

L-11
01.12.19

Prime Number (***)

→ only one even prime number exist

* There are infinitely many primes.

Proof: Let, There are numbers of prime numbers.

$$P_1, P_2, P_3, \dots, P_n$$

$$P = P_1 \times P_2 \times P_3 \times \dots \times \underline{P_n + 1}$$

$$\cancel{= 2 \times 3 \times 5 \times 7 + 1} = 2^{11}$$

$$2^{11} \nmid 2 = 1$$

$$2^{11} \nmid 3 = 1$$

$$2^{11} \nmid 5 = 1$$

$$\underline{\underline{2^{11} \nmid 7 = 1}}$$

Let;

$$P \nmid P_m = 1 [P \leq P_m \leq P_n]$$

∴ P is a prime number.

So, P is a prime number.
That's why there is infinitely prime Number.

L-U

GCD \rightarrow Greatest Common Divisor
LCM \rightarrow Lowest Common Multiple

$$24 = \cancel{1}, \cancel{2}, \cancel{3}, 4, 6, 8, \cancel{12}, \underline{\underline{24}}$$

$$18 = \cancel{1}, \cancel{2}, \cancel{3}, \cancel{6}, \cancel{9}, \cancel{18}$$

$$\text{GCD}(24, 18) = \cancel{1}, \cancel{2}, \cancel{3}, \underline{\underline{6}}$$

$$\text{GCD}(24, 18) = 6$$

$$\text{CMC}(24, 18) = \cancel{144}, 288, 432.$$

$$\text{LCM}(24, 18) = \cancel{144},$$

$$\text{LCM}(a, b) = \frac{a \times b}{\text{GCD}(a, b)}.$$

$$\text{LCM}(24, 18) = \frac{24 \times 18}{6}$$

Prime Factorization $= 72$

$$\begin{array}{l} 24 = 2^3 \times 3 \\ 18 = 2^1 \times 3^2 \end{array} \quad \begin{array}{l} \frac{24}{2} = \frac{1^2}{2} = \frac{6}{2} = \frac{3}{3} = 1 \\ \frac{18}{2} = \frac{9}{3} = \frac{3}{3} = 1 \end{array}$$

L-11

Prime Factorization

$$\text{GCD}(24, 18) = 2^1 \times 3^1 \rightarrow \text{minimum power should be taken} \\ = 6$$

$$\text{LCM}(24, 18) = 2^3 \times 3^2 \rightarrow \text{maximum} \\ = 72$$

* 65, 39, 130

$$65 = 5^1 \times 13^1$$

$$39 = 3^1 \times 13^1$$

$$130 = 2^1 \times 5^1 \times 13^1$$

$$\frac{65}{13} = \frac{5}{1} = 1$$

$$\frac{39}{13} = \frac{3}{1} = 1$$

$$\frac{130}{13} = \frac{10}{5} = \frac{2}{1} = 1$$

$$\text{GCD}(65, 39, 130) = 2^0 \times 3^0 \times 5^0 \times 13^1 = 13$$

$$\text{LCM}(65, 39, 130) = 2^1 \times 5^1 \times 3^1 \times 13^1 = 390$$

$$\text{LCM}(65, 13, 130) = 2^0 \times 3^0 \times 5^0 \times 13^1 = 130$$

L-11

Euclid - GCD(a, b)

if ($b > a$) return Euclid - GCD(b, a)
if ($b == 0$) return a
else return Euclid GCD($b, a \% b$)

* GCD(18, 24)

Step-1 Step-2 Step-3

$$\begin{array}{l} a = 18 \\ b = 24 \end{array}$$

$$\begin{array}{l} a = 24 \\ b = 18 \end{array}$$

$$\begin{array}{l} a = 18 \\ b = \frac{24}{18} = 6 \end{array}$$

$$\begin{array}{l} a = 18 \\ b = 24 \% 18 = 6 \end{array}$$

Step-4

$$a = 6$$

$$b = \cancel{18} \frac{18 - 1 \cdot 6}{\cancel{6}} = 0$$

* GCD(733, 855)

Step-1

$$a = 733$$

$$b = 855$$

Step-2

$$a = 855$$

$$b = 733$$

Step-3

$$a = 733$$

$$b = \frac{855}{73} 855 \% 733$$

$$= 122$$

Step-4

$$a = 122$$

$$b = 733 \% 122 = 1$$

Step-5

$$a = 1$$

$$b = 122 \% 1 = 0$$

Step-6

$$\text{GCD} = 1$$

L-11

Relatively Prime

$$\text{gcd}(a, b) = 1$$

L-13

08.12.19

Heap

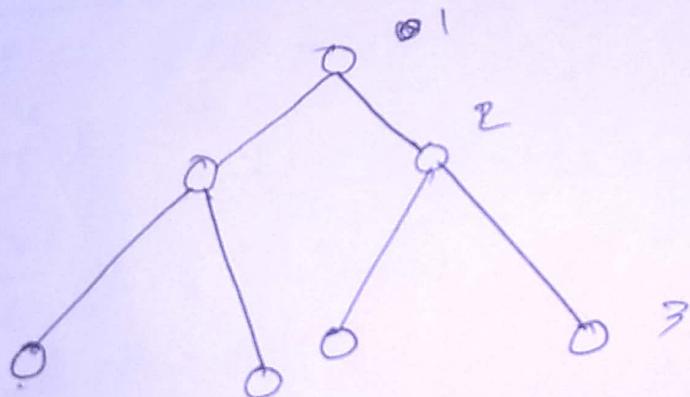
1	2	3	-3	6	8	-6
---	---	---	----	---	---	----

$$\frac{O(n)}{n \rightarrow \text{node}} \\ n \rightarrow \log_2 n$$

① Insert $\rightarrow O(\log_2 n)$

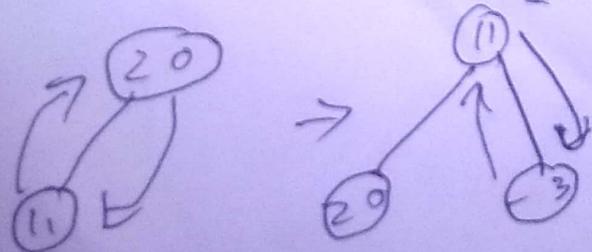
② Find $\rightarrow O(1)$

③ Delete $\rightarrow O(\log_2 n)$



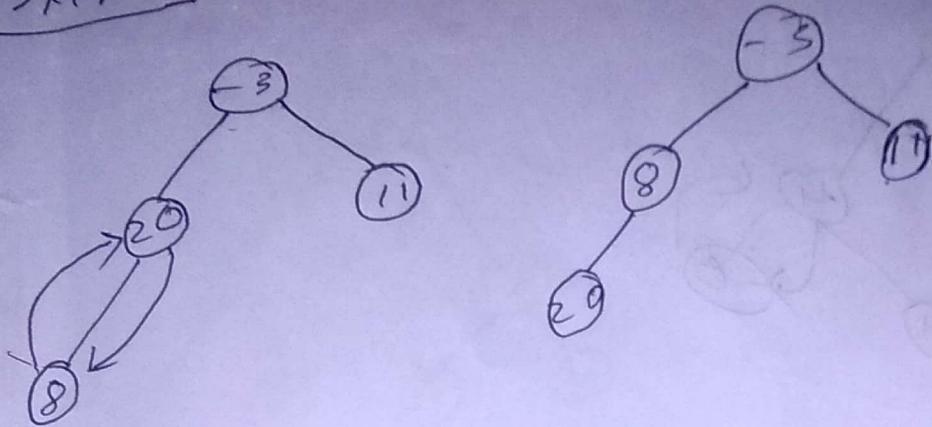
20	11	-3	8	7	4	=6	
----	----	----	---	---	---	---------------	--

Step - 1 $\xrightarrow{g + e^{P-2}}$

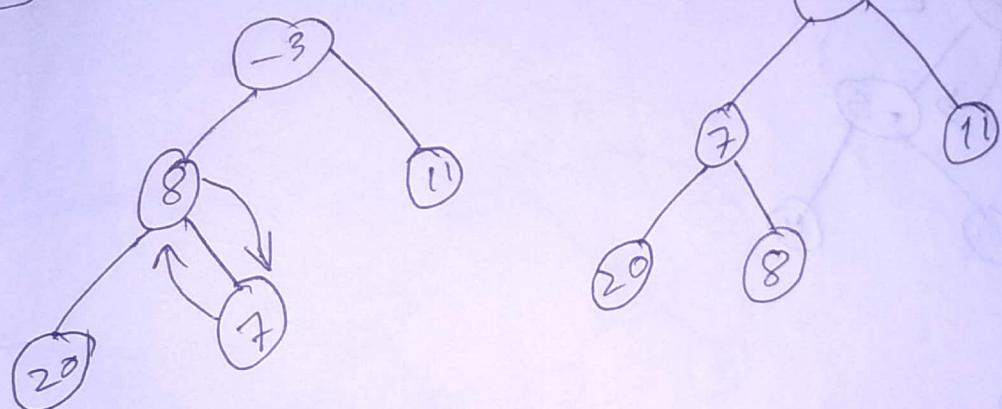


Step-3

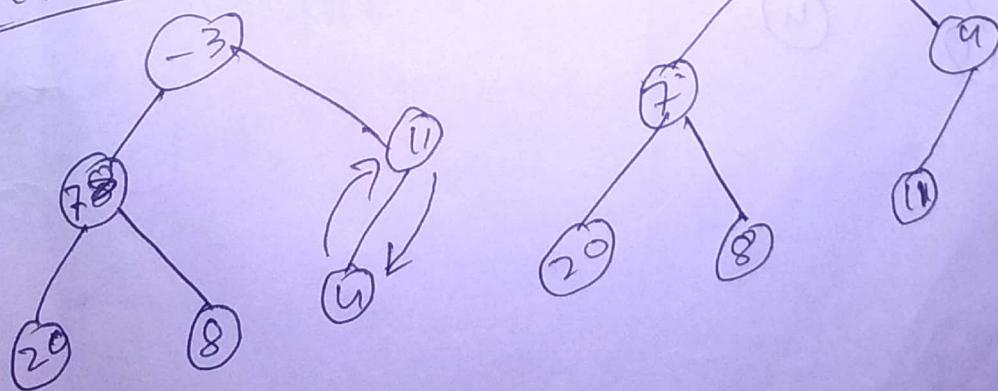
L-12



Step-4

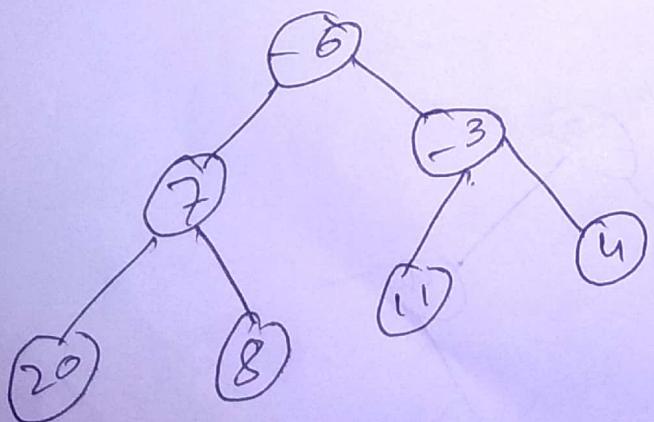
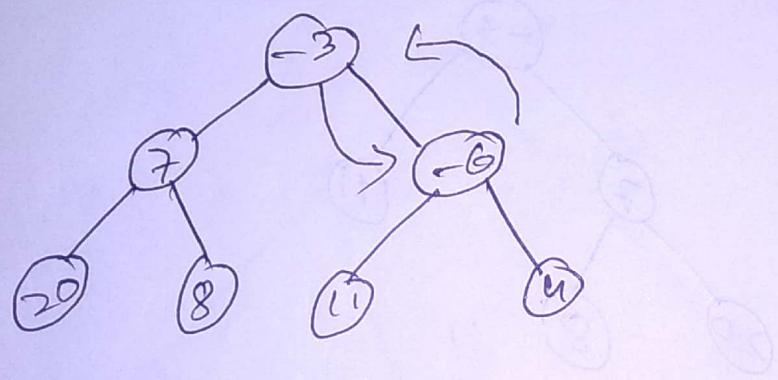
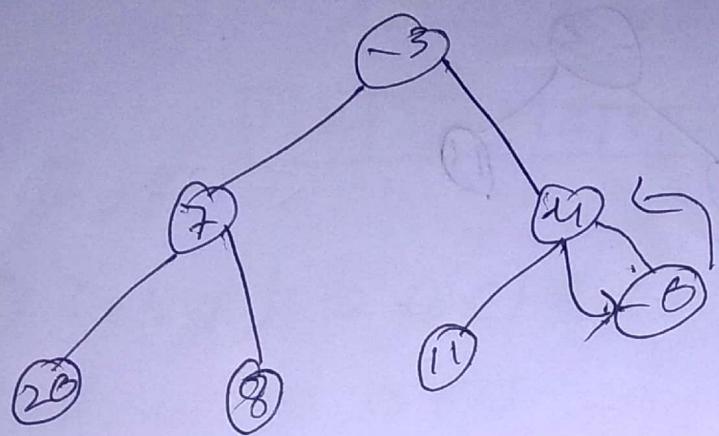


Step-5



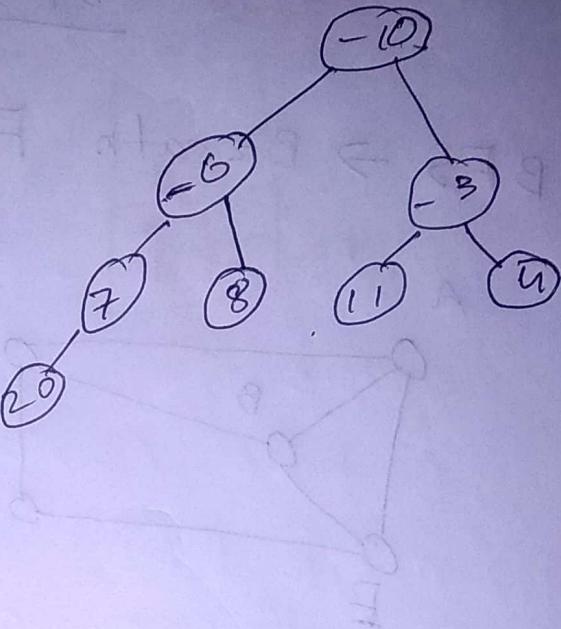
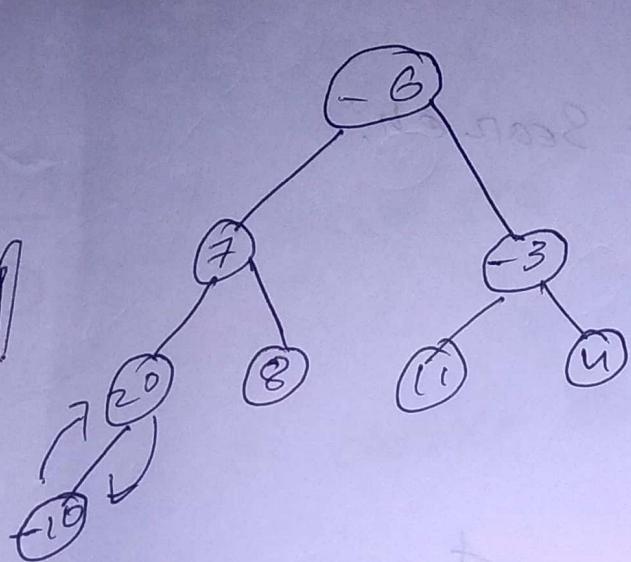
Step - 6

L - 12

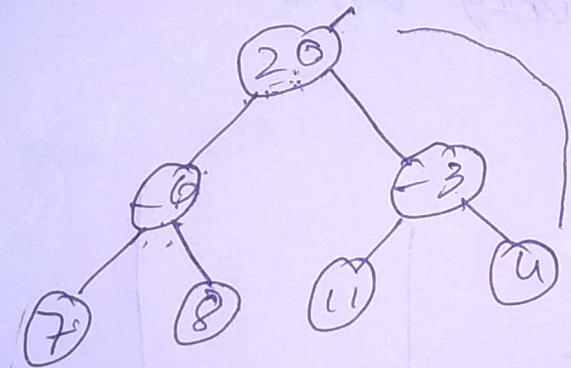


20	11	-3	8	7	9	-6	-10
----	----	----	---	---	---	----	-----

L - 12



Delete:



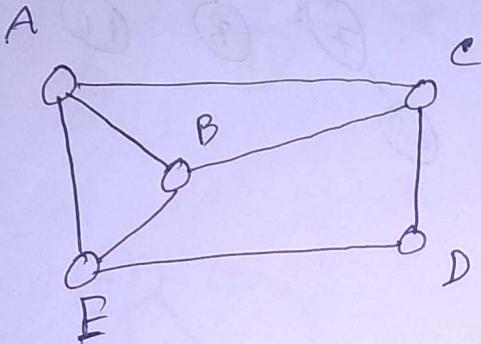
Why is the time complexity of Heap is $O(n \log_2 n)$?

[10-12-19]

[L-13]

BFS

① BFS → Breath First Search;



Rule: ① cost of every $\xrightarrow{\text{root}}$ nodes is same.

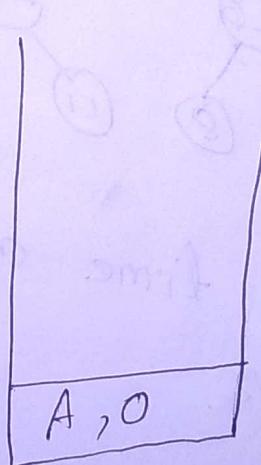
② FIFO

Tree:

① Push zero color
change zero zero

② Oray

③ Black



Step

'13'

Step

Step-5

L - 13

Step-1



Step-2

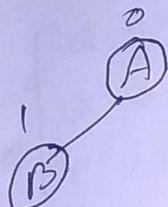
(i) Black zero 2²

E, 1
C, 1
B, 1

→ POP

A 2²

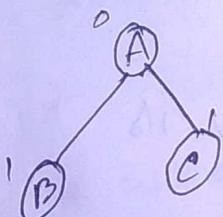
E, C, B change 2²



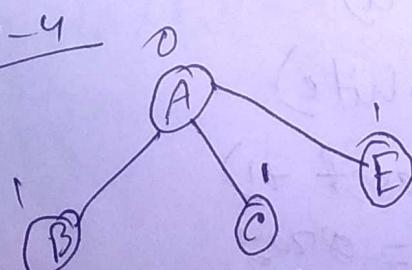
Step-3 D gray 2²

D - 2
E - 1
C, 1

→ POP



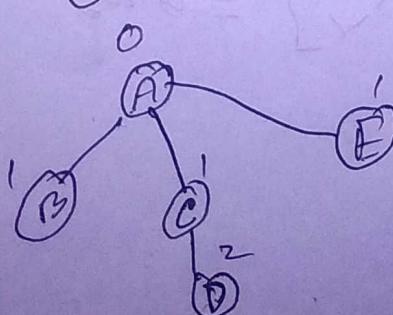
Step-4



D - 2
E - 1

→ POP

Step-5



D - 2
→ POP

BFS (s)

$\boxed{L-13}$
 $O(V+E)$

{

color [u] = white

distance [u] = 0

queue Q

Q . push ($s, 0$)

color [s] = gray

while (Q = empty)

{

$u = Q$. front;

Q . pop

distance [u] = u . cost

color [u] = black.

for all $v \in$ graph & if v is
connected with u)

if (color [v] == white)

Q . push (v, u . cost + 1)

color [v] = gray

}

Huffman coding:

$O(n \log_2 n)$

L-13

BTXE

B

BT

BTX

BTXE

E
EX
EXT
EXTB

linked List

* Set / Heap

* STL

~~prefix~~

a b c
a x
e d f
g h

abc
a
bx
b

00 11 } prefix
10 0011 } code
000

* what is prefix 2.

* what is prefix code 2.

* what is the time complexity of

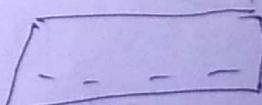
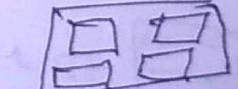
Huffman coding 2.

$$a = 0011$$

$$b = 0$$

$$c = 1$$

$$d = 011$$



$$A = 12$$

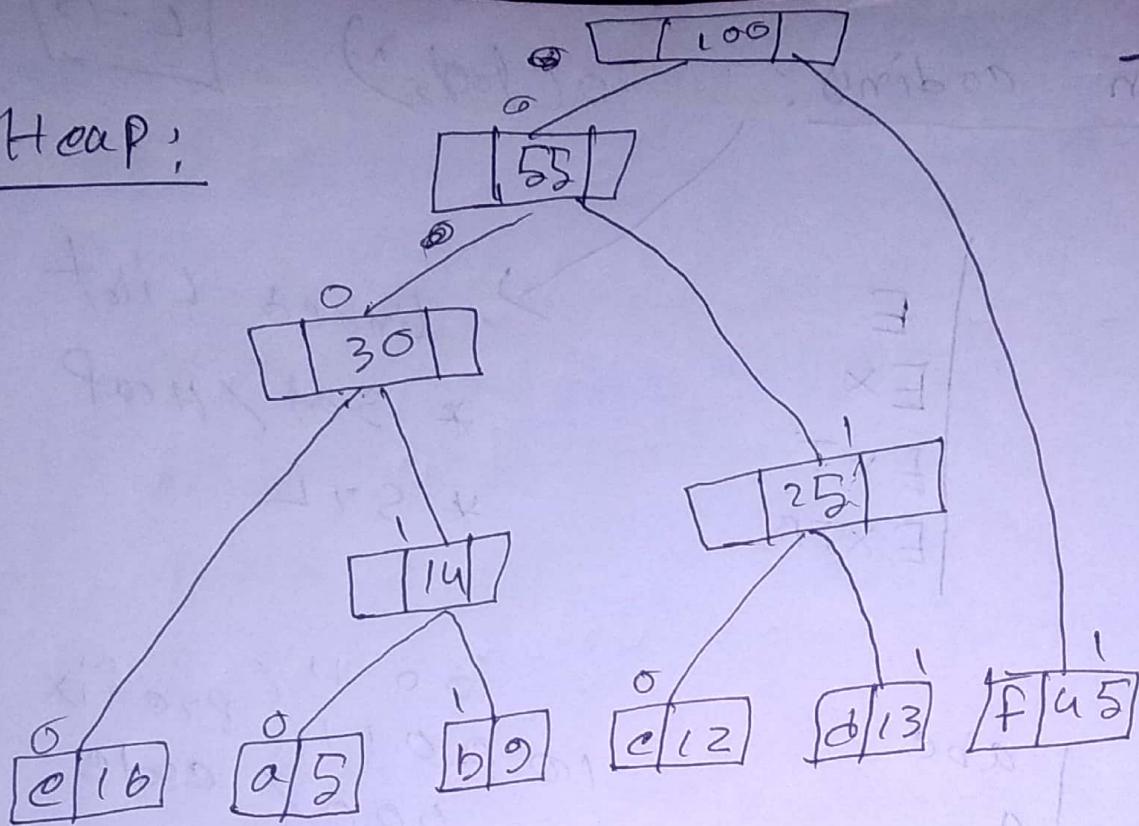
$$a = 13$$

$$; = 10$$

$$= 13$$

$$3 = 3$$

$$C = 8$$

Heap:

$$e = 000.$$

$$a = 0010$$

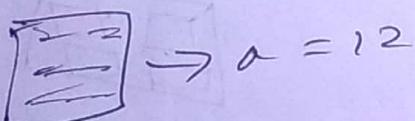
$$b = 0011$$

$$e = 010$$

$$d = 011$$

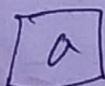
$$f = 1$$

$$acd = \underline{0010} \quad \underline{010} \quad \underline{011}$$



Huffman coding fig, Binary bet C&Y

28 27 20 22 21



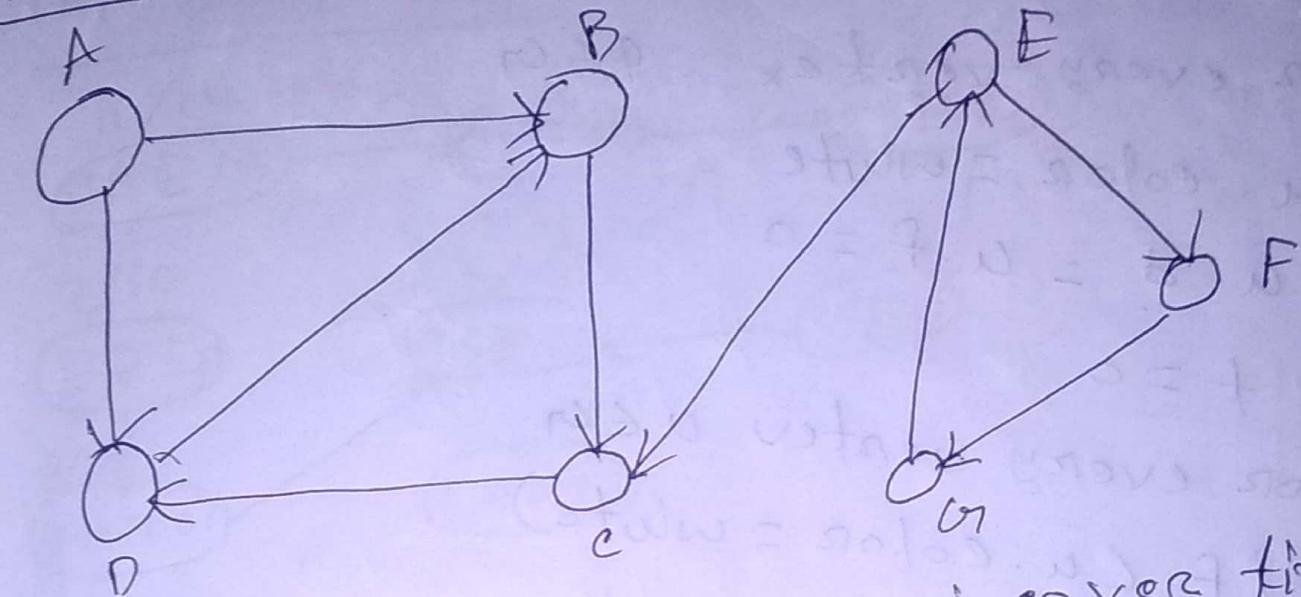
zip

b

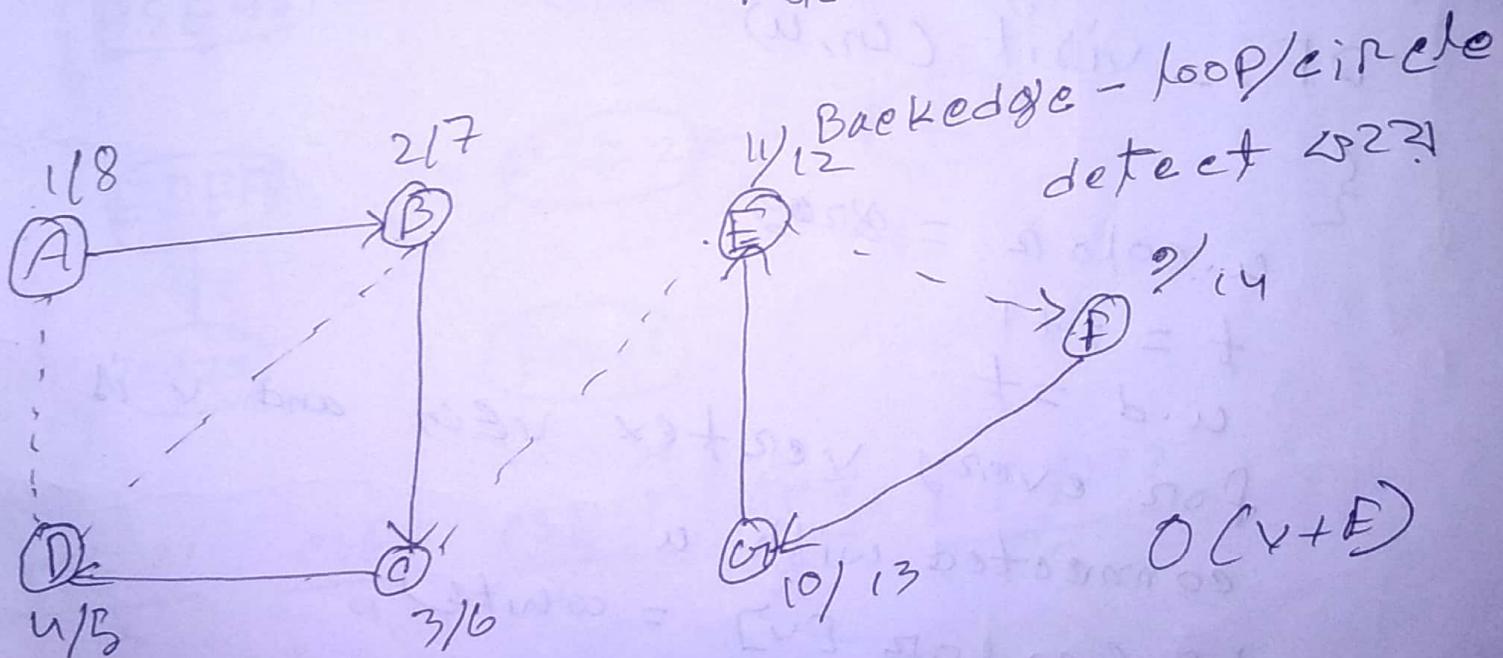
15.12.17

L - 14

DFS : Depth first Search



Start discover time End discover time =
forward edge.



AlgorithmDFS (u)for every vertex $q \in G$ $q.\text{color} = \text{white}$ $q.d = q.f = 0$ $t = 0$ for every vertex $u \in G$ if ($u.\text{color} = \text{white}$) DFS-visit (u) DFS-visit (G, u)

{

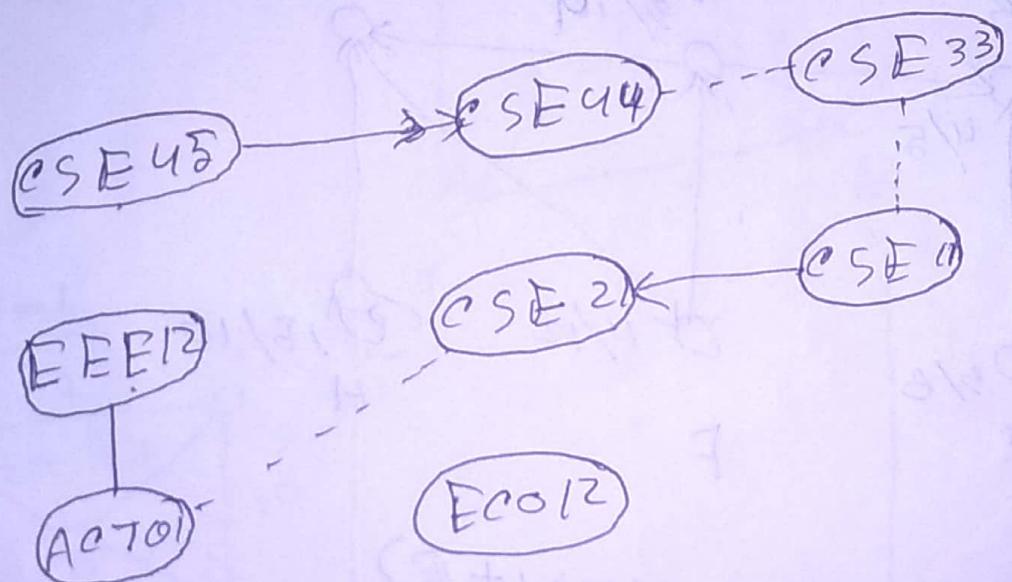
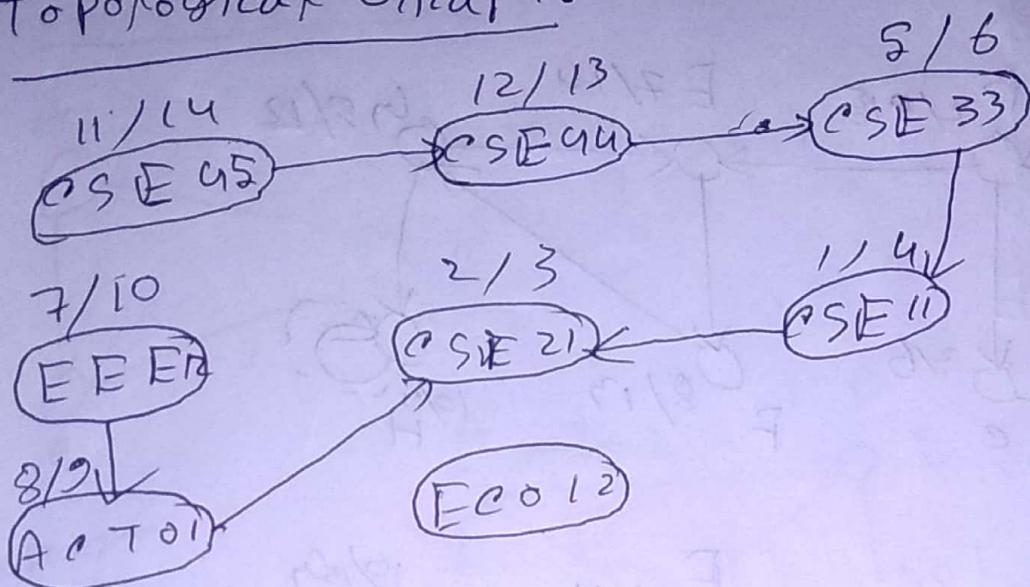
 $u.\text{color} = \text{gray}$ $t = t + 1$ $u.d = t$ for every vertex $v \in G$ and v is
 connected with u . if ($\text{color}[v] = \text{white}$) DFS-visit (G, v) $t = t + 1$ $u.f = t$ $u.\text{color} = \text{black}$

S

22.12.19

L-15

Topological Graph:



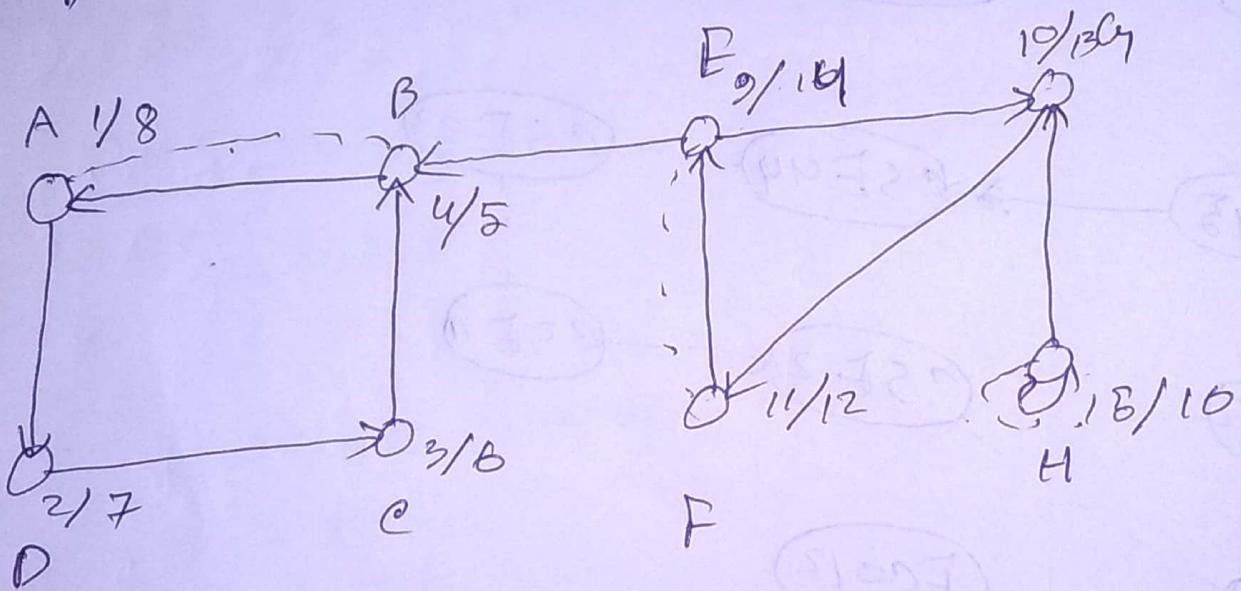
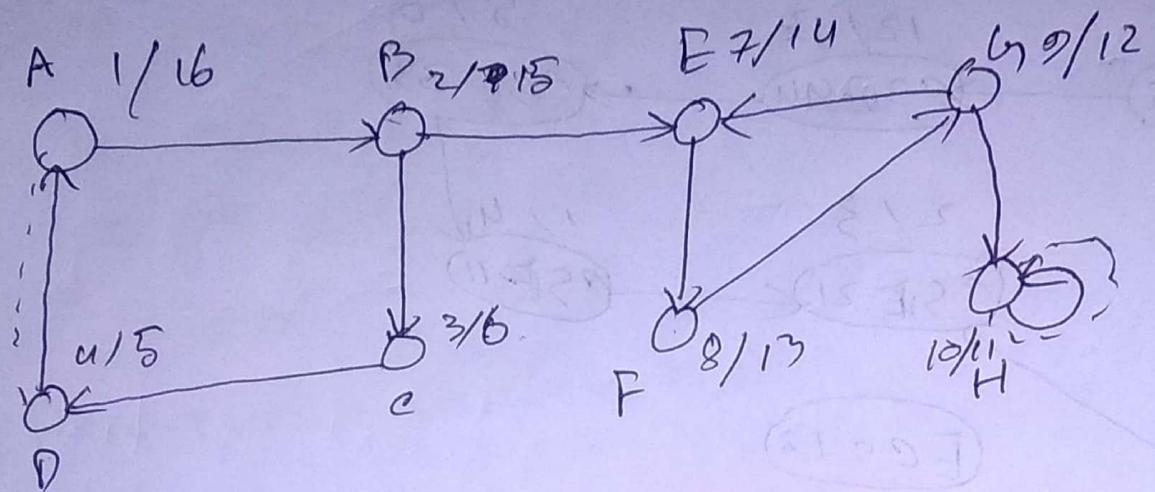
$$\begin{array}{c}
 \text{CSE } 21 \quad \text{CSE } 11 \quad \text{CSE } 33 \quad \text{ACT } 10 \quad \text{EEE } 12 \\
 \hline
 3 \qquad \qquad 4 \qquad \qquad 6 \qquad \qquad 9 \qquad \qquad 10
 \end{array}$$

$$\begin{array}{c}
 \text{CSE } 44 \quad \text{CSE } 45 \quad \text{ECO } 12 \\
 \hline
 13 \qquad \qquad 14 \qquad \qquad 16
 \end{array}$$

If backedge found in DFS, Answer is not valid.

L-15

Strongly Connected Component:

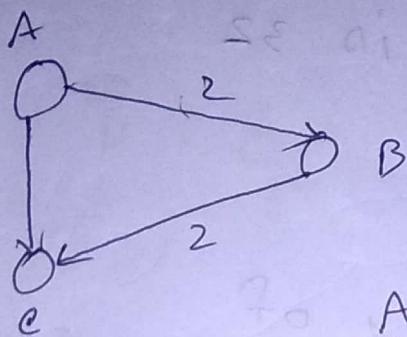


$$2 \times (V + E) \\ = V + E$$

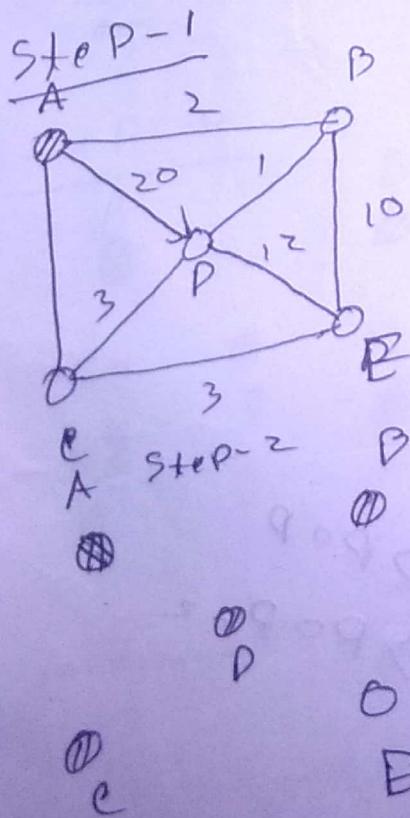
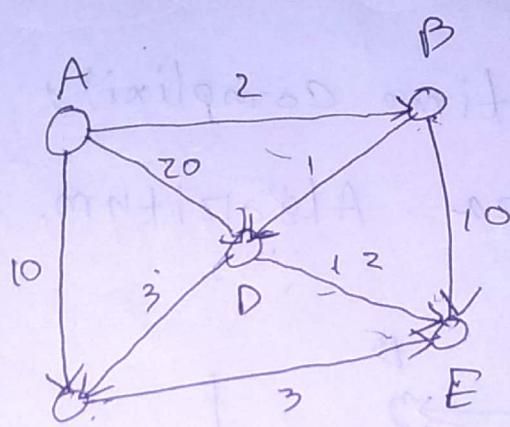
L-16

24.12.19

Dijkstra Algorithm



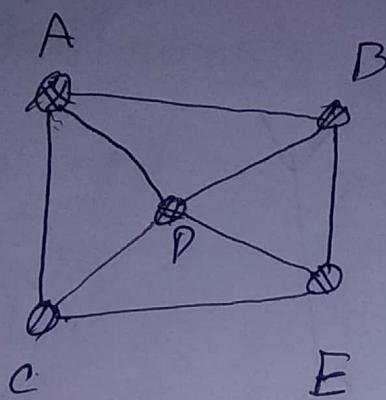
~~Step 1~~



D - B, 12	→ P ₀
D - C, 3	
A - C, 10	
A - B, 2	→ P ₂
A - D, 20	→ P ₁
A, ∞	

Step-3

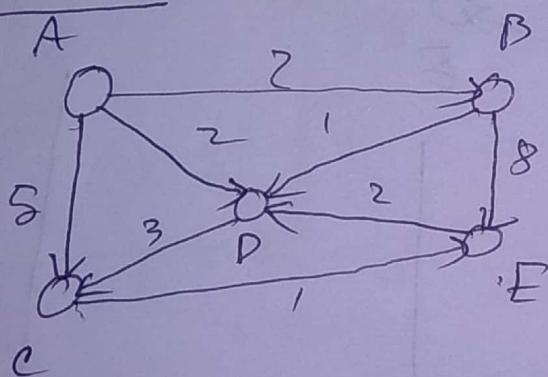
L-16



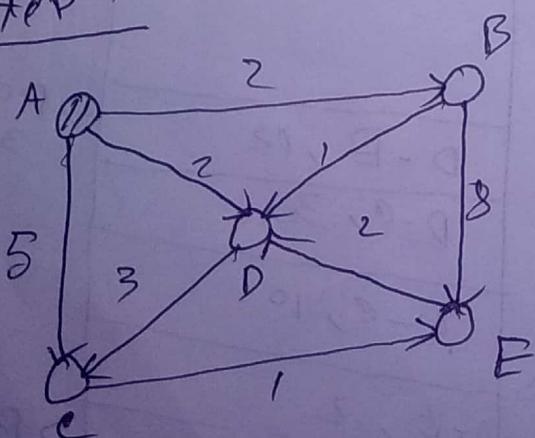
Maximum is 32

What is the time complexity of
~~Dijkstra~~ Dijkstra Algorithm.

Practice:



Step-1



E-B, 8
C-E, 1
D-E, 3
A-D, 2
A-C, 5
A-B, 2
A, 0

→ POP

→ POP - 2

POP - 1

Step - 2

A
⊗

B
⊗

D
⊗

E
⊗

C
⊗

Step - 3

A
⊗

B

C
⊗

D
⊗

E
⊗

Step - 4

A
⊗

B

C
⊗

D
⊗

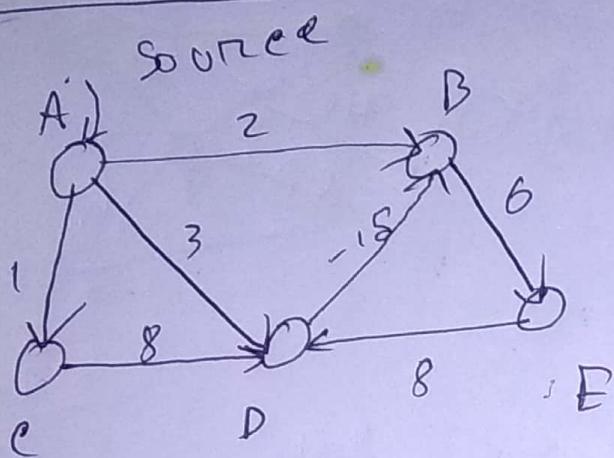
E
⊗

minimum is 6

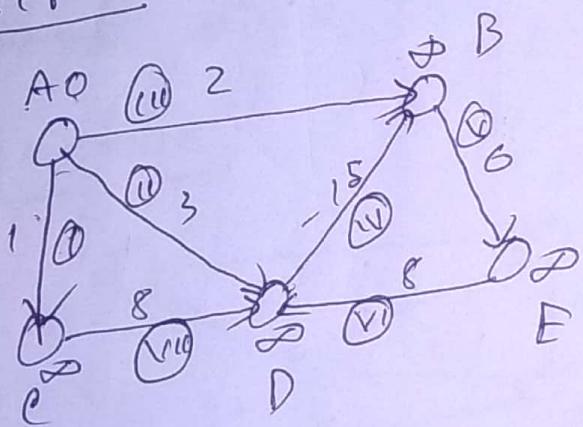
L-17

Bellman - Ford

$O(V^2)$



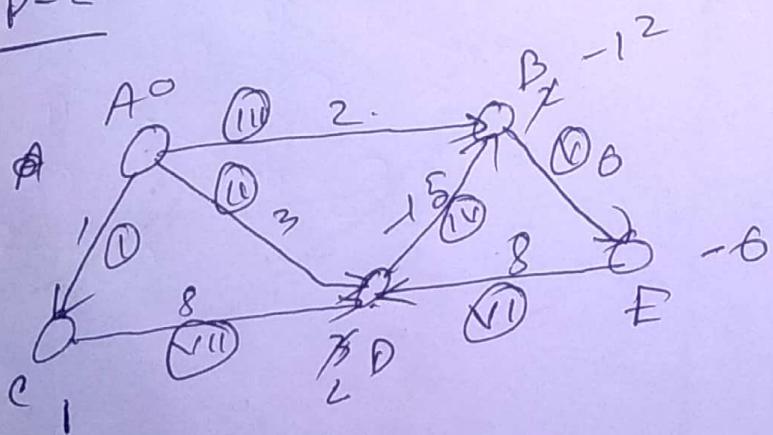
Step-1



Relax

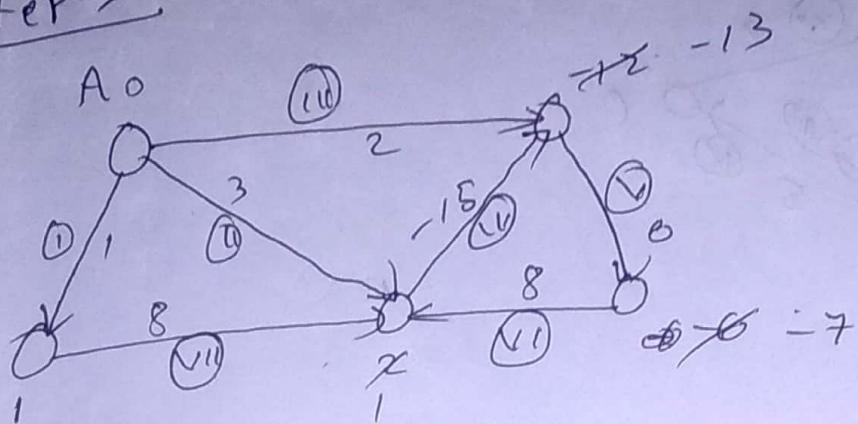
$$\begin{aligned} \text{dist}[u] + \text{cost}[u][v] &< \text{dist}[v] \\ \text{dist}[v] &< \text{dist}[u] + \text{cost}[u][v] \end{aligned}$$

Step-2

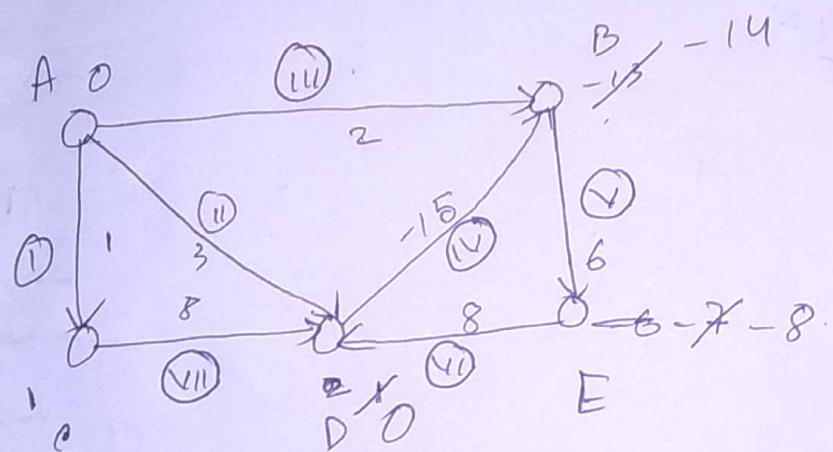


L-17

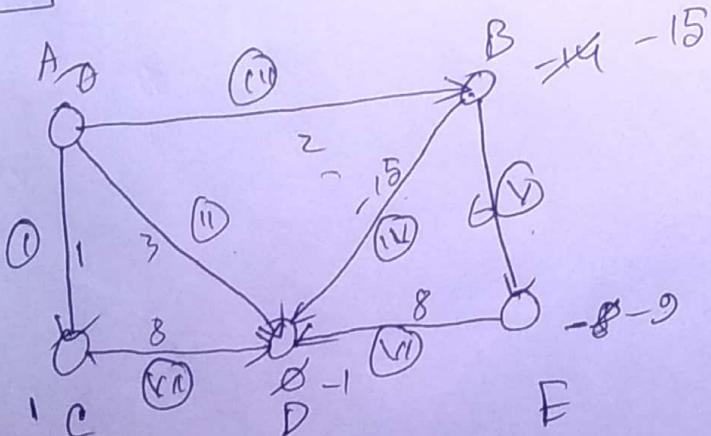
Step-3



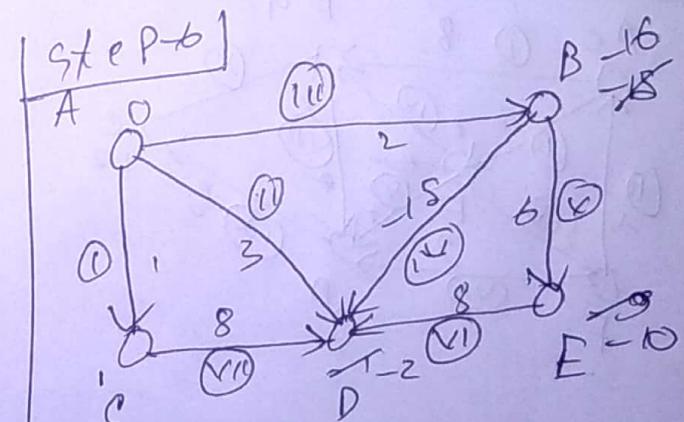
Step - 4



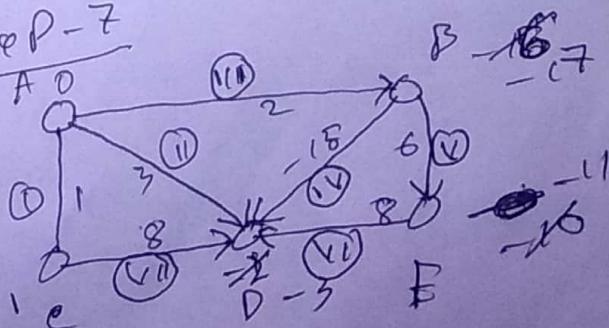
Step-5]



There is an infinity loop.

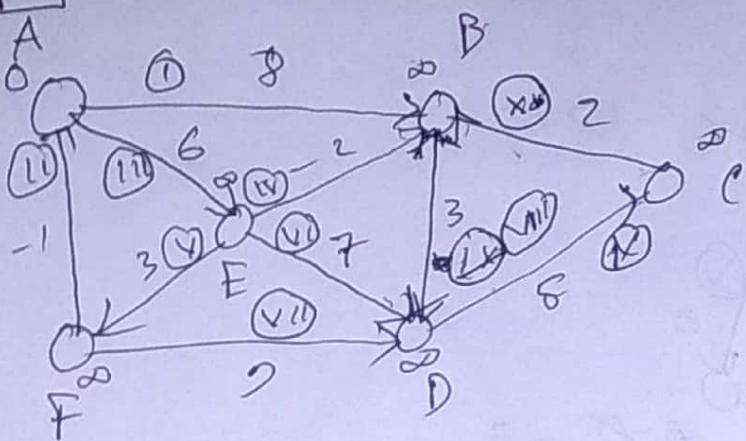


Step P-7

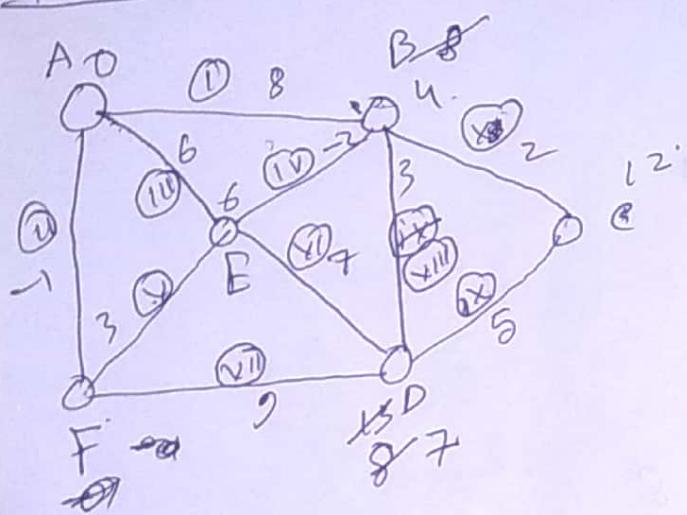


L-17

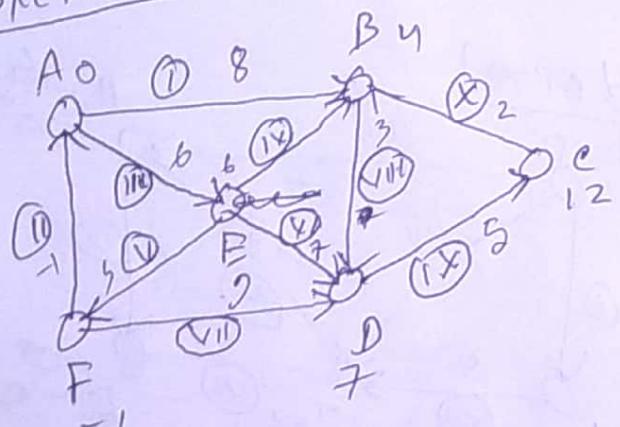
Step-1



Step-2



Step-3



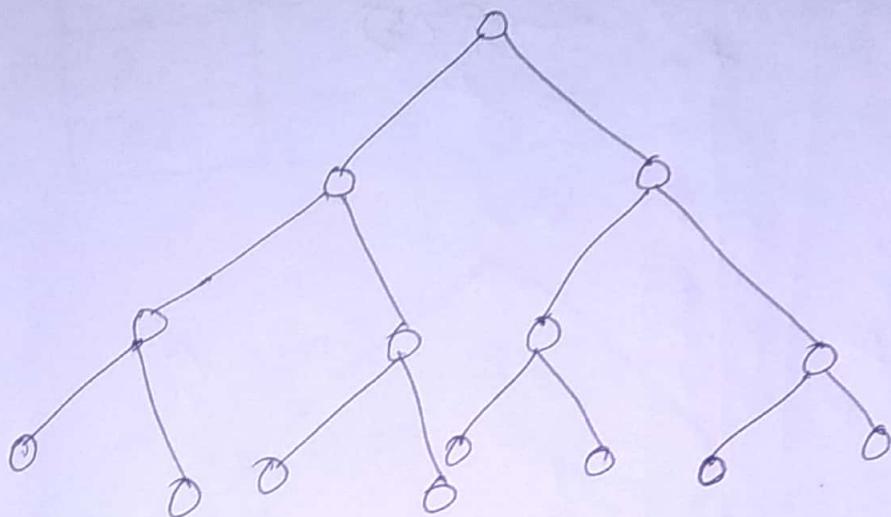
L-18

05.01.2020

Dynamic Programming:

$$(1+2) + (5+6) + (2+1)$$

① overlapping subproblem



② optimal Substructure:

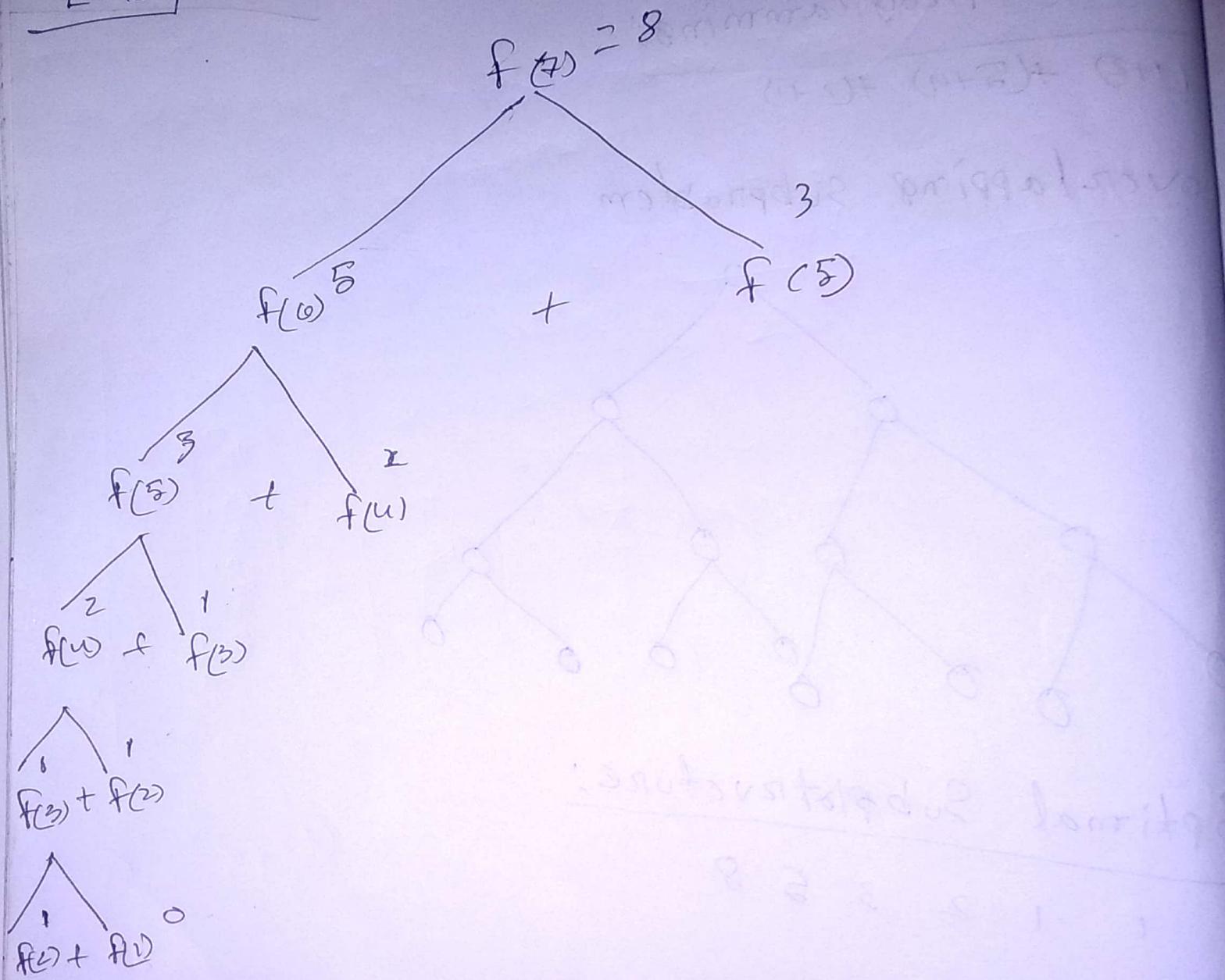
$$0 \ 1 \ 1 \ 2 \ 3 \ 5 \ 8$$

$$f(1) = 0$$

$$f(2) = 1$$

$$f(n) = f(n-1) + f(n-2) \quad n \geq 3$$

$L = 10$

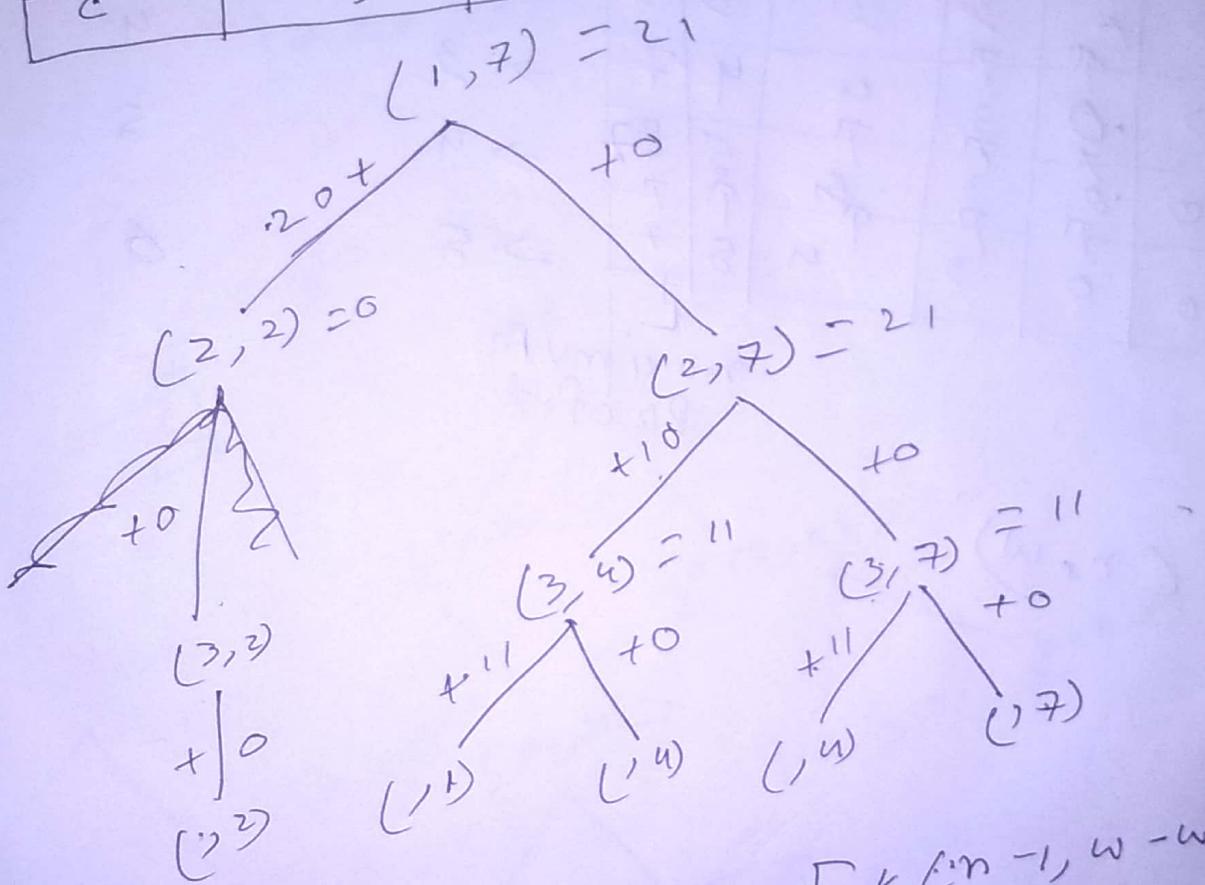


L-18

0/1 Knapsack

Item	weight	cost
A	5	20
B	3	10
C	3	11

max weight = 7
 $\sum A_3$, $\{B_3\}$, $\{C_3\}$ $\sum A, B_3$ $\sum B_3, C_3$
 $\sum B_3, C_3 \sum A, B, C_3 \sum D_3$
 $\sum C_3$, $\{D_3\}$ $\sum D_3$
 $\sum A_1$, $\{B_1\}$, $\{C_1\}$ $\sum A_1, B_1$
 $\sum B_1$, $\{C_1\}$ $\sum C_1$



$$\begin{aligned} k(n, 0) &> 0 \\ k(0, w) &= 0 \\ k(n, w) &= \end{aligned}$$

$$\max \left[k^{Cn-1}, w^{-w[n]} \right] + P$$

$$K^{(n+13, \omega)}]$$

L-19

07.01.2020

Dynamic Fa-Program Table

O/I Knapsack

n\w	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	3	3	3	3	3
2	0	3	4	4	7	
3	0	3	4	5	7	
4	0	3	4	5	7	

n = 4
weight

→ 2

→ 3

→ 4

→ 5

w = 5
Profit

3

4

5

6

maximum profit

O(nw)

L-20

09.01.2020

Longest Common Subsequence

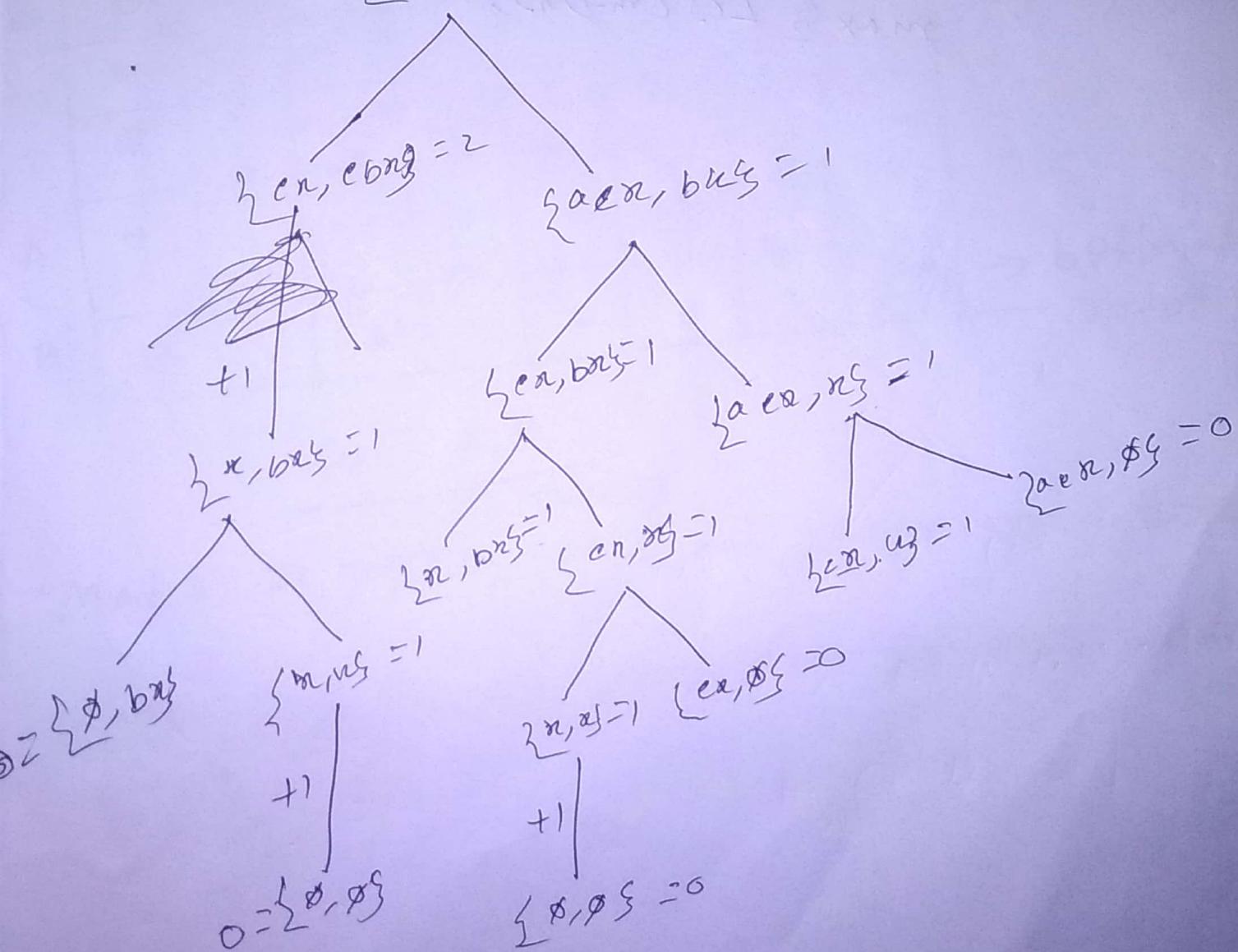
a c a

$\{a\}$, $\{c\}$, $\{a\}$ $\{a, c\}$
 $\{a, c\}$ $\{c, a\}$ $\{a, c\}$ $\{\emptyset\}$

c b a

$\{c\}$, $\{b\}$, $\{a\}$ $\{c, b\}$, $\{b, a\}$, $\{c, a\}$ $\{\emptyset\}$

$\{a c a\}, \{c b a\} = 2$



$$LCS(m, \emptyset) = 0$$

$$LCS(\emptyset, n) = 0$$

$$LCS(m, n) =$$

$$\text{if } m[0] == n[0]$$

$$1 + LCS(m-1, n-1)$$

else

$$\max \{ LCS(m-1, n), LCS(m, n-1) \}$$

start 1 = AGGTAB

start 2 = GXTXAYB

	O	G	X	T	X	A	Y	B
O	0	0	0	0	0	0	0	0
G	0	1	0	1	0	=1	0	1
X	0	0	1	1	1	1	1	1
T	0	0	1	1	1	1	1	1
A	0	1	1	2	2	2	2	2
Y	0	1	1	2	2	=3	3	3
B	0	1	1	2	2	3	3	=3

→ optimal solution

GTAB

arrow

Match এর স্থান n Row, col কেও স্টোর করা হবে।

Match করার পর একটা বিশেষ রীতে স্টোর করা হবে।
path + 1.