Data Structures

Lecture 8
Recursion

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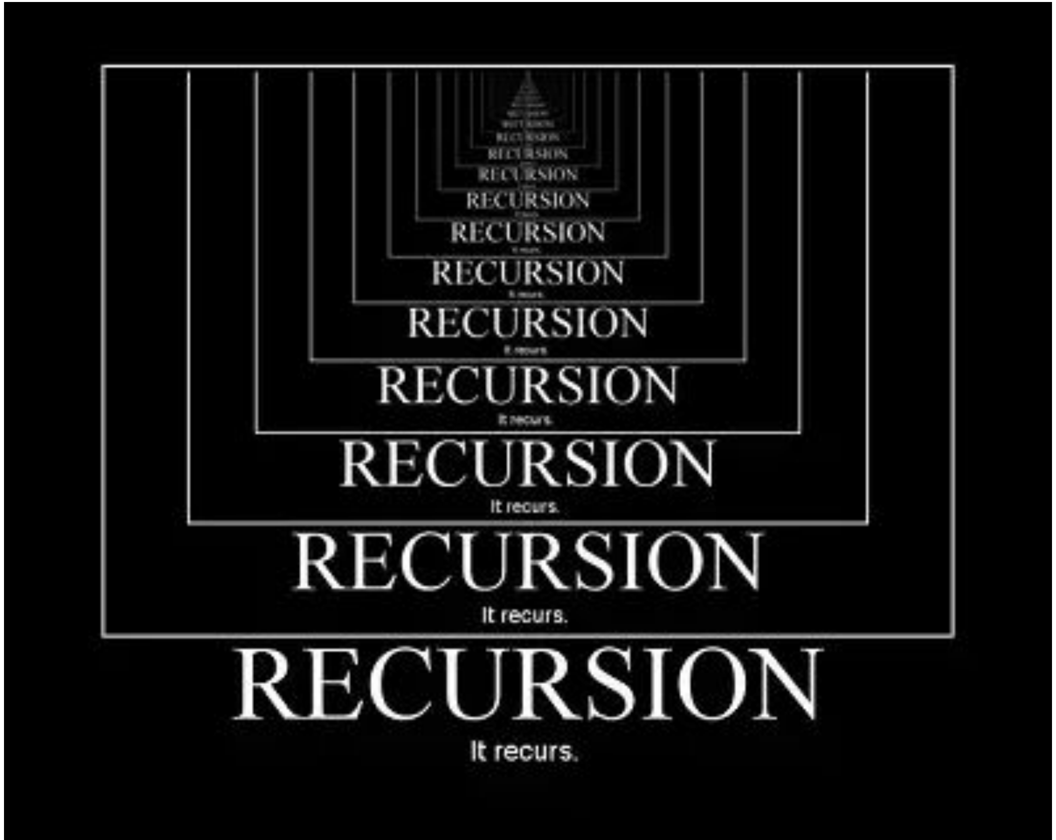
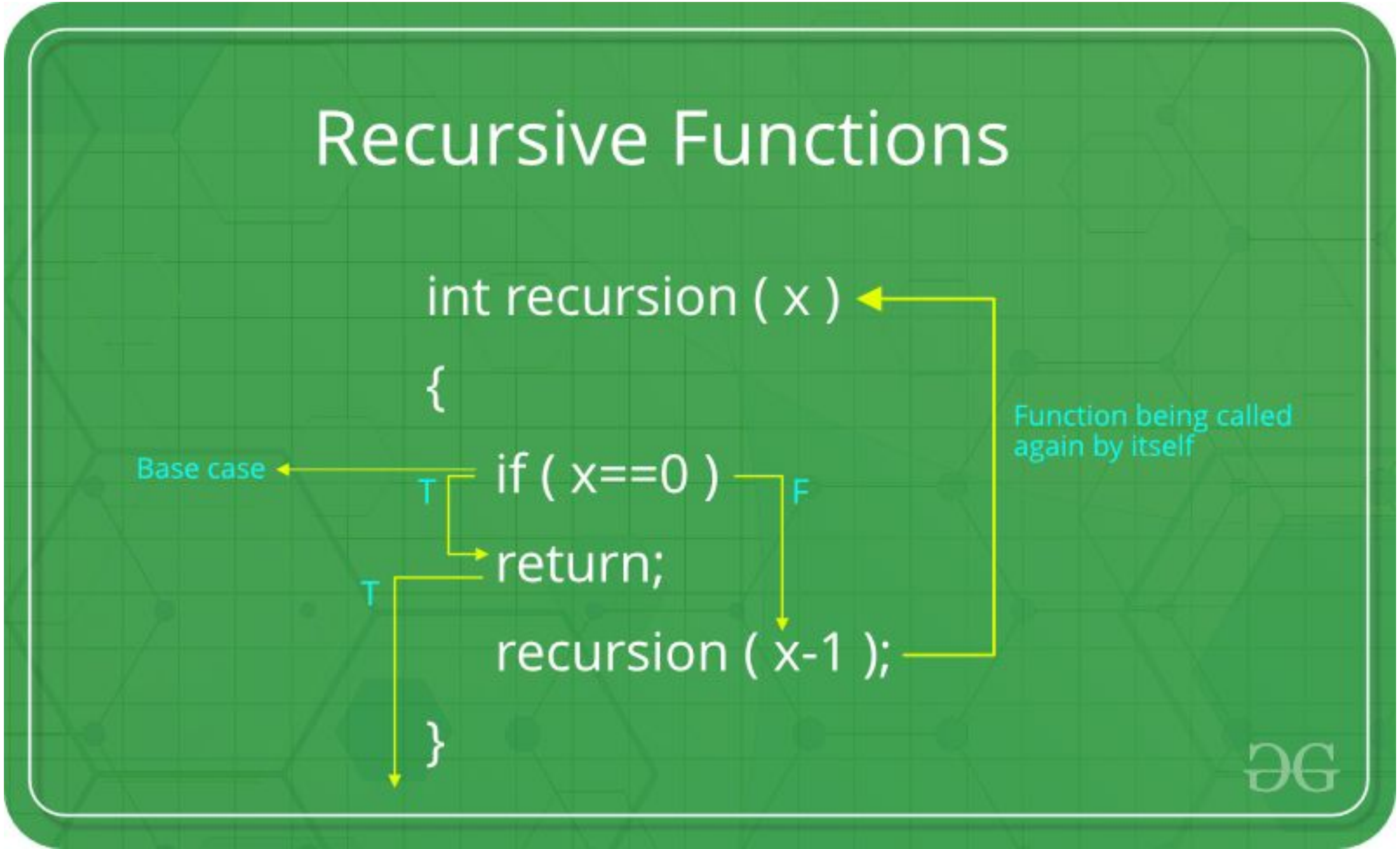
Recursion



RECURSION
It recurs.
RECURSION
It recurs.
RECURSION
It recurs.
RECURSION

Recursive Definition



Recursive Definition (Factorial)

$$
\begin{aligned}
1! &= 1 \\
2! &= 2 \times 1 \\
3! &= 3 \times 2 \times 1 \\
4! &= 4 \times 3 \times 2 \times 1 \\
5! &= 5 \times 4 \times 3 \times 2 \times 1
\end{aligned}
$$

...

and so on.

Recursive Definition (Factorial)

$$n! = \begin{cases} 1 & \text{if } n = 0 \\ 1 & \text{if } n = 1 \\ n \times (n-1)! & \text{if } n > 1 \end{cases}$$

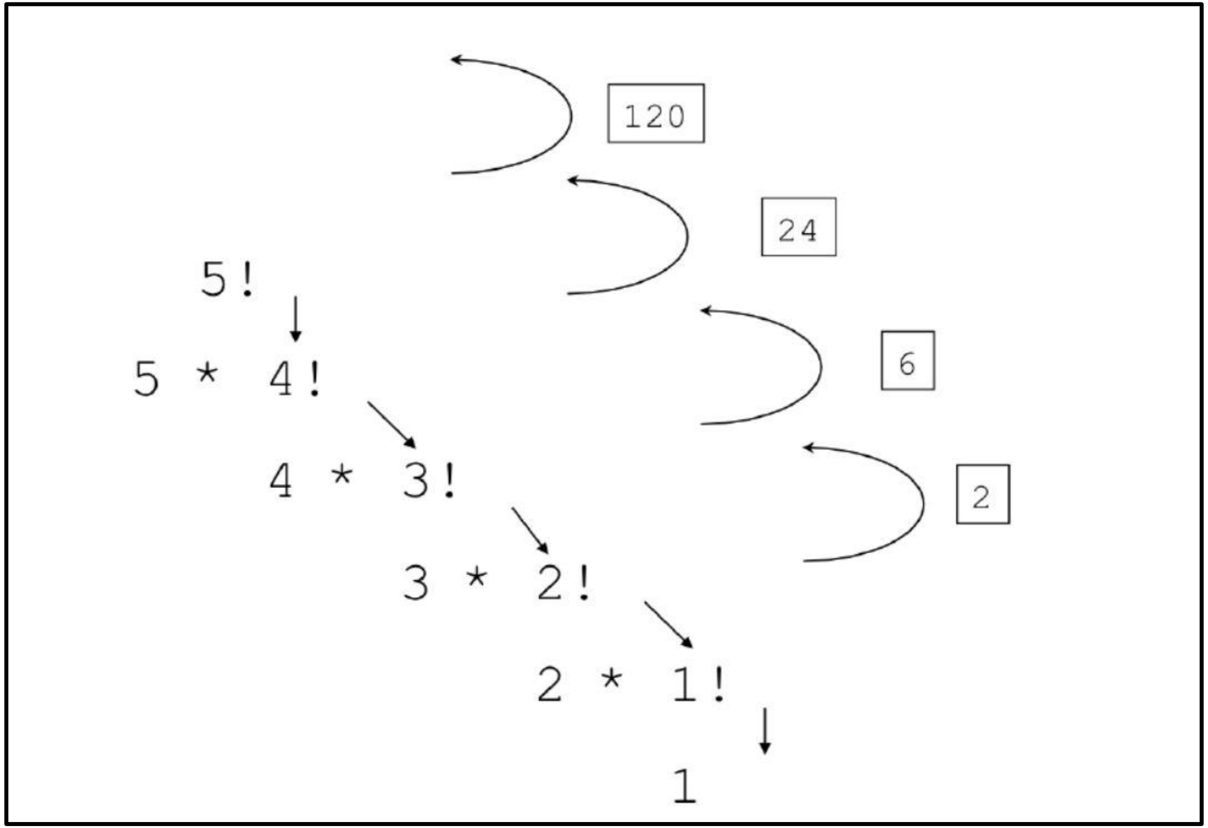
Recursive Definition (Factorial)

$$
\begin{array}{llllllllll}
5! = & 4 \times 4! & & & \\
& & 4 \times 3! & & \\
& & & 3 \times 2! & \\
& & & & 2 \times 1! \\
& & & & \quad\quad\; 1
\end{array}
$$

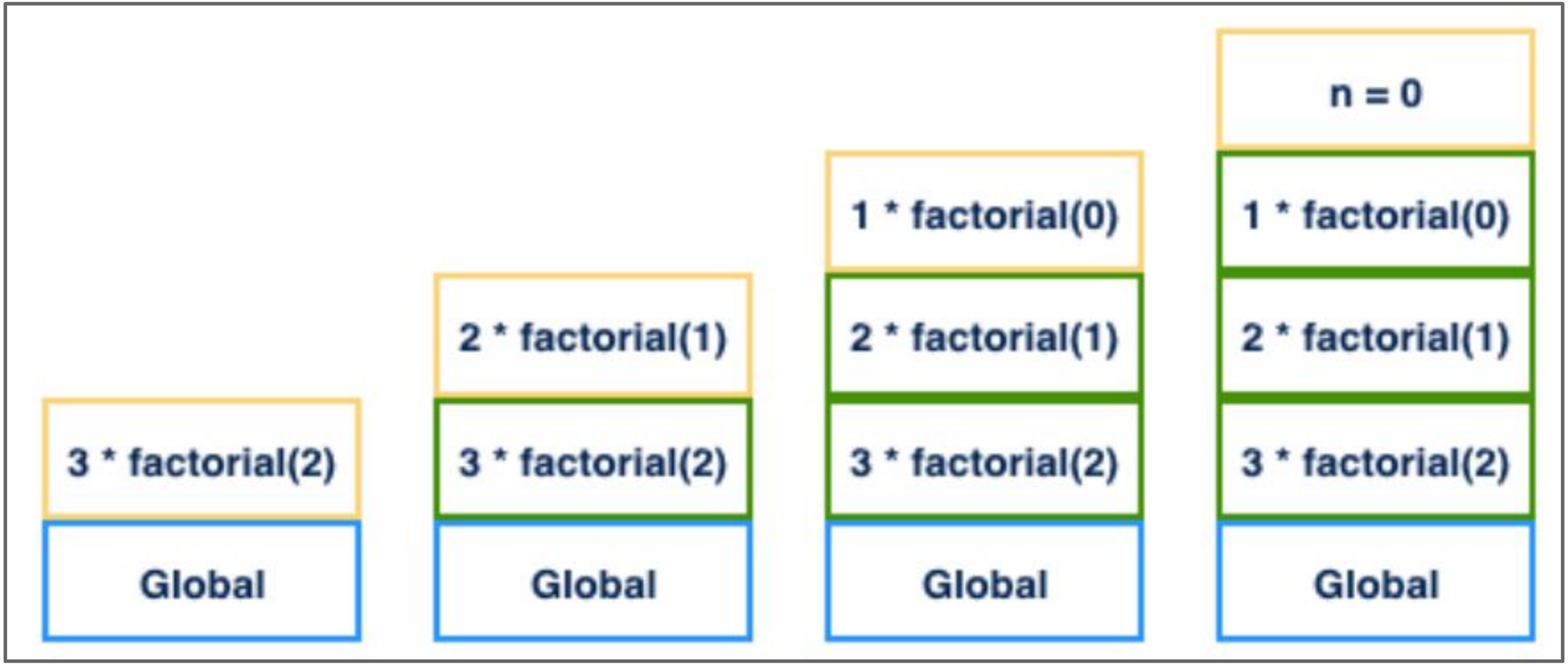
Recursive Programming (Factorial)

```
def fact(n):
  if n==0 or n==1:
    return 1    #Base Case
  else:
    return n * fact(n-1)  #recursive part
```

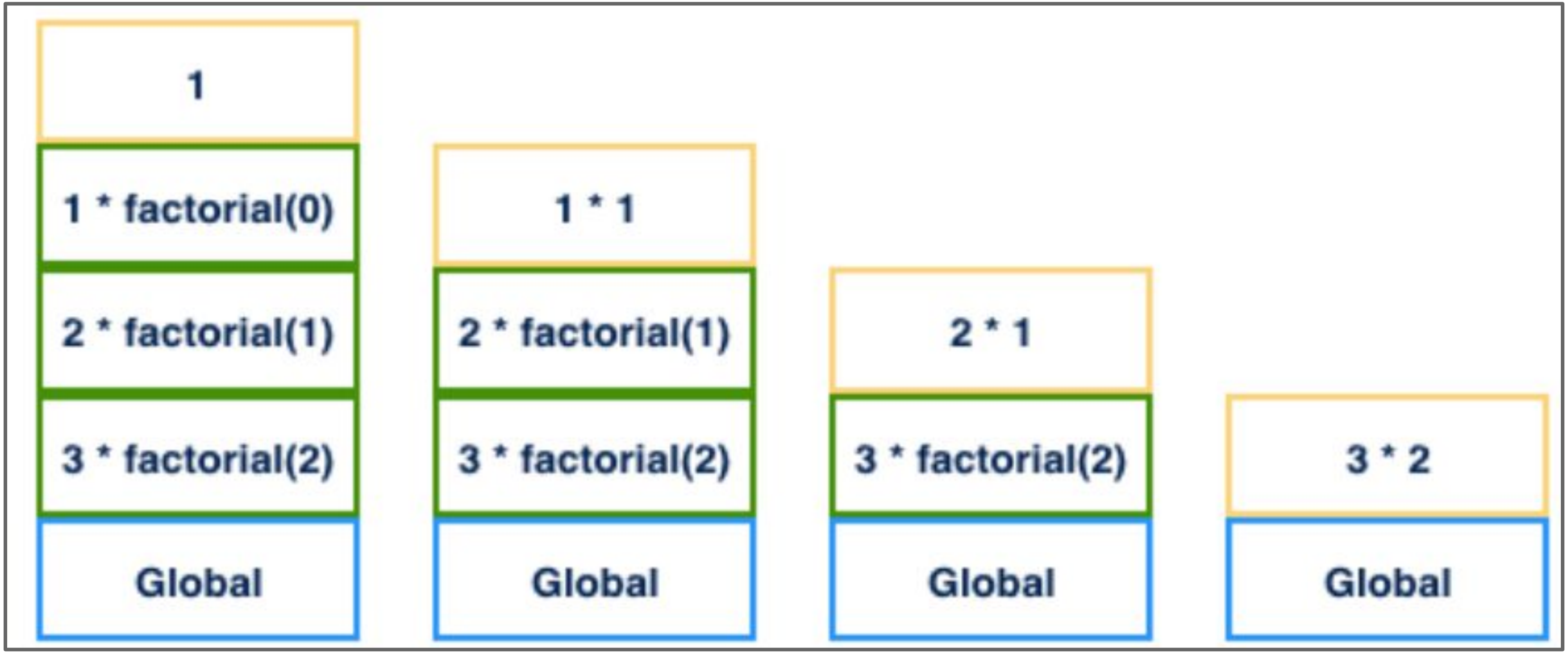
Recursion Tree (Factorial)



Recursion Stack (Factorial)



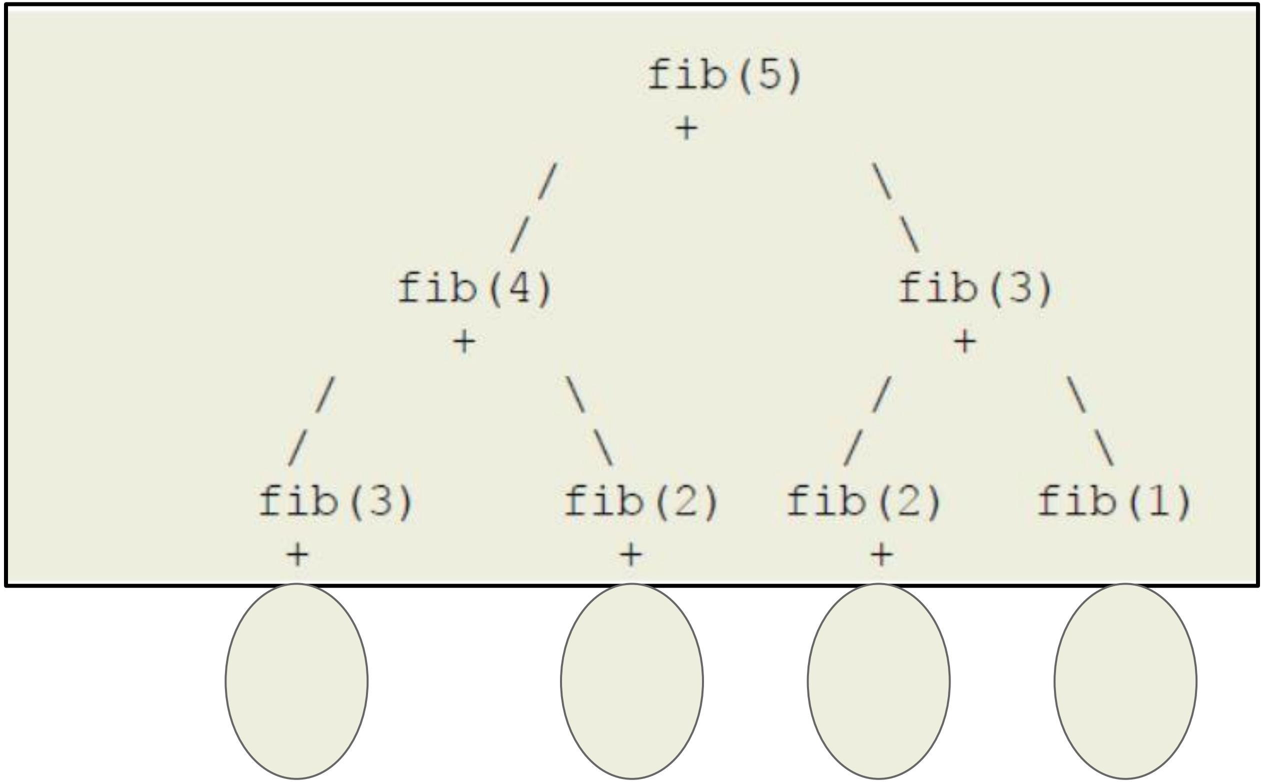
Recursion Stack (Factorial)



Recursive Definition (Fibonacci)

$$
\mathrm{fib}(n) = \begin{cases} 0 & \text{if } n = 0 \\ 1 & \text{if } n = 1 \\ \mathrm{fib}(n-1) + \mathrm{fib}(n-2) & n \geq 2 \end{cases}
$$

Recursion Tree (Fibonacci)



Sum of numbers

```
def iterativeSum(head):
    sum=0
    temp=head
    while temp!=None:
        sum+=temp.element
        temp=temp.next
    return sum
```

Recursive Definition (Sum of numbers)

```
        -
        | k                  k is the only element
sum =   |
        | k + sum(n.next)    otherwise
        -
```

Recursive Programming (Sum of numbers)

```
def recursiveSum(head):
  if head.next==None:
    return head.element  #Base case: Linked list
  else:
    return head.element+recursiveSum(head.next)
```

Recursive Definition (Length of Linked List)

```
              -
              | 0                if l is an empty list
len(l)  =     |
              | 1 + len(rest) otherwise
              -
```

Recursive Programming (Length of Linked List)

```
def listLength(head):
  if head==None:  #empty linked list
    return 0      #base case
  else:
    return 1+listLength(head.next)  #Recursive part
```

Recursive Definition (Sequential Search LL)

```
                 -
                 | false              if list l is empty
                 | true               if l.item = k
contains(l,k) =  |
                 | contains(l.next,k) otherwise
                 -
```

Recursive Programming (Sequential Search LL)

```
def contains(head,key):
  if head==None:
    return False  #base case
  elif head.element==key:
    return True   #base case
  else:
    return contains(head.next,key)  #recursive part
```

Recursive Programming (Sequential Search Array)

```
def contains(arr,key):
  if len(arr)==0:
    return False     #base case
  elif arr[0]==key:
    return True      #base case
  else:
    return contains(arr[1: ],key)   #recursive part
```

Recursive Definition (Seq Search : Left Index)

```
                   -
                   | false                       if l ≥ a.length
                   | true                        if a[l] = k
contains(a,l,k) =  |
                   | return contains(a,l+1,k) otherwise
                   -
```

Recursive Programming (Seq Search : Left Index)

```
def contains(arr,left,key):
  if left>=len(arr):
    return False     #base case
  elif arr[left]==key:
    return True      #base case
  else:
    return contains(arr,left+1,key)   #recursive part
```

Recursive Programming (Find Maximum - Linear LL)

```
def maximum(a,b):
  return a if a>=b else b

def findMax(head):
  if head.next==None:
    return head.element   #base case
  else:
    maxRest=findMax(head.next)     #recursive part
    return maximum(head.element,maxRest)
```

Recursive Programming (Find Maximum - Linear Array)

```
def maximum(a,b):
  return a if a>=b else b

def findMax(arr, left):
  if left == len(arr)-1:
    return arr[left]     #base case
  else:
    maxRest=findMax(arr, left+1)     #recursive part
    return maximum(arr[left], maxRest)
```

Recursive Definition (Exponentiation)

$$a^n = \begin{cases} 1 & n = 0 \\ a \times a^{n-1} & n > 0 \end{cases}$$

Recursive Programming (Exponentiation)

```
def exp(a, n):
    if n==0:
        return 1        #base case
    else:
        return a * exp(a, n-1)      #recursive part
```

Recursive Definition (Binary Search)

```
if the array is empty (if l > r that is):
  return false
else:
  Find the position of the middle element: mid = (l + r)/2
  If key == data[mid], then return true
  If key > data[mid], the search the right half data[mid+1..r]
  If key < data[mid], the search the left half data[l..mid-1]
```

Recursive Definition (Binary Search)

```
                     -
                     | false                   if l > r
                     | true                    if k = a[mid]
contains(a,l,r,k) =  |
                     | contains(a,mid+1,r,k)   if k > a[mid]
                     | contains(a,l,mid-1,k)   if k < a[mid]
                     -
```

Recursive Programming (Binary Search)

```
def contains(arr, left, right, key):
  if left > right:
    return False     #base case
  else:
    mid=(left+right)//2
    if key==arr[mid]:
      return True    #base case
    elif key > arr[mid]:
      return contains(arr, mid+1, right, key) #recursive part
    else:
      return contains(arr, left, mid-1, key)  #recursive part
```

Recursive Programming (Find Maximum - Binary)

```
def maximum(a,b):
  return a if a>=b else b

def findMax(arr, left, right):
  if left == right:
    return arr[left]     #base case
  else:
    mid = (left+right)//2
    maxLeftHalf=findMax(arr, left, mid)      #recursive part
    maxRightHalf=findMax(arr, mid+1, right) #recursive part
    return maximum(maxLeftHalf, maxRightHalf)
```

Recursive Definition (Exponentiation Efficient)

$$a^n = \begin{cases} 1 & n = 0 \\ a^{n/2} \times a^{n/2} & n \text{ is even} \\ a^{(n-1)/2} \times a^{(n-1)/2} \times a & n \text{ is odd} \end{cases}$$

Recursive Programming (Exponentiation Efficient)

```
def exp(a, n):
  if n == 0:
    return 1      #base case
  elif n % 2 == 0:
    return exp(a, n/2) * exp(a, n/2)              #recursive part
  else:
    return exp(a, (n-1)/2) * exp(a, (n-1)/2) * a    #recursive part
```

Recursive Programming (Exponentiation Efficient)

```
def exp(a, n):
  if n == 0:
    return 1     #base case
  elif n % 2 == 0:
    temp = exp(a, n/2)          #recursive part
    return  temp * temp
  else:
    temp = exp(a, (n-1)/2)      #recursive part
    return  temp * temp * a
```

Recursive Programming (Problems)

- Inefficient Recursion
- Space for Activation Frames
- Infinite Recursion