ECE661: Homework 3

Fall 2022
Due Date: 1:30pm, Sep 20, 2022

In this homework, you will explore and compare different approaches for removing the perspective and affine distortions in images. Note that you can NOT use any built-in functions to compute homography matrices or for image warping.

Turn in typed solutions via BrightSpace. Additional instructions can be found at BrightSpace.

1 Introduction

The goal of this homework is to remove projective and affine distortions in the given images. This is referred as *metric rectification* in the text [1] (Second Edition). For your programming tasks, you will implement the approaches given the textbook for metric rectification. Make sure to read the description for the given tasks carefully. This homework is based on the concepts covered in Lectures 4 and 5.

You will show results after the distortions removal on the given input images as well as using your own images for the three approaches detailed in the rest of this section. Note that the set of points or lines you pick for estimating any homography should reside on the same planar surface in the scene.

1.1 Point-to-point correspondences

This approach is a trivial extension of what you implemented in your Homework 2. You will manually find point-to-point correspondences between the original undistorted scene as your domain and its photograph as your range which has projective and affine distortions. The inverse homography will eliminate these distortions. Obviously, using point-to-point correspondences is the most straightforward method compared to the other two. However, it often requires a large number of correspondences to give a numerically stable solution for estimating the homography.

1.2 The Two-Step Method

In the two-step method, you first remove the projective distortion using the Vanishing Line (VL) method discussed in Lecture 4. Subsequently, you remove the affine distortion by using the $\cos\theta$ expression with θ equal to 90°. Note that you must first remove the projective distortion before you can remove the affine distortion with the $\cos\theta$ based method. In the text [1], the two-step approach is termed as *stratified.*

Removing the Projective Distortion: The projective distortion can be eliminated by the homography that takes the VL back to $\mathbf{l}_\infty$. To do so, you first estimate a VL in the image plane by picking pixel coordinates of at least two pairs of parallel lines in the original scene. Taking the cross-product of two such pixels on any line in the image will give you the HC representation of that line. Taking the cross-product of the 3-vectors for two different lines (which are parallel in the original scene) will give you the HC representation for the Vanishing Point (VP) for those two lines. Then taking the cross-product of two such VPs for two different pairs of parallel lines will give you the VL you need for getting rid of the projective distortion.

Removing the Affine Distortion: The affine distortion can be eliminated by the homography that restores the angle between two orthogonal lines in the original scene back to 90°. As you learned in Lecture 5, you can obtain the expression for $\cos\theta$ in the form of Dual Degenerate Conic $\mathbf{C}_\infty^*$ as follows

$$\cos\theta = \frac{\mathbf{l}^T\mathbf{C}_\infty^*\mathbf{m}}{\sqrt{(\mathbf{l}^T\mathbf{C}_\infty^*\mathbf{l})(\mathbf{m}^T\mathbf{C}_\infty^*\mathbf{m})}}$$

where $\mathbf{C}_\infty^* = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 0\end{bmatrix}$. Substituting $\mathbf{l} = \mathbf{H}^T\mathbf{l}'$ and $\mathbf{m} = \mathbf{H}^T\mathbf{m}'$ in the nominator and setting it to zero, we can write the following constraint for estimating $\mathbf{H}$:

$$\cos\theta = \mathbf{l}'^T\mathbf{H}\mathbf{C}_\infty^*\mathbf{H}^T\mathbf{m}' = 0, \tag{1}$$

where $\mathbf{H}$ is an affine transformation matrix:

$$\mathbf{H} = \begin{bmatrix}\mathbf{A} & \mathbf{t}=\mathbf{0}\\ \mathbf{0}^T & 1\end{bmatrix}.$$

For affine distortion removal, you need to identify orthogonal line pairs, i.e., angle-to-angle correspondence between orthogonal lines in the original scenes and in the image planes. You will need at least two such orthogonal line pairs in order to solve the unknowns in the affine transformation matrix.

1.3 The One-Step Method

The one-step method removes both the projective and affine distortions in one step. Let $\mathbf{C}^{*'}$ be a projection of the dual degenerate conic $\mathbf{C}^*_\infty$, then the one-step method eliminates both distortions using the homography that maps $\mathbf{C}^{*'}$ back to $\mathbf{C}^*_\infty$.

Recall that the projection of the dual degenerate conic can be written as follows:

$$\mathbf{C}^{*'} = \mathbf{H}\mathbf{C}^*_\infty\mathbf{H}^T. \tag{2}$$

Substituting the above representation into eq. (1), we arrive at:

$$\cos\theta = \mathbf{l'}^T\mathbf{C}^{*'}\mathbf{m'} = 0. \tag{3}$$

Now, let

$$\mathbf{C}^{*'} = \begin{bmatrix} a & b/2 & d/2 \\ b/2 & c & e/2 \\ d/2 & e/2 & f \end{bmatrix},$$

then we can estimate the unknowns $a, b, c, d,$ and e by solving eq. (3) with multiple pairs of $\mathbf{l'}$ and $\mathbf{m'}$.

To do so, you need to identify lines $\mathbf{l'}$ and $\mathbf{m'}$ in an image such that their corresponding lines $\mathbf{l}$ and $\mathbf{m}$ in world coordinates are orthogonal. You need to identify at least five such pairs. Note that since we're working with HC where only ratios matter, you can fix $f = 1$.

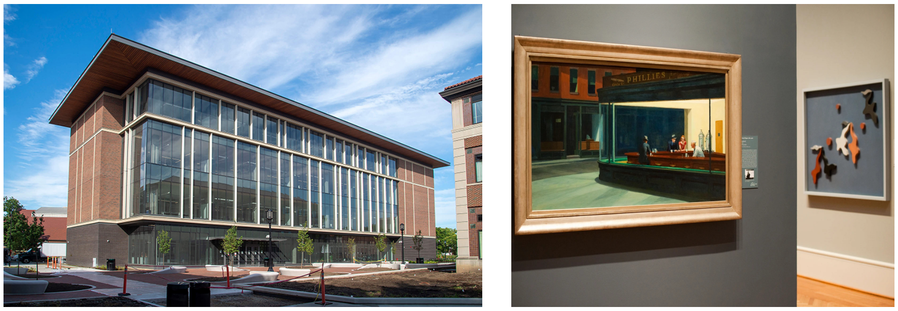
After $\mathbf{C}^{*'}$ has been fully estimated, our task now is to derive the homography matrix $\mathbf{H}$ that satisfies eq. (4). Let's start by expanding eq. (4):

$$\begin{aligned} \mathbf{C}^{*'} &= \mathbf{H}\mathbf{C}^*_\infty\mathbf{H}^T \\ &= \begin{bmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{v}^T & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{A}^T & \mathbf{v} \\ \mathbf{0}^T & 1 \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{A}\mathbf{A}^T & \mathbf{A}\mathbf{v} \\ \mathbf{v}^T\mathbf{A}^T & \mathbf{v}^T\mathbf{v} \end{bmatrix}. \end{aligned} \tag{4}$$

Subsequently, we can establish the following relationships for solving $\mathbf{A}$ and $\mathbf{v}$:

$$\mathbf{A}\mathbf{A}^T = \begin{bmatrix} a & b/2 \\ b/2 & c \end{bmatrix}, \tag{5}$$

$$\mathbf{A}\mathbf{v} = \begin{bmatrix} d/2 \\ e/2 \end{bmatrix}. \tag{6}$$



(a) Image 1

(b) Image 2

Figure 1: Input images for Task1

Once you have solved $\mathbf{A}$ and $\mathbf{v}$, the resulting matrix $\mathbf{H}^{-1}$ will be your desired homography to rectify both projective and affine distortions in one step. For further information on the one-step approach, see pages 42, 55 and 56 of the text [1].

2 Programming Tasks

2.1 Task 1

Download the input images, shown in Fig. 1 and their world coordinates (height and width measurements of some planar object in the scenes), separately.

1. Show results after distortion corrections using **point-to-point correspondences**, this is a trivial extension of Homework 2. With the given height and width, your points in the undistorted image should be (0, 0), (0, width), (height, 0), and (height, width). After you have found the correspondences you simply need to apply the homography to the input image to remove distortion.
2. Show results after distortion correction using the **two-step** and **one-step** approaches.
3. Outline your observations on the results obtained using the above three methods.

2.2 Task 2

Repeat the steps, outlined in Task 1, on at least two of your own images. Usually, images with repetitive patterns on planar surfaces such as facades, walls with portraits, etc have many parallel or orthogonal line features and are easier to work with. Make sure to use images with significant projective and affine distortions. You can use approximate estimations of world coordinates, however state your assumptions clearly.

3 Submission Instructions

Include a typed report explaining how did you solve the given programming tasks.

1. Turn in a zipped file, it should include (a) a typed pdf report with source code files and results, (b) source code files **(only .py files are accepted — convert your .ipynb files to .py)**, (c) Rename your .zip file as hw3_<First Name><Last Name>.zip and follow the same file naming convention for your pdf report too.

2. Your pdf must include a description of

   - The logic that you used to solve the given tasks.
   - The steps that you used for each of the tasks with relevant equations.
   - The input and output images for each task. Clearly show the plotted parallel / orthogonal lines that you chose in the input images.
   - At least some use vectorized numpy operations, if not fully optimized, is expected in your Python code.
   - Your observations on the output quality and performance of each approach.
   - Your source code. Make sure that your source code files are adequately commented and cleaned up.

3. In order to avoid large file size of your submission, include JPEG images in your report for showing your results and your input images for Task2.

4. The sample solutions from previous years are for reference only, it's important not to get too biased by those solutions. **Your code and final report must be your own work.**

References

[1] Richard Hartley and Andrew Zisserman. *Multiple view geometry in computer vision.* Cambridge university press, 2003.