I Intro to Mechatronics

1 (GB)

(1.1) (D) mechatronics = study of devices that simultaneously combine at least 2 fields of engineering science that that keeps interacting

↳ Mech. engineering (continuum mechanics / fluids / solids)
(Mécan. du C)

↳ ~~Electrotechnic~~ Electrical (Electricity, magnetism ...)
(Electrotechnique)

↳ Process Engineering (chemistry, thermodynamics, $t^{?}$, NRJ)

↳ Science & technology of information & communication)
(control, computer science, network, binary / logic)

Tendency: high ↑ of mechatronic devices

↳ smarter system.

↳ autonomous —

↳ internet of things / connected objects.

↳ increase of functionalities (↑ density)

↳ — performances (control)

↳ Artificial intelligence.

↳ systems able to learn.

Needs investigate mechatronics with "adapted Tools" (not combining monodisciplinary objects)

Aim Finding equilibrium / trade off / optimal behavior at the general level not at the local / elementary level
How to find the good Balance? Advantage of synergetic effects?

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⑧ Modelling

<u>Model</u> = Mathematical description of a physical system.

↳ correspond more or less to the real behavior of the system (depend on hypothesis done)

↳ trade off between model complexity / trueness

<u>interest for modelling</u> → behavior understanding → explanation of behavior → prediction of behavior | Modelling / control / Simulation

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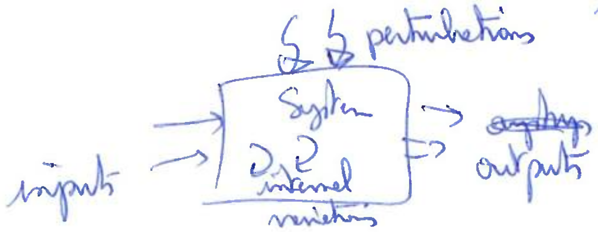
⑧ <u>System</u> : Set of object combined for a given purpose.

- The behavior of every object can be described by general laws (mechanics, electricity, thermodynamics) linking different physical quantities.

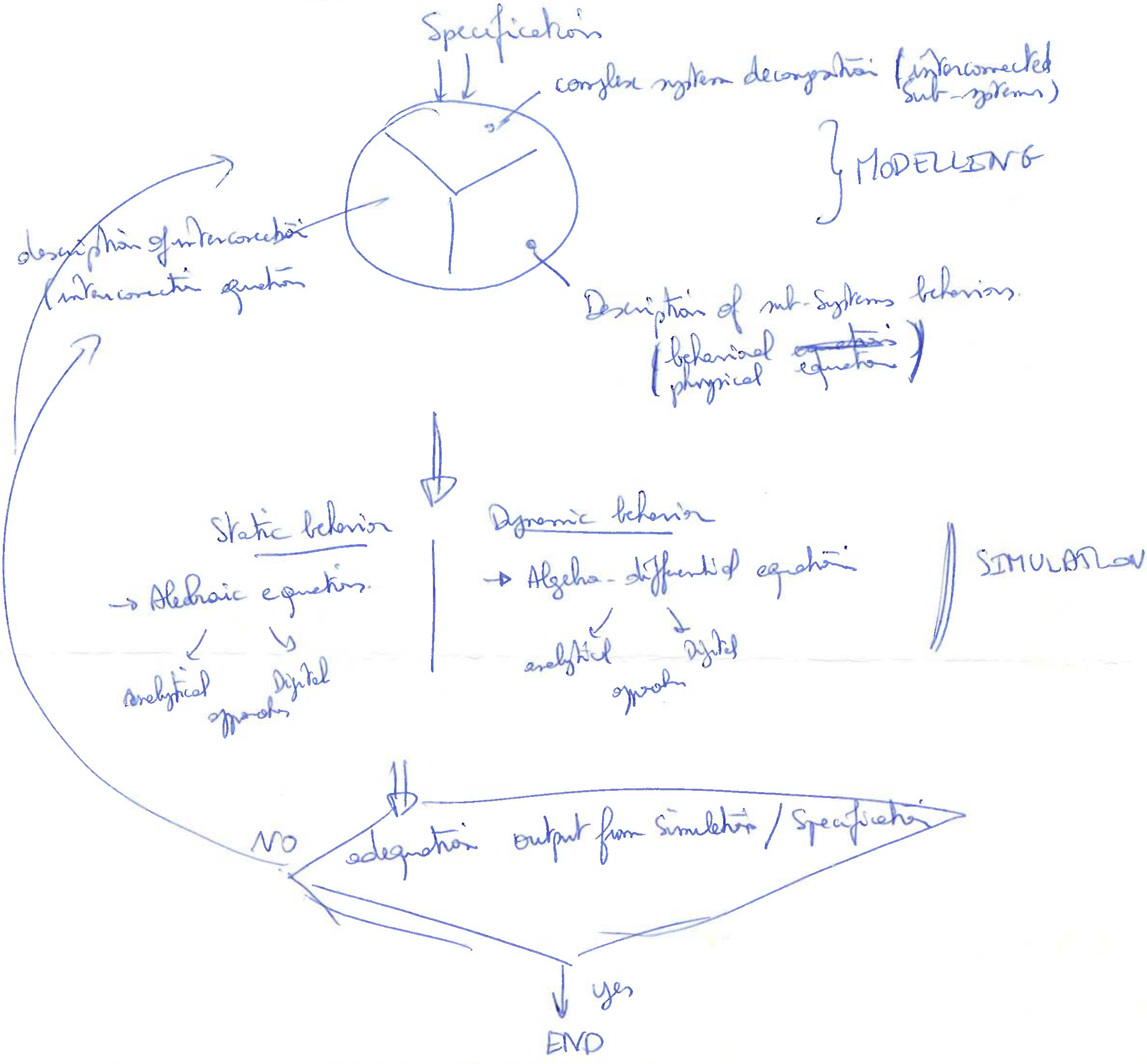
- one define a system by its constituents (what belongs to or not) ~~+ its limits~~ + its interactions (inputs / outputs) with the environment and other systems.



- holistic approach (decomposition of complex systems into simpler ones / or / study of the behavior of a complex system based on the behavior of sub-systems

1.2 Mechatronics based on system analysis

(a) Aproch for the design of Mechatronic System.



Systems can be classified into 2 main categories!

Systèmes localisés

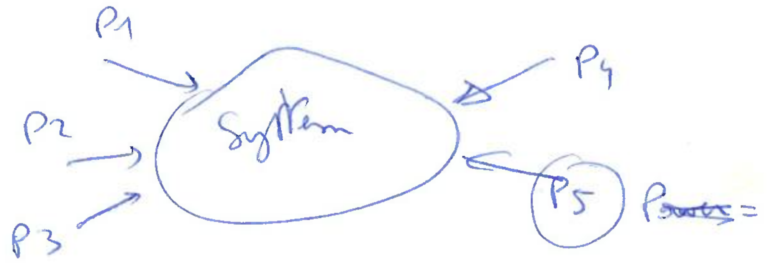
↳ Lumped parameters system. (equations = ~~differential~~ finite differential eq.)
(finite dimensional syst) → described by a finite number of signals.
⇒ System evolution / changes only depends on time

↳ distributed parameters system (equations = partial differential eq)
(infinite dimensional syst) → ∞ number of signals / continuous spatial distribution
→ System evolution / changes depends on time + space

II Energy Based approach of systems

① general approach

"ideal" point of view : a system exchange NRJ with its environm^t (environm^t includes sub systems interacting with the system) through a finite number of ports (NRJ & Power).



$P(t) = \dfrac{d(E(t))}{dt}$ — instantaneous ~~stored~~ NRJ stored in the system.

instantaneous power flow across the boundary
↳ ⊕ if brought to the system (input)
↳ ⊖ if "taken" —— (output)

$P_\cdot(t) = P_1(t) + P_2(t) \ldots$

= flow variable × effort variable.

Variables can be distinguished into 2 groups:

⊛ variables associated to conservative internal quantities.
(their change induces a ~~flow~~ flow)

ex Force / Torque / current / Volume flow rate

$F = \dfrac{dp}{dt}$ $C = \dfrac{d\beta}{dt}$ $i = \dfrac{dq}{dt}$ Q.

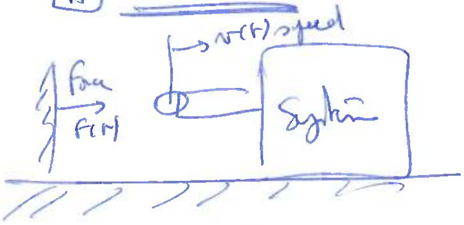
they are associated with their variations (charge q, linear momentum
qtité de m^t)

⊛ variables associated to ~~q~~ continuous quantities ~~ov~~ over time
→ use of potential variables (electrical potential, position)
→ use of variables = potential difference (Voltage, speed)

5 (GB)

2 Mechanical systems

A Translation



Power – $P(t) = F(t) \cdot v(t) = \frac{dE(t)}{dt}$ — NRJ.

Pt

- $v(t) = \frac{dx(t)}{dt} \rightarrow \frac{dE(t)}{dt} = F(t)\frac{dx(t)}{dt} \Rightarrow F(t) = \frac{dE(t)}{dx(t)}$.

mechanical work.

potential NRJ (mechanical work is due to ~~dong~~ NRJ changes, themselves due to displacem^t changes).

qtité m^t / linear momentum

- $F(t) = \frac{dp}{dt} \Rightarrow \frac{dE(t)}{dt} = \frac{dp(t)}{dt} \cdot V(t) \Rightarrow V(t) = \frac{dE(t)}{dp(t)}$.

Pk

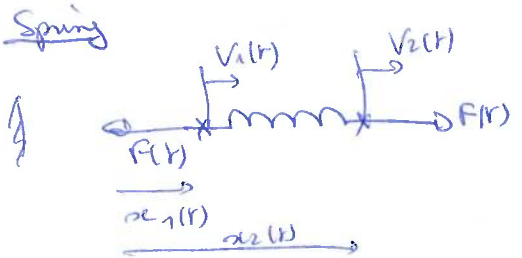
kinetic NRJ

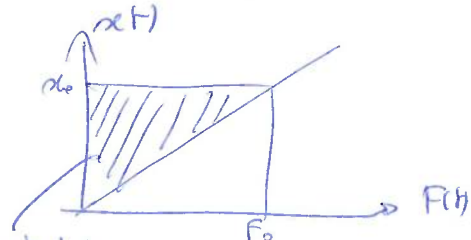
- $\frac{dE}{dt} = F(t) \cdot v(t)$ dissipated NRJ

* NRJ Storage elements

→ element that can store energy and to restore it (cannot provide NRJ to the environm^t in a infinite way).

- Spring





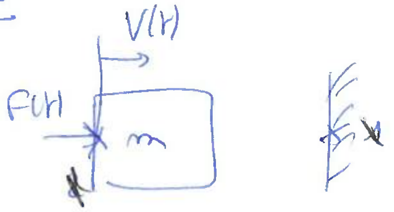
NRJ stored at x_0.

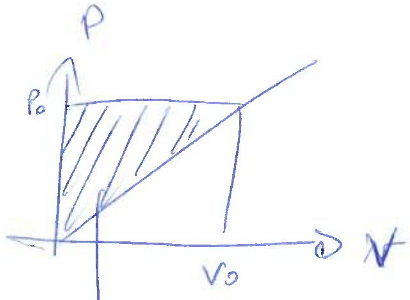
$E = \int_0^x F dx$ with $F = Kx$

$\Rightarrow E = \int_0^x K \cdot x \cdot dx = \frac{1}{2K} F^2$ (always > 0)

NRJ stored.

• Mass





Stored NRJ at Po.

linear momentum $p = m \cdot V$

↓

differentiating

$\frac{dp}{dt} = F = m\frac{dV}{dt}$ (Newton law) linking Force F and acceleration $a = \frac{dV}{dt}$

linear momentum (qtité de mt)

$E = \int_0^p V\,dp$ with $V = \frac{p}{m}$.

$\Rightarrow E = \int_0^p \frac{p}{m}\,dp = \frac{1}{2m}p^2 = \frac{1}{2}mV^2$

always > 0.

* NRJ dissipation elements

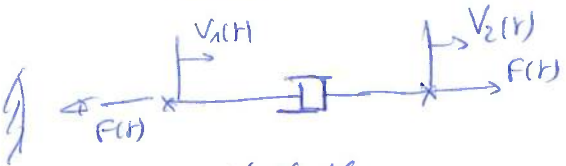
From the NRJ point of view = element that receives NRJ (as a work for ex) and transforms a part of it into heat.

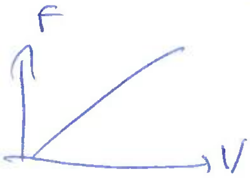
$\Rightarrow$ NRJ is stored and recovered by another element (as work for ex)

Stored NRJ always $>$ recovered NRJ.

These elements highlight NRJ loss ~~in~~ for a system $\Rightarrow$ To be minimized

• Damper





$V = V_2 - V_1$.

perfect damper $F = \alpha \cdot V$

(no ~~friction, no aerodynamic forces~~).

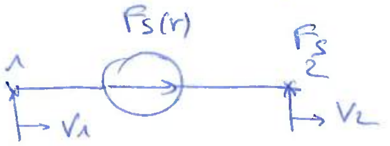
$P = FV = \alpha \cdot V^2 = \frac{F^2}{\alpha}$

↓

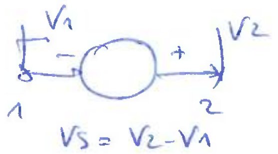
Power dissipated

* Sources

↳ Forces Sources = Force generated independently to Speed V
Speed depends on the system it is connected to.



↳ Speed Source



Speed generation independent to force
Force ——————

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example

- Car — Mass m
  — initial Speed V_0
  — friction (~~wheel~~ tire - Road) $\vec{F_r} = \mu . \vec{P}$ (μ: friction coeff.)

- Det the distance required by the car to stop.

$$F_r = \mu m g.$$

from the NRJ prof view:

↳ Stored NRJ (Mass / linear momentum) $p = mV$. $E_c = \frac{1}{2} p . V$.

$$E_c = \frac{1}{2} m V_0^2 \quad (\text{at } t = 0).$$

↳ NRJ dissipated through friction

$$E_d = \int P(t) dt \quad \text{with } P(t) = F_r . V(t).$$
$$= F_r . x(t)$$

the car will stop when $E_c = E_d$

$$\frac{1}{2} m V_0^2 = F_r x(t) \Rightarrow \boxed{x(t) = \frac{1}{2} \frac{m V_0^2}{F_r}}$$

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1 11 points
fait → Page 7 inclus

A faire p8

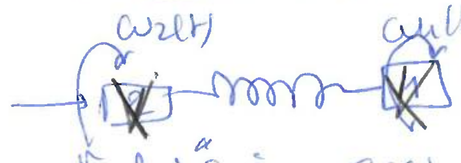
B) Rotation

Torque $C(t)$ | Rotation Speed $\omega(t)$

Power flow $\longrightarrow$ $P(t) = C(t).\omega(t)$

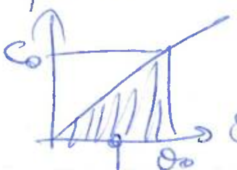
⊛ NRJ Storage Elements

• Rotational Spring



perfect spring $\quad C(t) = K_r . \theta(t)$ ↳ angular displacement. $\theta(t) = \theta_2(t) - \theta_1(t)$

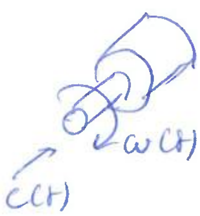
↓ Rotational Spring constant.



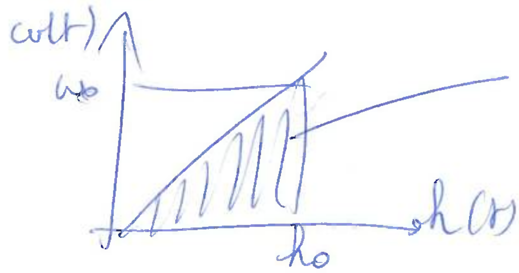
NRJ stored by the spring $\quad E = \int_0^{\theta} C\, d\theta = \int_0^{\theta} K_r \theta(t)\, d\theta$

$$\Rightarrow \boxed{E = \frac{1}{2} K_r \theta^2 = \frac{1}{2K_r} C^2}$$

• Rotational inertia element



angular momentum (moment d'inertie) $\quad h(t) = J.\omega(t)$



NRJ stored $\quad E = \int_0^{h} \omega\, dh$

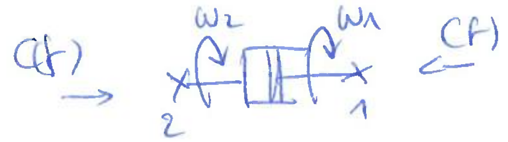
$$= \frac{1}{J} \int_0^{h} h(t)\, dh$$

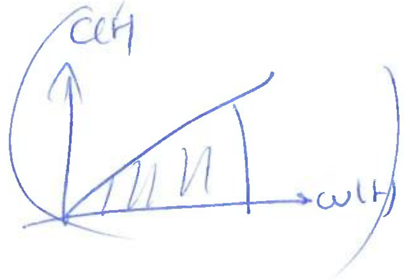
$$\boxed{E = \frac{h_0^2}{2J} = \frac{J\omega_0^2}{2}}$$

Remark by differentiating $\quad \frac{dh}{dt} = J\frac{d\omega}{dt} \Rightarrow C(t) = J\frac{d\omega}{dt}$

* **Energy dissipation elements**

viscous friction = rotational damper
(frottement visqueux)





$C(t) = D \cdot \omega(t)$

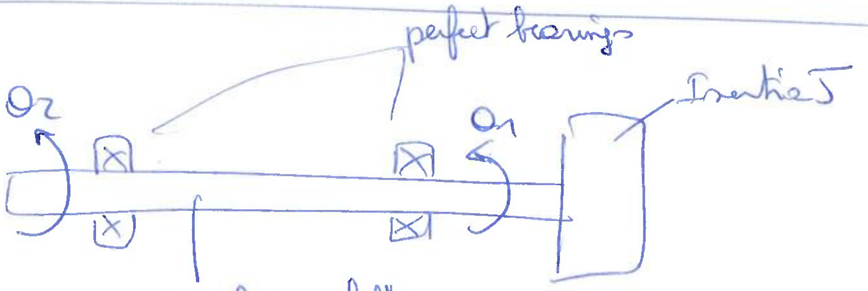
↓ viscous friction / rotary damper constant

$P(t) = C(t) \cdot \omega(t) = D\omega^2 = \dfrac{C^2}{D}$

↓ Power dissipated

* **Sources**
  - ↳ Torque
  - ↳ Speed

(exe)



Rotational Spring Constant K_r

- initial state : the whole system rotates – constant speed ω_0
- the input of the shaft is suddenly stopped ($\theta_2 = 0$)

⟹ What is the maximum orientation / torsion of the shaft

NRJ stored (inertia) $E_c = \frac{1}{2} J\omega_0^2$
NRJ dissipated (friction) $E_d = \frac{1}{2} K_r \cdot \theta^2$

$\theta = \theta_{max}$ when $E_c = E_d$

$\frac{1}{2} J\omega_0^2 = \frac{1}{2} K_r \theta_{max}^2$

$\Rightarrow \boxed{\theta_{max} = \sqrt{\frac{J}{K_r}} \cdot \omega_0}$

2.3 Generalisation

⊛ Classification of variables

Several classifications may be used

2 Most widespread ones.

↳ Accross & through Variables → our case.
↳ Force & flow Variables → Bond Graph.

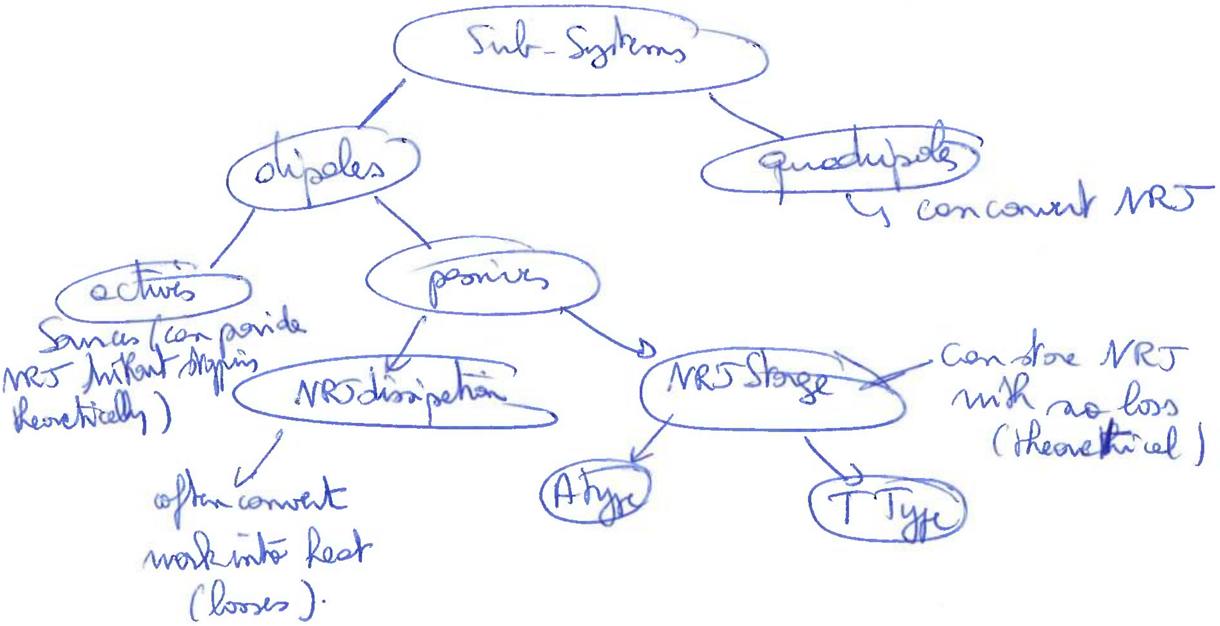
Variables	mechanics: Translation	mechanics: Rotation	Electricity	fluidic
through (flux) f	Force $F = \frac{dp}{dt}$	Torque $C = \frac{dh}{dt}$	Current $I = \frac{dq}{dt}$	Flow rate (débit) Q
Accross (potential) v	Speed $V(t)$	Rotational Speed $\omega(t)$	Voltage (U)	Pressure P.

linear momentum (quantité M^t)

angular momentum (moment d'inertie)

charge.

⊛ Classification of systems



M (GB)

- Sources

→ $\vec{F}$, $\vec{v}$ (Force / Speed)

→ U, i (Voltage / Current)

→ P, Q (Pressure / Flow Rate)

- Passive / dissipative

their internal equation directly links flow & potential variable
(f) (v)

(ex) Damper
(friction' viscous)

$$F_{(t)} = \cancel{D} . V_{(t)}$$

$F_{(t)}$ ↓ Force; D ↓ constant; $V_{(t)}$ speed.

electrical Resistor $U = R . I$

U: voltage; R: cte; I: current.

- Passive / Storage

through   Across.

(A type)
their internal equation is like $f = C . \frac{dv}{dt}$

(ex) Mass / inertia $F = m \frac{dV}{dt}$

electrical capacitor $i = C . \frac{dU}{dt}$

Tank
(reservoir) $Q = \frac{S}{\rho g} \frac{dP}{dt}$

Q: flow Rate

Surface Tank

$$Q = \frac{dV}{dt} = S \frac{dh}{dt} = \frac{S}{\rho g} \frac{dP}{dt}$$

S: Surface Tank

T type

internal equation such as $\underset{\text{Accross}}{v} = L \frac{d\overset{\text{through}}{f}}{dt}$

ex Spring $F = kx$ $\left(V = L\frac{dF}{dt} \Rightarrow \frac{dx}{dt} = L\frac{dF}{dt} \right)$

coil
inductance
$U = L\frac{di}{dt}$

Rq | Passive storage element do have a "natural" causality
= link between the variables that is a source / input & the consequence / output

ex T type / coil → causality = U (= U is the control input) V Accross.
A — / electrical capacitor → causality = I (= I ———) f through
Mass → ——— = Force

$F \rightarrow \boxed{M} \rightarrow$ derivative causality
$F = m\frac{dV}{dt}$

Sometimes a Mass may have a integer ($\int$) causality
(Speed = V = input) $\Rightarrow$ $V = \frac{1}{m}\int F dt$

III Intro to the linear graph

3.1 General principle

linear graph = graphical representation of a dynamic system.
(~~Specifically~~ Especially relevant for multi-Pc systems)

= generalized / systematic way of Modelling (no forgetting of equations + non-redundancy)

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graph = Set of branches & nodes

branches: oriented line

nodes: point of interconnection between sub-systems.

Across Variables.

→ node

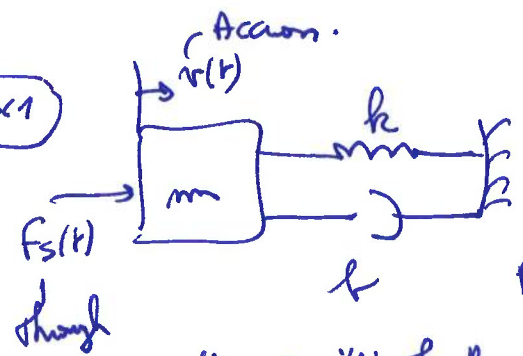
branche ↓ Arrow = orientation for the through Variable.

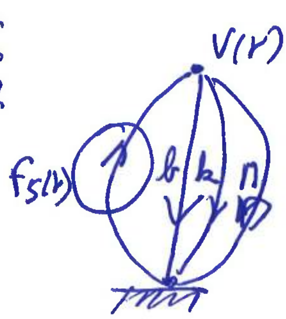
→ node

linear graph ⇒ 2 set of equations:

- between Across Variables (compatibility laws)
- between Through Variables. (continuity laws)

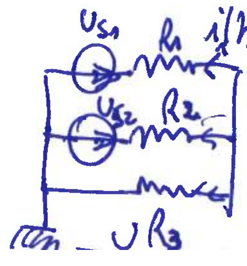
ex 1

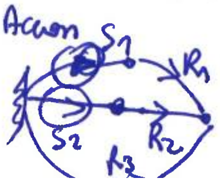




$F_s - k\int V\,dt - b\,V(t) - m\frac{dV}{dt} =$

ex 2





$V = V_{S1} + R_1\, i_1(t)$
$V = V_{S2} + R_2\, i_2(t)$
$V = R_3\, i_3(t)$

3.2 Causal based Modelling of interconnections

Interconnection of a graph can be written by ~~Alog~~ Algebraic equations under a Matrix form.

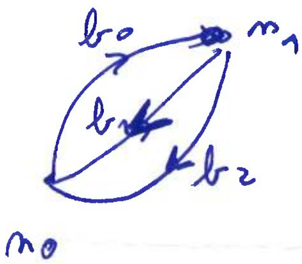
pb (to carefull): Algebraic eq => no hypothesis dealing with causality!

a) Incidence Matrix (A_e)

Considering a ~~G~~ Graph comprising n_n nodes, n_b branches

The incidence Matrix (A_e) will have n_n lines, n_b column

ex



$$\Rightarrow A_e = \begin{array}{c c} & \begin{array}{ccc} b_0 & b_1 & b_2 \end{array} \\ \begin{array}{c} n_0 \\ n_1 \end{array} & \begin{pmatrix} 1 & -1 & -1 \\ -1 & 1 & 1 \end{pmatrix} \end{array}$$

b_0 starts in n_0 (+1) and ends in n_1 (-1)

Rq: a 0 is written when a node does not belong to a branch.

Reduced incidence Matrix (A)

Rq: Rank of A_e Matrix $= n_n - 1$

↳ 1 line is a linear combination of others

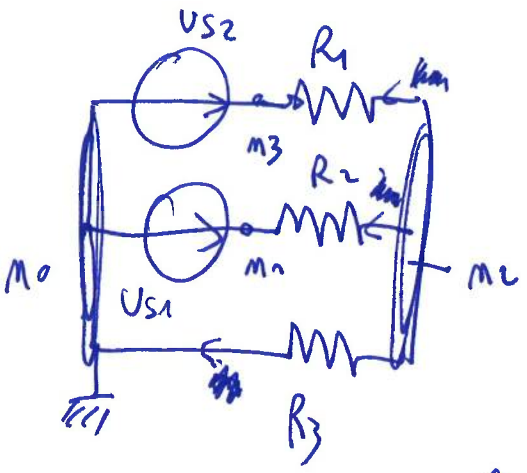
=> 1 line can be removed without information loss

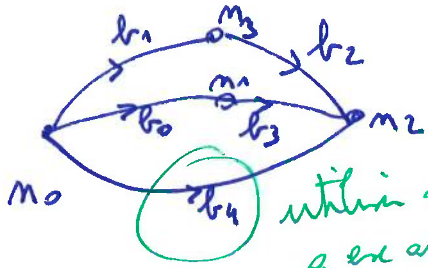
if n_0 = reference node (potential = 0 for ex) => we can remove the first line

ex: $A = (-1 \ 1 \ 1)$

15

ex Det – Graph for the following circuit
– A_e & A





utilizăm un nou q ex avant tîm numerotat.

$$A_e = \begin{array}{c} \\ n_0 \\ n_1 \\ n_2 \\ n_3 \end{array} \begin{array}{c} \begin{array}{ccccc} b_0 & b_1 & b_2 & b_3 & b_4 \end{array} \\ \begin{pmatrix} 1 & 1 & 0 & 0 & 1 \\ -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & -1 & -1 \\ 0 & -1 & 1 & 0 & 0 \end{pmatrix} \end{array}$$

$\hookrightarrow A$ (on enlève la ligne de n_0)

$$\Rightarrow A = \begin{pmatrix} -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & -1 & -1 \\ 0 & -1 & 1 & 0 & 0 \end{pmatrix}$$

(*) generalised Kirchhoff equation / law for currents (electrical currents) through variable

$$A \cdot i = 0 \quad \text{with } i = \begin{pmatrix} i_0 \\ i_1 \\ i_2 \\ i_3 \\ i_4 \end{pmatrix}$$ — current in every Branch (Through Variable)

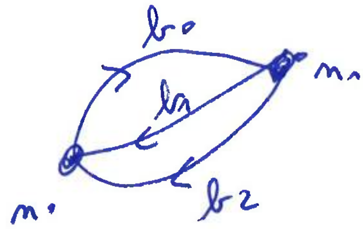
$$\begin{pmatrix} -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & -1 & -1 \\ 0 & -1 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} i_0 \\ i_1 \\ i_2 \\ i_3 \\ i_4 \end{pmatrix} = 0$$

$$\Rightarrow \begin{cases} -i_0 + i_3 = 0 \\ -i_2 - i_3 - i_4 = 0 \\ -i_1 + i_2 = 0 \end{cases}$$

b | Mesh Matrix
Matrice de mailles

1δ Considering a $\mathcal{G}$ graph

ex



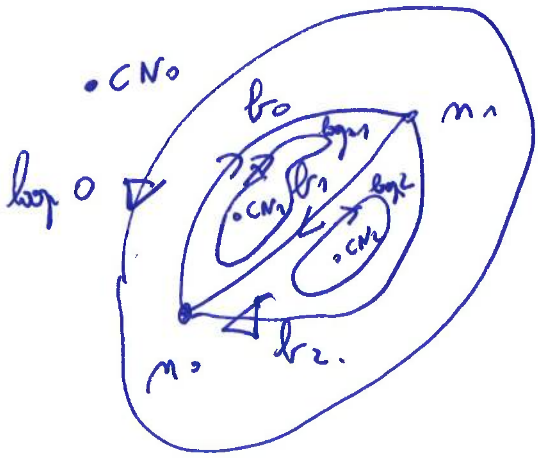
A Mesh (loop) = sub-graph of $\mathcal{G}$ whose nodes only have 2 branches

ex

1 Mesh/loop has an orientation

Co-Node = area in the middle of an elementary Mesh/loop

in general : loop whose co-node is inside have clockwise orientation (and reversely).



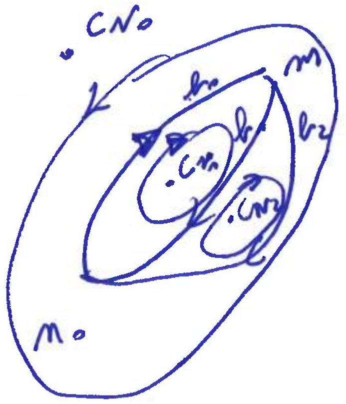
(*) Matrix of loops for the graph G.

(M_e)

m_{CS} = nb of co. nodes = nb of lines for M_e

m_b = — branches = — columns for M_e

$br_0 \in$ loop 0 but the orientation of loop 0 is opposite with the orientation of br_0.



$$M_e = \begin{array}{c|ccc|} & br_0 & br_1 & br_2 \\ \text{loop } 0 & -1 & 0 & -1 \\ \text{loop } 1 & 1 & 1 & 0 \\ \text{loop } 2 & 0 & -1 & 1 \end{array}$$

belongs to ~~the~~ loop 1 / same orientation

does not belongs to ~~the~~ loop 2

* Reduced Matrix of loops (M)

$\boxed{Rq}$ Rank of M_e = $m_{CS} - 1$

↳ 1 line is a linear combinat° of others

↳ — can be removed without loss of info

⇒ removing of the external loop (CN outside)

(ex) $M = \begin{pmatrix} 1 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix}$

exemple (P16 F2) puis P18 F2 = TD.

GB

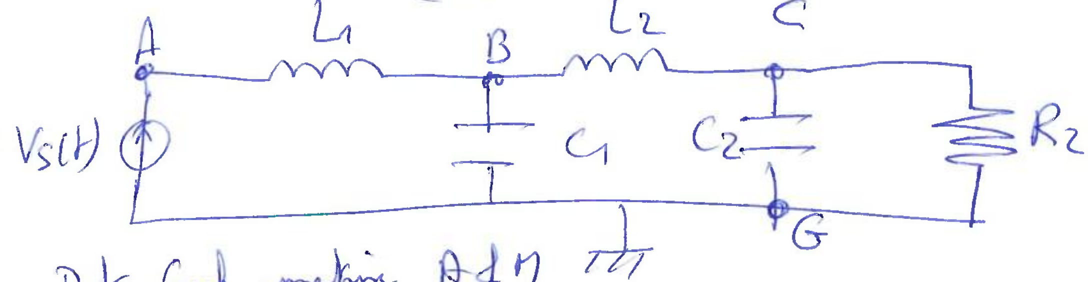
19 Bis (TD)

Exo 1

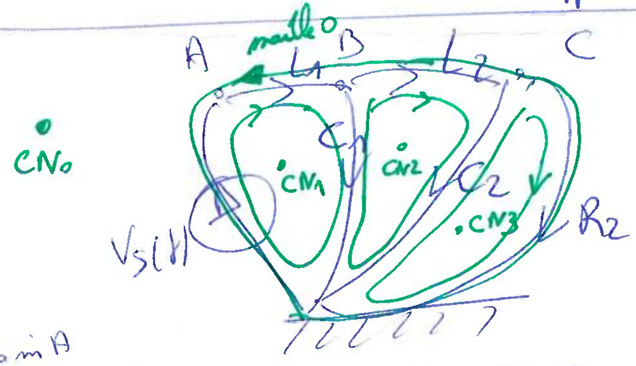
Study & modelling of an analog filter ~~to prevent~~
pb: an alternator provides electric NRJ to an instrument
but ~~the~~ its signal has high frequency ~~freq~~
objective :
↳ preventing/reducing from high frequency noise to be transmitted to the instrument

⟺

constant signal + noise →



Det Graph, matrix A & M + Kirchoff



Vs ends in A

L1 starts in A

incidence

$$A_E = \begin{array}{c|cccccc} & V_S & L_1 & C_1 & L_2 & C_2 & R_2 \\ \hline A & -1 & 1 & 0 & 0 & 0 & 0 \\ B & 0 & -1 & 1 & 1 & 0 & 0 \\ C & 0 & 0 & 0 & -1 & 1 & 1 \\ \cancel{G} & \cancel{1} & \cancel{0} & \cancel{-1} & \cancel{0} & \cancel{-1} & \cancel{-1} \end{array} \Rightarrow A = \text{reste}$$

mettre G en 1er

Mesh/loop

$$M_e = \begin{array}{c|cccccc} & V_S & L_1 & C_1 & L_2 & C_2 & R_2 \\ \hline \cancel{C_{N_0}} & \cancel{-1} & \cancel{-1} & \cancel{0} & \cancel{-1} & \cancel{0} & \cancel{-1} \\ C_{N_1} & 1 & 1 & 1 & 0 & 0 & 0 \\ C_{N_2} & 0 & 0 & -1 & 1 & 1 & 0 \\ C_{N_3} & 0 & 0 & 0 & 0 & -1 & 1 \end{array} \Big\} \text{reste} = M.$$

Kirchoff I

$$A \cdot i = 0$$

$$\begin{pmatrix} -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 1 \end{pmatrix} \begin{pmatrix} i_{V_S} \\ i_{L_1} \\ i_{C_1} \\ i_{L_2} \\ i_{C_2} \\ i_{R_2} \end{pmatrix} = 0$$

$$\Rightarrow \begin{cases} -i_{V_S} + i_{L_1} = 0 \\ -i_{L_1} + i_{C_1} + i_{L_2} = 0 \\ -i_{L_2} + i_{C_2} + i_{R_2} = 0 \end{cases}$$

~~Kirchoff U~~

$M \cdot V = 0$ ~~avec~~

* Generalized Kirchhoff laws for Across Variables.

within a linear graph G:

- the algebraic ~~sum~~ sum of across variables within a loop $= 0$

- $M \cdot V = 0$

Across variables (in branches) $\longrightarrow$ Voltage here.

$$V = A_e^T \cdot U_s$$

~~through~~ Potential of a node.

$$\begin{pmatrix} V_0 \\ V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ -1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} U_{s0} \\ U_{s1} \end{pmatrix}$$

$$\Rightarrow \begin{cases} V_0 = U_{s0} - U_{s1} \\ V_1 = -U_{s0} + U_{s1} \\ V_2 = -U_{s0} + U_{s1} \end{cases}$$

$$M \cdot V = \begin{pmatrix} 1 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix} \begin{pmatrix} V_0 \\ V_1 \\ V_2 \end{pmatrix} = 0$$

$$\begin{cases} V_0 + V_1 = 0 \\ -V_1 + V_2 = 0 \end{cases}$$

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ex précédent (filtre)

$M \cdot V = 0$ ~~avec~~ with $V = A_e^T \cdot U_s$

$$\begin{array}{c} M \\ \begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{pmatrix} \end{array} \begin{pmatrix} V_S \\ V_{L1} \\ V_{C1} \\ V_{L2} \\ V_{C2} \\ V_{R2} \end{pmatrix} \Rightarrow \begin{cases} V_S + V_{L1} + V_{C1} = 0 \\ -V_{C1} + V_{L2} + V_{C2} = 0 \\ -V_{C2} + V_{R2} = 0 \end{cases}$$

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C Theorem Tellegen

Considering a Mechatronic System modelled by a linear Graph following interconnection rules of Kirchhoff, the Vector Space (espace Vectoriel) of Across variables is perpendicular to the Vector Space of Through variables.

variables (Vectors) + operation / operator méthodes + or X

proof

$i \rightarrow$ Through Variables $\Rightarrow A \cdot i = 0$.

$V \rightarrow$ Across — $\Rightarrow V = A^T \cdot V_S$.

Scalar product

$$\langle i, v \rangle = \langle i, A^T \cdot V_s \rangle$$

$$= \langle A i, A \cdot A^T V_s \rangle$$

$$= \langle 0, V_s \rangle$$

$$= 0 \quad \forall i \text{ et } V \Rightarrow i \perp V$$

The Tellegen theorem is a Topological property of linear Graphs.

It enables to prove the NRJ conservation within the system.

↳ It induces the orthogonality between incidence of Mesh / Matrice

$$M \cdot A^T = 0$$

$$A \cdot M^T = 0$$

[3.3] Causal Modelling of interconnections

↳ Easier solving of the pb
↳ Takes into account the required causality (active elements) and preferred / "natural" causality (~~conservative~~ passive storage elements)

Modelling

- without / No causality
  - Matrix.
    - incidence
    - Mesh / loops
- Causality
  - Matrix
    - cycles ⇓ Needs of Defining Tree / co-Tree
    - co-cycles.

[a] Tree / co-Tree

[D] **Tree** A Tree "A" belong to a graph G
↳ = sub-graph of G that includes all nodes but no complete cycle
↳ (= closed loop of the graph)

nb of Branches of the Tree = Rank of the graph ($p = n_n - 1$)
↳ nb of nodes

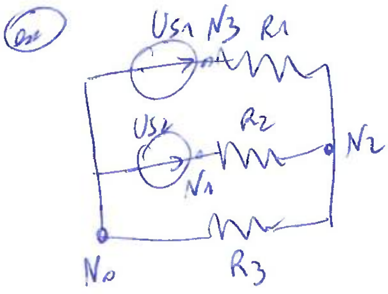
[D] **Co-Tree** The co-Tree "$\bar{A}$" of a graph G is the complementary of A

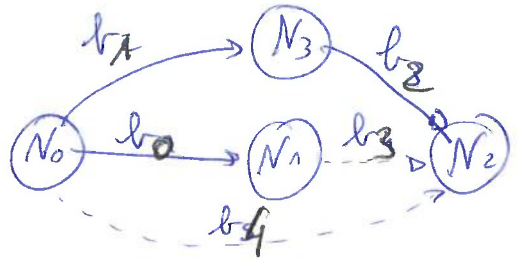
↳ Branch of a co Tree = link (clavier)

[Rq] Considering a graph G → one can choose several Trees
→ the choice has to be done considering causalities: required, "natural"
→ Sources are part of the Tree
→ ~~num~~ Rule for numbering: first for branches (Tree), after for links (coTree)

(NB) Relevant choice ⇒ better design of simulation especially for the dynamic behaviour using the State Space representation

(Ex)





→ Tree
- - → Co-Tree

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b) ~~Cycle~~ Matrix of Cycles

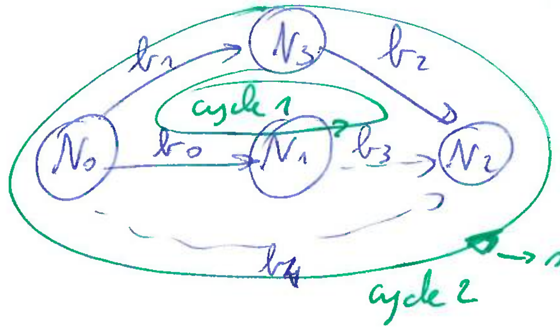
b') cycle

A cycle = adding 1 link (of the co-Tree) to the tree A
→ a closed loop is created

nbr of cycles = nb of links

for every cycle, ~~we~~ is provided → a current (= current along the added link) Through variable
+ an orientation (= the one of the link)

(ex)



b'') Matrix of cycles B

$$B = \begin{array}{c} \\ \text{cycle 1} \\ \text{cycle 2} \end{array} \begin{array}{c} \begin{array}{ccccc} b_0 & b_1 & b_2 & b_3 & b_4 \end{array} \\ \left(\begin{array}{ccc|cc} 1 & -1 & -1 & 1 & 0 \\ 0 & -1 & -1 & 0 & 1 \end{array} \right) \end{array}$$

B_A Matrix $B_{\bar{A}}$ Matrix $= I_B$

$=1$ because $b_0 \in$ cycle 1 + same orientation

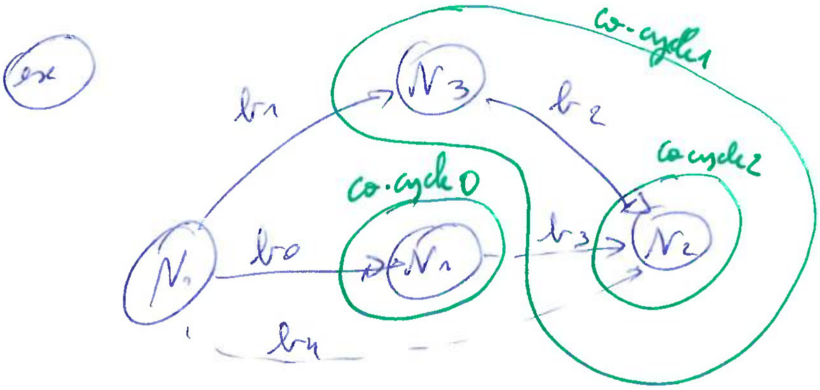
decomposition only possible when good choices are done before

C Matrix of Co-Cycles

D co-cycle

↳ duality of cycles
↳ a co-cycle crosses one ~~link~~ Branch of a tree, everything else belongs to the co-tree
↳ nbe co-cycles = nbe of ~~branches~~ ~~link~~ Branch
↳ number of a co-cycle = number of the branch crossed.

ex



D Matrix of co-cycles

nbe lines = nbe of co-cycles = nbe Branches of the Tree (3 for the ex)
nbe column = nbe of links + branches. (5 for the ex)

$$Q = \begin{array}{c|ccc|cc|} & b_0 & b_1 & b_2 & b_3 & b_4 \\ \hline \text{co cycle } 0 & 1 & 0 & 0 & -1 & 0 \\ \text{co cycle } 1 & 0 & 1 & 0 & 1 & 1 \\ \text{co cycle } 2 & 0 & 0 & 1 & 1 & 1 \end{array}$$

Q_A Matrix = I_a identity $Q_{\bar{A}}$ Matrix

co cycle 0 crossed b0   + b0 oriented to the inside part of co cycle 0

24 (GB)

d) Theorem Tellegen 2

The Vector Space of Across Variables (readings of the links / branches) is $\perp$ to Through

$$B Q^T = 0 = Q B^T$$

Matrix cycles, cocycles.

=> there is no need to calculate both B & Q => one can be calculated while the second one is deduced.

$$\begin{cases} B = \begin{pmatrix} -Q_{\bar{A}}^T & I_B \end{pmatrix} \\ Q = \begin{pmatrix} I_Q & -B_A^T \end{pmatrix} \end{cases}$$

e) Generalized Current / Through Kirchhoff law

$$Q \cdot i = 0.$$

$$\begin{pmatrix} 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} i_0 \\ i_1 \\ i_2 \\ i_3 \\ i_4 \end{pmatrix} = 0 \Rightarrow \begin{cases} i_0 - i_3 = 0 \\ i_1 + i_3 + i_4 = 0 \\ i_2 + i_3 + i_4 = 0 \end{cases}$$

it is also possible to use $i_A = B_{\bar{A}}^T \cdot i_{\bar{A}}$

$$\begin{pmatrix} i_0 \\ i_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -1 & -1 \\ -1 & -1 \end{pmatrix} \begin{pmatrix} i_3 \\ i_4 \end{pmatrix} \Rightarrow \begin{cases} i_0 = i_3 \\ i_1 = -i_3 - i_4 \\ i_2 = -i_3 - i_4 \end{cases}$$

f) Generalized Voltage / Across Kirchhoff law

$$B \cdot V = 0$$

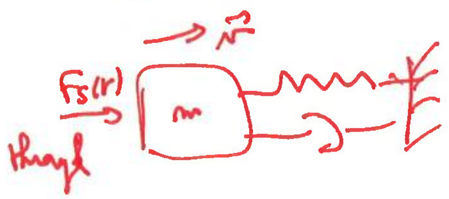
$$\begin{pmatrix} 1 & -1 & -1 & 1 & 0 \\ 0 & -1 & -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} V_0 \\ V_1 \\ V_2 \\ V_3 \\ V_4 \end{pmatrix} = 0 \Rightarrow \begin{cases} V_0 - V_1 - V_2 + V_3 = 0 \\ -V_1 - V_2 + V_4 = 0 \end{cases}$$

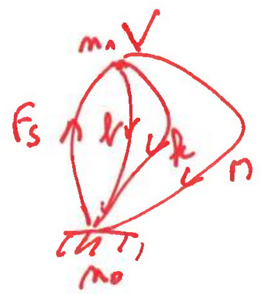
~~But~~, it is also possible to: $V_{\bar{A}} = -B_A \cdot V_A$

$$\begin{pmatrix} V_3 \\ V_4 \end{pmatrix} = - \begin{pmatrix} 1 & -1 & -1 \\ 0 & -1 & -1 \end{pmatrix} \begin{pmatrix} V_0 \\ V_1 \\ V_2 \end{pmatrix}$$

$$\Rightarrow \begin{cases} V_3 = -V_0 + V_1 + V_2 \\ V_4 = V_1 + V_2 \end{cases}$$

Système méca



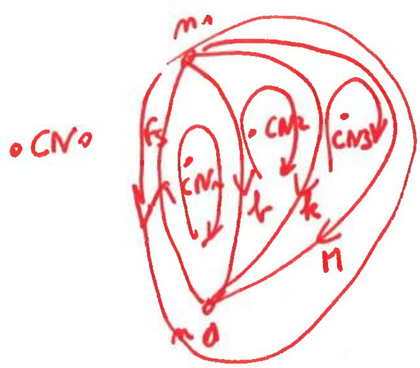


Matrice des ~~…~~ d'incidence

Fs sort de m0

$$A_c = \begin{array}{c} \\ m_0 \\ m_1 \end{array}\begin{array}{c} \begin{array}{cccc} F_s & b & k & M \end{array} \\ \begin{pmatrix} 1 & -1 & -1 & -1 \\ -1 & 1 & 1 & 1 \end{pmatrix} \end{array} \longrightarrow A = \begin{pmatrix} -1 & 1 & 1 & 1 \end{pmatrix}$$

Matrice des mailles



$$M_c = \begin{array}{c} \\ \text{maille } 0 \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{cccc} F_s & b & k & M \end{array} \\ \begin{pmatrix} -1 & 0 & 0 & -1 \\ 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{pmatrix} \end{array}$$

$$\hookrightarrow M = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{pmatrix}$$

Matrice des cycles B

$$B = \begin{array}{c} \\ \text{cycle } 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{cccc} F_s & b & k & M \end{array} \\ \left(\underbrace{\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}}_{B_A} \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{B_{\bar{A}} = I_B}\right) \end{array}$$

méca 2

Matrice co-cycle Q

$$Q = \begin{array}{c} \begin{array}{cccc} F_S & b & k & M \end{array} \\ \begin{pmatrix} 1 & -1 & -1 & -1 \end{pmatrix} \end{array}$$

$Q_A = I_Q$ $\quad$ $Q_{\bar{A}}$

Fs entre dans co-cycle

Loi Kirchoff $\bar{I}$

$$Q \cdot i = 0$$

$$\begin{pmatrix} 1 & -1 & -1 & -1 \end{pmatrix} \begin{pmatrix} F_s \\ F_b \\ F_k \\ F_M \end{pmatrix} = 0 \Rightarrow F_s - F_b - F_k - F_M = 0.$$

or $i_A = B_A^T i_{\bar{A}}$

$$(F_S) = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} F_b \\ F_k \\ F_M \end{pmatrix}$$

$$\Rightarrow F_S = F_b + F_k + F_M$$

Loi Kirchoff U

$$B \cdot V = 0$$

$$\begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} V_{FS} \\ V_b \\ V_k \\ V_M \end{pmatrix} = 0$$

$$\begin{cases} V_{FS} + V_b = 0 \\ V_{FS} + V_k = 0 \\ V_{FS} + V_M = 0 \end{cases}$$

$$V_{FS} = V_1 - V_0 = V_1$$
$$V_b = V_0 - V_1 = -V_1$$

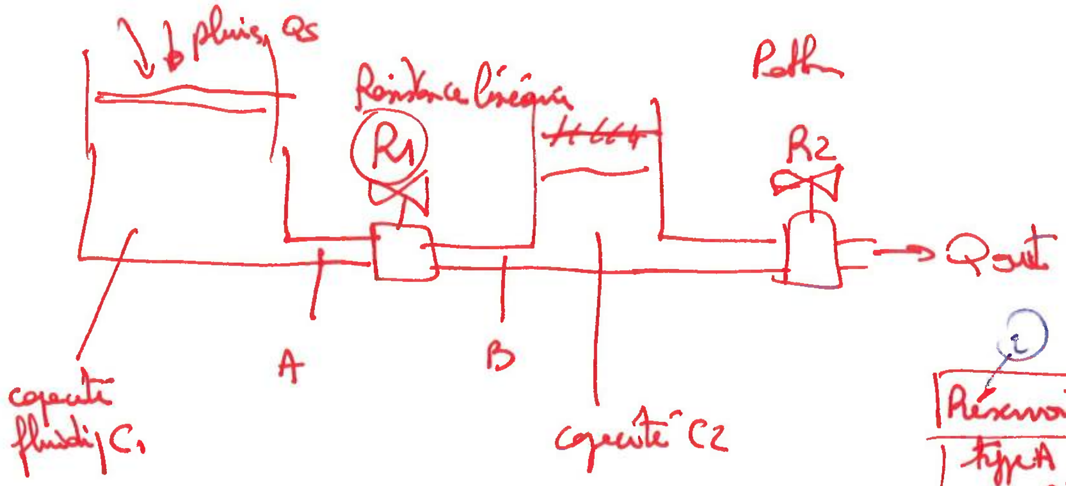
$$V_{\bar{A}} = -B_A \cdot V_A$$

$$\begin{pmatrix} V_b \\ V_k \\ V_M \end{pmatrix} = -\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} (V_{FS})$$

$$\Rightarrow \begin{cases} V_b = -V_{FS} \\ V_k = -V_{FS} \\ V_M = -V_{FS} \end{cases}$$

exo 2

Syst récupération d'eau



Through → débit

Accross → Pression

Through

Accross → Pression — Patm, P_A, P_B ⇒ 3 nœuds.

① $\boxed{\text{Réservoir}}$ = passif conservatif

type A, le stockage

$$\text{Through} = C\,\frac{d\,\text{Accross}}{dt}$$

$$Q = \text{débit} = C\,\frac{dP}{dt} = \frac{dV}{dt}$$

$$Q = \frac{S}{\rho g}\,\frac{dP}{dt} = \frac{dV}{dt}$$

$$V = \frac{S}{\rho g}\cdot P$$

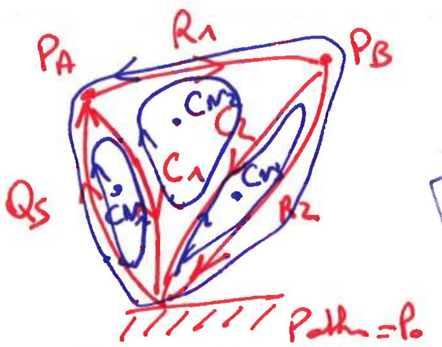
C_1 / C_2

② $\boxed{\text{Vanne}}$ = dissipatif

lie Q et P

$$P = RQ$$

CN0



Source Qs

Qs part de P0

$$A_e = \begin{array}{c|ccccc} & Q_S & C_1 & R_1 & C_2 & R_2 \\ \hline P_0 & 1 & -1 & 0 & -1 & -1 \\ P_A & -1 & 1 & 1 & 0 & 0 \\ P_B & 0 & 0 & -1 & 1 & 1 \end{array} \Rightarrow A = \begin{pmatrix} -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 \end{pmatrix}$$

$R_2 \in$ Maille 0 sens inverse

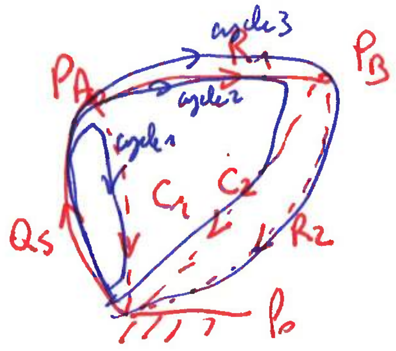
$$M_e = \begin{array}{c|ccccc} & Q_S & C_1 & R_1 & C_2 & R_2 \\ \hline \text{maille } 0 & -1 & 0 & -1 & 0 & -1 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 2 & 0 & -1 & 1 & 1 & 0 \\ 3 & 0 & 0 & 0 & -1 & 1 \end{array} \Rightarrow M = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{pmatrix}$$

exo 2 suite

Arbre / co-arbre

↳ nbr branche = nbr noeuds − 1

= 2



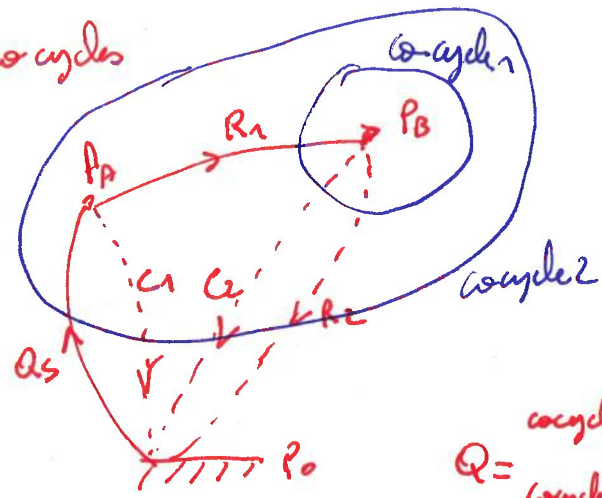
nbr cycle = nbr de chainons

↳ (Branches du co-arbre)

Et même sens

$$B = \begin{array}{c|ccccc} & Q_S & C_1 & R_1 & C_2 & R_2 \\ \hline \text{cycle 1} & 1 & 1 & 0 & 0 & 0 \\ \text{cycle 2} & 1 & 0 & 1 & 1 & 0 \\ \text{cycle 3} & 1 & 0 & 1 & 0 & 1 \end{array}$$

co-cycles



$$Q = \begin{array}{c|ccccc} & Q_S & C_1 & R_1 & C_2 & R_2 \\ \hline \text{cocycle1} & 0 & 0 & 1 & -1 & -1 \\ \text{cocycle2} & 1 & -1 & 0 & -1 & -1 \end{array}$$

Q_1 sr ds co cycle2

Tellegen / Kirchhoff

I $\quad Q \cdot i = 0 \quad \begin{pmatrix} 0 & 0 & 1 & -1 & -1 \\ 1 & -1 & 0 & -1 & -1 \end{pmatrix} \begin{pmatrix} Q_S \\ Q_{C1} \\ Q_{R1} \\ Q_{C2} \\ Q_{R2} \end{pmatrix} \Rightarrow \begin{array}{l} Q_{R1} - Q_{C2} - Q_{R2} = 0 \\ Q_S - Q_{C1} - Q_{C2} - Q_{R2} = 0 \end{array}$

J $\quad B \cdot V = 0 \quad \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} P_0 \\ P_A \\ P_B \end{pmatrix} \Rightarrow P_0$