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21. 求满足 $2|n, 3|(n+1), 4|(n+2), 5|(n+3), 6|(n+4)$ 的最小整数 $n(>2)$.

解：即解方程组：

$$\begin{cases} n \equiv 0 \pmod 2 \equiv 2 \pmod 2 \\ n \equiv 2 \pmod 3 \\ n \equiv 2 \pmod 4 \\ n \equiv 2 \pmod 5 \\ n \equiv 2 \pmod 6 \end{cases}$$

那么根据同余式性质：

$$\begin{aligned} n &\equiv 2 \pmod{[2,3,4,5,6]} \\ &\equiv 2 \pmod{60} \end{aligned}$$

故大于2的最小整数 n 为 62

22. 计算 $\phi(42), \phi(420), \phi(4200)$.

解：$\phi(42) = \phi(2\times3\times7)$ 由于2.3.7互素. 故 $\phi(42) = \phi(2)\cdot\phi(3)\cdot\phi(7)$

即 $\phi(42) = (2-1)\times(3-1)\times(7-1) = 12$

$$\begin{aligned} \phi(420) &= \phi(2^2\times3\times5\times7) \\ &= 420\times(1-\tfrac{1}{2})\times(1-\tfrac{1}{3})\times(1-\tfrac{1}{5})\times(1-\tfrac{1}{7}) = 96 \end{aligned}$$

$$\begin{aligned} \phi(4200) &= \phi(2^3\times3\times5^2\times7) \\ &= 420\times(1-\tfrac{1}{2})\times(1-\tfrac{1}{3})\times(1-\tfrac{1}{5})\times(1-\tfrac{1}{7}) = 96 \end{aligned}$$

故 $\phi(42) = 12$ $\phi(420) = 96$ $\phi(4200) = 96$

27. 314^{159}除以7的余数是多少？

解：因为$(7,314)=1$，故由欧拉定理：$314^{6}\equiv 1 \pmod 7$

$$(314)^{159}=(314^{6})^{26}\cdot(314)^{3}\equiv(314)^{3}=2^{3}\times 157^{3}\equiv 157^{3}$$

$$\equiv(-4+23\times 7)^{3}\equiv(-4)^{3}\equiv(14+2)\cdot(-4)$$

$$\equiv -8\equiv 6 \pmod 7$$