BRAC UNIVERSITY
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Quiz 2 Solution

Date : ....

1. $T(n) = T\left(\frac{n}{2}\right) + T\left(\frac{n}{4}\right) + n$

$T_1(n) = T\left(\frac{n}{2}\right) + n$ — (i)

$T_2(n) = T\left(\frac{n}{4}\right) + n$ — (ii)

solve these two and then add. Ignore the terms except the dominating term.

2. $T(n) = 6T\left(\frac{n}{3}\right) + \frac{n^2}{\log n}$

$a = 6, \quad b = 3, \quad f(n) = \frac{n^2}{\log n}$

set B

$n^{\log_b a} = n^{1.63}$

$f(n) < n^{1.63}$ [check for several values of n]

$\therefore T(n) = \Theta(n^{1.63})$

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3.

$$T(n) = \begin{cases} 1, & n = 1 \\ T\left(\frac{n}{2}\right) + 1 & n > 0 \end{cases}$$

$$\therefore T(n) = T\left(\frac{n}{2}\right) + 1$$

$$= T\left(\frac{n}{4}\right) + 2$$

$$= T\left(\frac{n}{8}\right) + 3$$

$$\vdots$$

$$\frac{n}{2^k} = 1$$

$$k = \log n$$

$$= T\left(\frac{n}{2^k}\right) + k$$

$$\cancel{n-k=0}$$

$$\cancel{k=n}$$

$$= T(1) + \log n$$

$$= 1 + \log n$$

$$= \log n$$

$$\therefore T(n) = O(\log n)$$

4. Max Sum Subarray

$O(n \log n)$

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2. $T(n) = 64T(n/8) + n^2 \log n$

set A

$a = 64, \quad b = 8, \quad f(n) = n^2 \log n$

$n^{\log_b a} = n^{\log_8 64} = n^2$

$f(n) > n^2$

$\therefore \quad T(n) = \Theta(n^2 \log n)$

5.(a) $f_1(n) = (\log n)^{2023} = O((\log n)^{2023})$

$f_2 = n^2 \log_n n^n = n^3 \log_n n$

$= O(n^3)$

$f_3 = n^3 + 7n^2 = O(n^3)$

$f_4 = (2.023)^n = O(2.023)^n$

$f_5 = n \log n = O(n \log n)$

$f_6 = n \sqrt[3]{n^2} = n^{5/2}$

(b) $f_1 < f_5 < f_6 < f_3 = f_2 < f_4$