MCQ: Choose Only One Answer.

1. A function has roots at -3, 2 and 5. If start the iteration using the iteration formula $x_{k+1} = g(x_k)$ with $x_0 = 0$, then to which root the iteration most likely converges to? Assuming it is converging.

   A. -3. **B.** 2. **C.** 5. **D.** It can not be determined.

   1. **B**

2. Which of the following should be avoided if we do not fall into an infinite loop while finding a root of a function?

   A. There should not be any critical point. **B.** The critical point must be a negative number. **C.** The critical point should not be in between two successive iterations. **D.** The critical must not be at $x = 0$.

   2. **C**

3. Consider a linear system which has only one unique solution. Which of the following is the condition for the existence of a unique solution?

   A. $\det A \neq 0$ if you use Gaussian method. **B.** $\det U \neq 0$(if you use LU-decomposition method.
   C. The matrix Q must be a square matrix if you use QR-decomposition method. **D.** All of the above.

   3. **D**

4. Compute upper bound of error of numerical integration of the function e^x for the interval $[0, 1]$ using the Trapezium rule. Consider up to two significant figure.

   A. 0.90. **B.** 0.23. **C.** 0.84. **D.** 0.63.

   4. **B**

5. In composite Newton-Cotes formula, we

   A. for each sub-interval, apply the trapezoidal rule, and multiply them.
   B. for one sub-interval, apply closed Newton-Cotes formula and then for the next we apply open Newton-Cotes formula. Thus we get result for all the sub-intervals and we multiply all of them.
   C. for each sub-interval, apply the trapezoidal rule, and add them up.
   D. for one sub-interval, apply closed Newton-Cotes formula and then for the next we apply open Newton-Cotes formula. Thus we get result for all the sub-intervals and we add all of them.

   5. **C**

Problems: Marks are as indicated

6. Consider the function $f(x) = e^{0.5x} + \dfrac{1}{30}x^2$ which is continuous on the interval $[0, 2]$.

   (a) (2 marks) Compute the exact value of integration $I(f)$.

   Solution: $I(f) = \displaystyle\int_0^2 \Big[e^{0.5} + \frac{1}{30}x^2\Big]dx = \Big[\frac{e^{0.5x}}{0.5} + \frac{x^3}{30 \times 3}\Big]_0^2 \implies I(f) = 3.52545.\checkmark$

   (b) (3 marks) Compute the approximate integral $C_{1,2}$ using the Composite Newton-Cotes formula.
   Here $m = 2$, $a = 0$, $b = 2$ and $h = (b - a)/m = (2 - 0)/2 = 1$. Hence, $x_1 = 1$. Therefore, by the closed Composite Newton-Cotes rule, we find,

$$\begin{aligned} C_{1,2} &= \frac{h}{2}\big[f(a) + 2f(x_1) + f(b)\big] \,, \\ &= \frac{1}{2}\big[f(0) + 2f(1) + f(2)\big] \,, \\ &= 0.50\big[1 + 2(1.68205) + 2.85162\big] \,, \\ \therefore\ C_{1,2} &= 3.60786 \,. \checkmark \end{aligned}$$