Study of effect of presence of Moon on a geo-stationary orbit

Nagendra K. Panduranga

April 20, 2018

A geo-stationary satellite is a satellite whose time period of revolution is equal to one siderial day. In this report, we study the effect of presence of moon on the keplerian orbit of a geo-stationary satellite.

1 Program

A fortran 90 programme was written to integrate Newton's equations of motion based on fourth-order Runge-Kutta(RK) scheme. The x,y,z components of the acceleration of the satellite due to gravitational forces due to earth and moon were defined to be

$$a_i = -\frac{GM_e x_i}{r^3} - \frac{GM_m x_{sm,i}}{r_{sm}^3} \tag{1}$$

where i =1,2,3 represent the x,y,z components. M_e and M_m are the masses of the earth and moon respectively. r and r_{sm} are the distance of the satellite from Earth and Moon respectively.

The time period of the keplerian orbit of the satellite T_{sat} was defined in terms of the time period of revolution of a geostationary satellite as

$$T_{sat} = a * T_s \tag{2}$$

where T_s is one siderial day expressed in seconds.

The radius of the Keplerian orbit with the time period T_{sat} is defined by

$$r_{sat} = (\frac{GM_e T_{sat}^2}{4\pi^2})^{1/3} \tag{3}$$

The position of the satellite was expressed in terms of the r,ϕ and θ and their relation with the x,y,z co-ordinates was defined by

$$x = r * Cos(\theta) * Cos(\phi); y = r * Cos(\theta) * Sin(\phi); z = r * Sin(\theta). \tag{4}$$

The Programme integrated the equations of motion for a total time of $tmax$ at time steps of $\delta_t = T_s/N_t$.

2 Results and Discussion

The effect of moon on the orbit of the satellite was studied for two different cases.

2.1 Effect of "equatorial moon"

The motion of the geostationary satellite was studied for the case when the orbit of the moon is assumed to be in the same plane as that of the satellite. The deviation of the orbit of the satellite for 100 day period, characterized by the quantities $\Delta\phi$ and Δr, are shown in figs.(1). The plots clearly show that the angular deviation of the satellite, from that of an ideal geostationary orbit, increases continuously with time and hence the effect of moon on the orbit of a geo-stationary orbit cannot be ignored for longer time-scales, of the order of 100 days.

The plots shown in the figs.(1) were produced with time step = 1 second. The choice for this time step is justified by the figs.2 which show that the plots cannot be resolved for the cases of $\delta_t = 1$ and $\delta_t = 0.1$ second.

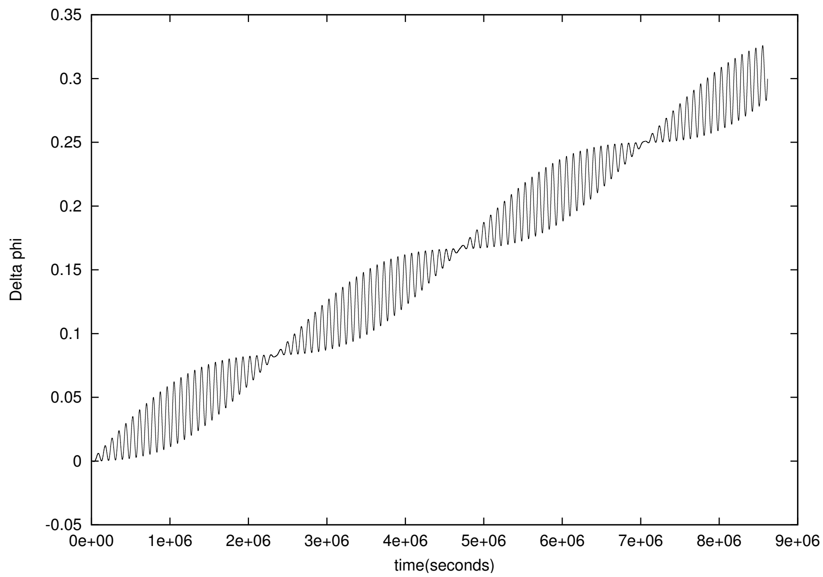
The programme was run for different values of a and the optimum value that minimizes the angular deviation of the orbit of the satellite from that of an ideal geostationary satellite was found to be $a = 1.000482$. Hence the Keplerian radius r was found to be 1.00289 times that of the radius of a Keplerian geostationary orbit.The plots of angular deviation for the orbit with optimum a are plotted in figs.3.

The reason for the existence of an optimum value of a can be understood by noticing that the average effect of the presence of moon is to reduce the effective radial component of gravitational force acting on the satellite. Therefore the centripetal acceleration of the satellite reduces. Hence, the initial radius of the orbit should be larger and the initial velocity of the satellite should be smaller compared to the corresponding values for that of the ideal geostationary orbit, in the absence of moon, for the satellite to go around earth in one siderial day. This is what is acheived when we choose a to be greater than 1. The initial radius of the orbit is greater than the geostationary orbit and the initial velocity of the satellite is smaller because it is taken to be proportional to $\frac{1}{\sqrt{r_{sat}}}$.

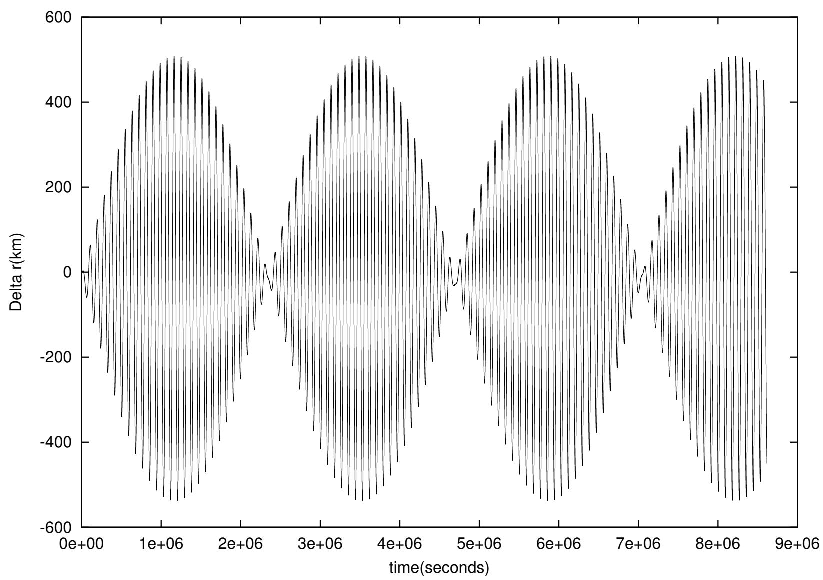
2.2 Effect of "non-equatorial" moon

The perturbations of the orbit of the satellite was studied for the case when the orbit of the moon was inclined to the oribit of the satellite by 25degrees. The plots of deviation of azimuthal angle and the angle that satellite makes with the equatorial plane are shown in the plots 4.

The programme was run for different values of a and the optimum a that minimizes the angular drift was found to be 1.00044. The initial radius is found to be 1.0026 times the radius of the geostationary orbit, i.e. $4.2188 * 10^4$ km. The plots of the angular deviation and the deviation of the satellite from the equatorial plane for 500 day period are plotted in figs.5.

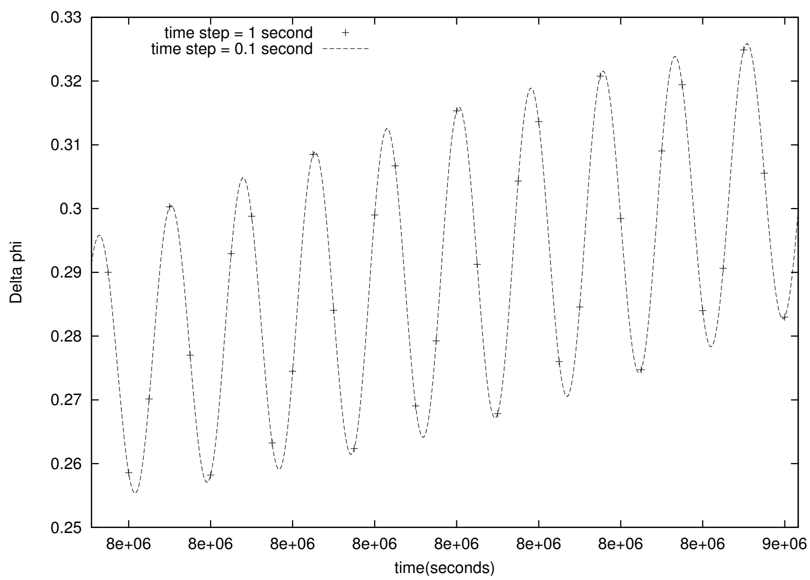


(a)

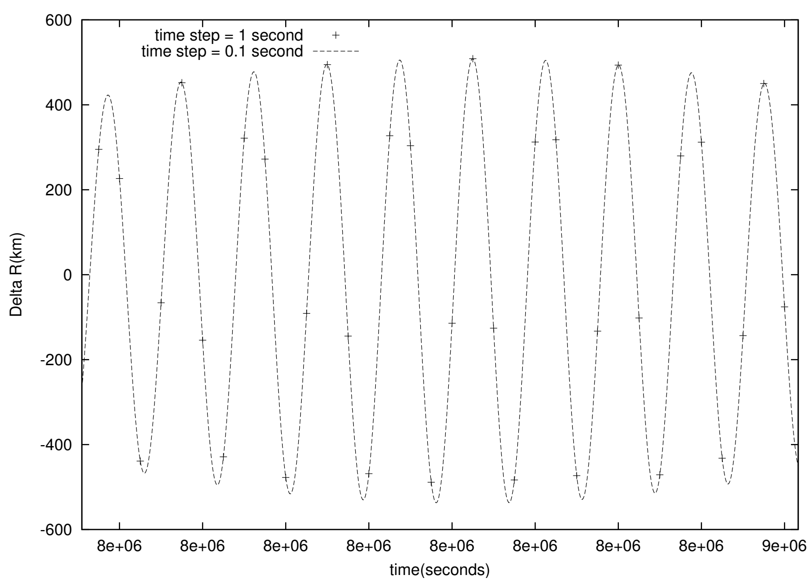


(b)

Figure 1: The deviation of the orbit of the satellite from that of an ideal geostationary orbit for 100 day period.

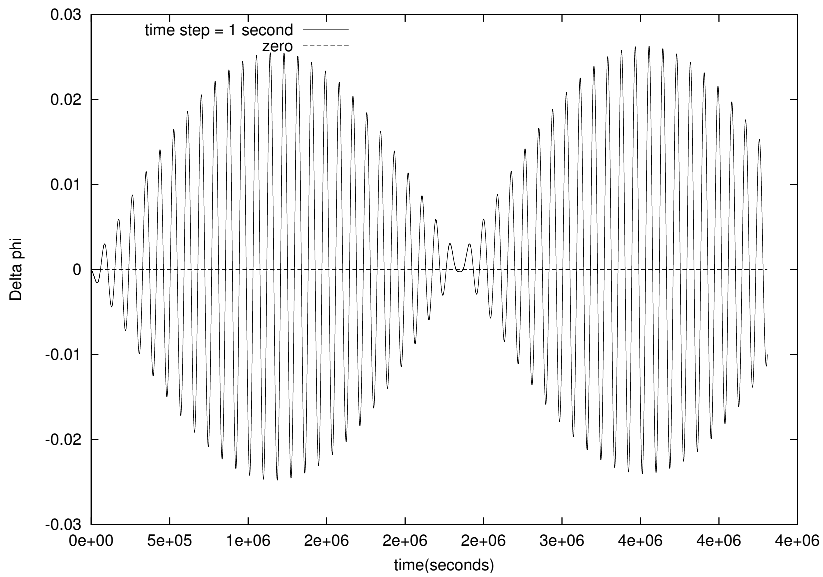


(a)

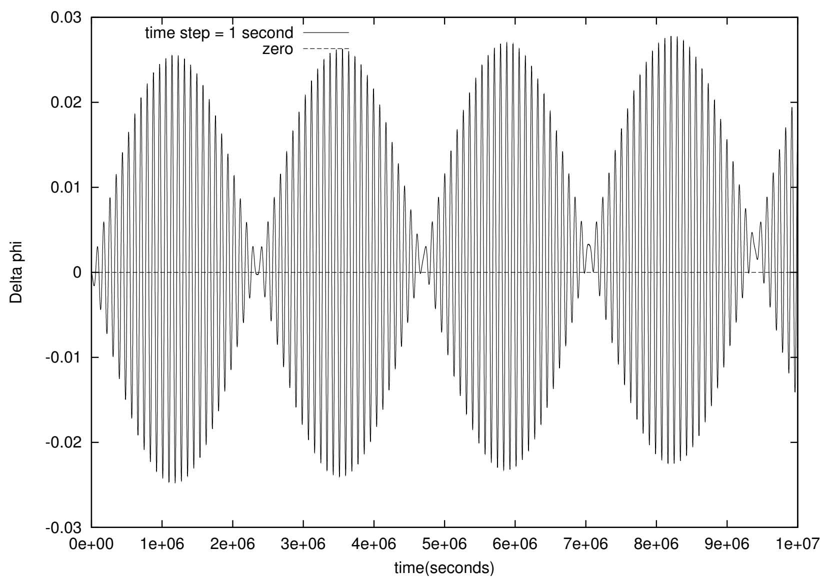


(b)

Figure 2: Plots of angular and radial deviations for time step of 0.1 and 1 seconds.

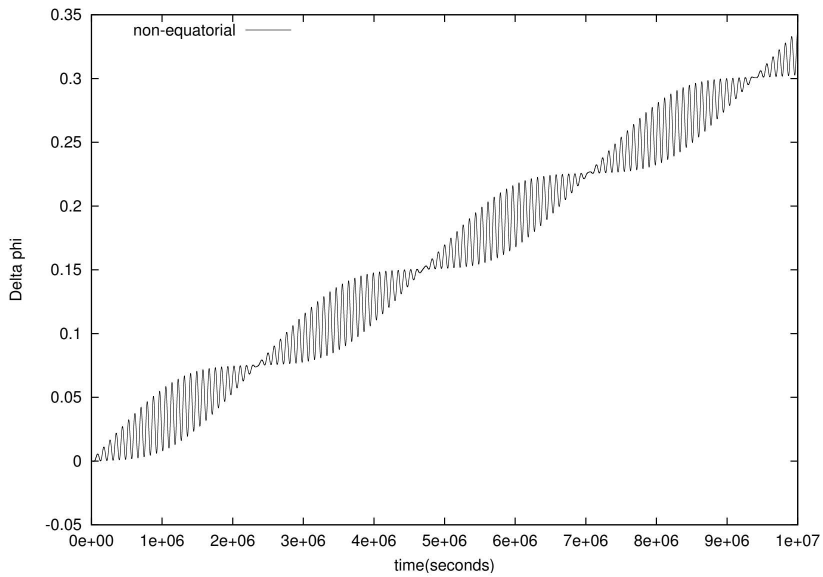


(a)

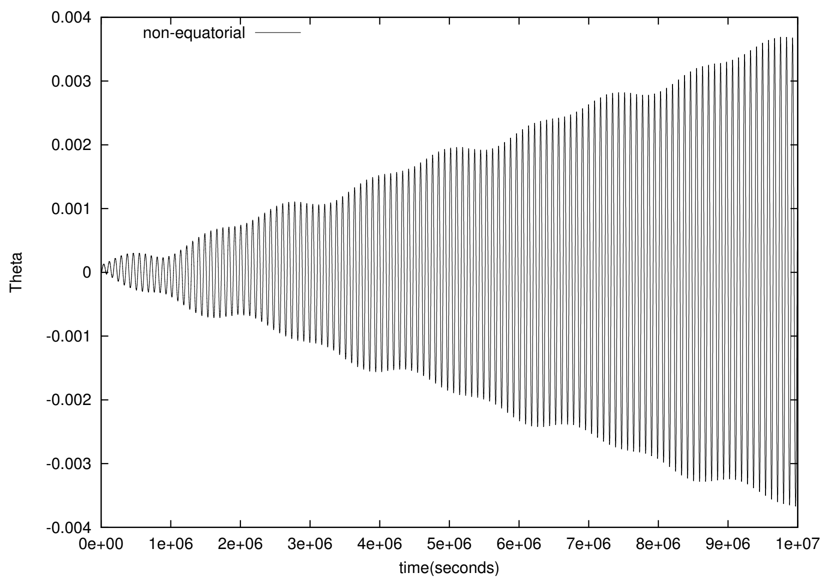


(b)

Figure 3: The angular deviation of the satellite in an orbit that minimizes the angular deviation from that of a true geostationary orbit.

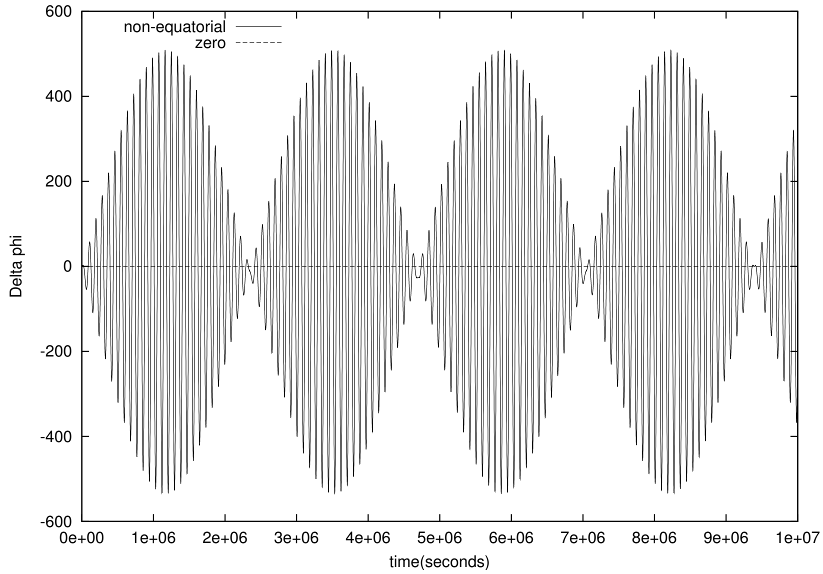


(a)



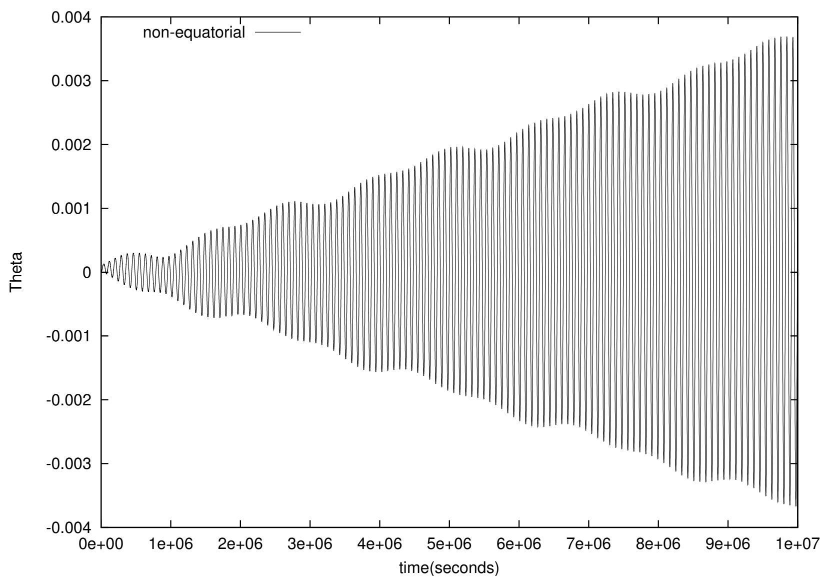
(b)

Figure 4: Deviation of the satellite for the case of 'non-equatorial' moon over a 200 day period.



600
400
200
0
-200
-400
-600
Delta phi
non-equatorial
zero
0e+00
1e+06
2e+06
3e+06
4e+06
5e+06
6e+06
7e+06
8e+06
9e+06
1e+07
time(seconds)

(a)



0.004
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-0.004
Theta
non-equatorial
0e+00
1e+06
2e+06
3e+06
4e+06
5e+06
6e+06
7e+06
8e+06
9e+06
1e+07
time(seconds)

(b)

Figure 5: