

## Quick Sort

Example: Split is  $\frac{1}{10} : \frac{9}{10}$

$\therefore$  one portion is  $\frac{n}{10}$  and

another portion is  $\frac{9n}{10}$ .

$$\therefore T(n) = T\left(\frac{n}{10}\right) + T\left(\frac{9n}{10}\right) + n$$

$$T_1(n) = T\left(\frac{n}{10}\right) + n$$

$$T_2(n) = T\left(\frac{9n}{10}\right) + n$$

$$T(n) = T_1(n) + T_2(n)$$

$$\therefore T_1(n) = T_1\left(\frac{n}{10}\right) + n$$

$$= T_1\left(\frac{n}{10^2}\right) + 2n$$

$$= T_1\left(\frac{n}{10^3}\right) + 3n$$

$$\frac{n}{10^k} = 1 \quad = T_1\left(\frac{n}{10^k}\right) + kn$$

$$10^k = n$$

$$\log_{10} n = k$$

$$= T_1(1) + n \log_{10} n$$

$$\therefore T_1(n) = O(n \log_{10} n)$$

$$T_2(n) = T_2\left(\frac{9n}{10}\right) + n$$

$$= T_2\left(\frac{9^2 n}{10^2}\right) + 2n$$

$$= T_2\left(\frac{9^3 n}{10^3}\right) + 3n$$

$$\vdots$$

$$= T_2\left(\frac{9^k n}{10^k}\right) + kn$$

$$\frac{9^k n}{10^k} = 1 \quad \rightarrow \quad = T_2(1) + n \log_{10/9} n$$

$$n = \frac{10^k}{9^k}$$

$$n = \left(\frac{10}{9}\right)^k$$

$$\Rightarrow k = \log_{\frac{10}{9}} n$$

$$\therefore T_2(n) = O(n \log_{\frac{10}{9}} n)$$

$$\therefore T(n) = O(n \log_{10} n)$$

$$+ O(n \log_{\frac{10}{9}} n)$$

$$= O(n \log_{\frac{10}{9}} n)$$

[taking max]