

CSE260

Assignment 02

This assignment must be hand-written. Show ALL steps in ALL questions.

Marks: **5+5+5 = 15**

Graded Questions [1,2,3]:

1. Simplify the following boolean expressions to minimum number of literals:

$$(A \oplus B)(C+D)(A \odot C) + (A+B)(C'+D)(A' + CC')$$

$$\begin{aligned}
 F(A, B, C, D) &= (A \oplus B)(C+D)(A \odot C) + (A+B)(C'+D)(A' + CC') \\
 &= (AB' + A'B)(C+D)(AC + A'C') + (A+B)(C'+D)A' \\
 &= (AB' + A'B)(ACC + ACD + A'C'C + A'C'D) + (AA' + A'B)(C'+D) \\
 &= (AB' + A'B)(AC + ACD + A'C'D) + A'B'C' + A'BD \\
 &= (AB' + A'B)(AC + A'C'D) + A'B'C' + A'BD \\
 &= (AB' + A'B)(AC + A'C'D) + A'BC' + A'BD \\
 &= AAB'C + AB'A'C'D + ABA'C + A'B'A'C'D + A'BC' + A'BD \\
 &= AB'C + 0 + 0 + A'B'C'D + A'BC' + A'BD \\
 &= AB'C + A'B'C'D + A'BC' \\
 &\rightsquigarrow = AB'C + A'BC' + A'BD
 \end{aligned}$$

2. Find the complement of the following expression:

$$\begin{aligned}
 &(x' + y + z')(x + y')(x' + z) \\
 &= (x'.y.z') + (x.y') + (x'.z) \\
 &= (x.y'.z) + (x'.y) + (x.z')
 \end{aligned}$$

3. Find out SOP and POS for the following [You cannot use truth table to solve these]:

$$F(V, W, X, Y, Z) = WY + WX + X'Y$$

You must show procedure for both. You must use distributive formula for POS.

$$\sum(2, 3, 10, 11, 12, 13, 14, 15, 18, 19, 26, 27, 28, 29, 30, 31)$$

$$\prod(0, 1, 4, 5, 6, 7, 8, 9, 16, 17, 20, 21, 22, 23, 24, 25)$$

SOP

$$\begin{aligned} F(V,W,X,Y,Z) &= WY + WX + X'Y \\ &= WY(V+V')(X+X')(Z+Z') + WX(V+V')(Y+Y')(Z+Z') + X'Y(V+V')(W+W')(Z+Z') \\ &= WY(VX+VX'+V'X+V'X')(Z+Z') + WX(VY+VY'+V'Y+V'Y')(Z+Z') + X'Y(VW+VW'+V'W+V'W')(Z+Z') \\ &= WY(VXZ+VXZ'+VX'Z+VX'Z'+V'XZ+V'XZ'+V'X'Z+V'X'Z') + WX(VYZ+VYZ'+VYZ+VYZ'+VYZ'+VYZ'+VYZ') + X'Y(VWZ+VWZ'+VW'Z+VW'Z'+V'WZ+V'WZ'+V'W'Z+V'W'Z') \\ &= WYVXZ + WYVXZ' + WYVX'Z + WYVX'Z' + WYV'XZ + WYV'XZ' + WYV'X'Z + WYV'X'Z' + WXVYZ + WXVYZ' + WXVY'Z + WXVY'Z' + WXV'YZ + WXV'YZ' + WXV'Y'Z + WXV'Y'Z' + X'YVWZ + X'YVWZ' + X'YVW'Z + X'YVW'Z' + X'YV'WZ + X'YV'WZ' + X'YV'W'Z + X'YV'W'Z' \\ &= VWXYZ + VWXYZ' + VWX'YZ + VWX'YZ' + V'WXXYZ + V'WXXYZ' + V'WX'YZ + V'WX'YZ' + VWXYZ + VWXYZ' + VWXY'Z + VWXY'Z' + V'WXXYZ + V'WXXYZ' + V'WX'YZ + V'WX'YZ' + V'WX'YZ + V'WX'YZ' + VWX'YZ + VWX'YZ' + VWX'YZ + VWX'YZ' + VWX'YZ + VWX'YZ' + VWX'YZ + VWX'YZ' + V'WX'YZ + V'WX'YZ' + V'WX'YZ + V'WX'YZ' + V'WX'YZ + V'WX'YZ' + V'WX'YZ + V'WX'YZ' \\ &= \sum(11111, 11110, 11011, 11010, 01111, 01110, 01011, 01010, 11111, 11110, 11101, 11100, 01111, 01110, 01101, 01100, 11011, 11010, 10011, 10010, 01011, 01010, 00011, 00010) \\ &= \sum(31, 30, 27, 26, 15, 14, 11, 10, 31, 30, 29, 28, 15, 14, 13, 12, 27, 26, 19, 18, 11, 10, 3, 2) \\ &= \sum(2, 3, 10, 11, 12, 13, 14, 15, 18, 19, 26, 27, 28, 29, 30, 31) \end{aligned}$$

POS

$$\begin{aligned} F(V,W,X,Y,Z) &= WY + WX + X'Y \\ &= (WY + WX + X')(WY + WX + Y) \\ &= (WY + (W + X')(X + X')) (WY + (W + Y)(X + Y)) \\ &= (WY + W + X') (WY + (W + Y)(X + Y)) \\ &= (W + X' + W)(W + X' + Y)((W + Y)(X + Y) + W)(W + Y)(X + Y) + Y \\ &= (W + X')(W + X' + Y)(W + Y)(W + X + Y)(X + Y) \\ &= (W + X' + VV')(W + X' + Y + VV')(W + Y + VV')(W + X + Y + VV')(X + Y + VV') \\ &= (V + W + X')(V' + W + X')(V + W + X' + Y)(V' + W + X' + Y)(V + W + Y)(V' + W + Y)(V + W + X + Y)(V' + W \end{aligned}$$

$$\begin{aligned}
& +X+Y)(V+X+Y)(V'+X+Y) \\
& = \\
& (V+W+X'+Y)(V+W+X'+Y')(V'+W+X'+Y)(V'+W+X'+Y')(V+W+X'+Y+Z)(V+W+X'+Y+Z') \\
& (V'+W+X'+Y+Z)(V'+W+X'+Y+Z')(V+W+X+Y)(V+W+X'+Y)(V'+W+X+Y)(V'+W+X'+Y)(\\
& V+W+X+Y+Z)(V+W+X+Y+Z')(V+W+X+Y+Z)(V'+W+X+Y+Z')(V+W+X+Y)(V+W'+X+Y) \\
& (V'+W+X+Y)(V'+W'+X+Y) \\
& = \\
& (V+W+X'+Y+Z)(V+W+X'+Y+Z')(V+W+X'+Y'+Z)(V+W+X'+Y'+Z')(V'+W+X'+Y+Z)(V'+W \\
& +X'+Y+Z)(V'+W+X'+Y+Z')(V'+W+X'+Y'+Z')(V+W+X'+Y+Z)(V+W+X'+Y+Z')(V+W+X'+Y+ \\
& Z')(V'+W+X+Y+Z)(V'+W+X+Y+Z')(V'+W+X'+Y+Z)(V'+W+X'+Y+Z')(V+W+X+Y+Z)(V+ \\
& W+X+Y+Z')(V'+W+X+Y+Z)(V'+W+X+Y+Z')(V+W+X+Y+Z)(V+W+X+Y+Z')(V+W'+X+Y \\
& +Z)(V+W'+X+Y+Z')(V'+W+X+Y+Z)(V'+W+X+Y+Z')(V'+W'+X+Y+Z)(V'+W'+X+Y+Z') \\
& = \prod (00100, 00101, 00110, 00111, \\
& 10100, 10101, 10110, 10111, 00100, 00101, 10100, 10101, 00000, 00001, 00100, 00101, 10000, 100 \\
& 01, 10100, 10101, 00000, 00001, 10000, 10001, 00000, 00001, 01000, 01001, 10000, 10001, 11000, \\
& 11001) \\
& = \prod (4, 5, 6, 7, 20, 21, 22, 23, 4, 5, 20, 21, 0, 1, 16, 17, 1, 0, 4, 5, 16, 17, 20, 21, 0, 1, 8, 9, 16, 17, 24, 25) \\
& = \prod (0, 1, 4, 5, 6, 7, 8, 9, 16, 17, 20, 21, 22, 23, 24, 25)
\end{aligned}$$

Ungraded Questions [4, 5, 6, 7]:

4. Simplify the following boolean expressions to minimum number of literals:

$$\begin{aligned}
 & (A'B+B')BCC' + CB'C + D \\
 & = A'BBCC' + B'BCC' + CB'C + D \\
 & = 0 + 0 + B'C + D \\
 & = B'C + D
 \end{aligned}$$
5. Draw the following expression using NAND gate only. You cannot use more than 2 inputs in the gates: $F(A,B,C,D) = AB + C'D$
 Do it yourself.
6. Draw the following expression using NAND gate only. You cannot use more than 2 inputs in the gates: $F(A,B,C,D) = AB + C'D$
 Do it yourself.
7. Find out SOP and POS for the following [You cannot use truth table to solve these]:

$$F(A,B,C,D,E) = A + B'CD'$$

SOP

$$\begin{aligned}
F(A,B,C,D,E) &= A + B'CD' \\
&= A(B+B')(C+C')(D+D')(E+E') + B'CD'(A+A')(E+E') \\
&= (AB + AB')(CD+CD'+C'D+C'D')(E+E') + B'CD' (AE+AE'+A'E+A'E') \\
&= (AB + AB')(CDE+CD'E+C'DE+C'D'E) + CDE'+CD'E'+C'DE'+C'D'E' + \\
&\quad AEB'CD'+AE'B'CD'+A'E'B'CD' \\
&= ABCDE+ABCD'E+ABC'DE+ABC'D'E + ABCDE'+ABCD'E'+ABC'DE'+ABC'D'E' + \\
&\quad AB'CDE+AB'CD'E+AB'C'DE+AB'C'D'E + AB'CDE'+AB'CD'E'+AB'C'DE'+AB'C'D'E'+ \\
&\quad AB'CD'E+AB'CD'E'+A'B'CD'E+A'B'CD'E' \\
&= \sum(11111, 11101, 11011, 11001, 11110, 11100, 11010, 11000, 10111, 10101, 10011, 10001, \\
&\quad 10110, 10100, 10010, 10000, 10101, 10100, 00101, 00100) \\
&= \sum(00100, 00101, 10000, 10001, 10010, 10011, 10100, 10101, 10110, 10111, 11000, 11001, \\
&\quad 11010, 11011, 11100, 11101, 11110, 11111) \\
&= \sum(4, 5, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31)
\end{aligned}$$

POS

$$\begin{aligned}
F(A,B,C,D,E) &= A + B'CD' \\
&= (A+B')(A+CD') \\
&= (A+B')(A+C)(A+D') \\
&= (A+B'+CC')(A+C+BB')(A+D'+BB') \\
&= (A+B'+C)(A+B'+C')(A+C+B)(A+C+B')(A+D'+B)(A+D'+B') \\
&= (A+B'+C+DD')(A+B'+C'+DD')(A+C+B+DD')(A+C+B'+DD') \\
&\quad (A+D'+B+CC')(A+D'+B'+CC') \\
&= (A+B'+C+D)(A+B'+C+D')(A+B'+C'+D)(A+B'+C'+D') \\
&\quad (A+C+B+D)(A+C+B+D')(A+C+B'+D)(A+C+B'+D') \\
&\quad (A+D'+B+C)(A+D'+B+C')(A+D'+B'+C)(A+D'+B'+C') \\
&= (A+B'+C+D+EE')(A+B'+C+D'+EE')(A+B'+C'+D+EE')(A+B'+C'+D'+EE') \\
&\quad (A+C+B+D+EE')(A+C+B+D'+EE')(A+C+B'+D+EE')(A+C+B'+D'+EE') \\
&\quad (A+D'+B+C+EE')(A+D'+B+C'+EE')(A+D'+B'+C+EE')(A+D'+B'+C'+EE') \\
&= (A+B'+C+D+E)(A+B'+C+D+E')(A+B'+C+D'+E)(A+B'+C'+D+E)(A+B'+ \\
&\quad C'+D+E')(A+B'+C'+D'+E)(A+B'+C'+D'+E')(A+B+C+D+E)(A+B+C+D+E')(A+B+C+D'+ \\
&\quad E)(A+B+C+D'+E')(A+B'+C+D+E)(A+B'+C+D+E')(A+B'+C+D'+E)(A+B'+C+D'+E')(A+ \\
&\quad B+C+D'+E)(A+B+C+D'+E')(A+B+C'+D'+E)(A+B+C'+D'+E')(A+B'+C+D'+E)(A+B'+C \\
&\quad +D'+E')(A+B'+C'+D'+E)(A+B'+C'+D'+E') \\
&= \prod(01000, 01001, 01010, 01011, 01100, 01101, 01110, 01111, 00000, 00001, 00010, 00011, 01000, 0 \\
&\quad 1001, 01010, 01011, 00010, 00011, 00110, 00111, 01010, 01011, 01110, 01111) \\
&= \prod(8, 9, 10, 11, 12, 13, 14, 15, 0, 1, 2, 3, 8, 9, 10, 11, 2, 3, 6, 7, 10, 11, 14, 15) \\
&= \prod(0, 1, 2, 3, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15)
\end{aligned}$$

