Set-A

Diss-Kanekt is a platform used by university students to manage course registration, view grades, and access their academic dashboard. Following a major issue, *MD IT service* started tracking how many students accessed the platform during course registration period and how many student support tickets were generated in that period.

The table below presents the recorded data for the first three days of the course registration week, where x represents the number of platform visits (in thousands), and y represents the number of support tickets generated that day:

x	5	10	15
y	400	500	600

Now, your task is to find the equation of a best-fit straight line that gives the least error while fitting the data above, by answerring the following:

1. (2 marks) Represent the above information of the over-determied system using matrices A, x and b.
2. (7 marks) Use the QR-decomposition method to find the equation of the best-fit straight line that gives the least error while fitting the data above.
3. (1 mark) For upcoming course registration phase, platform usage is expected to rise to 1,500,000. Estimate how many support tickets the *MD IT service* should expect during that phase on your model.

Set-B

Diss-Kanekt is a platform used by university students to manage course registration, view grades, and access their academic dashboard. Following a major issue, *MD IT service* started tracking how many students accessed the platform during course registration period and how many student support tickets were generated in that period.

The table below presents the recorded data for the first three days of the course registration week, where x represents the number of platform visits (in thousands), and y represents the number of support tickets generated that day:

x	10	20	30
y	700	800	900

Now, your task is to find the equation of a best-fit straight line that gives the least error while fitting the data above, by answerring the following:

1. (2 marks) Represent the above information of the over-determied system using matrices A, x and b.
2. (7 marks) Use the QR-decomposition method to find the equation of the best-fit straight line that gives the least error while fitting the data above.
3. (1 mark) For upcoming course registration phase, platform usage is expected to rise to 1,800,000. Estimate how many support tickets the *MD IT service* should expect during that phase on your model.

Set-A

1.

$straight\ line \rightarrow a_0 + a_1 x$

$P_1(x_0) = a_0 + a_1(x_0) \Rightarrow f(x_0) = a_0 + a_1(5) = 400$

$P_1(x_1) = a_0 + a_1(x_1) \Rightarrow f(x_1) = a_0 + a_1(10) = 500$

$P_1(x_2) = a_0 + a_1(x_2) \Rightarrow f(x_2) = a_0 + a_1(15) = 600$

$$A = \begin{bmatrix} 1 & 5 \\ 1 & 10 \\ 1 & 15 \end{bmatrix}$$

$$x = \begin{bmatrix} a_0 \\ a_1 \end{bmatrix}$$

$$b = \begin{bmatrix} 400 \\ 500 \\ 600 \end{bmatrix}$$

2.

$$A = \begin{array}{c} \begin{bmatrix} 1 & 5 \\ 1 & 10 \\ 1 & 15 \end{bmatrix} \\ \begin{array}{cc} u_1 & u_2 \end{array} \end{array}$$

$$p_1 = u_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$q_1 = \frac{p_1}{|p_1|} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{bmatrix}$$

$$p_2 = u_2 - \left(u_2^T q_1\right) q_1 = \begin{bmatrix} 5 \\ 10 \\ 15 \end{bmatrix} - \begin{bmatrix} 5 & 10 & 15 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{bmatrix} q_1$$

$$p_2 = \begin{bmatrix} 5 \\ 10 \\ 15 \end{bmatrix} - 10\sqrt{3} \begin{bmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{bmatrix}$$

$$p_2 = \begin{bmatrix} 5 \\ 10 \\ 15 \end{bmatrix} - \begin{bmatrix} 10 \\ 10 \\ 10 \end{bmatrix}$$

$$p_2 = \begin{bmatrix} -5 \\ 0 \\ 5 \end{bmatrix}$$

$$q_2 = \frac{p_2}{|p_2|} = \frac{1}{5\sqrt{2}} \begin{bmatrix} -5 \\ 0 \\ 5 \end{bmatrix} = \begin{bmatrix} -\frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$Q = \begin{bmatrix} \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$R = Q^T A = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} 1 & 5 \\ 1 & 10 \\ 1 & 15 \end{bmatrix}$$

$$R = \begin{bmatrix} \sqrt{3} & 10\sqrt{3} \\ 0 & 5\sqrt{2} \end{bmatrix}$$

$$R\,x = Q^T\,b$$

$$\begin{bmatrix} \sqrt{3} & 10\sqrt{3} \\ 0 & 5\sqrt{2} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} 400 \\ 500 \\ 600 \end{bmatrix}$$

$$\begin{bmatrix} \sqrt{3} & 10\sqrt{3} \\ 0 & 5\sqrt{2} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 500\sqrt{3} \\ 100\sqrt{2} \end{bmatrix}$$

$$a_1 = \frac{100\sqrt{2}}{5\sqrt{2}} = 20$$

$$a_0 = \frac{500\sqrt{3} - \left(10\sqrt{3}\right)20}{\sqrt{3}} = 300$$

$$\therefore P_1(x) = 300 + 20x$$

3.

$$P_1(x) = 300 + 20x$$

$$P_1(1500) = 300 + 20 \times 1500 = 30300$$

Set-B

4.

$straight\ line \rightarrow a_0 + a_1 x$

$P_1(x_0) = a_0 + a_1(x_0) \Rightarrow f(x_0) = a_0 + a_1(10) = 700$

$P_1(x_1) = a_0 + a_1(x_1) \Rightarrow f(x_1) = a_0 + a_1(20) = 800$

$P_1(x_2) = a_0 + a_1(x_2) \Rightarrow f(x_2) = a_0 + a_1(30) = 900$

$$A = \begin{bmatrix} 1 & 10 \\ 1 & 20 \\ 1 & 30 \end{bmatrix}$$

$$x = \begin{bmatrix} a_0 \\ a_1 \end{bmatrix}$$

$$b = \begin{bmatrix} 700 \\ 800 \\ 900 \end{bmatrix}$$

5.

$$A = \begin{array}{c} \begin{bmatrix} 1 & 10 \\ 1 & 20 \\ 1 & 30 \end{bmatrix} \\ \begin{matrix} u_1 & u_2 \end{matrix} \end{array}$$

$$p_1 = u_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$q_1 = \frac{p_1}{|p_1|} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{bmatrix}$$

$$p_2 = u_2 - \left(u_2^{\ T} q_1\right) q_1 = \begin{bmatrix} 10 \\ 20 \\ 30 \end{bmatrix} - \begin{bmatrix} 10 & 20 & 30 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{bmatrix} q_1$$

$$p_2 = \begin{bmatrix} 10 \\ 20 \\ 30 \end{bmatrix} - 20\sqrt{3} \begin{bmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{bmatrix}$$

$$p_2 = \begin{bmatrix} 10 \\ 20 \\ 30 \end{bmatrix} - \begin{bmatrix} 20 \\ 20 \\ 20 \end{bmatrix}$$

$$p_2 = \begin{bmatrix} -10 \\ 0 \\ 10 \end{bmatrix}$$

$$q_2 = \frac{p_2}{|p_2|} = \frac{1}{10\sqrt{2}} \begin{bmatrix} -10 \\ 0 \\ 10 \end{bmatrix} = \begin{bmatrix} -\frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$Q = \begin{bmatrix} \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$R = Q^T A = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} 1 & 10 \\ 1 & 20 \\ 1 & 30 \end{bmatrix}$$

$$R = \begin{bmatrix} \sqrt{3} & 20\sqrt{3} \\ 0 & 10\sqrt{2} \end{bmatrix}$$

$$R\,x = Q^T\,b$$

$$\begin{bmatrix} \sqrt{3} & 20\sqrt{3} \\ 0 & 10\sqrt{2} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} 700 \\ 800 \\ 900 \end{bmatrix}$$

$$\begin{bmatrix} \sqrt{3} & 20\sqrt{3} \\ 0 & 10\sqrt{2} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 800\sqrt{3} \\ 100\sqrt{2} \end{bmatrix}$$

$$a_1 = \frac{100\sqrt{2}}{10\sqrt{2}} = 10$$

$$a_0 = \frac{800\sqrt{3} - \left(20\sqrt{3}\right)10}{\sqrt{3}} = 600$$

$$\therefore P_1(x) = 600 + 10x$$

6.

$$P_1(x) = 600 + 10x$$

$$P_1(1800) = 600 + 10 \times 1800 = 18600$$