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Hamming Distance

tags: Data communication

main concept

The number of 1's in the result of a XOR operation on two bits is the hamming distance.

example:

$$10101$$

$$00100$$

between this two stream of bits the hamming distance is :

$$10101 \oplus 00100$$

the result is

$$10001$$

here the number of 1's is 2

So the hamming distance here is 2

minimum hamming distance

for a given a set of bits, the lowest hamming distance between each possible combination of pairs from the set is minimum hamming distance.

example :

dataword	codeword
00	00000
01	01011
10	10101
11	11110

here codeword are the set of bits.

all possible hamming distances :

$$
\begin{aligned}
d(00000, 01011) &= 00000 \oplus 01011 = 01011 \qquad (\text{distance} = 3) \\
d(00000, 10101) &= 00000 \oplus 10101 = 10101 \qquad (\text{distance} = 3) \\
d(00000, 11110) &= 00000 \oplus 11110 = 11110 \qquad (\text{distance} = 4) \\
d(01011, 10101) &= 01011 \oplus 10101 = 11110 \qquad (\text{distance} = 4) \\
d(01011, 11110) &= 01011 \oplus 11110 = 10101 \qquad (\text{distance} = 3) \\
d(10101, 11110) &= 10101 \oplus 11110 = 01011 \qquad (\text{distance} = 3)
\end{aligned}
$$

the least distance here is 3, hence minimum hamming distance or $d_{min} = 3$

d_{min} notation is used in Block Code Parameters