Project of Financial Optimization and Asset Management

Sapienza Università di Roma

Group- B

Max-Min Model using Excel

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Introduction

Controlling risk and quantifying losses are, in managing a portfolio, two fundamental problems that an investor would easily want to solve. This process is very difficult to quantify accurately, and even investment management companies have focused on the importance of efficient risk management within the portfolio.

The goal of improving the performance of the Markowitz model has led many researchers to formulate different portfolio optimization models with the introduction of new alternative measures of risk.

In fact, we present the **MaxMin model (MM) – Young, 1988.**

Max-Min Model

To find the optimal solution, the minimum portfolio returns rather than the variance (as per Markowitz) is defined as the measure of risk. This model attempts to show that this asymmetric measure of risk is more appropriate for asymmetric distributions of returns.

In particular, the selected portfolio is the one that minimizes the maximum possible loss with respect to all past periods, and with the restriction of a certain minimum average return level defined as acceptable throughout the observation period.

This model is also known as the MaxMin model since maximizing the minimum portfolio return corresponds to minimizing the maximum loss.

First, suppose we have the observed historical data for N assets, in the time period t = 1,…, T, then we can define the following variables:

$\boldsymbol{Xj}$ portfolio fraction invested in j, j=1,2,...,n

$\boldsymbol{Rjt}$ asset j return at time t, j=1,2,...,n; t=1,2,...,T

$\boldsymbol{E(Rj)}$ asset j expected return, j=1,2,...,n

$\boldsymbol{R^*}$ fixed value for the portfolio expected return

$\boldsymbol{Z}$ minimum portfolio return

The risk of a portfolio ($X1$, $X2$,..., Xn) is evaluated by measuring the minimum return of the portfolio observed in the past time periods t = 1,...,T.

The MaxMin portfolio maximizes the amount Z, subject to the restriction that the average return $E(Rj)$ exceeds a certain threshold, desirable by the investor and that the sum of the capital invested in each individual asset does not exceed the budget, in this case equal to 1.

R min

Represents the minimum return maximized that we could take (it can also be seen as a maximum loss minimized by changing the sign). We firstly calculated the Expected Return with respect to each asset by doing the average of each one for the whole time period

$$E(R_i) = \frac{\sum_{i=1}^{n} Xi}{T}$$

Then, we set the decision variables as fraction of the capital Xi invested in the 30 assets. At this point, we impose the constraints to our model, starting from the portfolio constraint, in which the sum of the weight of the assets must be equal to 1

$$\sum_{j=1}^{n} Xj = 1$$

Finally, to maximize the minimum loss, we need to define Z:
the Z Value is calculated by taking the minimum value of the sum of each return multiplied by their weight for each time

$$z = min_t \sum_{j=1}^{n} XjRjt$$

Once we found the Z Value, we imposed that the Z Value (which is in the RHS) must be lower or equal (<=) than the LHS

$\sum_{j=1}^{n} XjRjt \geq z$ t = 1,…,T

We use the Excel Solver to maximize the objective function (z), then we set the assets weight as the decision variables and by adding all the previous mentioned constraints, we maximize the objective function (as a linear model)

$$\max z$$

Once we optimize, we obtain the optimal portfolio which maximizes the minimum loss.

R max

Now our purpose is to find out the maximum expected return (Rmax) of the portfolio, by multiplying the weight of each asset per the expected return of each asset. In this case the only constraint is the one related to the portfolio

$$\sum_{j=1}^{n} Xj = 1$$

We used the Excel Solver to maximize the objective function (expected return of the portfolio), then we set the assets weight as the decision variables and by adding the portfolio constraint, we maximize the objective function (as a linear model)

$$\max_{X} \sum_{j=1}^{n} E(R_j)X_j$$

Efficient frontiers

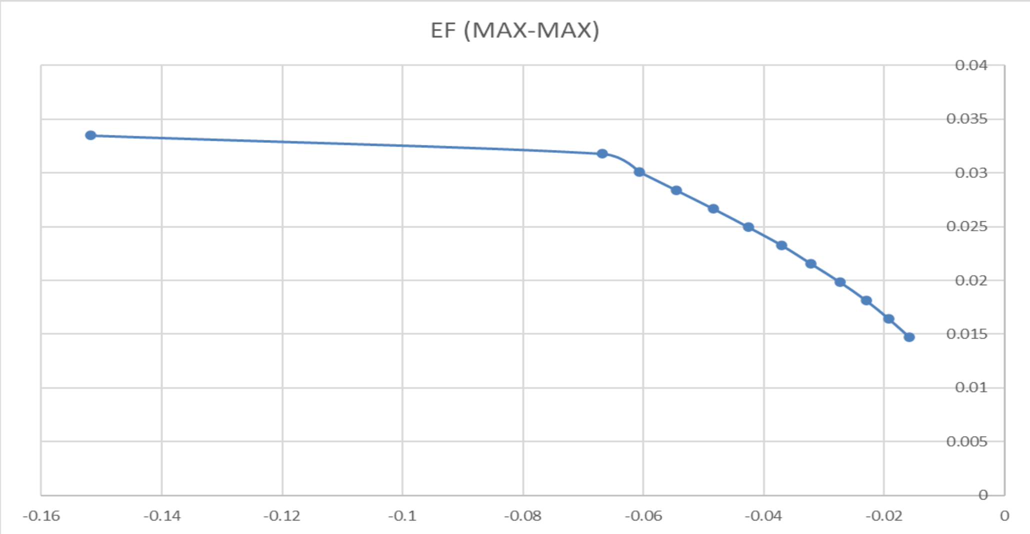
When choosing a portfolio, the investor prefers a high return and a low risk. Given the above principle, it is possible to reduce the set of available portfolios to a smaller subset by eliminating those portfolios which are surely worse than others available in the same market.

The expected return constraint that we use, is given by imposing the sum of the weight per their expected return equal to the target value

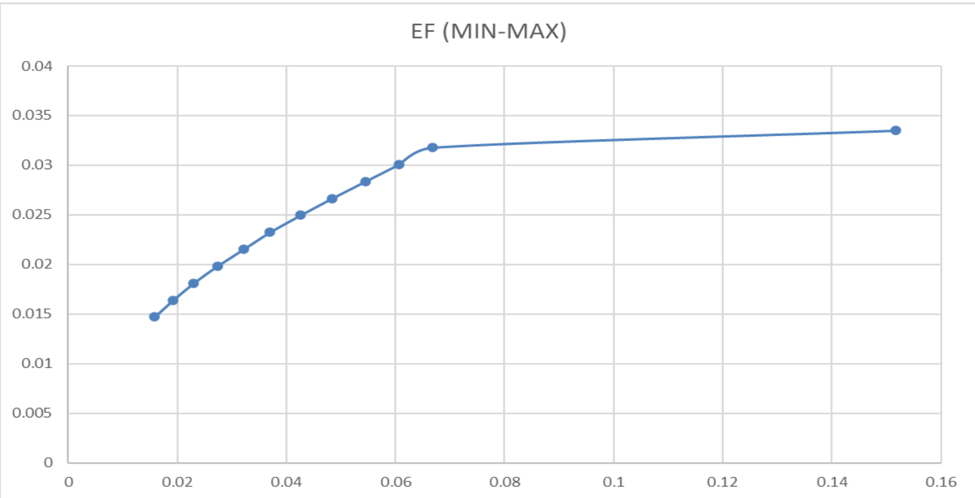
$$\sum_{j=1}^{n} E(R_j)X_j = R^*$$

The purpose of this step is to fix ten equally spaced values between Rmin and Rmax and compute the MM Model. Each time we compute the model by changing the target value in the expected return constraint, replacing It with one of the ten equally spaced values.

In this way we can plot the efficient frontier, related to the minimum return maximized against the maximized expected return. By plotting the graph, we should expect the results in the Northwest box.



Instead, if we want to analyze it from a loss point of view, we can change the sign of our minimum return maximized so that we have a loss and the plot as we are used to (Northeast box).



As we can see in the plot, we have an increase of 10% between each point starting from Rmin to P10 and then a bigger gap between P10 and Rmax, obviously because the MinRisk difference between these last two points Is significant.

Cardinality and Buy-in threshold constraint

We are going to introduce the Cardinality and Buy-in threshold constraints into our model, considering just the period up to end of 2005.

By Mathematical Programming we are able to model conditions on how many and which assets are included in the selected portfolio. We can fix a precise or a maximum portfolio cardinality.

Cardinality:

This model is more sophisticated than the MM one, since we need an additional binary variables Y , i = 1,2,...,n.

We define:

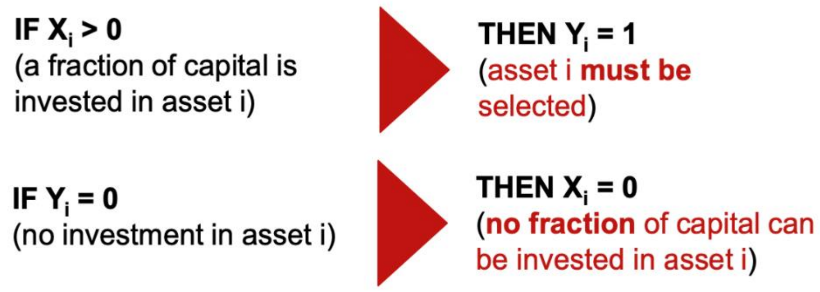
$$Yi \begin{cases} 1 \text{ if asset is included in the portfolio} \\ 0 \text{ otherwise} \end{cases}$$

$$\sum_{j=1}^{n} Yi \leq K$$

$$Xi \leq Yi \quad i = 1,2,\ldots,n$$

$$Xi \geq 0 \quad i = 1,2,\ldots,n$$

$$Yi\ binary \quad i = 1,2,\ldots,n$$



Buy-in threshold constraint

By exploiting binary variables, buy-in threshold constraints can be formulated to model a double decision:

1) selecting or not an asset;

2) bounding the fraction invested in a selected asset.

We still need the additional binary variables Y , i = 1,2, . . . , n

$$Xi \leq ubi \cdot Yi \quad i = 1,2,\dots,n$$
$$Xi \geq lbi \cdot Yi \quad i = 1,2,\dots,n$$
$$Yi \; binary \quad i = 1,2,\dots,n$$

When asset i is selected $(X_i > 0)$

→$Y_i = 1$

And both bounds on X_i are activated; $lb_i \leq X_i \leq ub_i$

On the other hand, is asset i is not selected $(Y_i = 0)$

→$X_i = 0$

In fact $0 \leq X_i \leq 0$

Due to the presence of the new binary variables Y, i = 1,2,...,n, these constraints are frequently included together with cardinality constraints.

In our model we set the limit of assets that we should include in our portfolio equal to five, while the Buy-in threshold determines a range between 0.1 < Xi < 0.4 in which our assets must be included.

With regard to the buy-in threshold we faced the problem concerned the excel solver, which has a maximum capacity of two-hundred decision variable cells and one-hundred constratints cells. For that reason we could not include the upper bound constraint (0.4) into the solver. In any case, in our portfolio we did not have any assets that exceeded the upper bound, so we could say that the constraint was respected.

In order to calculate the bounds, we have to multiply the values of the Yi per the thresholds that were given

$$Xi \leq 0.4 \cdot Yi \quad i = 1,2,\dots,30$$

$$Xi \geq 0.1 \cdot Yi \quad i = 1,2,\dots,30$$

By imposing the cardinality constraint, we set thirty new binary variables which can only take either zero (if the asset is not included) or one (if the asset is included) as value. In order to activate them, we also imposed another constraint which is given by

$$Y_i = \{0,1\}$$

$$X_i \leq Y_i$$

The last constraint to be considered, concerned the target value, which should be imposed as the expected return. The target was defined as

$$\sum_{j=1}^{n} E(R_j)X_j = (\rho_{\min} + \rho_{max})/2$$

We use the Excel Solver in order to maximize the Objective Function, and thus we found the optimal portfolio up to the end of 2005 and will be used later to compute the returns up to 11/2006.

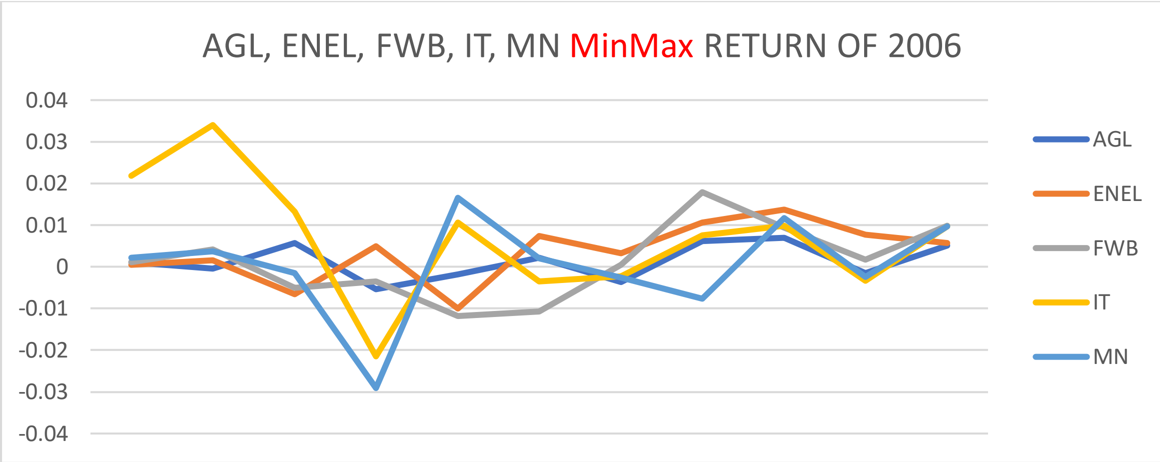
As expected our optimal portfolio includes five assets that respect all of our constraints.

Returns

We used the portfolio fractions found in the previous section to compute the portfolio's returns which the optimal portfolio would have realized with the return data of the year 2006.

By applying the formula below, we obtained the returns that the optimal portfolio would have realized for 2006.

Once we found the optimal results for the five assets in our portfolio, we proceeded to plot them.



We have also plotted each asset Individually and Included the returns of the Initial portfolio to compare them with the optimal portfolio that we found.