



OXFORD MAN INSTITUTE, UNIVERSITY OF OXFORD
SUMMER RESEARCH PROJECT

Price Impact and Optimal Execution Strategy

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Abstract

Price impact refers to the change in the price of a security in response to an incoming market order. A buy trade should increase the price and a sell order should do the opposite. The main purpose of this paper is to study the correlation between the volume of the order and its subsequent price movement, as well as investigate how will this correlation alter with time. After identifying the power law and linear relationship, this paper then takes a step further to model the optimal execution strategy that would minimize the expected cost.

1. Price Impact Curve

Introduction:

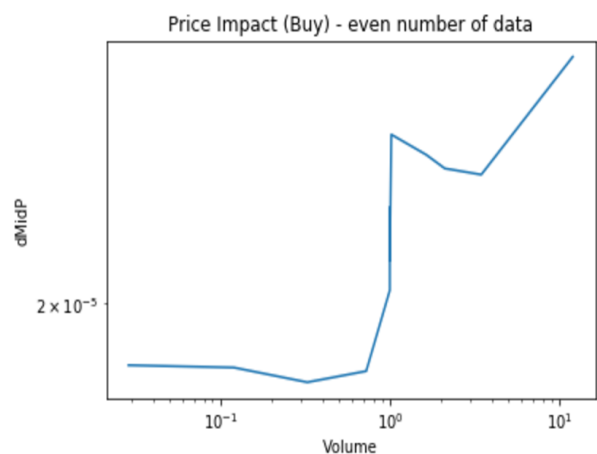
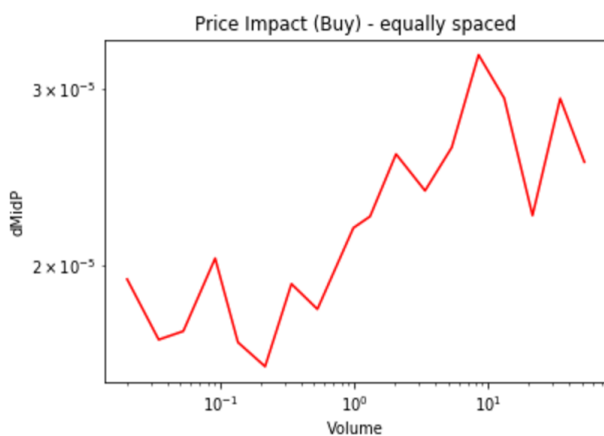
There are many factors which have an impact on the price of a security, such as its **volatility**, **trading volume and liquidity**. This sector will focus on monitoring the **volume dependence impact**. **Modelling this correlation could help the trader to split and spread the orders**

Data:

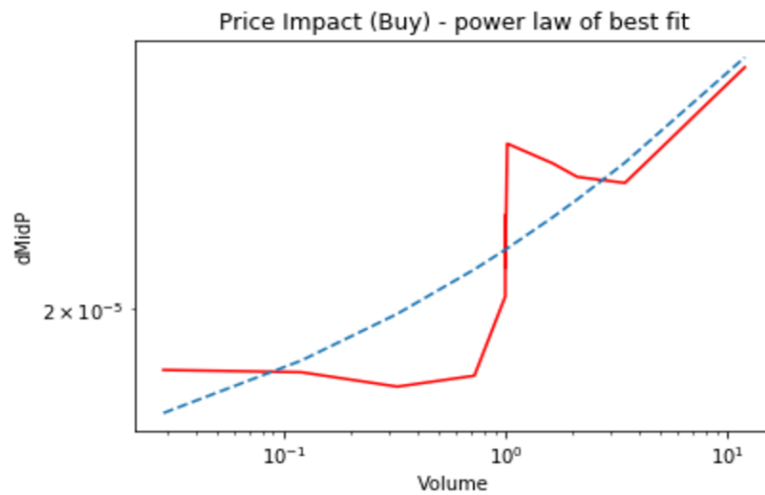
- Stock Name: Exxon Mobil Corporation XOM
- Market: US, trading hours: 8:30 AM CT to 3:15 PM
- Trading time period used: 2016-10-03 to 2016-10-28, everyday 11am to 3pm
- **Quote data: Date time, Bid quote, Ask quote**
- **Trade data: Date time, transaction price, transaction volume**

Methodology: (Code – see Python Jupyter)

1. Import the Quote and Trade data, change the time to DateTime format, only use the data between 11am and 3pm during the market hours
2. Compute Mid Price from the Quote data
3. **Identify 'Buy' or 'Sell' data by applying Lee & Ready algorithm to the trade data**
4. Arrange Quote ('Buy' and 'Sell') and Trade data in time order, combine consecutive trade by adding up the transaction volume.
5. **Compute the change in log Mid Price before and after a transaction takes place, normalize the transaction volume by the median volume**
6. Plot the price impact curve in logarithm scale by:
 - 6.1 plot the scatter graph of volume VS dMidPrice
 - 6.2 group the data in such a way that the bins in the x-axis (Volume) are equally spaced.
 - 6.3 group the data in such a way that each bin contains even number of data

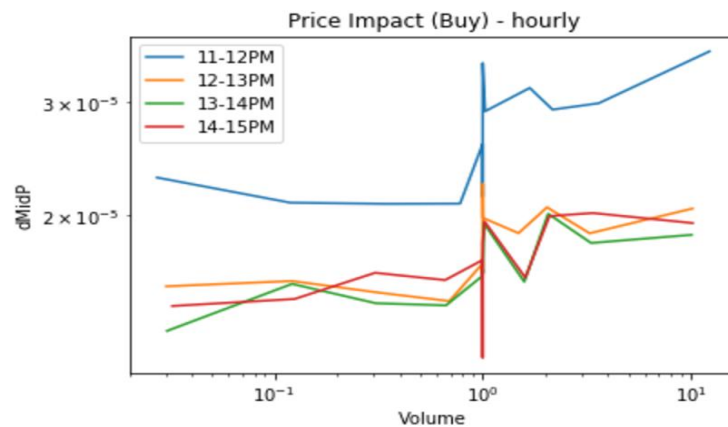


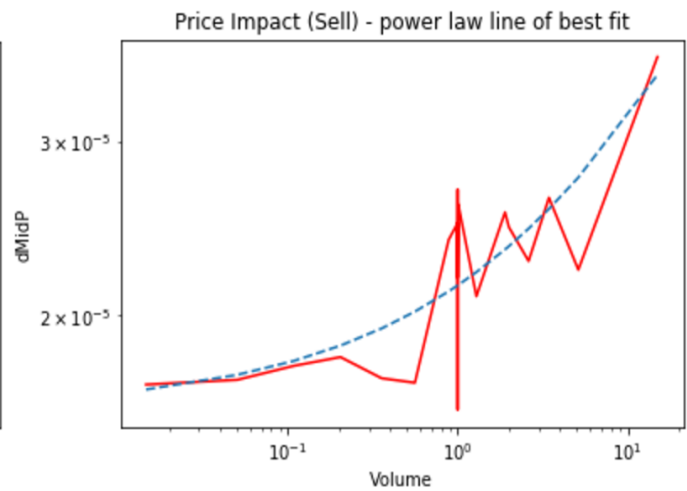
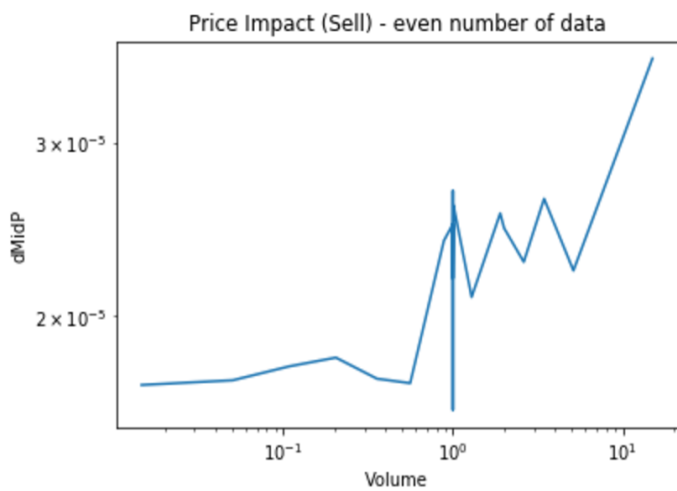
7. Model the power law fitting function: number of bins to group the data in section 6.3 is determined by applying the KS test to identify the correlation between the raw data plot and the power law fitting function



Equation: $y = 7.54 \cdot 10^{-6} x^{0.264} + 1.42 \cdot 10^{-5}$

8. Apply the same method to model the time dependence of the price impact curve (i.e. plot the hourly curves)

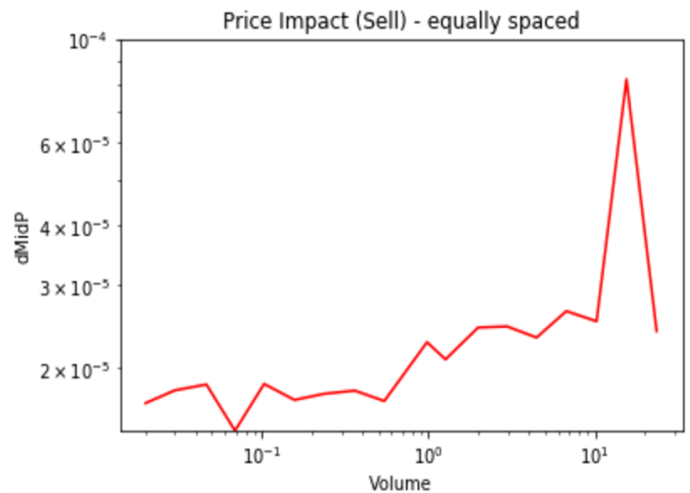




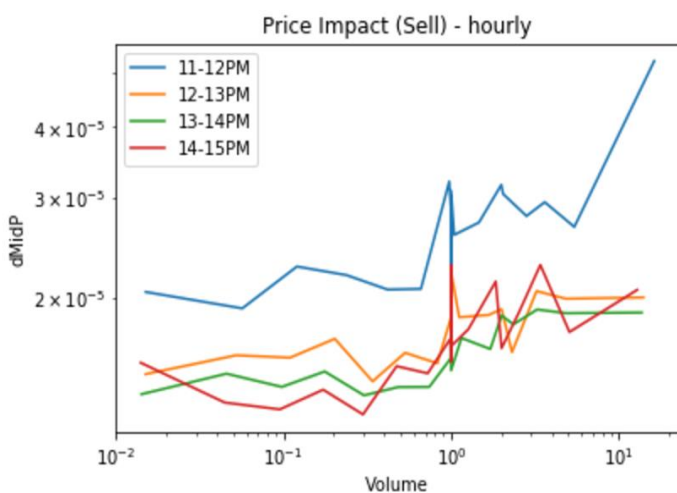
Then apply the same method to model the price impact of the volume sold

Observations:

The graphs indicate that there is a positive correlation between the trade volume and the change in price for both buy and sell orders.



However, it is difficult to determine the exact impact function due to insufficient data. It would be possible to reach a more accurate conclusion if 1-Year trade data are provided instead of 1-Month.



trajectories

Furthermore, there are other factors, such as volatility and liquidity, which also have an impact on the change in price.

2. Capture and cost of trading

The second half of this paper will be focused on the implementation of the observed price impact function model. According to the research on optimal execution of portfolio transactions by Robert Almgren and Neil Chriss [1], the aim of the optimal execution is to minimize the combination of volatility (standard deviation) and transaction costs (expected value) arising from permanent and temporary market impact.

$$\min_x (E(x) + \lambda V(x)).$$

Where E = expected transaction costs

$$E(x) = \frac{1}{2}\gamma X^2 + \epsilon \sum_{k=1}^N |n_k| + \frac{\tilde{\eta}}{\tau} \sum_{k=1}^N n_k^2$$

V = Variance

$$V(x) = \sigma^2 \sum_{k=1}^N \tau x_k^2.$$

When a trader executes a series of trades, the second trade is always more expensive than the first trade as the market impact induces extra costs. Therefore, it is important to determine the rate of trading by finding a transaction path that maximizes the risk-reward.

We define a trading strategy to be optimal if it has the lowest variance for the same level of expected transaction costs, or, equivalently, the lowest expected transaction costs for the same level of variance.

Optimal execution under the assumption that price dynamics follow an arithmetic random walk with zero drift

Optimize function:

When optimizing the Utility function $E(\mathbf{x}) + \lambda V(\mathbf{x})$:

the efficient trading trajectory according to Almgren/Chriss is:

$$x_j = \frac{\sinh(\kappa(T - t_j))}{\sinh(\kappa T)} X \quad j = 0, \dots, N,$$

where x_j is the number of units that we plan to hold at time t_j , and

$$\tilde{\kappa}^2 = \frac{\lambda \sigma^2}{\tilde{\eta}} = \frac{\lambda \sigma^2}{\eta \left(1 - \frac{\gamma \tau}{2\eta}\right)}$$

Parameters:

Initial stock price: S - from the quote data

Initial holdings: $X = 5 \cdot 10^4$

Liquidation time: T - 60 minutes (1 hour)

Unit time: τ - every 5 minutes

Number of time periods: $N = 60/5=12$

Bid-Ask Spread: ϵ - Average(Ask price – Bid price)

Volatility: standard deviation on the return $((\text{previous price} / \text{new price}) - 1)$

Unit volatility: $\sigma = \text{Volatility} * (\text{number of seconds})^{0.5}$

Impact at 1% of market: η : Theoretical = $\epsilon / (0.01 * \text{average trading volume in 5 mins})$

Daily volume shares: $\gamma = \eta * 10$

Risk parameter: λ Risk value chosen by the trader using the efficient frontier [2]

[2] Efficient frontier: help to identify the minimal level of cost for its level of variance

Value of the parameters:

Initial stock price: S

Initial holdings: $X = 5 \cdot 10^4$

Liquidation time: $T = 60$ minutes (1 hour)

Unit time: $\tau = 5$ minutes

Number of time periods: $N = 12$

Bid-Ask Spread: $\epsilon = 0.011$

Volatility: $= 0.013\%$

Unit volatility: $\sigma = 0.22\%$

Impact at 1% of market: η (Theoretical) $= 4.85 \cdot 10^{-5}$

Daily volume shares: γ (Theoretical) $= 4.85 \cdot 10^{-6}$

Risk parameter: $\lambda = 10^{-6}$

Implementation shortfall is the total cost of trading, which is defined as the difference between the initial book value and the capture:

$$XS_0 - \sum n_k \tilde{S}_k$$

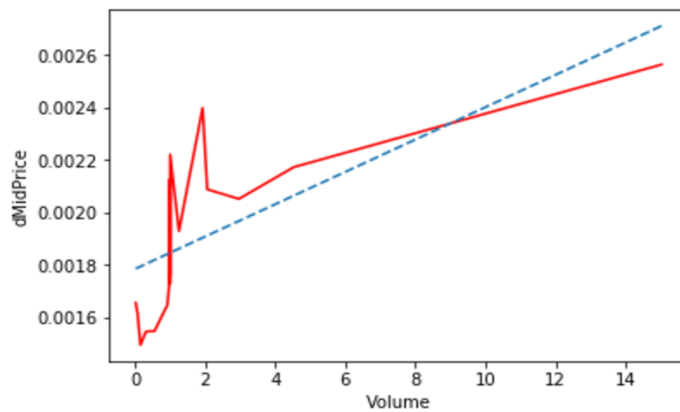
Computing implementation shortfall with Theoretical η and γ :

Assume the participation rate is 50%

- 1) Group market depth data into 5 minute sections
- 2) Compute volume of stock bought in each 5 minute period
- 3) Buy a holding at the cheapest price first (level 1 Ask Price), selecting from the available volumes (Ask size)
- 4) Compute the cost by multiplying the volume bought at a price and the ask price
- 5) Model Implementation shortfall using the above formula
- 6) Repeat step 3 to 5 over the next time range (e.g. start 5 minutes later and end 5 minutes later)

Computing implementation shortfall with η and γ derived from the linear price-impact curve:

η = the gradient of the linear price impact curve of volume traded VS change in mid-price

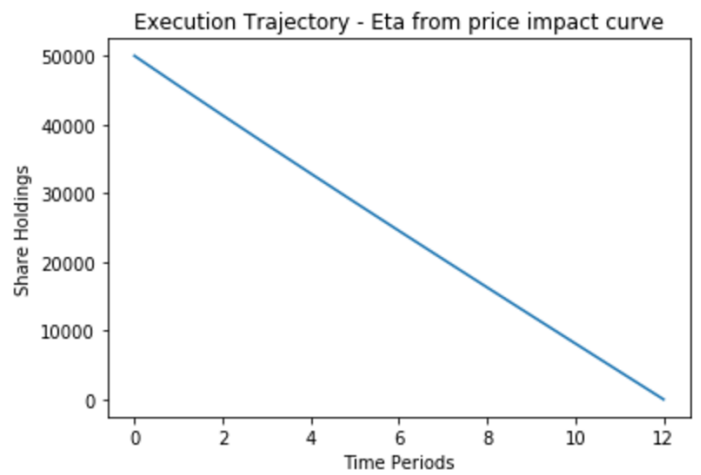
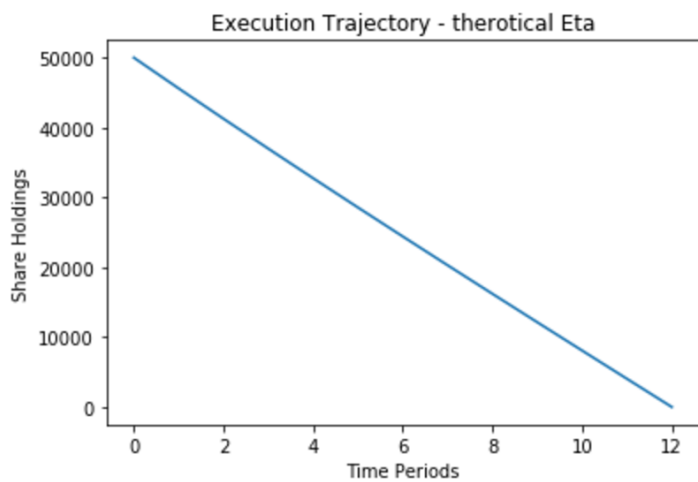


$$\eta = \text{Gradient} = 6.6 \times 10^{-5}$$

$$\text{Y interception} = 1.78 \times 10^{-3}$$

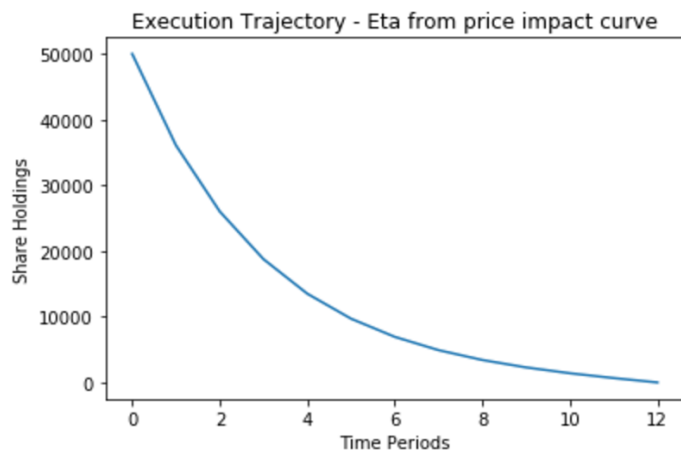
$$\gamma = \eta \times 10 = 6.6 \times 10^{-4}$$

Then use the same method as described above to compute implementation shortfall



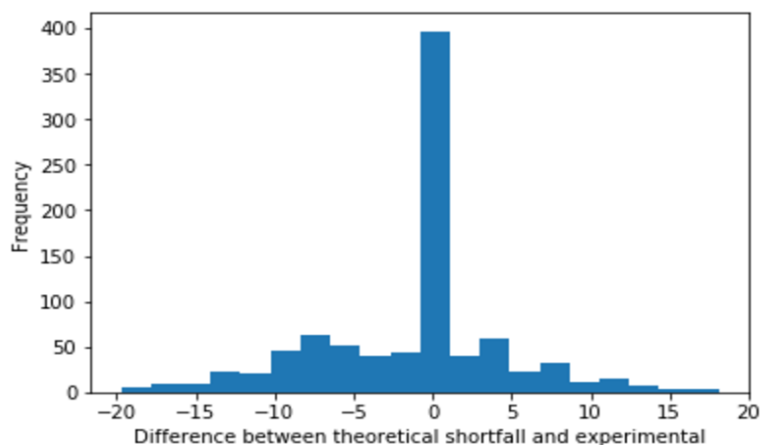
Comparing Trajectory:

The two trajectories are both close to linear – equal volume of shares execute in each time period

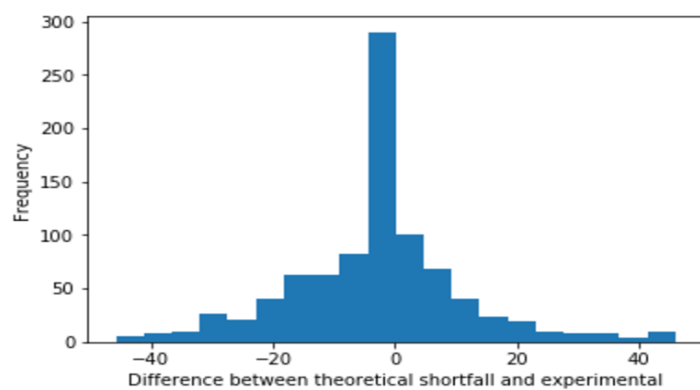


For non-linear trajectories that take variance into account so that the initial execution volume \gg final execution volume, we need to either increase volatility (σ) or decrease η

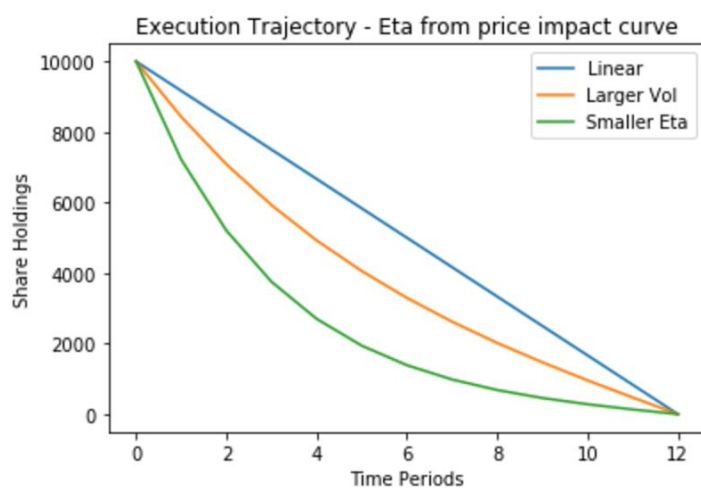
The difference in implementation shortfalls computed using the Eta values described above (Theoretical VS Gradient of the price impact curve) is shown as a histogram below:

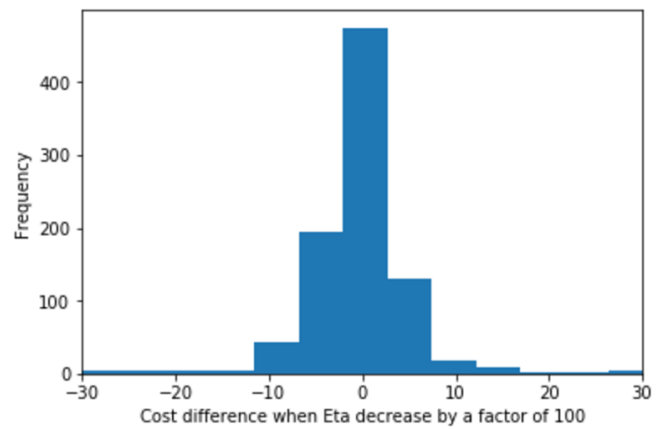
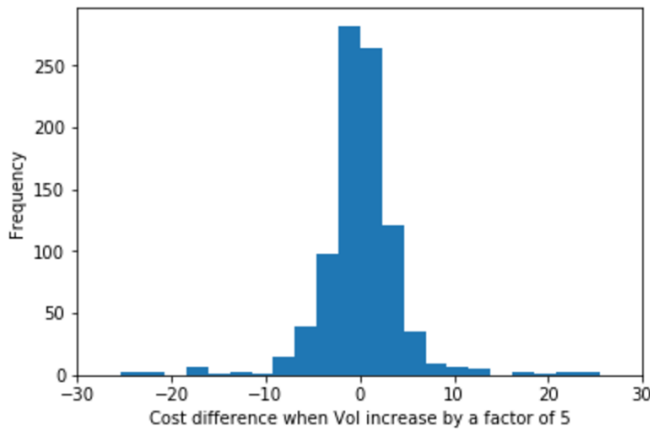


Then apply the same method to the volume sold, using bid data instead of ask data



At the end of the paper, we investigate different execution trajectories by increasing the volatility (by a factor of 5) or decreasing Eta (by a factor of 100).





Conclusion:

In this paper, we firstly model the market impact and the dynamic of this price impact, then we study its impact on investment decisions by examining the execution trajectories.

However, it is hard to draw a conclusion based on the implementation shortfall values computed using different methods. There is an equal likelihood for the implementation shortfalls modelled using price impact curve to be greater or smaller than the theoretical value.

One of the reasons is due to the small impact of volatility and large value of Eta. Furthermore, it is hard to determine the gradient of the price impact curve (i.e. Eta) due to the limited range of data and uncertainties associated with the plot.