'Assignment - 5'

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Ans. to the que. no-01

Given linear systems,

$$2x+y-z=1$$
$$x+2y+z=0$$
$$-x-y+2z=2$$

We identify,

$$A=\begin{pmatrix} 2 & 1 & -1 \\ 1 & 2 & 1 \\ -1 & -1 & 2 \end{pmatrix}, \quad b=\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}, \quad x=\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

here, $\det(A)=6$. As $\det(A)\neq 0$, there exists a solution.

Constructing the augmented matrix,

$$Aug(A)=(A|b)=\left(\begin{array}{ccc|c} 2 & 1 & -1 & 1 \\ 1 & 2 & 1 & 0 \\ -1 & -1 & 2 & 2 \end{array}\right)$$

as matrix A has 3x3 dimension, we have to $(n-1)=2$ row operations to form,

$$Aug(A) \longrightarrow (U|b')$$

1st row operation,

$$m_{21} = \frac{a_{21}}{a_{11}} = \frac{1}{2}$$

$$m_{31} = \frac{a_{31}}{a_{11}} = \frac{-1}{2}$$

$$r_1 \to r_1' = r_1$$

$$r_2 \to r_2' = r_2 - m_{21} r_1$$

$$= \left(1 \quad 2 \quad 1 \mid 0\right) - \frac{1}{2}\left(2 \quad 1 \quad -1 \mid 1\right)$$

$$= \left(0 \quad \frac{3}{2} \quad \frac{3}{2} \mid -\frac{1}{2}\right)$$

$$r_3 \to r_3' = r_3 - m_{31} r_1$$

$$= \left(-1 \quad -1 \quad 2 \mid 2\right) + \frac{1}{2}\left(2 \quad 1 \quad -1 \mid 1\right)$$

$$= \left(0 \quad -\frac{1}{2} \quad \frac{3}{2} \mid \frac{5}{2}\right)$$

therefore,

$$Aug(A) \xrightarrow{\text{1st row operation}} \left(\begin{array}{ccc|c} 2 & 1 & -1 & 1 \\ 0 & \frac{3}{2} & \frac{3}{2} & -\frac{1}{2} \\ 0 & -\frac{1}{2} & \frac{3}{2} & \frac{5}{2} \end{array}\right)$$

applying 2nd row operation,

$$m_{32} = \frac{a_{32}}{a_{22}} = \frac{-\frac{1}{2}}{\frac{3}{2}} = -\frac{1}{3}$$

$r_1 \rightarrow r_1' = r_1$

$r_2 \rightarrow r_2' = r_2$

$r_3 \rightarrow r_3' = r_3 - m_{32} r_2$

$$= \left(0 \quad -\frac{1}{2} \quad \frac{3}{2} \;\middle|\; \frac{5}{2}\right) + \frac{1}{3}\left(0 \quad \frac{3}{2} \quad \frac{3}{2} \;\middle|\; -\frac{1}{2}\right)$$

$$= \left(0 \quad 0 \quad 2 \;\middle|\; \frac{7}{3}\right)$$

therefore, after 2nd row operation,

$$Aug(A) \longrightarrow \left(\begin{array}{ccc|c} 2 & 1 & -1 & 1 \\ 0 & \frac{3}{2} & \frac{3}{2} & -\frac{1}{2} \\ 0 & 0 & 2 & \frac{7}{3} \end{array}\right) = (u \,|\, b')$$

So, we get the upper triangular matrix,

$$u = \begin{pmatrix} 2 & 1 & -1 \\ 0 & \frac{3}{2} & \frac{3}{2} \\ 0 & 0 & 2 \end{pmatrix}$$

(Ans.)

Answer to the que. no - 02

from (1), we get

$$(u|b') = \left(\begin{array}{ccc|c} 2 & 1 & -1 & 1 \\ 0 & \frac{3}{2} & \frac{3}{2} & -\frac{1}{2} \\ 0 & 0 & 2 & \frac{7}{3} \end{array}\right)$$

So, we identify,

$$u = \begin{pmatrix} 2 & 1 & -1 \\ 0 & \frac{3}{2} & \frac{3}{2} \\ 0 & 0 & 2 \end{pmatrix}, \quad b' = \begin{pmatrix} 1 \\ -\frac{1}{2} \\ 7/3 \end{pmatrix}$$

Now, using backward substitution method,

$$2x_3 = 7/3$$

$$\Rightarrow x_3 = \frac{7}{6}$$

and, $\frac{3}{2}x_2 + \frac{3}{2}x_3 = -\frac{1}{2}$

$$\Rightarrow \frac{3}{2}x_2 = -\frac{1}{2} - \frac{3}{2} \times \frac{7}{6}$$

$$\Rightarrow x_2 = -\frac{9}{4} \times \frac{2}{3}$$

$$\Rightarrow x_2 = -\frac{3}{2}$$

and, $2x_1 + x_2 - x_3 = 1$

$$\Rightarrow 2x_1 = 1 + x_3 - x_2$$

$\Rightarrow 2x_1 = 1 + \frac{7}{6} + \frac{3}{2}$

$\Rightarrow \quad x_1 = \frac{11}{6}$

So, the solution, $x_1 = \frac{11}{6}$, $x_2 = -\frac{3}{2}$, $x_3 = \frac{7}{6}$ and the matrix,

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \frac{11}{6} \\ -\frac{3}{2} \\ \frac{7}{6} \end{pmatrix}$$

(Ans)

Ans. to the que. no- 03

Given linear systems,

$$2x + y - z = 1$$
$$x + 2y + z = 0$$
$$-x - y + 2z = 2$$

we identify,

$$A = A^{(1)} = \begin{pmatrix} 2 & 1 & -1 \\ 1 & 2 & 1 \\ -1 & -1 & 2 \end{pmatrix}, \quad b = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$$

Since, $\det(A)=6$ and so, there exists a solution as

$\det(A) \neq 0$.

Now, row multipliers,

$$m_{21} = \frac{a_{21}}{a_{11}} = \frac{1}{2}$$

$$m_{31} = \frac{a_{31}}{a_{11}} = \frac{-1}{2}$$

So, frobenius matrix, $F^{(1)} = \begin{pmatrix} 1 & 0 & 0 \\ -m_{21} & 1 & 0 \\ -m_{31} & 0 & 1 \end{pmatrix}$

$$\Rightarrow F^{(1)} = \begin{pmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 \\ \frac{1}{2} & 0 & 1 \end{pmatrix}$$

$$\therefore A^{(2)} = F^{(1)} \times A^{(1)}$$

$$\Rightarrow A^{(2)} = \begin{pmatrix} 1 & 0 & 0 \\ -1/2 & 1 & 0 \\ 1/2 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 2 & 1 & -1 \\ 1 & 2 & 1 \\ -1 & -1 & 2 \end{pmatrix}$$

$$\Rightarrow A^{(2)} = \begin{pmatrix} 2 & 1 & -1 \\ 0 & 3/2 & 3/2 \\ 0 & -1/2 & 3/2 \end{pmatrix}$$

Now, $m_{32} = \dfrac{a_{32}}{a_{22}} = \dfrac{-\frac{1}{2}}{\frac{3}{2}} = -\dfrac{1}{3}$

So, frobenius matrix, $F^{(2)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -m_{32} & 1 \end{pmatrix}$

$\Rightarrow F^{(2)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \frac{1}{3} & 1 \end{pmatrix}$

Finally, the frobenius matrices are,

$F^{(1)} = \begin{pmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 \\ \frac{1}{2} & 0 & 1 \end{pmatrix}$, $F^{(2)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1/3 & 1 \end{pmatrix}$

(Ans.)

Ans. to the que. no-4

from (3), we got,

$m_{21} = \frac{1}{2}$, $m_{31} = -\frac{1}{2}$, $m_{32} = -\frac{1}{3}$

Now, we know that, the unit lower triangular matrix, $L = F^{-1}$. So,

$$L = \begin{pmatrix} 1 & 0 & 0 \\ m_{21} & 1 & 0 \\ m_{31} & m_{32} & 1 \end{pmatrix}$$

$$\Rightarrow L = \begin{pmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ -1/2 & -1/3 & 1 \end{pmatrix}.$$ (Ans.)

Ans. to the que. no-05

from (3), we have,

$$A^{(2)} = \begin{pmatrix} 2 & 1 & -1 \\ 0 & 3/2 & 3/2 \\ 0 & -1/2 & 3/2 \end{pmatrix}, \quad F^{(2)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \frac{1}{3} & 1 \end{pmatrix}$$

so,

$$A^{(3)} = F^{(2)} \times A^{(2)}$$

$$\Rightarrow A^{(3)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1/3 & 1 \end{pmatrix} \times \begin{pmatrix} 2 & 1 & -1 \\ 0 & 3/2 & 3/2 \\ 0 & -1/2 & 3/2 \end{pmatrix}$$

$$\Rightarrow A^{(3)} = \begin{pmatrix} 2 & 1 & -1 \\ 0 & 3/2 & 3/2 \\ 0 & 0 & 2 \end{pmatrix} = U$$

from (4), (3)

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ -1/2 & -1/3 & 1 \end{pmatrix}, \quad b = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$$

from LU decomposition method, $A = LU$

so, $Ax = b$

$\Rightarrow LUx = b$

So, Lower triangular equation,

$$Ly = b$$

$$\Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ -1/2 & -1/3 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$$

using forward substitution method,

$$y_1 = 1$$

and, $\frac{1}{2}y_1 + y_2 = 0$

$\Rightarrow y_2 = -\frac{1}{2}y_1 = -\frac{1}{2} \times 1$

$\Rightarrow y_2 = -\frac{1}{2}$

and, $-\frac{1}{2}y_1 - \frac{1}{3}y_2 + y_3 = 2$

$\Rightarrow y_3 = 2 + \frac{1}{2}y_1 + \frac{1}{3}y_2$

$\Rightarrow y_3 = 2 + \frac{1}{2} \times 1 + \frac{1}{3} \times \left(-\frac{1}{2}\right)$

$\Rightarrow y_3 = \frac{7}{3}$

So, we get,

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 1 \\ -1/2 \\ 7/3 \end{pmatrix}$$

Then again,

$$Ux = y$$

$$\begin{pmatrix} 2 & 1 & -1 \\ 0 & 3/2 & 3/2 \\ 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ -1/2 \\ 7/3 \end{pmatrix}$$

using backward substitution,

$$2x_3 = 7/3$$

$$\Rightarrow x_3 = 7/6$$

and, $\frac{3}{2} x_2 + \frac{3}{2} x_3 = -\frac{1}{2}$

$$\Rightarrow \frac{3}{2} x_2 = -\frac{1}{2} - \left(\frac{3}{2} \times \frac{7}{6}\right)$$

$$\Rightarrow x_2 = -\frac{9}{4} \times \frac{2}{3}$$

$$\Rightarrow x_2 = -\frac{3}{2}$$

and, $2x_1 + x_2 - x_3 = 1$

$\Rightarrow 2x_1 = 1 + x_3 - x_2$

$\Rightarrow 2x_1 = 1 + \frac{7}{6} + \frac{3}{2}$

$\Rightarrow x_1 = \frac{11}{6}$

So, the solutions are, $x_1 = \frac{11}{6}$, $x_2 = -\frac{3}{2}$, $x_3 = 7/6$ and the matrix,

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 11/6 \\ -3/2 \\ 7/6 \end{pmatrix}$$

we can see that, it is similar to the one we found using gaussian elimination method.

(Ans.)