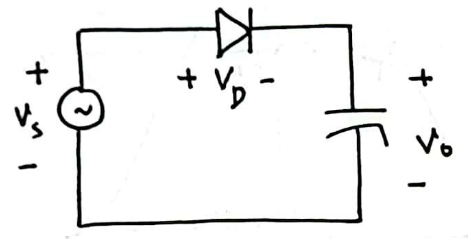
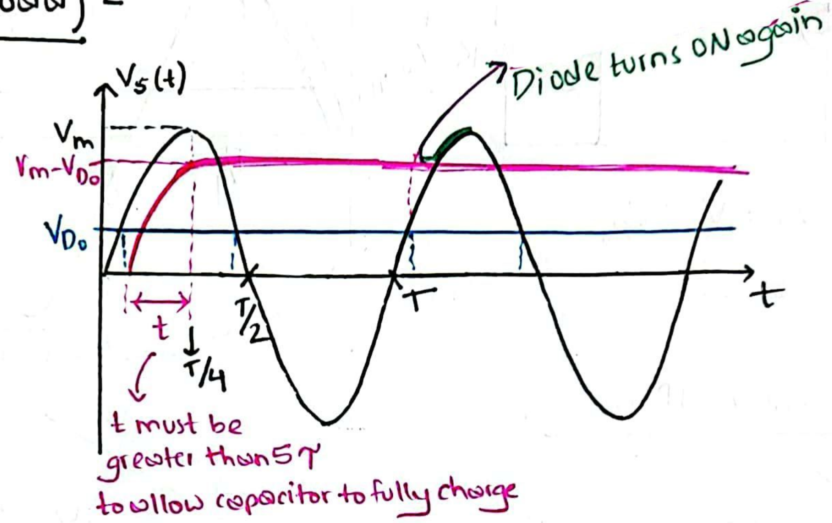
2. Diode Rectifiers with Capacitors (as filter)

* Let's look at an impractical circuit to understand how the capacitor works in a rectifier setup.

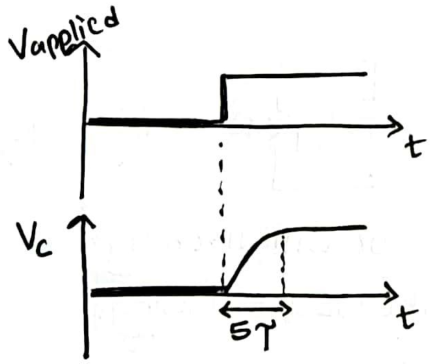
HW rectifier with C (No load) -





ON: $V_o = V_s - V_{D_0}$

* a capacitor requires 5τ amount of time for its voltage to reach a steady state value, where $\tau = RC$.

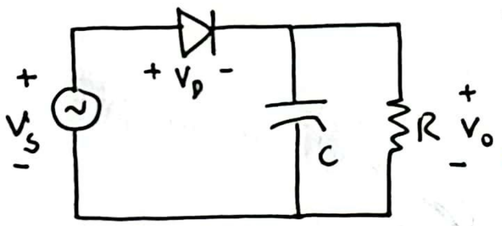


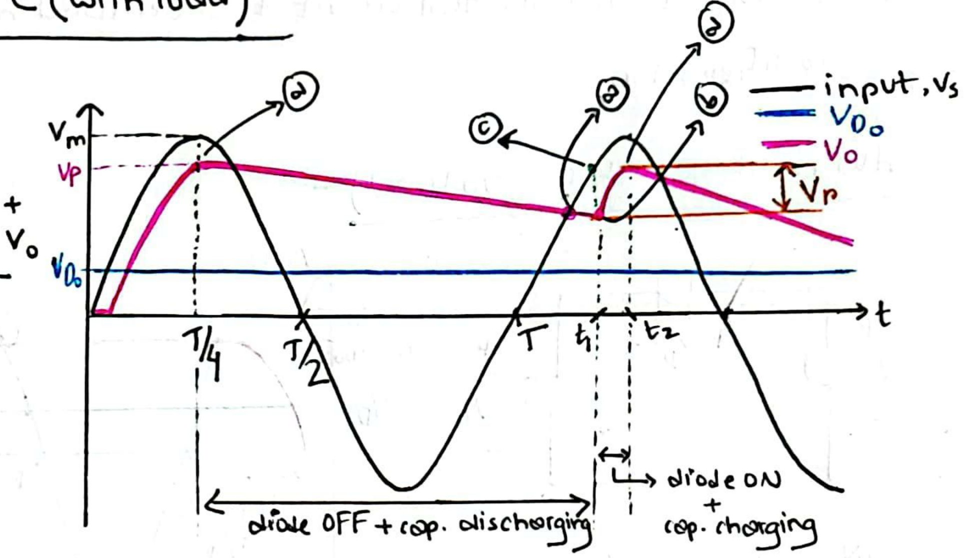
* If the capacitor is not given enough time, it will fail to fully follow the applied voltage.
* For the given circuit, $R_{load} = 0$
  $\therefore RC \approx 0$
  The capacitor will be able to follow in no time!
* $t > 5\tau$ usually, as $V_s(t)$ is usually at 50 Hz, and RC is sufficiently smaller.

* After T/4, i/p voltage starts decreasing.
* At and upto T/4, $V_D = V_s - V_o > 0 \longrightarrow$ diode ON
* After T/4, $V_D = V_s - V_o < 0$, as V_s starts decreasing and V_o is held constant by the capacitor $\longrightarrow$ diode slowly turns OFF.
* As diode turns OFF, there will be no more current flow. Capacitor is unable to discharge as there is no load connected. $V_o \rightarrow$ remains constant.

* connecting a resistive load will affect the capacitor voltage graph.

Half-wave rectifier with C (with load) -



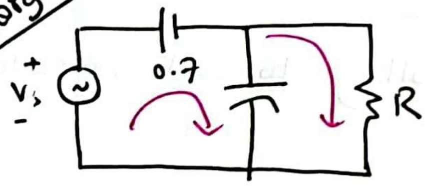


$V_o = V_s - V_{D_o}$

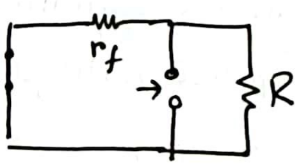
$\Delta t = t_2 - t_1$ (graph)

For +ve half cycle,

charging



Req:



* r_f is the diode resistance (<50Ω)
* all voltage sources shorted
* Req is seen from capacitor terminals.

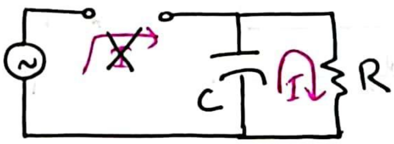
$\therefore R_{eq} = r_f \parallel R$

$R_{eq} < r_f$ (given that $\underset{\Omega}{r_f} << \underset{k\Omega}{R}$)

* Req is a very small value.
* Charging time constant, τ_{ch}, is thus very small, so capacitor will charge fully by T/4.

discharging

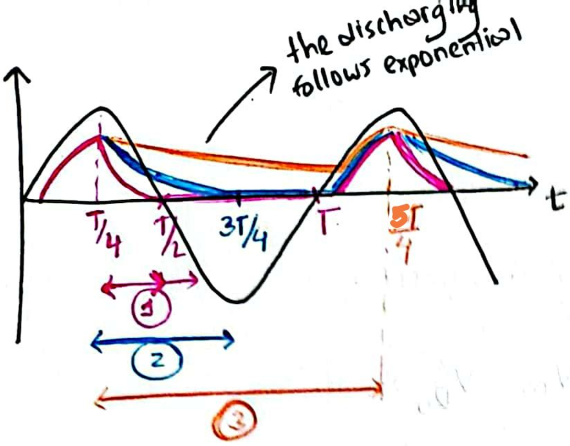
* At/After T/4, $V_s - V_o < V_{D_o}$
* The diode will be reverse biased, thus rendering the circuit -



($R_{eq} = R$)

* The capacitor will discharge through the load following - $5\tau_d = 5RC$.
* One important condition to be met is that $5\tau_d >> T$.
* The capacitor voltage will decrease following exponential rule.

$V_c(t) = V_p e^{-t/\tau}$



If $5\tau_d \gg T$ condition is not met, consider the following scenarios -

1. $5\tau_d = \frac{T}{4}$
2. $5\tau_d = T/2$
3. $5\tau_d = T$ ⟶ here, the input will charge capacitor again before the capacitor fully manages to discharge

---

discharging continued (points have been marked on the graph)

* At point ⓐ, $V_o > V_{D_o}$ and $V_s = V_o$ (they intersect) + diode still off.
* After point ⓐ, as V_s increases (while V_o decreases), the diode will eventually become forward biased, turning it on again.
* This happens when $V_s > V_o + V_{D_o}$ ⟶ this occurs at point ⓑ
* point ⓒ highlights (in green) that the input at that moment is greater than the ~~oup~~ output by diode voltage drop. ($V_s > V_o + V_{D_o}$)

(* point ⓑ ~~and~~ and ⓒ are the same, just marked on different lines)

* From point ⓑ to ⓓ, the diode is biased <u>ON</u> ⟶ capacitor will ~~d~~charge
* After point ⓓ, the capacitor will discharge, as the diode is biased <u>OFF</u>. The capacitor will continue discharging until the point ⓑ is reached in next cycle.
* The output now is relatively better, as it does not completely go down to 0 V, like it did without a capacitor.
* The output fluctuates between a max and min level (voltage at point ⓓ and ⓑ respectively). This is called the ripple voltage, V_r.

* V_r should be as small as possible.

Some points to note -

(i) $\Delta t = t_2 - t_1 \longrightarrow$ Diode conducts during this period

(ii) at $t_1 \longrightarrow V_o = V_p - V_r$ (point ⓑ), where $V_p = V_m - V_{D_o}$

(iii) at $t_2 \longrightarrow V_s = V_m$ and $V_o = V_p$ (both i/p and o/p reach their peak)

$$* \quad I = \frac{V_s - V_{D_o} - V_o}{R} = \frac{V_m - V_{D_o} - V_p}{R} = \frac{V_m - V_{D_o} - (V_m - V_{D_o})}{R} = 0$$

(iv) When the diode is off, capacitor discharges through load in an exponenti[al] manner.

(v) During discharging phase (at point ⓑ), $V_o = V_p e^{-(T-\Delta t)/\tau}$, $t = T - \Delta t$ = discharging duration

$$\Rightarrow V_p - V_r = V_p e^{-(T-\Delta t)/RC}$$

$$\Rightarrow V_p - V_r = V_p \Big[\underbrace{1 - \frac{T-\Delta t}{RC} + \left(\frac{T-\Delta t}{RC}\right)^2 \cdot \frac{1}{2!} - \cdots}_{\text{Taylor series}} \Big]$$

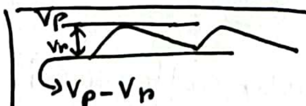
using the condition/approximations : $RC \gg T$
$\Delta t \ll T$

we can rewrite:

$$\Rightarrow V_p - V_r \simeq V_p \left(1 - \frac{T - \overset{0}{\cancel{\Delta t}}}{RC}\right)$$ [ignore higher order terms]

$$\therefore V_r = \frac{V_p}{fRC}$$ [f, R $\longrightarrow$ not changeable]

(vi) $V_{dc} = V_{avg} = V_p - \frac{V_r}{2}$

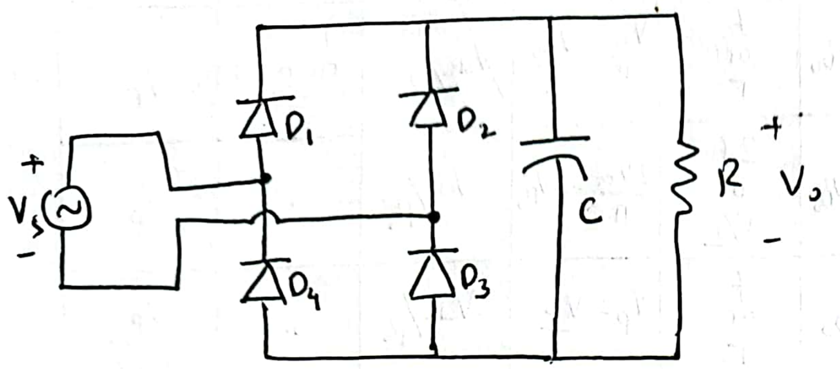


$$avg = \frac{V_p + V_p - V_r}{2} = V_p - \frac{V_r}{2}$$

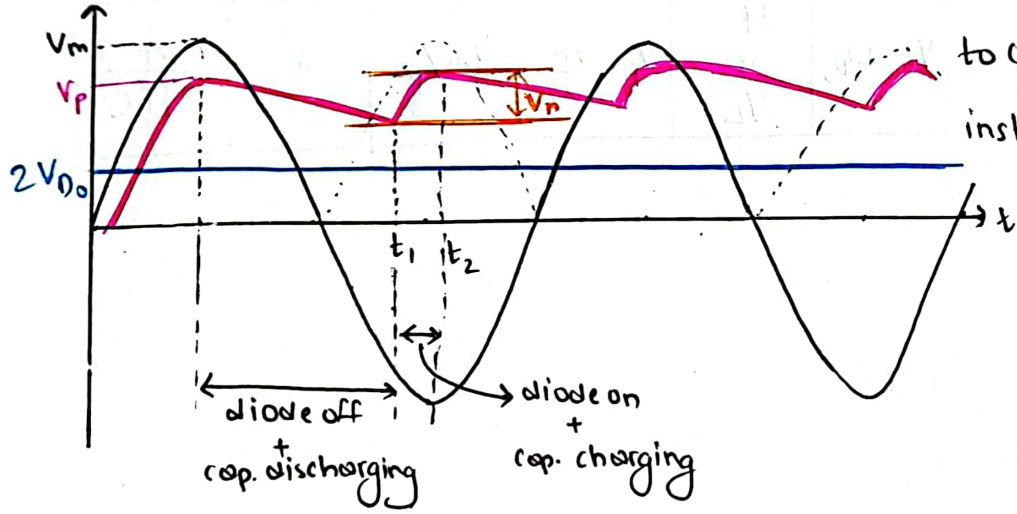
[C↑, V_r↓ ... but Δt↑]

↓ we don't want, so we have to limit how high C is.

Full-wave rectifier with C (with load) -



* the analysis is similar to that of half wave rectifier, i.e, capacitor will have charging and discharging phase
* Unlike with HW rectifier, in FW rectifier, capacitor will get to charge every half cycle, instead of every other cycle.



$$V_o = V_s - 2V_{D_o}$$

* During discharging phase, time taken = $T/2 - \Delta t$.

$$\Rightarrow V_p - V_r = V_p\, e^{-(T/2 - \Delta t)/RC}$$

$$\Rightarrow V_p - V_r \simeq V_p\, e^{-T/2RC} \quad [T >> \Delta t]$$

$$\Rightarrow V_p - V_r \simeq V_p \left[1 - T/2RC\right] \quad [RC >> T]$$

$$\therefore V_r = \frac{V_p}{2fRC} \longrightarrow V_r(FW) < V_r(HW)$$

* output for this ckt is better with lower fluctuations.
* $V_{dc} = V_p - \frac{V_r}{2}$ (same logic as HW)

Summary:

		i/p voltage peak	i/p v. freq/period	o/p v. peak	o/p v. freq/period	o/p V_{dc}	o/p I	V_{rms}	o/p V_r
without C	HW	V_m	f or, T	$V_m - V_{D_0}$	f or T	$\frac{V_m}{\pi} - \frac{V_{D_0}}{2}$	V_{dc}/R	no need	V_p
	FW	V_m	f or, T	$V_m - 2V_{D_0}$	$2f$ or $T/2$	$\frac{2V_m}{\pi} - 2V_{D_0}$	V_{dc}/R	"	V_p
with C	HW	V_m	f or T	$V_m - V_{D_0}$	f or T	$V_p - \frac{V_r}{2}$	V_{dc}/R	"	$\frac{V_p}{fRC}$
	FW	V_m	f or T	$V_m - 2V_{D_0}$	$2f$ or $T/2$	$V_p - \frac{V_r}{2}$	V_{dc}/R	"	$\frac{V_p}{2fRC}$

↓

Approximation for $V_D << V_m$.

Otherwise, $V_{dc} = \frac{1}{T}\int_0^T v(t)\,dt$