4b-1. 次の速度分布に対する排除厚さおよび運動量厚さを求めよ．

$$(1)\ \frac{u}{U} = 2\frac{u}{\delta} - \frac{y^2}{\delta^2}, \quad (2)\ \frac{u}{U} = (\frac{y}{\delta})^{1/7}$$

（解）

$$(1)\ \delta^* = \int_0^\delta (1 - \frac{u}{U})dy = \int_0^\delta (1 - \frac{2y}{\delta} + \frac{y^2}{\delta^2})dy = \frac{\delta}{3}, \quad \frac{\delta^*}{\delta} = \frac{1}{3}$$

$$(2)\ \delta^* = \int_0^\delta \{1 - (\frac{y}{\delta})^{1/7} dy = \frac{\delta}{8}, \quad \frac{\delta^*}{\delta} = \frac{1}{8}$$

$$(1)\ \theta = \int_0^\delta \frac{u}{U}(1 - \frac{u}{U})dy = \delta\int_0^1 (2\eta - \eta^2) - (2\eta - \eta^2)^2 d\eta$$

$$= \delta\int_0^1 (2\eta - 5\eta^2 + 4\eta^3 - \eta^4) = \delta(1 - \frac{5}{3} + 1 - \frac{1}{5}) = \frac{2}{15}\delta, \quad \frac{\theta}{\delta} = \frac{2}{15}$$

$$(2)\ \theta = \delta\int_0^1 (\eta^{1/7} - \eta^{2/7}) = \delta(\frac{7}{8} - \frac{7}{9}) = \frac{7}{72}\delta, \quad \frac{\theta}{\delta} = \frac{7}{72}$$

4b-2. 平板が $5m/s$ の水 (5^oC) の流れの中に流れに平行におかれている．次の値を求めよ．(1) 平板の先端から層流の部分の距離．(2) またその点における境界層の厚さ．(3) 平板の長さ $3.2m$ としたときの全抵抗．ただし層流境界層の速度分布は次の通りとする．(生井，水力学演習, p190)

$$\frac{u}{V} = \sin\frac{\pi}{2}\frac{y}{\delta}$$

（解）

$$(1)\ R_{ecx} = \frac{x_o V}{\nu} = 5 \times 10^5, \quad x_o = \frac{1.519 \times 10^{-6} \times 5 \times 10^5}{5} = 0.152m$$

$$(2)\ \frac{u}{V} = \sin\frac{\pi}{2}\frac{y}{\delta} = \sin\frac{\pi}{2}\eta$$

$$\tau_o = \rho V^2 \frac{d\delta}{dx}\int_0^1 (1 - \frac{u}{V})\frac{u}{V}d\eta = \rho V^2 \frac{d\delta}{dx}\int_0^1 (1 - \sin\frac{\pi}{2}\eta)\sin\frac{\pi}{2}\eta d\eta$$

$$= \rho V^2 \frac{d\delta}{ds}\{-\frac{-2}{\pi} - \frac{1}{\pi}\frac{\pi}{2} - 0\} = 0.137\rho V^2 \frac{d\delta}{dx}$$

$$\text{Note:} \int \sin^2 mx = \frac{1}{2m}(mx - \sin mx \cos mx)$$

$$\tau_o = \mu\left(\frac{du}{dx}\right)_{y=0} = \frac{\pi}{2}\mu\frac{V}{\delta} = 0.137\rho V^2 \frac{d\delta}{dx}$$

$$\delta d\delta = 11.46\frac{\mu dx}{\rho V}, \quad \frac{\delta^2}{2} = 11.46\frac{\nu}{V}x + C$$

$$\frac{\delta}{x} = 4.78\sqrt{\frac{\nu}{Vx}}$$

$$\delta = 4.78\sqrt{1/(5 \times 10^5)} \times 0.152 = 1.03 \times 10^{-3}m$$

$$(3)\ R_{el} = \frac{5 \times 3.2}{1.519 \times 10^{-6}} = 1.05 \times 10^7, \quad C_f = 0.455(\log R_{el})^{-2.58} = 2.98 \times 10^{-3}$$

$$D_f = 2C_f A\frac{\rho}{2}V^2 = 2 \times 2.98 \times 10^{-3} \times (3.2 \times 1) \times \frac{10^3}{2} \times 5^2 = 476.8N\ (24.kgf)$$

4b-3. 乱流境界層の速度分布が次式で与えられるとき，排除暑さと運動量厚さをそれぞれ境界層厚さの比で表せ．(宮井, 水力学 p155)

$$\frac{u}{U} = (\frac{y}{\delta})^{1/n}$$

（解）

$$(2)\ \delta^* = \int_0^\delta \{1 - (\frac{y}{\delta})^{1/n} dy = y - \frac{n}{n+1}(\frac{y}{\delta}y\Big|_0^\delta = \delta - \frac{n}{n+1}\delta = \frac{1}{n+1}\delta$$

$$\frac{\delta^*}{\delta} = \frac{1}{n+1}$$

$$(1)\ \theta = \int_0^\delta (\frac{y}{\delta}^{1/n}\{1 - (\frac{y}{\delta})^{1/n}\} = \frac{n}{n+2}(\frac{y}{\delta})^{2/n}y\Big|_0^\delta$$

$$= \frac{n}{n+1}\delta - \frac{n}{n+2}\delta = \frac{n}{(n+1)(n+2)}\delta, \quad \frac{\theta}{\delta} = \frac{n}{(n+a)(n+2)}$$

4b-4. 流速の一様流れに平行におかれた平板において，層流境界層内の速度分布が次式であらわされつとき，排除厚さ，運動量厚さ，形状係数，壁面せｎ断応力および平板の摩擦抗力係数を求めよ．(富田, 水力学, p118; 池森，水力学, 279)

$$\frac{v}{V} = \frac{2}{3}\eta - \frac{1}{2}\eta^3$$

（解）

$$\frac{v}{V} = \frac{2}{3}\eta - \frac{1}{2}\eta^3, \quad \frac{y}{\delta} = \eta, \quad dy = \delta d\eta$$

$$\delta^* = \int_0^\delta (1 - \frac{v}{V})dy = \delta\int_0^1 (1 - \frac{3}{2}\eta - \frac{1}{2}\eta^3)d\eta = \frac{3}{8}\delta$$

$$\theta = \delta\int_0^1 (1 - \frac{3}{2}\eta - \frac{1}{2}\eta^3)(\frac{3}{2}\eta - \frac{1}{2}\eta^3)d\eta = 0.1393\delta$$

$$H = \frac{\delta^*}{\theta} = 2.69$$

$$\tau_o = \mu(\frac{dv}{dy})_{y=0} = \frac{3}{2}\frac{\mu V}{\delta}$$

$$\tau_o = \rho V^2\frac{\theta}{dx} = 0.139\rho V^2\frac{d\delta}{dx}$$

$$\delta d\delta = 10.79\frac{\nu}{V}dx, \quad \frac{\delta^2}{2} = 10.79\frac{\nu}{V} + c$$

$$\frac{\delta}{2} = 4.65\sqrt{\frac{\nu}{Vx}} = \frac{4.65}{\sqrt{R_{ex}}}, \quad R_{ex} = \frac{Vx}{\nu}$$

$$\tau_o = 0.323\sqrt{\frac{\mu\rho V^3}{x}}$$

$$D = \int_0^l \tau_o dx = \rho V^2\theta = 0.645\rho V^2\sqrt{\frac{\nu l}{V}}$$

$$C_f = \frac{D}{(1/2)\rho V^2 l} = \frac{1.292}{\sqrt{R_{el}}}, \quad R_{el} = \frac{Vl}{\nu}$$

4b-5. 滑らかな平板上に生じた層流境界層の速度分布が次式で示されるとき，次の値を求めよ．(1) 係数 C_1，C_2，(2) δ，δ^*, τ_o を表す式．(森川，流れ学, p92)

$$\frac{u}{U} = C_1\frac{y}{\delta} + C_2(\frac{y}{\delta})^3$$

（解）

$$(1)\ u = U:\ y = \delta:\ 1 = C_1 + C_2$$
$$\frac{du}{dy})_{y=\delta} = 0:\ y = \delta:\ 0 = C_1 + 3C_2$$
$$C_1 = \frac{3}{2},\quad C_2 = -\frac{1}{2}$$
$$\frac{u}{U} = \frac{3}{2}\eta - \frac{1}{2}\eta^3,\quad \frac{y}{\delta} = \eta,\quad dy = \delta d\eta$$
$$(2)\delta^* = \int_0^\delta (1 - \frac{u}{U})dy = \delta\int_0^1 (1 - \frac{3}{2}\eta - \frac{1}{2}\eta^3)d\eta = \frac{3}{8}\delta$$
$$\frac{\delta^*}{x} = \frac{1.74}{\sqrt{R_{ex}}}$$
$$\theta = \delta\int_0^1 (1 - \frac{3}{2}\eta - \frac{1}{2}\eta^3)(\frac{3}{2}\eta - \frac{1}{2}\eta^3)d\eta = 0.139\delta$$
$$H = \frac{\delta^*}{\theta} = 2.69$$
$$\tau_o = \mu(\frac{du}{dy})_{y=0} = \frac{3}{2}\frac{\mu U}{\delta}$$
$$\tau_o = \rho U^2\frac{d\theta}{dx} = 0.139\rho U^2\frac{d\delta}{dx}$$
$$\delta d\delta = 10.79\frac{\nu}{U}dx,\quad \frac{\delta^2}{2} = 10.79\frac{\nu}{U}x + c$$
$$\frac{\delta}{x} = 4.65\sqrt{\frac{\nu}{Ux}} = \frac{4.65}{R_{ex}},\quad R_{ex} = \frac{Ux}{\nu}$$
$$\tau_o = 0.323\sqrt{\frac{\mu\rho U^3}{x}}$$
$$(3)D = \int_0^l \tau_o dx = \rho U^2\theta = 0.645\rho U^2\sqrt{\frac{\nu l}{U}}$$
$$C_f = \frac{D}{(1/2)\rho U^2 l} = \frac{1.292}{\sqrt{R_{el}}},\quad R_{el} = \frac{Ul}{\nu}$$

4b-6. 流速 $3m/s$ の気流中に $30cm \times 60cm$ の平板を次の二つの状態にして平行におくとき，平板の両面に働く摩擦抵抗を求め，それぞれの抵抗比を求めよ．ただし流れは層流とし速度分布は次式で表されるものとする．また気体の密度を $1.2kkg/m^3$, 動粘性係数 $1.48 \times 10^{-5}m^2/s$ をとする．(1) $30cm$ の底辺を流れ方向におく，(2) $60cm$ の底辺を流れ流れ方向におく．(森川，流れ学, p92)

（解）

$$D_1 = 0.73 \times \rho\sqrt{\nu V^3 l_1}2b_1,\quad D_2 = 0.73 \times \rho\sqrt{\nu V^3 l_2}2b_2$$
$$D_1 = 0.73 \times 1.2\sqrt{1.48 \times 10^{-5} \times 3^2 \times 0.3} \times 2 \times 0.6 = 5.76 \times 10^{-3}N$$
$$D_2 = 4.07 \times 10^{-3}N$$
$$\frac{D_1}{D_2} = \frac{\sqrt{l_2} \times b_1}{\sqrt{l_2} \times b_2} = \frac{\sqrt{0.3} \times 0.6}{\sqrt{0.6} \times 0.3} = 1.414$$

4b-8. 長さ $0.8m$, 幅 $0.4m$ の薄い滑かな平板を水温 20℃ の水中を平行に速度 $0.5m/s$ で板の長さ方向に引く．板を引くのに必要な力を求めよ．(加藤，流れの力学, p61)

（解）

$$
\begin{aligned}
&\nu = 10^{-6} m^2/s \ at \ 20^o C \\
&R_e = \frac{0.5 \times 0.8}{10^{-6}} = 4 \times 10^5 < 5 \times 10^5 \ (laminar) \\
&C_f = \frac{1.328}{R_e^{1/2}} \\
&D = 2C_f(\frac{1}{2})\rho V^2 lb = 0.18N
\end{aligned}
$$

4-16. 流速の一様流れに平行のにおかれた平板において，層流境界層内の速度分布ががが次式で表されるとき，排除厚さ，運動量厚さ，形状係数，壁面せん断応力および平板の摩擦抗力を求めよ．

$$
\frac{u}{V} = sin\frac{\pi}{2}\frac{y}{\delta}
$$

（解）

$$
\begin{aligned}
&\frac{u}{V} = sin\frac{\pi}{2}\frac{y}{\delta}, \quad \frac{y}{\delta} = \eta, \quad dy = \delta d\eta \\
&\delta^* = \int_0^\delta (1 - sin\frac{\pi}{2}\frac{y}{\delta})dy = \delta\int_0^1 (1 - sin\frac{\pi}{2}\eta)d\eta \\
&= \delta(\eta + \frac{2}{\pi}cos\eta)\Big|_0^1 = \delta(1 + 0 - 0 - \frac{2}{\pi}) = 0.363\delta \\
&\theta = \delta\int_0^1 sin\frac{\pi}{2}\eta(1 - sin\frac{\pi}{2}\eta)d\eta \\
&= \delta - \frac{2}{\pi}cos\eta - \frac{1}{\pi}(\frac{\pi}{2} - sin\frac{\pi}{2}cos\frac{\pi}{2})\Big|_0^1 = \delta(\frac{2}{\pi} - \frac{1}{2}) = 0.137\delta \\
&H = \frac{\delta^*}{\theta} = 2.65 \\
&\tau_o = \mu(\frac{dv}{dy})_{y=0} = \frac{\mu V\pi}{2\delta} \\
&\tau_o = \rho V^2\frac{d\theta}{dx} = 0.137\rho V^2\frac{d\delta}{dx} = \frac{\mu V\pi}{2\delta} \\
&2\delta d\delta = \frac{\pi}{0.137}\frac{\nu}{V}dx \\
&\delta^2 = 22.93\frac{\nu}{V}x + c, \quad \delta = 4.79\sqrt{\frac{\nu x}{V}} \\
&D = \int_0^l \tau_o dx = \rho V^2\theta = 0.137\rho V^2(4.79\sqrt{\frac{\nu l}{V}}) = 0.656\rho V^2\sqrt{\frac{\nu l}{V}} \\
&C_f = \frac{D}{(1/2)\rho V^2 l} = \frac{1.312}{\sqrt{R_{el}}}
\end{aligned}
$$

4-16-2. 流速の一様流れに平行のにおかれた平板において，層流境界層内の速度分布ががが次式で表されるとき，排除厚さ，運動量厚さ，形状係数，壁面せん断応力および平板の摩擦抗力を求めよ.（富田，水力学，p118；池森，水力学，p279）

$$\frac{v}{V} = \frac{3}{2}\eta - \frac{1}{2}\eta^3$$

（解）

$$\frac{v}{V} = sin\frac{3}{2}\eta - \frac{1}{2}\eta^3, \quad \frac{y}{\delta} = \eta, \quad dy = \delta d\eta$$

$$\delta^* = \int_0^\delta (1 - \frac{v}{V})dy = \delta \int_0^1 (1 - \frac{3}{2}\eta - \frac{1}{2}\eta^3)d\eta = \frac{3}{8}\delta$$

$$\theta = \delta \int_0^1 (1 - \frac{3}{2}\eta - \frac{1}{2}\eta^3)(\frac{3}{2}\eta - \frac{1}{2}\eta^3)d\eta = 0.139\delta$$

$$H = \frac{\delta^*}{\theta} = 2.69$$

$$\tau_o = \mu(\frac{dv}{dy})_{y=0} = \frac{3}{2}\frac{\mu V}{\delta}$$

$$\tau_o = \rho V^2 \frac{d\theta}{dx} = 0.139\rho V^2 \frac{d\delta}{dx} = \frac{3}{2}\frac{\mu V}{\delta}$$

$$\delta d\delta = 10.79\frac{\nu}{V}dx, \quad \frac{\delta^2}{2} = 10.79^f rac\nu Vx + c$$

$$\frac{\delta}{x} = 4.69\sqrt{\frac{\nu}{Vx}} = \frac{4.65}{\sqrt{R_{ex}}}, \quad R_{ex} = \frac{Vx}{\nu}$$

$$\tau_o = 0.323\sqrt{\frac{\mu\rho V^3}{x}}$$

$$D = \int_0^l \tau_o dx = \rho V^2\theta = 0.645\rho V^2\sqrt{\frac{\nu l}{V}}$$

$$C_f = \frac{D}{(1/2)\rho V^2 l} = \frac{1.292}{\sqrt{R_{el}}}, \quad R_{el} = \frac{Vl}{\nu}$$

4-17. 内径 $20cm$ の滑らかな円管に 10^oC の水を平均で流す場合，最大流速，管壁面上の摩擦応力および粘性底層の厚さを求めよ．

（解）

$$V = u^*(5.75log\frac{Ru^*}{\nu} + 1.75)$$

$$0.6 = u^*\{5.75log(0.1 \times 10^6/1.3) + 1.75 + 5.75logu^*\}$$

$$= u^*(29.84 + 5.75logu^*), \quad u^* = 0.029m/s$$

$$U = V + 3.75u^* = 0.6 + 3.75 \times 0.029 = 0.71m/s$$

$$R_e = \frac{0.6 \times 0.2 \times 10^6}{1.3} = 9.23 \times 10^4 < 10^5, \quad \lambda = 0.3164R_e^{-1/4} = 0.01815$$

$$u^* = \sqrt{\frac{\lambda}{8}}V, \quad u^* = 0.029m/s, \quad U = V + 3.75u^* = 0.71m/s$$

$$\tau_o = \rho u^{*2} = 10^3 \times 0.029^2 = 0.84Pa, \quad \delta_s = \frac{5\nu}{u^*} = \frac{5 \times 1.3 \times 10^{-6}}{0.029} = 0.224mm$$