1a) Explain

- **i) Principle of Transmissibility of forces**
- **ii) Principle of Superposition of forces**
- **iii) Principle of Physical Independence of forces**

Sol^n: i) This principle states that a force can be transmitted from one point to another point along the same line of action such that the effect produced by the force on a body remains unchanged.

Let us consider a rigid body subjected to a force of F at point O as shown in figure. According to the principle of transmissibility, the force can be transmitted to a new point O' along the same line of action such that the net effect remains unchanged.

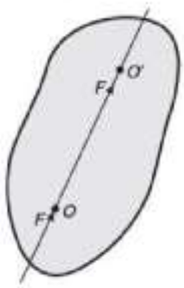
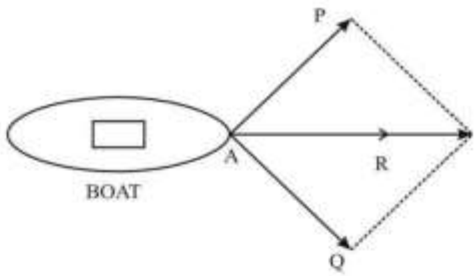


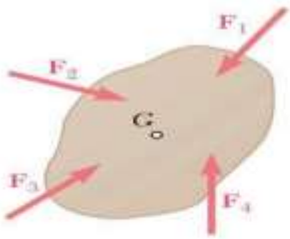
Figure 14 Transmissibility of force *F* from point *O* to *O'*.

ii) The principle stateds that the net effect of a system of forces on a body is same as that of the combined effect of individual forces on the body.



Principle of Superposition

iii) This principle states that the action of a force on a body is not affected by the action of any other force on the body.



1b) Solve for the magnitude and direction of the force system shown in Fig 1(b)

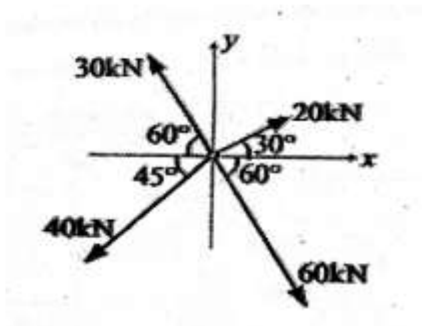


Fig. 1(b)

Sol^n:

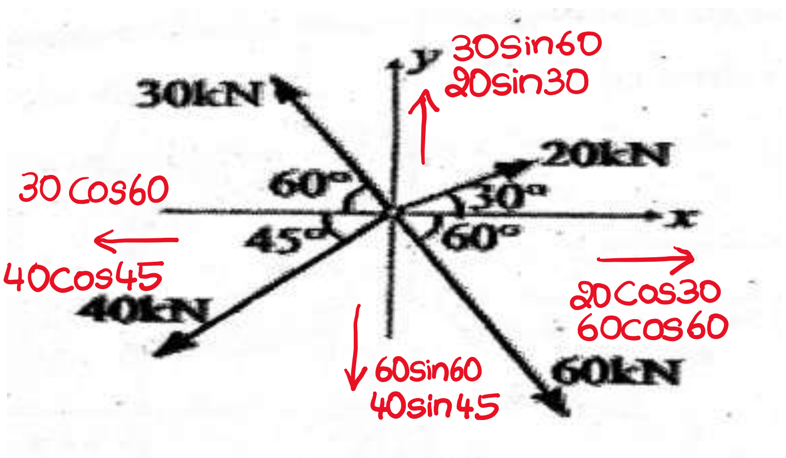


Fig. 1(b)

$\sum f_x$ = $20cos30 + 60cos60 - 30cos60 - 40cos45 = 4.036kN$

$\sum f_y$ = $20sin30 - 60sin60 + 30sin60 - 40sin45 = -44.265kN$

We know that,

$$R = \sqrt{\left(\sum f_x\right)^2 + \left(\sum f_y\right)^2}$$

$$R = \sqrt{(4.036)^2 + (-44.265)^2}$$

$\boxed{R = 44.449kN}$

$$\theta = tan^{(-1)}\left(\frac{\sum f_y}{\sum f_x}\right)$$

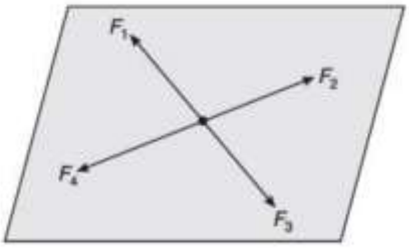
$$\theta = tan^{(-1)}\left(\frac{-44.265}{4.036}\right)$$

$\boxed{\theta = -84.79°}$

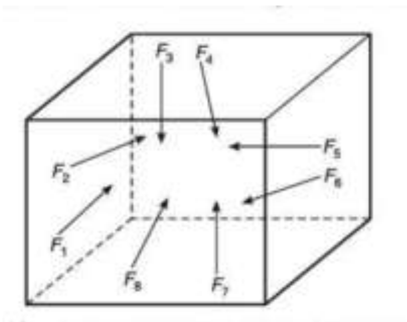
2a) Explain

i) **Coplanar concurrent force systems**
ii) **Non coplanar force system**
iii) **Like parallel force system**
iv) **Collinear force system**

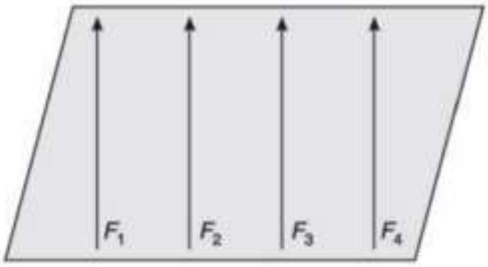
Sol^n: i) If two or more forces are acting in a single plane and their lines of action pass through a single point, then it is said to be a coplanar concurrent force system.



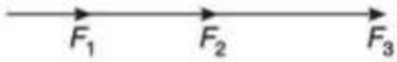
ii) If two or more forces are acting in different planes, the forces constitute a non – coplanar force system.



iii) All the forces act parallel to one another and are in the same direction.



iv) If the lines of action of two or more forces coincide with one another, it is called a collinear force system.



2b) Four forces acting on a hook are shown in Figure 2(b). Determine the direction of the force 125N such that the hook is pulled in the x – direction. Determine the resultant force in the x – direction.

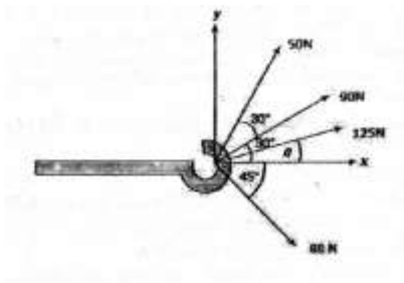


Fig 2(b)

Sol^n:

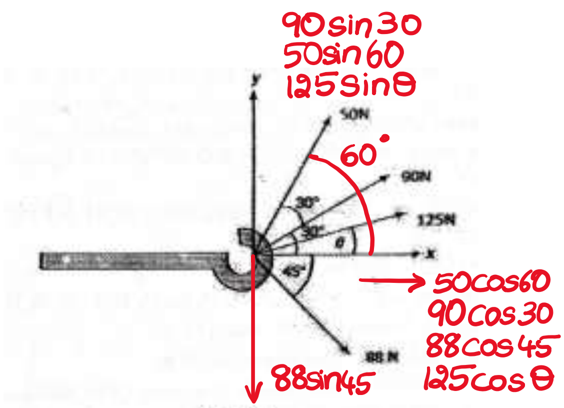


Fig 2(b)

$\sum f_y$ is zero in this case because the entire resultant force is acting in x – direction not in y – direction, so we equate $\sum f_y$ as 0 we get θ, hence we can write the equation as,

$$\sum f_y = 90sin30 + 50sin60 + 125sin\theta - 88sin45 \ = 0$$

By re-arranging the equation to sinθ we get,

$$sin\theta = \frac{88sin45 - 90sin30 - 50sin60}{125}$$

$$sin\theta = -0.208$$

Now,

$$\theta = sin^{(-1)}(-0.208)$$

We get,

$$\boxed{\theta = -12.005°}$$

Now take $\sum f_x$ as "R" (Resultant)

$$\sum f_x = 90cos30 + 50cos60 + 125\cos(-12.005) + 88cos45 \ = R$$

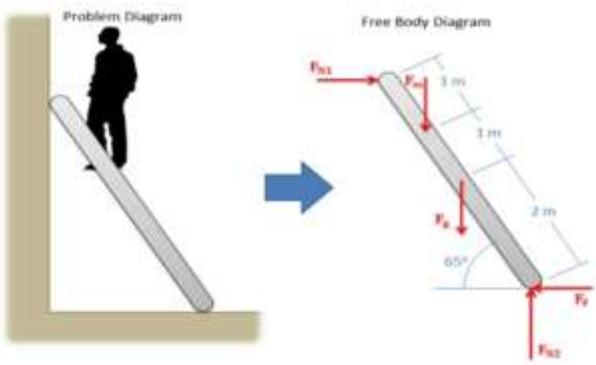
$$R = 90cos30 + 50cos60 + 125\cos(-12.005) + 88cos45$$

$$\boxed{R = 287.434N}$$

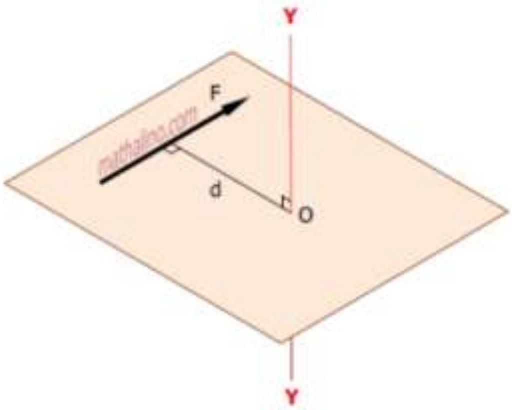
3a) Explain

i) Free body Diagram with a neat sketch
ii) Moment of a force
iii) Composition of forces

Sol^n: i) A free body diagram is a sketch of a body, a portion of a body, or two or more bodies completely isolated or free from all other bodies, showing the forces exerted by all other bodies on the one being considered.



ii) Moment of force is the measure of the capacity or ability to the force to produce twisting or turning effect about an axis. The axis is perpendicular to the plane containing the line of action of the force. The magnitude of moment is equal to the product of the force and the perpendicular distance from the axis to the line of action of the force. The intersection of the plane and the axis is commonly called the moment centre, and perpendicular distance from the moment centre to the line of action of the force is called moment arm.



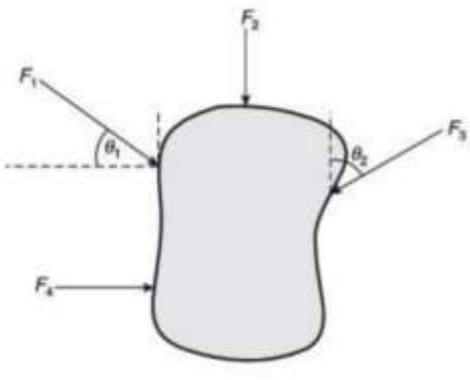
iii) It is the process of combining a number of forces into a single forces such that the net effect produced by single force is equal to the algebraic sum of effects produced by the individual forces.

$$\sum f_x = algebraic\ sum\ of\ the\ components\ of\ the\ forces\ along\ the\ x - axis$$

$$\sum f_x = f_4 + f_1 cos\theta_1 - f_3 sin\theta_2$$

$$\sum f_y = algebraic\ sum\ of\ the\ components\ of\ the\ forces\ along\ the\ y - axis$$

$$\sum f_y = -f_2 - f_1 sin\theta_1 - f_3 cos\theta_2$$



3b) Solve for the centroid for the area shown in the fig 3(b) with respect to the axis shown.

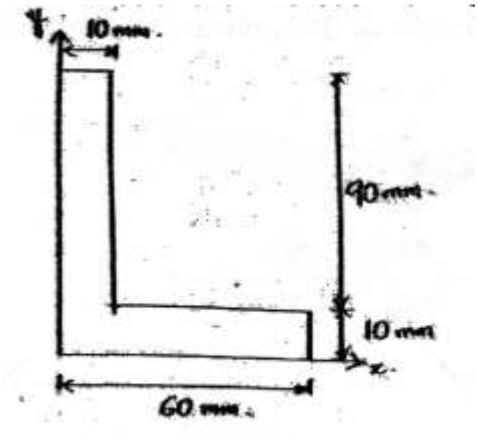


Fig 3(b)

Sol^n:

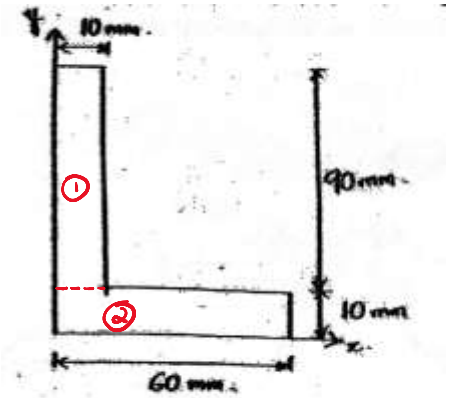


Fig 3(b)

Components	Area (mm^2) (A)	Centroidal Axis from y – axis (mm) (x)	Centroidal Axis from x – axis (mm) (y)	A.x (mm^3)	A.y (mm^3)
Rectangle 1	10×90	$\frac{10}{2}$	$\frac{90}{2} + 10$	4500	49500
Rectangle 2	60×10	$\frac{60}{2}$	$\frac{10}{2}$	18000	3000
	$\sum A = 1500$			$\sum A.x = 22500$	$\sum A.y = 52500$

$$\boxed{\bar{x} = \frac{\sum A.x}{\sum A} = \frac{22500}{1500} = 15mm}$$

$$\boxed{\bar{y} = \frac{\sum A.y}{\sum A} = \frac{52500}{1500} = 35mm}$$

4a) Explain

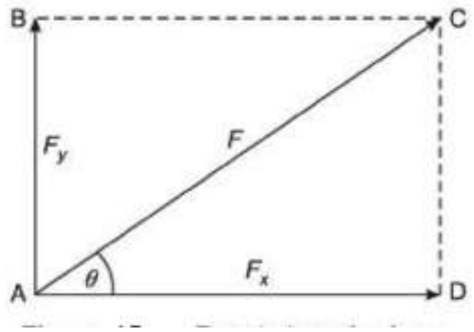
- **i) Characteristics of force**
- **ii) Resolution of force**

Sol^n: i) Characteristics of force are:

a) Magnitude: The length of the vector represents the magnitude of force, as shon in figure.

b) Direction: the direction of a force can be represented by an arrow head.
c) Line of action: It is the line along which force acts.
d) Point of application: It is the point at which the force acts.

ii) The process of splitting of a force into its two rectangular components (horizontal and vertical) is known as resolution of the forces, as shown in the Figure. In this figure, F is the force which makes an angle θ with the horizontal axis, and has been resolved into 2 components, namely F_x and F_y, along the x – axis and y – axis respectively.



$$cos\theta = \frac{AD}{AC} \Rightarrow \frac{F_X}{F} \Rightarrow\ F_x = Fcos\theta$$

$$sin\theta = \frac{AB}{AC} \Rightarrow \frac{F_y}{F} \Rightarrow\ F_y = Fsin\theta$$

If, on the other hand, θ is the angle made by the force F with the vertical axis, then

$$\boxed{F_y = Fcos\theta;\ F_x = Fsin\theta}$$

4b) Solve for the centroid for the area shown in the figure 4(b) with respect to the centroidal axis.

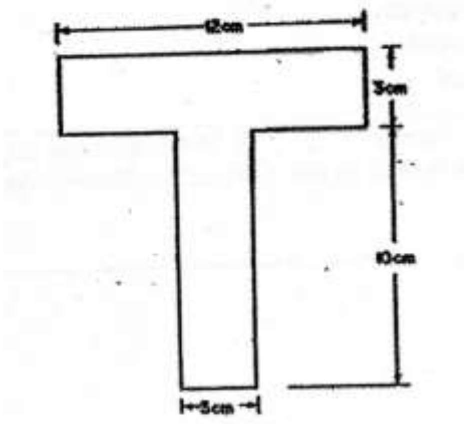


Fig 4(b)

Sol^n:

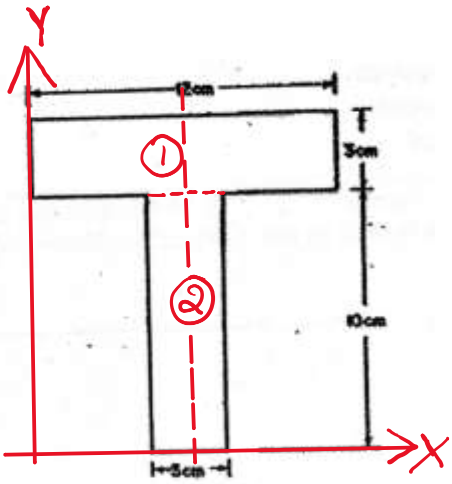


Fig 4(b)

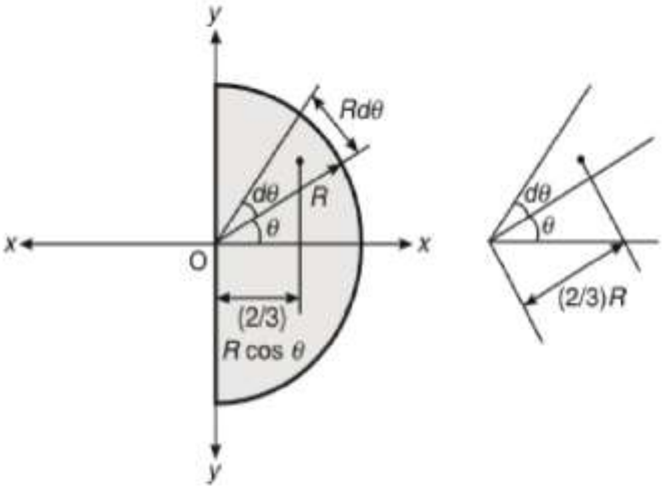
Components	**Area (cm^2) (A)**	**Centroidal Axis from y – axis (cm) (x)**	**Centroidal Axis from x – axis (cm) (y)**	**A.x (cm^3)**	**A.y (cm^3)**
Rectangle 1	12×3	$\frac{12}{2}$	$\frac{3}{2} + 10$	216	414
Rectangle 2	10×3	$\frac{12}{2}$	$\frac{10}{2}$	180	150
	$\sum A = 66$			$\sum A.x = 396$	$\sum A.y = 564$

$$\boxed{\bar{x} = \frac{\sum A.x}{\sum A} = \frac{396}{66} = 6cm}$$

$$\boxed{\bar{y} = \frac{\sum A.y}{\sum A} = \frac{564}{66} = 8.545cm}$$

5a) Apply the method of integration to derive the centroid of semicircle.

Sol^n:



Consider a semicircular lamina of area $\frac{\pi r^2}{2}$ as shown in figure. Now consider a triangular elementary strip of area $\frac{1}{2} \times R \times R \times d\theta$ at an angle of θ from the x – axis, whose centre of gravity is at a distance of $\frac{2}{3}R$ from O and its projection on the

x – axis = $\left(\frac{2}{3}\right) Rcos\theta$

Moment of area of elementary strip about the y – axis = $\frac{1}{2} \times R^2 \times d\theta \times \left(\frac{2}{3}\right) \times Rcos\theta$

$$= \frac{R^3.cos\theta.d\theta}{3}$$

Sum of moment of such elementary strips about the y – axis, = $\int_{-\pi/2}^{\pi/2} \frac{R^3}{3} cos.\,d\theta$

$$= \frac{R^3}{3}[sin\theta]_{-\pi/2}^{\pi/2}$$

$$= \frac{R^3}{3}\left(sin\frac{\pi}{2} + sin\frac{\pi}{2}\right) = \frac{2R^3}{3}$$

Moment of total area about the y – axis = $\frac{\pi R^2}{2} \times \bar{x}$

Using the principle of moments,

$$\frac{2R^3}{3} = \frac{\pi R^2}{2} \times \bar{x}$$

$$\bar{x} = \frac{2R.R^2 \times 2}{3R^2\pi}$$

$$\boxed{\bar{x} = \frac{4R}{3\pi}}$$

But in this case there is no base in x – direction,

Therefore,

$$\boxed{\bar{y} = 0}$$

5b) Solve for the centroid for the area shown in the fig 5(b) with respect to the axis shown.

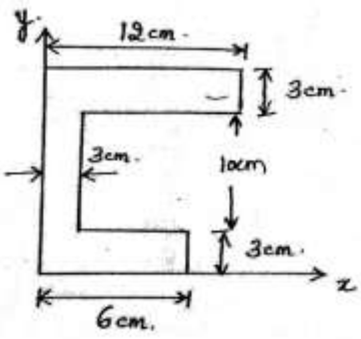


Fig 5(b)

Sol^n:

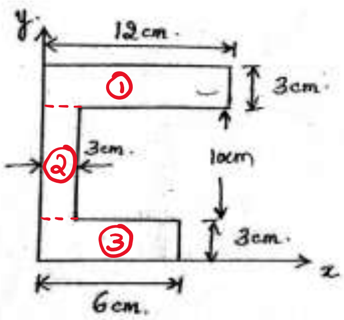


Fig 5(b)

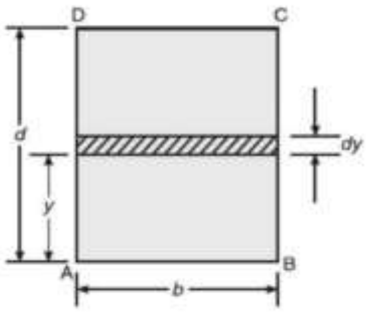
Components	Area (cm²) (A)	Centroidal Axis from y – axis (cm) (x)	Centroidal Axis from x – axis (cm) (y)	A.x (cm³)	A.y (cm³)
Rectangle 1	12×3	$\frac{12}{2}$	$\frac{3}{2} + 10 + 3$	216	522
Rectangle 2	10×3	$\frac{3}{2}$	$\frac{10}{2} + 3$	45	240
Rectangle 3	3×6	$\frac{6}{2}$	$\frac{3}{2}$	54	27
	$\sum A = 84$			$\sum A.x = 315$	$\sum A.y = 789$

$$\boxed{\bar{x} = \frac{\sum A.x}{\sum A} = \frac{315}{84} = 3.75cm}$$

$$\boxed{\bar{y} = \frac{\sum A.y}{\sum A} = \frac{789}{84} = 9.392cm}$$

6a) Apply the method of integration to derive the centroid of rectangle.

Sol^n:



Let us consider a rectangular lamina of area b × d as shown in figure. Now consider a horizontal elementary strip of area $b \times dy$, which is at a distance y from the reference axis AB.

Moment of area of elementary strip about AB = $b \times dy \times y$

Sum of moments of such elementary strips about AB is given by,

= $\int_0^d b \times dy \times y$

$=b\int_0^d y.dy$

$= b \times \left[\frac{y^2}{2}\right]_0^d$

$= \frac{bd^2}{2}$

Moment of total area about AB = b.d× $\bar{y}$

Apply the principle of moments about AB,

$\frac{bd^2}{2} = \mathrm{b.d} \times \bar{y} \Rightarrow \boxed{\bar{y} = \frac{d}{2}}$

By considering a vertical strip, similarly, we can prove that,

$$\boxed{\bar{x} = \frac{b}{2}}$$

6b) Solve for the centroid for the area shown in fig 6(b) with respect to the centroidal axis.

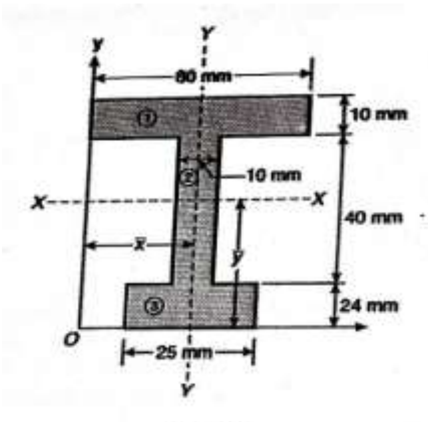


Fig 6(b)

Sol^n:

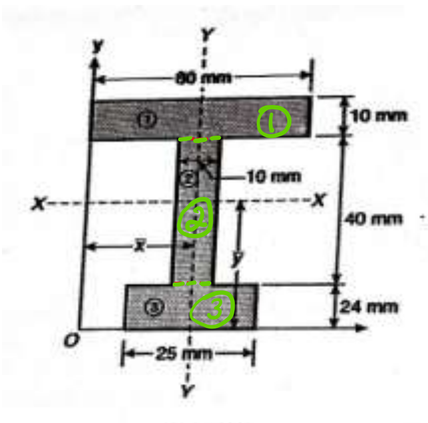


Fig 6(b)

Components	Area (mm^2) (A)	Centroidal Axis from y – axis (mm) (x)	Centroidal Axis from x – axis (mm) (y)	A.x (mm^3)	A.y (mm^3)
Rectangle 1	80×10	$\frac{80}{2}$	$\frac{10}{2} + 40 + 24$	32000	55200
Rectangle 2	10×40	$\frac{80}{2}$	$\frac{40}{2} + 24$	16000	17600
Rectangle 3	25×24	$\frac{80}{2}$	$\frac{24}{2}$	24000	7200
	$\sum A = 1800$			$\sum A.x = 72000$	$\sum A.y = 80000$

$$\bar{x} = \frac{\sum A.x}{\sum A} = \frac{72000}{1800} = 40mm$$

$$\bar{y} = \frac{\sum A.y}{\sum A} = \frac{80000}{1800} = 44.444mm$$