

Linear Regression

Task 1:

Given data points,

$$\text{Weight, } X = [2, 4, 5, 3, 6, 5, 7]$$

$$\text{Price, } Y = [35, 60, 20, 50, 50, 55, 60]$$

Mean of X and Y :

$$\bar{X} = \frac{2+4+5+3+6+5+7}{7} = 4.57$$

$$\bar{Y} = \frac{35+60+20+50+50+55+60}{7} = 47.14$$

• Slope (m):

$$m = \frac{\sum (X - \bar{X})(Y - \bar{Y})}{\sum (X - \bar{X})^2}$$

$$= \frac{(2-4.57)(35-47.14) + (4-4.57)(60-47.14) + (5-4.57)(20-47.14) + (3-4.57)(50-47.14) + (6-4.57)(50-47.14) + (5-4.57)(55-47.14) + (7-4.57)(60-47.14)}{17.71}$$

$$= \frac{46.49}{17.71}$$

$$= 2.62$$

Intercept, c :

$$\cdot c = \bar{Y} - mx$$

$$= 47.24 - (2.62 \times 4.57)$$

$$= 47.24 - 11.97$$

$$= 35.27$$

For $X = 6$,

$$Y = mx + c$$

$$= (2.62 \times 6) + 35.27$$

$$= 50.89$$

Task 2:

| Weight | Price | Predicted Price | Residual |
|--------|-------|-----------------|----------|
| 2 | 35 | 40.42 | -5.40 |
| 4 | 60 | 45.65 | 14.35 |
| 5 | 20 | 48.27 | -28.26 |
| 3 | 50 | 43.03 | 6.97 |
| 6 | 50 | 50.89 | -0.89 |
| 5 | 55 | 48.27 | 6.73 |
| 7 | 60 | 53.57 | 6.49 |

For $X = 2$:

$$\begin{aligned}
 Y &= mX + c \\
 &= (2.62 \times 2) + 35.27 \\
 &= 40.42
 \end{aligned}$$

For $X = 4$:

$$\begin{aligned}
 Y &= mX + c \\
 &= (2.62 \times 4) + 35.27 \\
 &= 45.65
 \end{aligned}$$

For $X = 5$:

$$\begin{aligned}
 Y &= mX + c \\
 &= (2.62 \times 5) + 35.27 \\
 &= 48.27
 \end{aligned}$$

For $X = 3$:

$$\begin{aligned}Y &= mx + c \\&= (2.62 \times 3) + 35.27 \\&= 43.03\end{aligned}$$

For $X = 6$:

$$\begin{aligned}Y &= mx + c \\&= (2.62 \times 6) + 35.27 \\&= 50.89\end{aligned}$$

For $X = 5$:

$$\begin{aligned}Y &= mx + c \\&= (2.62 \times 5) + 35.27 \\&= 48.27\end{aligned}$$

For $X = 7$:

$$\begin{aligned}Y &= mx + c \\&= (2.62 \times 7) + 35.27 \\&= 53.51\end{aligned}$$

• Residual = Price - Predicted Price

For $Y = 35$:

$$R = 35 - 40.42 \\ = -5.40$$

For $Y = 60$:

$$R = 60 - \cancel{40.42} \quad 45.65 \\ = 14.35$$

For $Y = 20$:

$$R = \cancel{20} - 48.27 \\ = -28.27$$

For $Y = 50$:

$$R = 50 - \cancel{43.03} \\ = 6.97$$

For $Y = 50$:

$$R = 50 - \cancel{50.89} \\ = -0.89$$

For $Y = 55$:

$$R = 55 - 48.27 = 6.73$$

For $Y = 60$:

$$n = 60 - 53.52 \\ = 6.49$$

Task 3:

MAE:

$$\text{MAE} = \frac{1}{N} \sum_{j=1}^N |y_j - \hat{y}_j|$$

$$\cancel{\frac{1}{N}}$$

For the values of Y , the absolute errors are:

$$|-5.40| = 5$$

$$|14.35| = 14$$

$$|-28.26| = 28$$

$$|6.97| = 6$$

$$|-0.89| = 0$$

$$|6.73| = 6$$

$$|6.49| = 6$$

• sum of absolute errors = 65

$$\therefore N = 7$$

$$\therefore \text{MAE} = \frac{65}{7} = 9.29$$

MSE:

$$\text{Mean Squared Error} = \frac{1}{N} \sum_{j=1}^N (y_j - \hat{y}_j)^2$$

• $N = 7$

• For the values of y , the squared errors are:-

$$(-5.40)^2 = 29.16$$

$$(24.35)^2 = 205.92$$

$$(-28.26)^2 = 798.63$$

$$(6.97)^2 = 48.58$$

$$(-0.89)^2 = 0.79$$

$$(6.73)^2 = 45.29$$

$$(6.49)^2 = 42.12$$

• sum of the squared errors: 1270.49

$$\therefore \text{MSE} = \frac{1270.49}{7} = 167.21$$