N}}$

$\cdot\, k = \frac{N}{5} = \frac{15}{5} = 3$

$15 - 3 = 12$

$c_{2k} = \begin{cases} \frac{1}{2j} ; & k = 3 \\ -\frac{1}{2j} ; & k = 12 \\ 0 ; & \text{otherwise} \end{cases}$

$$C_k = C_{1k} + C_{2k} = \begin{cases} \frac{1}{2j} & ; k=3 \\ \frac{1}{2} & ; k=5 \\ \frac{1}{2} & ; k=10 \\ -\frac{1}{2j} & ; k=12 \\ 0 & ; \text{otherwise} \end{cases}$$

(c) $x(n) = \{ \ldots\ldots -2, -1, \underset{\uparrow}{0}, 1, 2, -2, -1, 0, 1, 2, \ldots \}$

$N=5$

Sol $C_k = \frac{1}{5} \sum_{n=0}^{4} x(n) \, e^{\frac{-j2\pi kn}{N}}$

$= \frac{1}{5} \left[0 + e^{\frac{-j2\pi k}{5}} + 2e^{\frac{-j4\pi k}{5}} - 2e^{\frac{-j\pi 6k}{5}} - e^{\frac{-j8\pi k}{5}} \right]$

$= \frac{2j}{5} \left[-\sin\left(\frac{2\pi k}{5}\right) - 2\sin\left(\frac{4\pi k}{5}\right) \right]$

For putting k values.

$k=0$; $C_0 = 0$

$k=1$; $C_1 = \frac{2j}{5} \left(-\sin\frac{2\pi}{5} - 2\sin\frac{4\pi}{5} \right)$

$k=2$; $C_2 = \frac{2j}{5} \left[-\sin\frac{4\pi}{5} - 2\sin\frac{8\pi}{5} \right]$

$\therefore C_3 = -C_2$

$C_4 = -C_1$

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(e) $x(n) = \{ -1, 2, 1, 1, -1, 0, -1, 2, 1, 2 \cdots \}$

$\underset{N=6}{\longleftrightarrow}$

Sol $N = 6$

$$C_k = \frac{1}{6} \sum_{n=0}^{5} x(n) e^{\frac{-j2\pi kn}{6}}$$

$$= \frac{1}{6} \left[1 + 2e^{\frac{-j\pi k}{3}} - e^{\frac{-j2\pi k}{3}} - e^{\frac{-j4k}{3}} + 2e^{\frac{-j5\pi k}{3}} \right]$$

$$= \frac{1}{6} \left[1 + 4\cos\frac{\pi k}{3} - 2\cos\frac{2\pi k}{3} \right]$$

$c_0 = \frac{1}{2}$; $c_1 = \frac{2}{3}$; $c_2 = 0$; $c_3 = \frac{-5}{6}$; $c_4 = 0$; $c_5 = \frac{2}{3}$

(g) $x(n) = 1$, $-\infty < n < \infty$

Sol $N = 1$

$C_k = x(0) = 1$

$c_0 = 1$

(h) $x(n) = (-1)^n$, $-\infty < n < \infty$

Sol $N = 2$

$$C_k = \frac{1}{2} \sum_{n=0}^{1} x(n) e^{-j\pi nk}$$

$$= \frac{1}{2} (1 - e^{-j\pi k})$$

$c_0 = 0$; $c_1 = 1$

4.7

(a) $C_k = \cos\frac{nk\pi}{4} + \sin\left(\frac{3k\pi}{4}\right)$

Sol $x(n) = \sum_{n=0}^{7} C_k e^{\frac{j2\pi kn}{N}}$

let

$C_k = e^{\frac{j2\pi Pk}{N}}$

$\therefore \sum_{n=0}^{7} e^{\frac{j2\pi Pk}{N}} \cdot e^{\frac{j2\pi nk}{N}}$

$\sum_{n=0}^{7} e^{\frac{j2\pi(P+n)k}{N}}$

It gives 8 ; when $P=-n$

0 ; when $P \neq n$

$C_k = \frac{1}{2}\left[e^{\frac{j2\pi k}{8}} + e^{\frac{-j2\pi k}{8}}\right) - \frac{1}{2j}\left[e^{\frac{j6\pi k}{8}} - e^{\frac{-j\pi 6k}{8}}\right]$

$\therefore x(n) = 4\delta(n+1) + 4\delta(n-1) - 4j\delta(n-1) - 4j\delta(n+3) + 4j\delta(n-3)$

$; -3 \leq n \leq 5$

(b) $C_k = \begin{cases} \sin\frac{k\pi}{3} & , 0 \leq k \leq 6 \\ 0 & , k=7 \end{cases}$

$C_0 = 0, C_1 = \frac{\sqrt{3}}{2} ; C_2 = \frac{\sqrt{3}}{2}, C_3 = 0, C_4 = -\frac{\sqrt{3}}{2}, C_5 = -\frac{\sqrt{3}}{2}, C_6 = C_7 = 0$

$x(n) = \sum_{k=0}^{7} C_k e^{\frac{j2\pi nk}{8}}$

$= \frac{\sqrt{3}}{2}\left[e^{\frac{j\pi n}{4}} + e^{\frac{j\pi n2}{4}} - e^{\frac{-j4\pi n}{4}} - e^{\frac{j5\pi n}{4}}\right)$

$= \sqrt{3}\left[\frac{\sin\pi n}{2} + \frac{\sin\pi n}{4}\right] \cdot e^{\frac{j\pi(3n-2)}{4}}$

(c) $c_k = \{ \cdots 0, \frac{1}{4}, \frac{1}{2}, 1, 2, 1, \frac{1}{2}, \frac{1}{4}, 0, \cdots \}$

$$x(n) = \sum_{k=-3}^{4} c_k \, e^{\frac{j2\pi nk}{8}}$$

$$= 2 + e^{\frac{j\pi n}{4}} + e^{\frac{-j\pi n}{4}} + \frac{1}{2} e^{\frac{j\pi n}{2}} + \frac{1}{2} e^{\frac{-j\pi n}{2}}$$

$$+ \frac{1}{4} e^{\frac{j3\pi n}{4}} + \frac{1}{4} e^{\frac{-j3\pi n}{4}}$$

$$= 2 + 2\cos\frac{\pi n}{4} + \cos\frac{\pi n}{2} + \frac{1}{2}\cos\frac{3\pi n}{4}$$

* Determine the signals having the following Fourier transform.

(a) $X(\omega) = \begin{cases} 0 & 0 \leq |\omega| \leq \omega_0 \\ 1 & \omega_0 < |\omega| \leq \pi \end{cases}$

Sol $x(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\omega) e^{j\omega n} d\omega$

$$= \frac{1}{2\pi} \left[\int_{-\pi}^{\omega_0} X(\omega) e^{j\omega n} + \int_{\omega_0}^{\pi} X(\omega) e^{j\omega n} d\omega \right]$$

$$= \frac{1}{2\pi} \left[\frac{e^{-j\omega_0 n} - e^{-j\pi n}}{jn} + \frac{e^{j\pi n} - e^{j\omega_0 n}}{jn} \right]$$

$$= \frac{1}{2\pi} \left[2 \cdot \frac{e^{-j\omega_0 n} - e^{j\omega_0 n}}{2jn} + 2 \cdot \frac{e^{j\pi n} - e^{-j\pi n}}{2jn} \right]$$

$$= \frac{1}{2\pi} \left[\frac{2}{n} \cdot \sin\omega_0 n + \frac{2}{n} \cdot \sin\pi n \right]$$

$$= -\frac{\sin\omega_0 n}{n\pi} \quad n \neq 0.$$

for $n=0$

from eqn ①

$$=\frac{1}{2\pi}(\pi-\omega_0)+\frac{1}{2\pi}(\pi-\omega_0)$$

$$=\frac{(\pi-\omega_0)+(\pi-\omega_0)}{2\pi}=\frac{\cancel{2}(\pi-\omega_0)}{\cancel{2}\pi}=(\pi-\omega_0)\text{; when } n=0.$$

(b) $X(\omega)=\cos^2(\omega)$

$$=\left(\frac{e^{j\omega}+e^{-j\omega}}{2}\right)^2$$

$$=\frac{1}{4}\left[(e^{j\omega})^2+2\cdot e^{j\omega}\cdot e^{-j\omega}+(e^{-j\omega})^2\right]$$

$$=\frac{1}{4}\left[e^{j2\omega}+2+e^{-j2\omega}\right]$$

$$=\frac{1}{4}e^{j2\omega}+\frac{1}{2}+\frac{1}{4}e^{-j2\omega}$$

$\Downarrow$ I.F.T

$$=\frac{1}{4}\delta(n+2)+\delta(n)\frac{1}{2}+\frac{1}{4}\delta(n-2)$$

(c) $X(\omega)=\begin{cases}1; & \omega_0-\frac{\delta\omega}{2}\le|\omega|\le\omega_0+\frac{\delta\omega}{2} \\ 0; & \text{elsewhere}\end{cases}$

sol

$$\omega_0-\frac{\delta\omega}{2}\le|\omega|\le\omega_0+\frac{\delta\omega}{2}$$

$$\omega_0-\frac{\delta\omega}{2}\le-\omega\le\omega_0+\frac{\delta\omega}{2}$$

$$\omega_0-\frac{\delta\omega}{2}\le\omega\le\omega_0+\frac{\delta\omega}{2}$$

$$= \frac{1}{2\pi} \int_{\omega_0 - \frac{\delta\omega}{2}}^{\omega_0 + \frac{\delta\omega}{2}} 1 . e^{j\omega n} d\omega$$

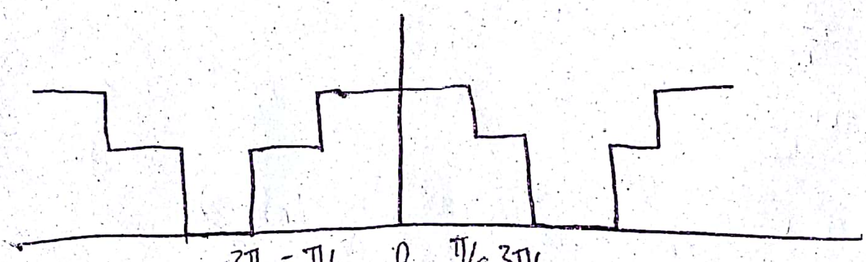
$$= \frac{1}{2\pi} \left[\frac{e^{j\omega n}}{jn} \right]_{\omega_0 - \frac{\delta\omega}{2}}^{\omega_0 + \frac{\delta\omega}{2}}$$

$$= \frac{2}{2\pi} \left[\frac{e^{j(\omega_0 + \frac{\delta\omega}{2})n} - e^{j(\omega_0 - \frac{\delta\omega}{2})}}{2jn} \right]$$

$$= \delta\omega \frac{2}{\pi} \left[\frac{\sin(\frac{\delta\omega}{2})n}{n \frac{\delta\omega}{2}} \right] e^{jn\omega_0}$$

$$= \delta\omega . \frac{2}{\pi} . sa(\frac{\delta\omega}{2} n) e^{jn\omega_0}.$$

d) The signal shown in fig



Sol $\frac{1}{2\pi} \int_{-\pi}^{\pi} X(\omega) e^{j\omega n} d\omega$

let consider limits 0 to π

$$= 2 \times \frac{1}{2\pi} \left[\int_0^{\pi/8} 2 e^{j\omega n} d\omega + \int_{\pi/8}^{3\pi/8} e^{j\omega n} d\omega + \int_{6\pi/8}^{7\pi/8} e^{j\omega n} d\omega \right.$$

$$+ \int_{7\pi/8}^{\pi} 2 e^{j\omega n} d\omega$$

$$= \frac{1}{\pi}\left[\int_{0}^{\pi/8} 2\cos\omega n\,d\omega + \int_{\pi/8}^{3\pi/8} \cos\omega n\,d\omega + \int_{6\pi/8}^{7\pi/8} \cos\omega n\,d\omega + \int_{7\pi/8}^{\pi} 2\cos\omega n\,d\omega\right.$$

$$= \frac{1}{\pi}\left[-2\sin\omega n\right]_{0}^{\pi/8} + \left[-\sin\omega n\right]_{\pi/8}^{3\pi/8} + \left[-\sin\omega n\right]_{6\pi/8}^{7\pi/8} + \left[-2\sin\omega n\right]_{7\pi/8}^{\pi}$$

$$= \frac{1}{\pi}\left[\sin\frac{7\pi}{8}n - \frac{\sin\pi n}{8} + \frac{\sin 6\pi}{8}n - \sin\frac{3\pi}{8}n\right]$$

$x(n) = \{1, 0, -1, 2, 3\}$

$X(\omega) = X_R(\omega) + j\,X_I(\omega)$

$X(\omega) = X_I(\omega) + X_R(\omega)\,e^{j2\omega}$

Sol $x_e(n) = \frac{x(n) + x(-n)}{2}$

$n=0$; $x_e(n) = \frac{x(0) + x(-0)}{2} = \frac{2+2}{2} = 2$

$n=1$; $\frac{x(1) + x(-1)}{2} = \frac{3+(-1)}{2} = 1$

$n=-1$; $\frac{x(-1) + x(1)}{2} = 1$

$n=2$; $\frac{x(2) + x(-2)}{2} = 0$

$n=3$; $\frac{x(3) + x(-3)}{2} = \frac{0+1}{2} = 1/2$

$n=-2$; $\frac{x(-2) + x(2)}{2} = 0+0 = 0$

$n=-3$; $\frac{x(-3) + x(3)}{2} = \frac{1+0}{2} = 1/2$

$x_e(n) = \{\frac{1}{2}, 0, 1, \underset{\uparrow}{2}, 1, 0, 1/2\}$

Similarly we get,

$$x_o(n) = \{\tfrac{1}{2}, 0, -2, 0, 2, 0, \tfrac{1}{2}\}$$

From $x_e(n) = \dfrac{x(n) - x(-n)}{2}$

$$x_R(\omega) = \sum_{n=-3}^{3} x_e(n)\, e^{-j\omega n}$$

$$J\, x_I(\omega) = \sum_{n=-3}^{3} x_e(n)\, e^{-j\omega n}$$

$$x(\omega) = x_I(\omega) + x_R(\omega)\, e^{j2\omega}$$

$= \dfrac{x_o(n)}{j} + x_e(n+2)$ → from IFT of $x(\omega)$

$$= -j\, x_o(n) + x_e(n+2)$$

$$= \{-\tfrac{1}{2}, 0, 1, \tfrac{-j}{2}, 2, 1, +\tfrac{j}{2}, 0, \tfrac{1}{2}, -2j, 0, \tfrac{j}{2}\}$$

*

Sol

$$x_1(\omega) = \sum_{n=0}^{m} 1 \cdot e^{-j\omega n}$$

$$= 1 + e^{-j\omega} + e^{-j\omega 2} + e^{-j\omega 3} + \dots - [e^{-j\omega(m+1)} + e^{-j\omega(m}$$

$$\frac{e^{-j\omega(m+1)}}{1 - e^{-j\omega}}$$

$$= \frac{1}{1 - e^{j\omega}} - \frac{e^{-j\omega(m+1)}}{1 - e^{-j\omega}}$$

$$= \frac{1 - e^{-j\omega(m+1)}}{1 - e^{-j\omega}}$$

$$x_2(\omega) = \sum_{n=-m}^{-1} e^{-j\omega m}$$

$$= \sum_{n=1}^{m} e^{j\omega m}$$

$$= \frac{1-e^{j\omega m}}{1-e^{j\omega}} e^{j\omega}$$

$$X(\omega) = X_1(\omega) + X_2(\omega)$$

$$= \frac{1-e^{j\omega}(m+1))}{1-e^{j\omega}} + \frac{1-e^{j\omega m}}{1-e^{j\omega}} e^{j\omega}$$

$$= \frac{1+e^{j\omega} - e^{j\omega} - 1 - e^{-j\omega(m+1)} - e^{j\omega(m+1)} + e^{j\omega m} + e^{-j\omega m}}{2-e^{-j\omega} - e^{j\omega}}$$

$$= \frac{2\cos\omega m - 2\cos\omega m(m+1)}{2-2\cos\omega}$$

$$= \frac{2\sin(\omega m + \frac{\omega}{2}) \cdot \cos\frac{\omega}{2}}{2\sin^2(\frac{\omega}{2})}$$

$$= \frac{\sin(m+\frac{1}{2})\omega}{\sin(\frac{\omega}{2})}$$

Prove that $1 + 2\sum_{n=1}^{m} \cos\omega n = \frac{(\sin(n) + \frac{1}{2})\omega}{\sin(\frac{\omega}{2})}$

14 $x(n) = \{-1, 2, -3, 2, -1\}$

(a) $x(0)$

Sol $X(\omega) = \sum_{\omega=0} x(n) e^{-j\omega n}$

$$x(0) = -3 \cdot e^{0} = -3$$

(b) $\angle x(\omega) = \pi$ for all ω

(c) $\int_{-\pi}^{\pi} X(\omega)\, d\omega$

$$x(0) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\omega)\, d\omega$$

$$\int_{-\pi}^{\pi} X(\omega)\, d\omega = 2\pi\, x(0)$$

$$= 2\pi(-3) = -6\pi$$

(d) $$X(\pi) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n}$$

$$= \sum_n e^{-jn\pi} \times x(n)$$

$$= \sum_n [\cos(n\pi) - j\sin(n\pi)]\, x(n)$$

$$= \sum_n (-1)^n x(n)$$

For $n = 0 = (-1)^0 x(n) = 1 \times -3 = -3$

$n = 1 = (-1)^1 x(1) = -1 \times 2 = 2$

$n = 2 = (-1)^2 x(2) = -1 = -1$

$n = -1 = (-1)^1 x(-1) \Rightarrow -2$

$n = -2 = (-1)^2 x(-2) \Rightarrow -1$

$\Rightarrow -3 \; -2 -1 \; -2 -1$

$\Rightarrow -9$

(e) $\int_{-\pi}^{\pi} |X(\omega)|^2\, d\omega$

Sol $$\frac{1}{2\pi} \int_{-\pi}^{\pi} |X(\omega)|^2\, d\omega = \sum_n |x(n)|^2$$

$$= (-1)^2 + (2)^2 + (-3)^2 + (2)^2 + (-1)^2$$

$$= 1+4+9+4+1 = 19$$

$$\therefore \int_{-\pi}^{\pi} X(\omega)^2 = 2\pi \cdot 19 = 38\pi$$

15 $$c = \frac{\sum_{n=-\infty}^{\infty} n\,x(n)}{\sum_{n=-\infty}^{\infty} x(n)}$$

(a) Express c in terms of $X(\omega)$

we know

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n}$$

$$X(0) = \sum_{n=-\infty}^{\infty} x(n)\, e^{0}$$

$$X(0) = \sum_{n=-\infty}^{\infty} x(n)$$

Now,

$$n\,x(n) \xrightarrow{F.T} j\frac{dX(\omega)}{d\omega}$$

$$-jn\,x(n) \xleftrightarrow{F.T} \frac{dX(\omega)}{d\omega}$$

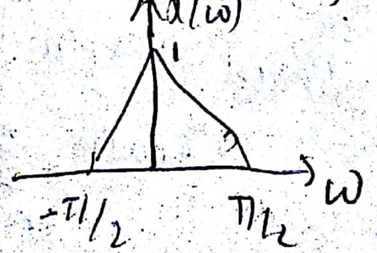
$$\frac{dX(\omega)}{d\omega} = \sum_{n=-\infty}^{\infty} -jn\,x(n)\, e^{-j\omega n} d\omega$$

$$= -j\sum_{n=-\infty}^{\infty} n\,x(n)\, e^{-j\omega n} d\omega$$

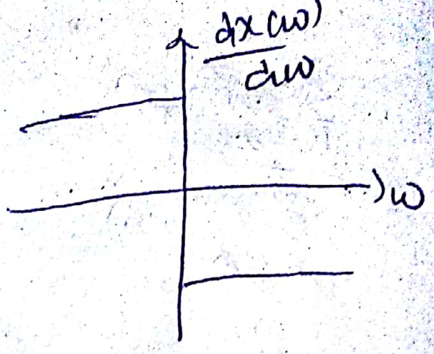
$$\frac{j\,dX(\omega)}{d\omega} = \sum_{n=-\infty}^{\infty} n\,x(n)\, e^{-j\omega n} d\omega$$

$$c = \frac{j\,dX(\omega)}{X(0)}\Big|_{\omega=0}$$

(B)



from given figure $x(0)=1$



$$c = \frac{\int dX(\omega)}{X(0)}$$

$$= \frac{0}{1}$$

$$= 0$$

16. $a^n u(n) \xleftrightarrow{F.T} \frac{1}{1-ae^{-j\omega}}$ $|a|<1$

$$x(n) = \frac{(n+l-1)!}{n!\,(l-1)!} a^n u(n) \xleftrightarrow{F} X(\omega) = \frac{1}{(1-ae^{-j\omega})^l}$$

Sol let $l = k+1$

$$x(n) = \frac{(n+k+1-1)!}{n!\,(k+1-1)!} a^n u(n)$$

$$= \frac{(n+k)!}{n!\,k!} a^n u(n)$$

$$= \frac{(n+k)(n+k-1)!}{n!\,(k-1)!} a^n u(n)$$

let $x_k(n) = \frac{(n+k-1)}{n!\,(k-1)!} a^n u(n)$

$$x_{k+1} = \sum_{n=-\infty}^{\infty} \frac{n+k}{k} x_k(n) e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} \left[\frac{n}{k} x_k(n) + x_k(n) \right] e^{-j\omega n}$$

$$= \frac{1}{k} \sum_{n=-\infty}^{\infty} n\, x_k(n)\, e^{-j\omega n} + \sum_{n=-\infty}^{\infty} x_k(n) e^{-j\omega n}$$

$$= \frac{1}{K} \sum_{n=-\infty}^{\infty} n x_K(n)\, e^{-j\omega n} + X_K(\omega)$$

$$= \frac{1}{K}\, j \frac{dX_K(\omega)}{d\omega} + X_K(\omega)$$

$$X_{K+1} = \frac{a e^{-j\omega}}{(1-a e^{-j\omega})^{K+1}} + \frac{1}{(1-a e^{-j\omega})^{K}}.$$

17

(a) $x^*(n)$

$$\sum_{n=-\infty}^{\infty} x^*(n) e^{-j\omega n}$$

$$\sum_{n=-\infty}^{\infty} [x(n) e^{j\omega n}]^*$$

$$X(-\omega)^*$$

(b) $x^*(-n)$

$$\sum_{n=-\infty}^{\infty} x^*(-n)\, e^{-j\omega n}$$

Replace '$-n$' with 'n'.

$$\sum_{n=-\infty}^{\infty} x^*(n)\, e^{j\omega n}$$

$$\sum_{n=-\infty}^{\infty} [x(n)\, e^{-j\omega n}]^*$$

$$X(\omega)$$

(c) $y(n) = x(n) - x(n-1)$

$$= \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n} - \sum_{n=-\infty}^{\infty} x(n-1)\, e^{-j\omega n}$$

$$= X(\omega) - \sum_{n=-\infty}^{\infty} x(n-1)\, e^{-j\omega n}$$

Let $l = n-1$ (dummy variable)

$$= X(\omega) - \sum_{n=\infty} x(l)\, e^{-j\omega(l+1)}$$

$$= X(\omega) - \sum_{n=-\infty}^{\infty} x(l)\, e^{-j\omega l} - e^{-j\omega}$$

Replace l by n

$$= X(\omega) - e^{-j\omega}\sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n}$$

$$= X(\omega) - e^{-j\omega} X(\omega)$$

$$= X(\omega)\left[1 - e^{-j\omega}\right].$$

(e) $y(n) = x(2n)$

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(2n)\, e^{-j\omega n}$$

let $l = 2n$

$$= \sum_{k=-\infty}^{\infty} x(l)\, e^{-j\frac{l}{2}\omega}$$

$$= \sum_{l=-\infty}^{\infty} x(l)\, e^{-j\frac{l\omega}{2}}$$

$$= X\left(\frac{\omega}{2}\right)$$

(f) $y(n) = \begin{cases} x\left(\frac{n}{2}\right) & ; \text{'n' even} \\ 0 & ; \text{'n' odd} \end{cases}$

$$X(\omega) = \sum_n x\left(\frac{n}{2}\right) e^{j\omega n}$$

let $n = 2l$

$$= \sum_n x\left(\frac{2l}{2}\right) e^{j2l\omega}$$

$$= \sum_n x(l)\, e^{j2l\omega}$$

$$= X(2\omega).$$

18

(a) $x_1(n) = \{1, 1, 1, 1, 1\}$

$\quad\quad -2 \quad \underset{0}{\uparrow} \quad 2$

$$\sum_{n=-\infty}^{\infty} x_1(n)\, e^{-j\omega n}$$

$$\sum_{n=-2}^{2} x_1(n)\, e^{-j\omega n}$$

For $n=-2$ $\quad 1 \cdot e^{+j2\omega} = e^{j2\omega}$

$n=-1 \quad 1 \cdot e^{j\omega} = e^{j\omega}$

$n=0 \quad 0 \cdot e^{0} = 1$

$n=1\,; \quad 1 \cdot e^{-j\omega} = \frac{1}{e^{j\omega}}$

$n=2\,; \quad 1 \cdot e^{-2j\omega} = \frac{1}{e^{j2\omega}}$

$= e^{j2\omega} + e^{j\omega} + 1 e^{-j\omega} + e^{-j2\omega}$

$= 2\cos(2\omega) + 2\cos(\omega) + 1$

(b) $x_2(n) = \{1, 0, 1, 0, 1, 0, 1, 0, 1\}$

$\quad -4 \quad\quad \underset{0}{\uparrow}\ 1\ 2\ 3\ 4$

for $n=-2\,; \quad e^{j2\omega}$

$n=-4\,; \quad e^{j4\omega}$

$n=0\,; \quad 1$

$n=2\,; \quad e^{-j2\omega}$

$n=4\,; \quad e^{-j4\omega}$

$\Rightarrow e^{j2\omega} + e^{j4\omega} + 1 + e^{-j2\omega} + e^{-j4\omega}$

$\Rightarrow 2\cos(2\omega) + 2\cos(4\omega) + 1$

(c) $x_3(n) = \{1,0,0,1,0,0,1,0,0,1,0,0,1\}$

$\quad\quad -6 \quad\quad\quad \uparrow \quad\quad\quad 6$

For

$n=-6\,;\ e^{j6\omega}$

$n=-3\,;\ e^{+j3\omega}$

$n=0\,;\ 1$

$n=3\,;\ e^{-j3\omega}$

$n=6\,;\ e^{-j6\omega}$

$\Rightarrow e^{j6\omega}+e^{j3\omega}+1+e^{-j3\omega}+e^{-j6\omega}$

$\Rightarrow 2\cos(6\omega)+2\cos(3\omega)+1$

(d) is there relation.

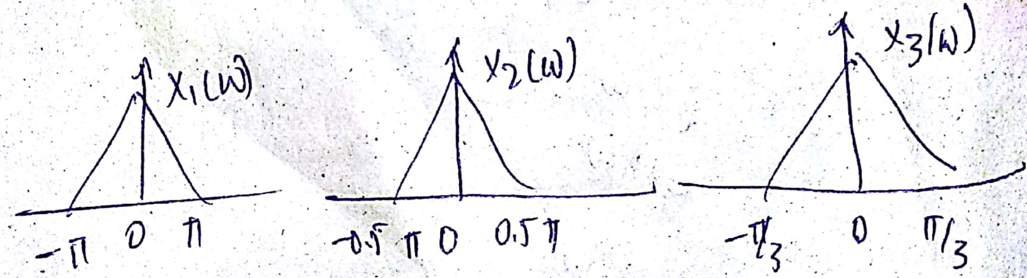
Sol $X_1(\omega) = 2\cos(2\omega)+2\cos(\omega)+1$

$X_2(\omega) = 2\cos(2\omega)+2\cos(4\omega)+1$

$X_2(\omega) = X_1(2\omega)$

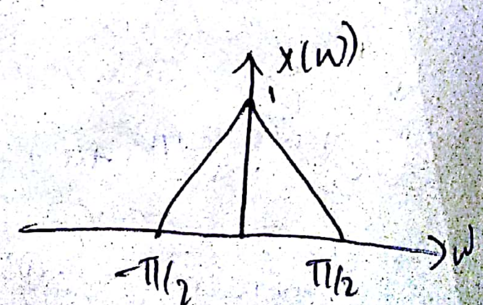
$X_3(\omega) = 2\cos(6\omega)+2\cos(3\omega)+1$

$X_3(\omega) = X_1(3\omega)$



19 let $x(n)$

(a) $x_1(n) = x(n)\cos\left(\frac{\pi n}{4}\right)$



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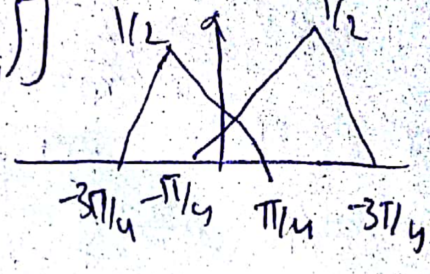
Sol we know

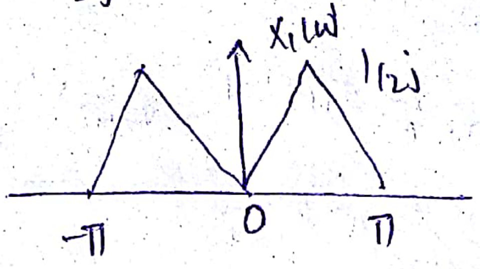
$$x(n)\cos(\omega_0 n) \xleftrightarrow{FT} \frac{1}{2}[X(\omega-\omega_0)+X(\omega+\omega_0)]$$

$$x_1(\omega) = \frac{1}{2}\left[X\left(\omega+\frac{\pi}{4}\right) + X\left(\omega-\frac{\pi}{4}\right)\right]$$



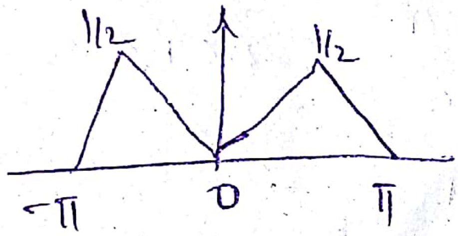
(b) $x_2(n) = x(n)\sin[\pi n/2]$

$$x_2(\omega) = \frac{1}{2j}[X(\omega+\pi/2) + X(\omega-\pi/2)]$$



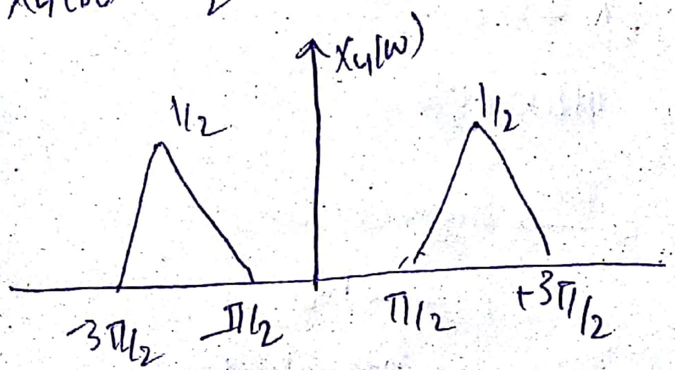
(c) $x_3(n) = x(n)\cos\left(\frac{-n}{2}\pi\right)$

$$x_3(n) = \frac{1}{2}[X(\omega-\pi/2) + X(\omega+\pi/2)]$$



(d) $x_4(n) = x(n)\cos(\pi n)$

$$X_4(\omega) = \frac{1}{2}[X(\omega-\pi) + X(\omega+\pi)]$$



20

$$y(n) = \sum_{l=-\infty}^{\infty} x(n-lN)$$

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Sol

$$C_k^y = \frac{1}{N} x\left(\frac{2\pi}{N}\right)k$$

$$C_k^y = \frac{1}{N}\sum_{m=0}^{N-1} y(n)\, e^{-j2\pi n/N}$$

$$= \frac{1}{N}\sum_{n=0}^{N-1}\left[\sum_{l=-\infty}^{\infty} x(n-lN)\, e^{-j2\pi kn/N}\right]$$

$$= \frac{1}{N}\sum_{l=-\infty}^{\infty}\sum_{m=0}^{N-1-lN} x(m)\, e^{-j2\pi k(m+lN)/N}$$

$$\text{But} \sum_{l=-\infty}^{\infty}\sum_{m=-lN}^{N-1-lN} x(m)\, e^{-j\omega(\omega+N)} = X(\omega)$$

$$C_k^y = \frac{1}{N} x\left(\frac{2\pi k}{N}\right)$$

21

Sol Let $x_N(n) = \frac{\sin \omega_c n}{\pi n}$; $-N \le n \le N$

$= x(n)\, w(n)$

where $x(n) = \frac{\sin \omega_c n}{\pi n}$ $\quad -\infty < n \le \infty$

$w(n) = 1$; $-N < n \le N$

0 ; otherwise

$$\frac{\sin \omega_c n}{\pi n} \xleftrightarrow{F.T} X(\omega)$$

$= 1$; $|\omega| < \omega_c$

$= 0$; otherwise

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$$x_N(\omega) = X(\omega) * W(\omega)$$

$$= \int_{-\pi}^{\pi} X(\theta)\, W(\omega-\theta)\, d\theta$$

$$= \int_{-\omega c}^{\omega c} \left[\frac{\sin(2N+1)\ (\omega-\theta/2)}{\sin(\omega-\theta/2)} \right] d\theta$$

22. $x(\omega) = \frac{1}{1-ae^{-j\omega}}$

(a) $x(2n+1)$

$$\sum_{n=-\infty}^{\infty} x(2n+1)\, e^{-j\omega n}$$

let $2n+1 = l$

$$\sum_{l=-\infty}^{\infty} x(l)\, e^{-j\omega\left(\frac{l-1}{2}\right)}$$

$$\sum_{l=-\infty}^{\infty} x(l)\, e^{-\frac{j\omega l}{2}} \cdot e^{\frac{j\omega}{2}}$$

$$e^{\frac{j\omega}{2}} \sum_{l=-\infty}^{\infty} x(l)\, e^{-\frac{j\omega l}{2}}$$

$$e^{\frac{j\omega}{2}} \sum_{l=-\infty}^{\infty} x(l)\, e^{-\frac{j\omega}{2}n}$$ replace $l = n$

$$e^{\frac{j\omega}{2}}\, X\left(\frac{\omega}{2}\right) \Rightarrow e^{\frac{j\omega}{2}} \cdot \frac{1}{1-ae^{-\frac{j\omega}{2}}} = \frac{e^{\frac{j\omega}{2}}}{1-ae^{-j\left(\frac{\omega}{2}\right)}}$$

(b) $e^{\frac{\pi j}{2}}$, $x(n+2)$

$$e^{j2\omega} \cdot X\left(\omega - \frac{\pi j}{2}\right)$$

$$x(n) \longleftrightarrow X(\omega)$$

$$x(n+2) \longleftrightarrow e^{j2\omega}\, X(\omega)$$

$e^{\frac{\pi}{2}j}$ $x(n+2) \longleftrightarrow e^{j2\omega} X(\omega - \frac{\pi j}{2})$

$\therefore e^{j2\omega} \cdot X(\omega - \frac{\pi j}{2})$

(c) $x(-2n)$

$x(n) \longleftrightarrow X(\omega)$

$x(2n) \longleftrightarrow X(\frac{\omega}{2})$

$x(-2n) \longleftrightarrow X(\frac{-\omega}{2})$

(d) $x(n) \cos(0.3\pi n)$

$x(n) \cos(\omega_0 n) \longleftrightarrow \frac{1}{2}[X(\omega+\omega_0) + X(\omega-\omega_0)]$

$x(n) \cos(0.3n) \longleftrightarrow \frac{1}{2}[X(\omega+0.3\pi) + X(\omega-0.3\pi)]$

(e) $x(n) * x(n-1)$

$X(\omega) \cdot e^{-j\omega} X(\omega)$

$X^2(\omega) e^{-j\omega}$

(f) $x(n) x\, x(-n)$

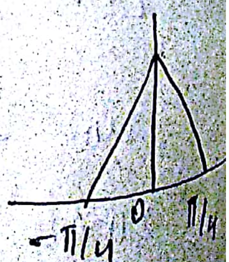
$X(\omega) * X^*(\omega)$

$\frac{1}{1-ae^{-j\omega}} \cdot \frac{1}{1-ae^{j\omega}}$

$\frac{1}{1-ae^{j\omega}-ae^{-j\omega}+a^2e^0} = \frac{1}{1+a^2-2\cos\omega}$

23 $y_1(n) = x(n)s(n)$

where $s(n) = \{\ldots 0, 1, 0, \underset{\uparrow}{1}, 0, 1, 0 \ldots\}$



Sol (a) $y_1(n) = \begin{cases} x(n), & \text{'n' even} \\ 0, & \text{'n' odd} \end{cases}$

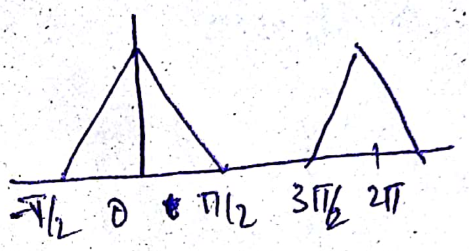
(b) $y_2(n) = x(2n)$

$y_2(n) = x(2n)$

$Y_2(w) = \sum_n y_2(n) e^{-jwn}$

$= \sum_n x(2n) e^{-jwn}$

$= X\left(\frac{w}{2}\right)$



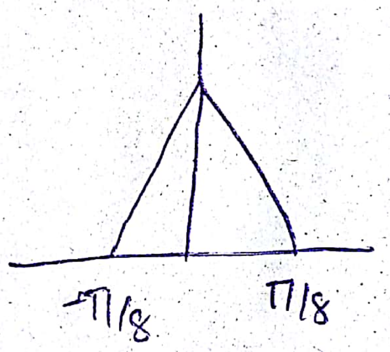
(c) $y_3(n) = \begin{cases} x(n/2) & ; \text{'n' even} \\ 0 & ; \text{'n' odd} \end{cases}$

$= \sum_n y_3(n) e^{-jwn}$

$= \sum_{n=\text{even}} x\left(\frac{n}{2}\right) e^{-jwn}$

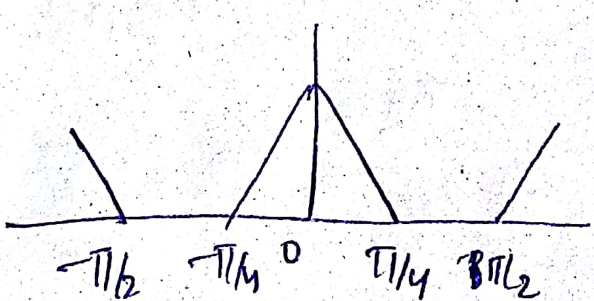
$= \sum_m x(m) e^{-j2wm}$

$= X(2w)$



(d) $y_1(n) = \begin{cases} y_2\left(\frac{n}{2}\right) & ; \text{'n' even} \\ 0 & ; \text{'n' odd} \end{cases}$

$X_1(w) = X_2(2w)$



END