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Time Complexity Slide 28

Big-O

1. $2n^2 = O(n^2)$

To prove,

$$2n^2 \leq cn^2 \quad , n > n_0$$

$$2 \leq c \quad \text{(assume } n > 0\text{)}$$

so, exist constant c and n_0 so that for all $n > n_0$, $2n^2$ is $O(n^2)$.

2. $2n^3 + 2 \neq O(n^2)$

To prove, let's say

$$2n^3 + 2 = O(n^2)$$

$$\therefore \quad 2n^3 + 2 \leq cn^2$$

$$\Rightarrow \quad 2(n^3 + 1) \leq cn^2$$

$$\Rightarrow \quad 2\left(n + \frac{1}{n^2}\right) \leq c$$

$$\Rightarrow \quad 2n \leq c \qquad n \to \infty, \quad \frac{1}{n^2} \to 0$$

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since, n grow indefinitely,

there will always be a c for which this inequality will not hold.

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1. $n = O(n^2) \neq \Theta(n^2)$

$n = O(n^2)$

$\Rightarrow n \leq cn^2$

$c = 1, \quad n_0 = 1$

$n \leq n^2$

true for all $n \geq 1$

$\therefore n = O(n^2)$

To prove $n = \Theta(n^2)$,

we have to show,

$n = \Omega(n^2)$

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$\therefore n \geq cn^2$

n grows linearly, how large

c is chosen. So, always be an n

for which $|n| < c \cdot |n^2|$

$\therefore n \neq \Omega(n^2)$

$\therefore n \neq \Theta(n^2)$

2. $200n^2 = O(n^2) = \Theta(n^2)$

$200n^2 = O(n^2)$

$200n^2 \leq cn^2$

$c = 200, \quad n_0 = 1$

true for all $n > n_0$

$\therefore 200n^2 = O(n^2)$

$200n^2 = \Theta(n^2)$

$c_1 n^2 \leq 200n^2 \leq c_2 n^2$

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$c_1 = 1,\ c_2 = 200,\ n_0 = 1$

$\therefore\ n^2 \le 200n^2 \le 200n^2$

true for all $n > n_0 = 1$

$\therefore\ 200n^2 = \Theta(n^2)$

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Θ notation:

1. $\frac{n^2}{2} - \frac{n}{2} = \Theta(n^2)$

$\frac{n^2}{2} - \frac{n}{2} = O(n^2)$

$\Rightarrow \frac{n^2}{2} - \frac{n}{2} \le cn^2$

$\Rightarrow \frac{1}{2}(n^2 - n) \le cn^2$

$= \frac{1}{2}\ \frac{n^2 - n}{n^2} \le c$

$\Rightarrow \frac{1}{2}\ \frac{n-1}{n} \le c$

$c = \frac{1}{2},\ n_0 = 1$

$n > 1$, this holds

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$$\frac{n^2}{2} - \frac{n}{2} = \Omega(n^2)$$

$$\Rightarrow \frac{n^2}{2} - \frac{n}{2} \geq cn^2$$

$$\Rightarrow \frac{1}{2} \frac{n^2 - n}{n^2} \geq c$$

$$\Rightarrow \frac{1}{2} \left(\frac{n-1}{n}\right) \geq c$$

$c = \frac{1}{4}$, $n_0 = 2$,

so, for $n \geq 2$, this

holds

$$\therefore \frac{n^2}{2} - \frac{n}{2} = \Theta(n^2)$$

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2. $\frac{(6n^3+1)}{n+1} \log n = \Theta(n^2 \log n)$

(a) $\frac{6n^3+1}{n+1} \log n = O(n^2 \log n)$

$\Rightarrow \frac{6n^3+1}{n+1} \log n \leq c n^2 \log n$

$\Rightarrow \frac{6n^3+1}{n+1} \cdot \frac{1}{n^2} \leq c$ [divide by $n^2 \log n$]

$\Rightarrow \frac{6n^3+1}{n^3+n^2} \leq c$

$\Rightarrow \frac{6+\frac{1}{n^3}}{1+\frac{1}{n}} \leq c$ [divide by n^3]

$\Rightarrow$ taking $\lim_{n \to \infty} \frac{6+0}{1+0} \leq c$

$\Rightarrow 6 \leq c$

$\therefore$ for some constant $c = 6$, $n_0 \geq 1$ this holds.

(b) $\frac{6n^3+1}{n+1} \log n = \Omega(n^2 \log n)$

$\Rightarrow \frac{6n^3+1}{n+1} \log n \geq c n^2 \log n$

$\Rightarrow \frac{6n^3+1}{(n+1)n^2} \geq c$ [divide by $n^2 \log n$]

$\Rightarrow \frac{6n^3+1}{n^3+n^2} \geq c$

So, how large c is chosen, there will always be some $n > n_0$, for which this will hold.

$\therefore$ the statement is true.

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1. $f(n) = \log n^2 \quad g(n) = \log n + 5$

\# $f(n) = O(g(n))$

$\Rightarrow 2\log n \le c(\log n + 5)$

$\Rightarrow c = 2, \quad n_0 = 1$

$2\log n \le 2\log n + 5$

$n \ge 1$

true

$\therefore f(n) = O(g(n))$

\# $f(n) = \Omega(g(n))$

$2\log n \ge c(\log n + 5)$

$c = \frac{1}{2}, \quad n_0 = 1$

$2\log n \ge \frac{1}{2}(\log n + 5)$

$n \ge 1$

true

$\therefore f(n) = \Omega(g(n))$

$\therefore f(n) = \Theta(g(n))$.

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2. $f(n) = n, \quad g(n) = \log n^2$

\# $f(n) = O(g(n))$

$\Rightarrow n \leq c \log n^2$

$\Rightarrow n \leq \cancel{2} c \log n^2$

$c = 1, \quad n_0 = 1$

$n \leq 1 \cdot \log n^2$ for $n \geq 1$

$\therefore f(n) = O(g(n))$

\# $f(n) = \Omega(g(n))$

$\Rightarrow n \geq c \log n^2$

$c = 1, \quad n_0 = \cancel{8}\, 4$

$n \geq 1 \cdot \log n^2$ for $n \geq 4$

$\therefore f(n) = \Omega(g(n))$

$\therefore f(n) = \Theta(g(n))$

3. $f(n) = \log\log n$, $g(n) = \log n$

$f(n) = O(g(n))$

$\Rightarrow \underbrace{\log\log n}_{\text{this is smaller, because}} \leq c\log n$

this is smaller, because taking log of ~~some~~ a value, makes that value even smaller,

$\therefore$ so this is true.

$\therefore$ ~~$f(n) = ($~~

$f(n) = \Omega(g(n))$

$\Rightarrow \log\log n \geq c\log n$

not true.

$\therefore f(n) \neq \Omega(g(n))$

$\therefore f(n) = O(g(n))$

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5. $f(n) = n\log n + n$ $g(n) = \log n$

If $f(n) = O(g(n))$

$n\log n + n \leq c\log n$

since $n > 0$

$n\log n$ is clearly greater

than $\log n$.

$\therefore n\log n + n > \log n$

$\therefore f(n) \neq O(g(n))$

$\therefore f(n) = \Omega(g(n))$

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6. $f(n) = 10, \quad g(n) = \log 10$

$f(n) = O(g(n))$

$10 \leq c \log 10$

$c = 10, \quad n_0 = 1$

this is true.

$f(n) = \Omega(g(n))$

$10 \geq c \log 10$

$c = 1, \quad n_0 = 1$

this is true

$\therefore f(n) = \Theta(g(n))$

7. $f(n) = 2^n$, $g(n) = 10n^2$

$f(n) = O(n^2)$

$2^n \leq c\,10n^2$

~~$\log(2^n) \leq \log 10c + \log n^2$~~

$\log(2^n) \leq \log(c.10n^2)$ [taking log]

$\Rightarrow n \leq \log(c.10n^2)$

n grows exponentially,

$\log(c.10n^2)$ grows

logarithmically, $\therefore$

n cannot be bound

by $\log(c.10n^2)$

$\therefore$ $f(n) = \Omega(g(n))$

$\log 2^n \geq \log(c.10n^2)$

$\therefore$ this is true.
same explanation.

$\therefore$ $f(n) = \Omega(g(n))$