Date:......................

Master Theorem

Exercises:

a. $T(n) = 2T(n/4) + 1$

$a = 2, \quad b = 4, \quad f(n) = 1$

$n^{\log_4 2} = n^{\log_4 4^{1/2}} = \frac{1}{n^2} =$

$n^{1/2} \Rightarrow n^{1/2 - 1/2} = n^0 = 1 = f(n)$

$f(n) = O(n^{1/2 - \varepsilon})$ for any $\varepsilon > 0$

$f(n) < n^{\log_b a - \varepsilon}$

$T(n) = \Theta(n^{\log_b a})$

$= \Theta(n^{1/2})$

Date:....................

(b) $T(n) = 2T(n/4) + \sqrt{n}$

$a = 2, \quad b = 4, \quad f(n) = \sqrt{n}$

$n^{\log_4 2} = n^{\log_4 4^{1/2}} = n^{1/2} = \sqrt{n}$

$\therefore f(n) = n^{\log_b a} = \Theta(n^{\log_b a})$

$\therefore T(n) = \Theta(n^{\log_b a} \log n)$

$= \Theta(\sqrt{n} \log n)$

(c) $T(n) = 2T(n/4) + n$

$a = 2, \quad b = 4, \quad f(n) = n$

$n^{\log_b a} = n^{\log_4 2} = n^{\log_4 4^{1/2}}$

$= n^{1/2}$

$n^{1/2+\varepsilon} = n^{1/2+1/2} = n = f(n)$

$\therefore f(n) = \Omega(n^{\log_b a + \varepsilon})$ for $\varepsilon > 0$

$\therefore f(n) > n^{\log_b a + \varepsilon}$

Date:.....................

$$af(n/b)$$

$$= 2 \frac{n}{b}$$

$$= \frac{2}{b} n$$

$$= \frac{2}{4} n$$

$$= \frac{1}{2} n = cf(n) \quad , c < 1$$

$$\therefore T(n) = \Theta(f(n))$$

$$= \Theta(n)$$

(d) $T(n) = 2T(n/4) + n^2$

$a = 2, \quad b = 4, \quad f(n) = n^2$

$n^{\log_b a} = \sqrt{n} = n^{1/2}$

$n^{\frac{1}{2} + 3/2} = n^2 = f(n)$

$f(n) > n^{\log_b a + \varepsilon} \quad , \varepsilon = 3/2$

$\therefore f(n) = \Omega(n^{\log_b a + \varepsilon})$ for $\varepsilon > 0$

$\therefore af(n/b) = a \frac{n^2}{b^2} = \frac{a}{b^2} n^2$

$= \frac{2}{16} n^2 = \frac{1}{8} n^2 = cf(n), \quad c < 1$

$\therefore T(n) = \Theta(f(n)) = \Theta(n^2)$