Assignment-4 Solution

#1 (a) one root is 1. Therefore, we can write

$$f(x) = x^2(x-1) - 4(x-1) = (x^2-4)(x-1)$$
$$= (x+2)(x-2)(x-1)$$

Remaining two roots are $x_* = \pm 2$ such that $f(x) = 0$. ✓

(b) $f(x) = x^3 - x^2 - 4x + 4 = 0 \Rightarrow x^3 + 4 = x^2 + 4x = x(x+4)$

$$\Rightarrow x = \frac{x^3+4}{x+4} = g_1(x). \checkmark$$

And $f(x) = x^3 - x^2 - 4x + 4 = 0 \Rightarrow x^3 - 4x + 4 = x^2$

$$\Rightarrow x = +\sqrt{x^3 - 4x + 4} = g_2(x). \checkmark$$

(c) $$\lambda = \left|\frac{dg_1}{dx}\right|_{x=x_*} = \left|\frac{2x^3+12x^2-4}{(x+4)^2}\right|_{x=+1,-2,2}$$

$$= \begin{cases} 0.4 \text{ for } x=1, \text{ converging to } x=1. \\ 7 \text{ for } x=-2, \text{ diverging} \\ 5/3 \text{ for } x=+2, \text{ diverging.} \end{cases}$$

Now $$\lambda = \left|\frac{dg_2}{dx}\right|_{x=x_*} = \left|\frac{\frac{3}{2}x^2-2}{\sqrt{x^3-4x+4}}\right|_{x=+1,-2,+2}.$$

$$= \begin{cases} 0.5 \text{ for } x=1; \text{ converging to } x=1 \\ 2 \text{ for } x=-2, \text{ diverging} \\ 2 \text{ for } x=2, \text{ diverging.} \end{cases}$$

#2 for superlinear, $g(x) = x - \frac{f(x)}{f'(x)} \Rightarrow g(x) = x - \frac{x^3 - x + \sin x}{3x^2 - 1 + \cos x}$

By inspection $f(x) = x^3 - x + \sin(x)$ has only one exact root at $x_* = 0$. In the following, we start iteration at $x_0 = 1.5$ and find x_n such that $|x_* - x_n| \le \delta = 10^{-5}$
$= 0.00001.$

k	x_k	$x_{k+1} = g(x_k)$	is $\lvert f(x_k) \rvert \le \delta = 1 \times 10^{-5}$
0	1.5	1.00651	$0.85812 \not\le \delta$
1	1.00651	0,67313	$0.25530 \not\le \delta$
2	0,67313	0,44941	$0.07579 \not\le \delta$
3	0,44941	0.29981	$0.02248 \not\le \delta$
4	0,29981	0,19993	$0.00666 \not\le \delta$
5	0,19993	0,13330	$0.00197 \not\le \delta$
6	0,13330	0,08887	$0.00058 \not\le \delta$
7	0.08887	0,05925	$0.00017 \not\le \delta$
8	0,05925	0.03950	$5 \times 10^{-5} \not\le \delta$
9	0.03950	0,02633	$2 \times 10^{-5} \not\le \delta$
10	0.02633	0,01755	$\boxed{0.5 \times 10^{-5} \le \delta}$

Hence for ~~n =~~ $n = k = 11$ iteration, we obtaind $f(x_{11}) \le \delta$ or $f(x_{11}) \approx 0$.

therefore $x_* \approx x_{11} = 0.01755$ with $\delta = 10^{-5}$.