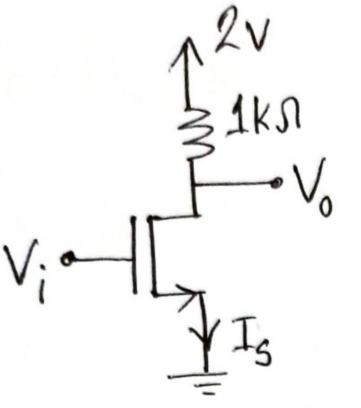
Lets start simple — one resistor connected to the 'Drain' terminal

Example 1



Find V_o & I_S when—

(i) $V_i = 0.1V$

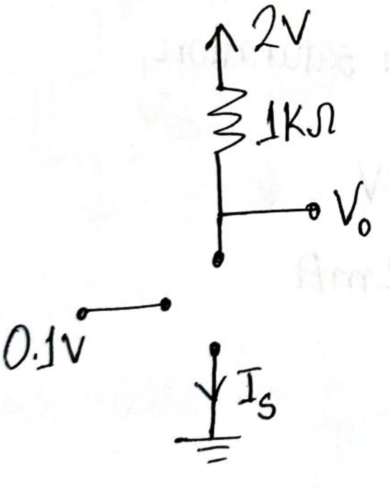
(ii) $V_i = 0.3V$

(iii) $V_i = 1V$

$$\boxed{\begin{array}{l} V_T = 0.2V \\ K = 4mA/V^2 \end{array}}$$

(i) for $V_i = 0.1V \rightarrow V_{GS} = 0.1V < V_T (0.2V)$

$\therefore$ MOSFET in cut off —



$\therefore V_o = 2V$

& $I_S = 0A$

(ii) for $v_i = 0.3V \rightarrow V_{GS} = 0.3V \geq V_T\ (0.2V)$

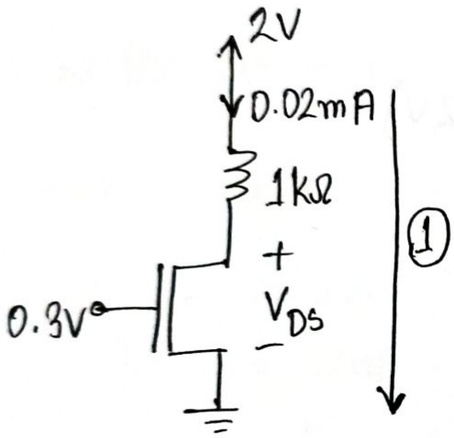
$\therefore$ MOSFET in either saturation or triode.

Assuming saturation $\rightarrow$ we have to show $V_{DS} \geq V_{OV}$ given $I_D = \frac{1}{2} k V_{OV}^2$

Here, $V_{OV} = V_{GS} - V_T = 0.1V$

$\therefore I_D = 0.02mA$

in the ckt —



KVL in line 1 —

$$0.02 + V_{DS} = 2$$

$$\Rightarrow V_{DS} = 1.98V \geq V_{OV}\ (0.1V)$$

$\therefore$ MOSFET is in saturation

$\therefore V_0 = V_{DS} = 1.98V$

& $I_S = I_D = 0.02mA$

(iii) for $v_i = 1V \rightarrow V_{GS} = 1V \geq V_T (0.2V)$

$\therefore$ MOSFET in either saturation or triode.

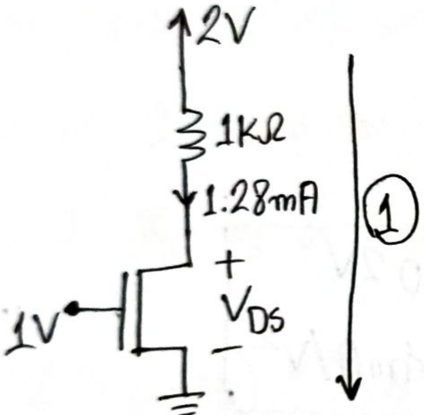
Assuming saturation —

$I_D = \frac{1}{2} V_{ov}^2 \cdot K$

$V_{ov} = V_{GS} - V_T = 0.8V$

$\therefore I_D = \frac{1}{2} \times 4 \times 0.8^2 = 1.28mA$

in the ckt —



KVL in line 1 —

$1.28 + V_{DS} = 2$

$\Rightarrow V_{DS} = 0.72V < V_{ov}(0.8V)$

$\therefore$ MOSFET cannot be in saturation, it has to be in triode mode.

In triode mode $\rightarrow I_D = K(V_{ov} - \frac{1}{2}V_{DS})V_{DS}$

in the ckt — $I_D = \frac{2 - V_{DS}}{1}$ [using Ohm's law across the 1kΩ]

$\therefore 2 - V_{DS} = K(V_{ov} - \frac{1}{2}V_{DS}) \cdot V_{DS}$ for this ckt

$\Rightarrow 2 - V_{DS} = 4(0.8 - \frac{1}{2}V_{DS})V_{DS}$

$\Rightarrow 2 - V_{DS} = (3.2 - 2V_{DS})V_{DS}$

$\Rightarrow 2V_{DS}^2 - 4.2V_{DS} + 2 = 0$

$\therefore V_{DS} = 0.73V$ or $1.37V$

Now the rule is to take the value that better fits your hypothesis.

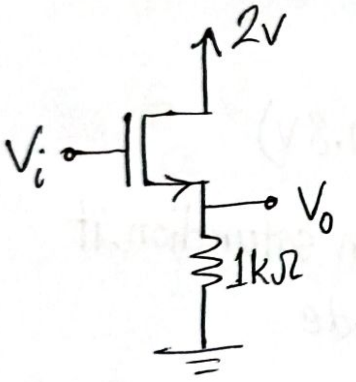
Here if you take $1.37V \geq V_{ov}$, your triode assumption fails.

So instead take $V_{DS} = 0.73V$

$\therefore V_o = 0.73V$ & $I_s = I_D = 2 - V_{DS} = 1.27mA$

Now lets try out one resistor connected to the 'Source' terminal.

Example 2



Find V_o for

(i) $V_i = 0.3V$

(ii) $V_i = 1V$

(iii) $V_i = 3V$

$\boxed{V_T = 0.2V \quad K = 4mA/V^2}$

(i) for $V_i = 0.3V$, lets assume cut-off $\rightarrow I_D = 0$

then $V_S = 0V$

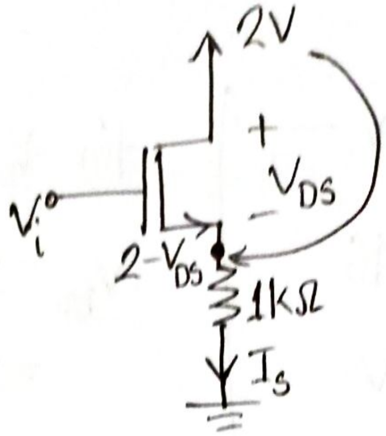
$\therefore V_{GS} = 0.3V \geq V_T\,(0.2V)$

So, MOSFET will not be in cut off.

Assuming saturation now —

$$I_D = \frac{1}{2} k V_{ov}^2$$

from the ckt —



$$I_D = I_S = \frac{2 - V_{DS} - 0}{1}$$

$$= 2 - V_{DS}$$

$$\therefore I_D = \frac{1}{2} k V_{ov}^2$$

$$\Rightarrow 2 - V_{DS} = \frac{1}{2} \times 4 \times (V_{GS} - V_T)^2$$

Now problem is that we do not have the values of V_{DS} or V_{GS}, but we do know the values of V_D & V_G. So, we have to split V_{DS} to $V_D - V_S$ & V_{GS} to $V_G - V_S$.

$$\Rightarrow 2 - (V_D - V_S) = \frac{1}{2} \times 4 \times (V_G - V_S - V_T)^2$$

$$\Rightarrow 2 - (2 - V_S) = 2(0.3 - 0.2 - V_S)^2$$

$$\Rightarrow V_S = 2(0.1 - V_S)^2$$

$$\Rightarrow V_S = 2V_S^2 - 0.4V_S + 0.02 \Rightarrow 2V_S^2 - 1.4V_S + 0.02 = 0$$

$$\therefore V_S = 0.0146\,V \text{ or } 0.685\,V$$

Now, we cannot directly say which on we should pick. So lets do a bit more work —

<table>
<tr>
<td>

for $V_S = 0.0146\,V$ —

$V_{DS} = 2 - 0.0146$

$= 1.9854\,V$

& $V_{GS} = 0.3 - 0.0146$

$= 0.2854\,V$

$\therefore V_{OV} = 0.0854\,V \leq V_{DS}$

So, $V_S = 0.0146\,V$ is the correct value

</td>
<td>

for $V_S = 0.685\,V$ —

$V_{DS} = 2 - 0.685$

$= 1.315$

& $V_{GS} = 0.3 - 0.685$

$= -0.385\,V$

We can discard $V_S = 0.685\,V$ directly from V_{GS} being $-ve$.

</td>
</tr>
</table>

$\therefore$ MOSFET is in saturation mode & $V_0 = V_S = 0.0146\,V$

(ii) for $V_i = 1V$, lets assume MOSFET is in saturation —

$I_D = \frac{1}{2} K V_{ov}^2$

$\Rightarrow 2 - V_{DS} = \frac{1}{2} \times 4 \times (V_{GS} - V_T)^2$ [Just like before]

$\Rightarrow 2 - 2 + V_S = 2(0.8 - V_S)^2$

$\Rightarrow V_S = 2V_S^2 - 3.2V_S + 1.28$

$\Rightarrow 2V_S^2 - 4.2V_S + 1.28 = 0$

$\therefore V_S = 0.37V$ or $1.73V$

for $V_S = 0.37V$

$V_{DS} = 2 - 0.37$
$= 1.63V$

& $V_{GS} = 1 - 0.37$
$= 0.63V$

$V_{DS} \geqslant V_{GS} - 0.2$

$\therefore$ correct value & correct assumption

for $V_S = 1.73V$

$V_{DS} = 2 - 1.73$
$= 0.27V$

& $V_{GS} = 1 - 1.73$
$= -0.73$

$\therefore$ wrong value

$\therefore V_o = V_S = 0.37V$

(iii) for $V_i = 3V \rightarrow$ lets assume saturation

$$I_D = \frac{1}{2} K V_{ov}^2$$

$$\Rightarrow 2 - V_{DS} = 2(V_{GS} - V_T)^2$$

$$\Rightarrow V_S = 2(2.8 - V_S)^2$$

$$\Rightarrow V_S = 2 \cdot (V_S^2 - 5.6V_S + 7.84)$$

$$\Rightarrow V_S^2 - 6.1V_S + 7.84 = 0$$

$$\Rightarrow V_S = 1.84\,V \text{ or } 4.26V$$

V_S cannot be $4.26\,V$ because the supply is $2V$.

$\therefore V_S = 1.84V$

$\therefore V_{DS} = 0.16V$

& $V_{GS} = 1.16\,V$

$\because V_{DS} < V_{GS} - V_T$

$\therefore$ assumption for saturation is wrong

$\therefore$ the MOSFET is in triode mode –

$I_D = K(V_{OV} - \frac{1}{2}V_{DS}) \cdot V_{DS}$

$\Rightarrow -V_{DS} + 2 = 4(V_{GS} - V_T - \frac{1}{2}V_{DS}) \cdot V_{DS}$

$\Rightarrow V_S = 4(2.8 - V_S - \frac{1}{2}(2 - V_S))(2 - V_S)$

$\Rightarrow V_S = 4(1.8 - 0.5V_S) \cdot (2 - V_S)$

$\Rightarrow V_S = (7.2 - 2V_S)(2 - V_S)$

$\Rightarrow V_S = 14.4 - 4V_S - 7.2V_S + 2V_S^2$

$\Rightarrow 2V_S^2 - 12.2V_S + 14.4 = 0$

$\Rightarrow V_S = 1.6V$ or $V_S = 4.5V$

V_S cannot be 4.5V because supply is 2V.

$\therefore V_S = 1.6V$

$\therefore V_{DS} = 0.4V$

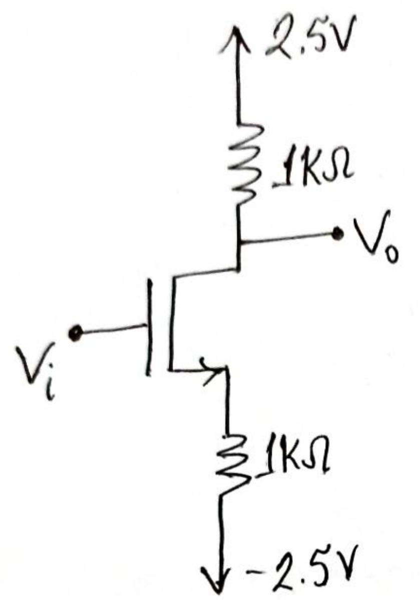
& $V_{GS} = 1.4V$

$\because V_{DS} < V_{GS} - V_T$

$\therefore$ assumption is correct. $\therefore V_O = V_S = 1.6V$

Lets make our problem harder. We are going to add one resistor to the 'Drain' and also one to the 'Source'.

Example 3



Find V_o for

(i) $V_i = -2V$

(ii) $V_i = 0V$

(iii) $V_i = 2V$

$$\boxed{\begin{array}{l} V_T = 0.7V \\ K = 4mA/V^2 \end{array}}$$

(i) for $V_i = -2V$ lets assume cut off.

then $I_D = I_S = 0A$

$\therefore V_D = 2.5V$ & $V_S = -2.5V$

$\therefore V_{GS} = -2-(-2.5) = 0.5V < V_T (0.7V)$

$\therefore$ assumption is correct.

(ii) for $V_i = 0V$, lets assume saturation —

$$I_D = \frac{1}{2} K V_{ov}^2$$

$$= \frac{1}{2} \times 4 \times (V_{GS} - V_T)^2$$

$$= 2(V_G - V_S - V_T)^2$$

$$= 2(-0.7 - V_S)^2$$

$$= 2V_S^2 + 2.8V_S + 0.98$$

Now we need I_D in terms of V_S to solve the problem —

$$I_D = I_S = \frac{V_S - (-2.5)}{1} = V_S + 2.5$$

$$\therefore V_S + 2.5 = 2V_S^2 + 2.8V_S + 0.98$$

$$\Rightarrow 2V_S^2 + 1.8V_S - 1.52 = 0$$

$$\therefore V_S = -1.43V \text{ or } 0.53V$$

for $V_S = 0.53 \rightarrow V_{GS} = -0.53V$ [not possible]

$$\therefore V_S = -1.43V$$

$$\therefore V_{GS} = 1.43V$$

Now, $I_D = \frac{2.5 - V_D}{1} = I_S = V_S + 2.5$

$\therefore V_D = -V_S = 1.43V \quad \therefore V_{DS} = 2.86V \geq V_{GS} - V_T \; (0.73V)$

So, our assumption is correct

$\therefore V_0 = V_D = 1.43V$

(iii) for $v_i = 2V$, lets assume saturation —

$$I_D = \frac{1}{2} K V_{ov}^2$$

$$\Rightarrow V_S + 2.5 = 2(2 - 0.7 - V_S)^2 \text{ [same as before]}$$

$$\Rightarrow V_S^2 - 3.1V_S + 0.44 = 0$$

$\therefore V_S = 0.15V$ or $2.95V$

$V_S = 2.95V$ isn't possible because the max node voltage in the main line is 2.5V.

$\therefore V_S = 0.15V \rightarrow V_{GS} = 1.85V$

$\therefore$ Ohm's law $\rightarrow I_D = \frac{2.5 - V_D}{1} = I_S = V_S + 2.5$

$$\Rightarrow V_D = -V_S \Rightarrow V_D = -0.15V$$

$\therefore V_{DS} = -0.3V < V_{GS} - V_T$

$\therefore$ saturation assumption is wrong; MOSFET has to be in triode mode.

$$I_D = K(V_{ov} - \frac{1}{2}V_{DS})V_{DS}$$

$$= 4(V_G - V_S - V_T - \frac{1}{2}V_D + \frac{1}{2}V_S)(V_D - V_S)$$

$$= 4(1.3 - \frac{1}{2}V_D - \frac{1}{2}V_S)(V_D - V_S)$$

Now, we know, $I_D = 2.5 - V_D = I_S = V_S + 2.5$

$$\therefore V_D = -V_S$$

$\therefore 2.5 - V_D = 4\left(1.3 - \frac{1}{2}V_D + \frac{1}{2}V_D\right)\cdot\left(V_D + V_D\right)$

$\Rightarrow 2.5 - V_D = 5.2 \times 2V_D$

$\Rightarrow 2.5 = 11.4\, V_D$

$\Rightarrow V_D = .22V$

$\therefore V_{DS} = 0.44V$

$\&\ V_{GS} = 2.22V$

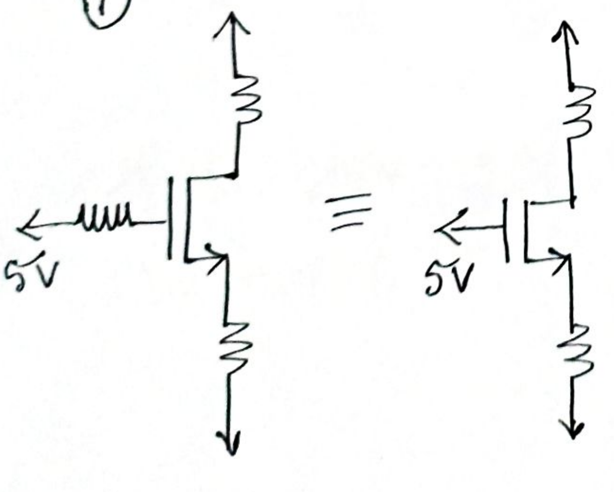
$\because V_{DS} < V_{GS} - V_T\ (1.52\,V)$

$\therefore$ assumption is correct

$\therefore V_o = V_D = 0.22\,V$

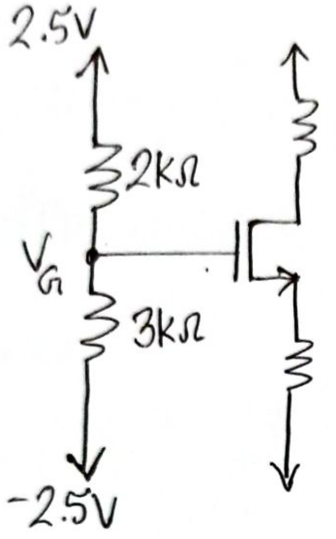
Special cases —

(i)



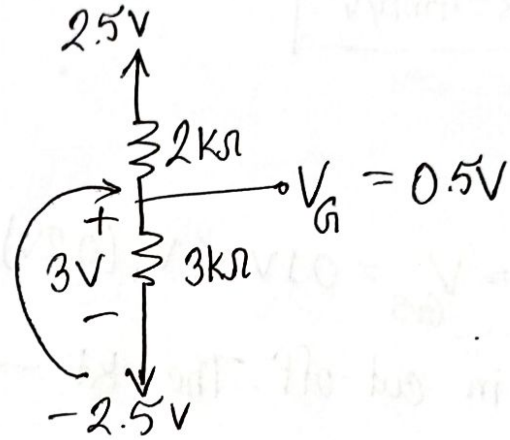
Resistance connected in series to gate terminal doesn't change anything since $I_G = 0$ all the time.

However, this property of $I_G = 0$ allows us to put complex ckts on the gate terminal from which we have to solve for V_G. The most common case —

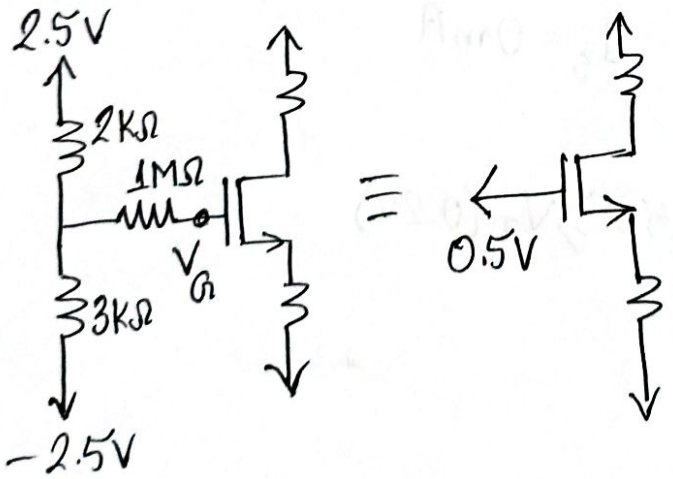


Find V_G.

→ Just a simple voltage divider suffices —

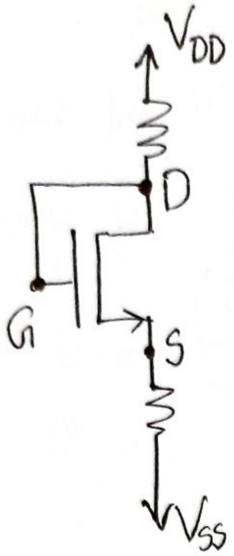


Same ckt with one small complication —



Just like before, the 1MΩ doesn't matter as $I_G = 0A$

(ii)



This type of residual connection from drain to gate forces the MOSFET to be in saturation mode, when $V_{DD} \geqslant V_T$.

In this config, $V_D = V_G$

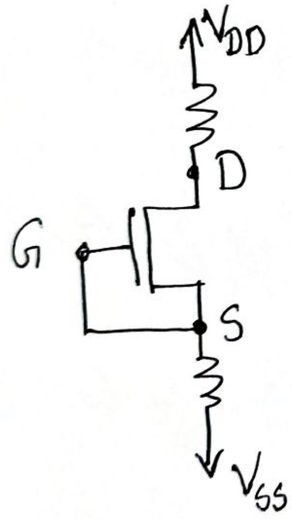
$\therefore V_{DS} = V_{GS}$

Since, V_T is always positive,

$V_{DS} \geqslant V_{GS} - V_T$ [criteria for saturation]

This proof was set in last final [Fall 24]

(iii)



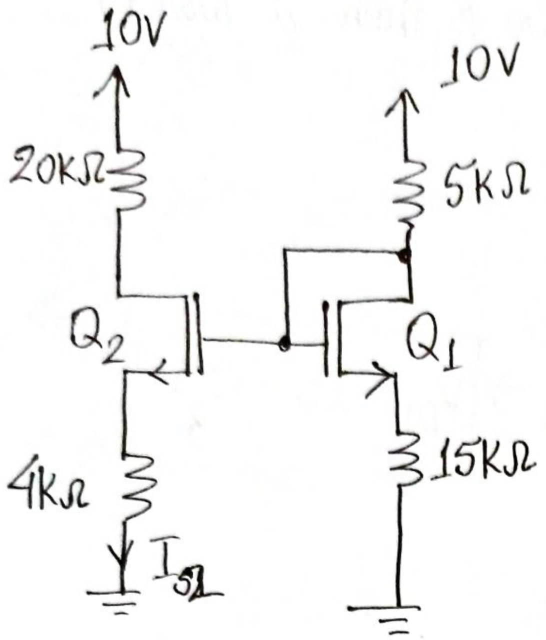
This config will always result in cut-off if $V_T > 0$.

Here, $V_G = V_S$

$\therefore V_{GS} = 0\,V$

So, unless $V_T = 0V$, the MOSFET will remain in cut-off.

Example 4



Find I_{S2}.

Assume $V_T = 1V$ & $K = 4mA/V^2$ for both MOSFETs.

Q_1 is in saturation because of the configuration. So it will be easier to work with Q_1 first.

$$I_{D1} = \frac{1}{2} K V_{ov_1}^2$$

$$\Rightarrow I_{D1} = 2 \cdot (V_G - V_{S1} - V_T)^2$$

$$= 2(V_G - V_{S1} - 1)^2$$

$$I_{D1} = I_{S1} = \frac{10 - V_{D1}}{5} = \frac{V_{S1}}{15}$$

$$\therefore \frac{V_{S1}}{15} = 2(V_G - V_{S1} - 1)^2$$

$$\Rightarrow V_{S1} = 30(V_G - V_{S1} - 1)^2 \quad \text{(i)}$$

Now, we have to understand that we cannot solve the equation (i) because we have two unknowns — V_G & V_{S1}. So, is there a way to replace V_G with V_{S1}?

Because of the residual connection, $V_G = V_{D1}$

and we know $\frac{10-V_{D1}}{5} = \frac{V_{S1}}{15}$ [From $I_{D1} = I_{S1}$]

$$\Rightarrow V_{D1} = 10 - \frac{V_{S1}}{3}$$

$$\therefore (i) \Rightarrow V_{S1} = 30\left(10 - \frac{V_{S1}}{3} - V_{S1} - 1\right)^2$$

$$\Rightarrow V_{S1} = 30\left(9 - \frac{4}{3}V_{S1}\right)^2$$

solving $\rightarrow$ $V_{S1} = 6.4V$ or $7.11V$

for $V_{S1} = 6.4V$

$V_{DS1} = 1.467V = V_{GS1}$

for $V_{S1} = 7.11V$

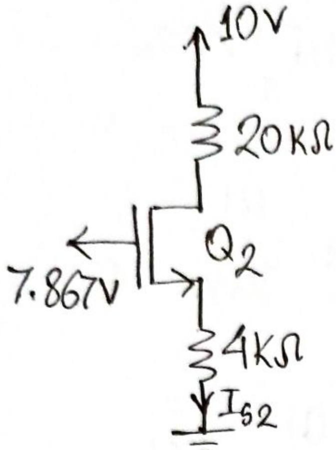
$V_{DS1} = 0.52V = V_{GS1} < V_T$

So this value is wrong

Now, $V_{D1} = 7.867V = V_G$

Now we move to Q2.

After solving for Q_1, our problem is simplified to —



Assuming Q_2 is in saturation—

$$I_{D2} = \frac{1}{2} k V_{ov_2}^2$$

$$\Rightarrow \frac{V_{S2}}{4} = 2(7.867 - 1 - V_{S2})^2$$

$$\Rightarrow \frac{V_{S2}}{8} = V_{S2}^2 + 61.889 - 15.734 V_{S2}$$

$$\Rightarrow V_{S2}^2 - 15.859 V_{S2} + 61.889 = 0$$

$$\therefore V_{S2} = 6.935V \text{ or } 8.92V$$

$V_{S2} \neq 8.92V$ [goes to cutoff, $V_{GS} < 0$]

$$\therefore V_{S2} = 6.935V$$

We know, $\frac{10 - V_{D2}}{20} = \frac{V_{S2}}{4}$

$\Rightarrow V_{D2} = 10 - 5V_{S2} = -24.675V$ [not possible, lowest voltage is 0V]

You can still go ahead and check if $V_{DS2} < V_{ov}$

Either ways, the MOSFET is in Triode mode

In triode mode —

$$I_{D2} = K\left(V_{ov2} - \frac{1}{2}V_{DS2}\right) \cdot V_{DS2}$$

$$\Rightarrow \frac{V_{S2}}{4} = 4\left(6.867 - V_{S2} - \frac{1}{2}V_D + \frac{1}{2}V_{S2}\right) \cdot \left(V_D - V_{S2}\right)$$

Now, $V_{D2} = 10 - 5V_{S2}$

$$\therefore \frac{V_{S2}}{16} = \left(6.867 - \frac{1}{2}V_{S2} - 5 + 2.5V_{S2}\right)\left(10 - 6V_{S2}\right)$$

$$\Rightarrow \frac{V_{S2}}{16} = \left(1.867 + 2V_{S2}\right)\left(10 - 6V_{S2}\right)$$

$$\Rightarrow \frac{V_{S2}}{16} = 18.67 + 20V_{S2} - 11.202V_{S2} - 12V_{S2}^2$$

$$\Rightarrow 12V_{S2}^2 - 8.7355V_{S2} - 18.67 = 0$$

$\therefore V_{S2} = -0.935V$ or $1.66V$

$V_{S2} \neq -0.935V$ [lowest voltage is 0V]

$\therefore V_{S2} = 1.66V$

$\therefore V_{D2} = 1.7V$

$\therefore V_{DS2} = 0.04V$

and $V_{GS2} = 6.207V$

$\therefore V_{DS2} < V_{GS2} - V_T$

So assumption is correct.

$\therefore I_{S2} = \frac{1.66}{4} = 0.415 mA$