Instructions for preparing the solution script:

- Write your name, ID#, and Section number clearly in the very front page.
- Write all answers sequentially.
- Start answering a question (not the part of the question) from the top of a new page.
- Write legibly and in orderly fashion maintaining all mathematical norms and rules. Prepare a single solution file.
- Start working right away. There is no late submission form. If you miss the deadline, you need to use the make-up assignment to cover up the marks.

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1. Consider the function, $f(x) = xe^{-x} + x^3 - \sin(x)$.
   (a) (6 marks) Verify by calculation that the following three unctions are the fixed point functions for $f(x)$:
   $$(i)\ g_1(x) = e^x\left(\sin x - x^3\right), \quad (ii)\ g_2(x) = \ln\left(\frac{x}{\sin x - x^3}\right) \quad \text{and} \quad (iii)\ g_3(x) = \sin^{-1}\left(xe^{-x} + x^3\right) .$$
   (b) (6 marks) Construct a super linear fixed point function from $f(x)$ and compute its root within 1.0×10^{-5}.
2. A student in the PHY111 Lab class has been asked to determine the spring constant of an ideal spring using the Hook's law: $F(x) = kx$ (the minus sign is ignored, because it is not relevant here). He collected the following data for x vs. $F(x)$: $F(0) = 0$, $F(0.05) = 1.3$, $F(0.10) = 2.5$, $F(0.15) = 3.8$ and $F(0.20) = 5.1$. Here x is in meters and $F(x)$ is in Newton per meters. But you can ignore all units for the sake simplicity. Just do the mathematics. Since the Hook's law is a linear equation, the student need to find the best-fit straight line for the above data set and find the slope of the line which will be the spring constant.
   (a) (5 marks) Using the Discrete-square approximate method, compute the best-fit straight line and the spring constant.
   (b) (5 marks) Construct the matrices Q and R using the QR-decomposition method. Also verify that $Q^{\mathrm{T}}Q = I$.
3. Consider the function, $f(x) = xe^{-x} + x^3 - \sin(x)$ and the interval $[-0.15, 0.25]$. Answer the following:
   (a) (2 marks) Compute the exact integral: $I(f) = \displaystyle\int_{-0.15}^{0.25} f(x)dx$.
   (b) (3 marks) Evaluate $I_2(f)$ using the Simpson's rule.
   (c) (3 marks) Evaluate $C_{1,4}(f)$ using the closed Newton-Cotes formula.

Motto: Mathematics is NOT difficult, but what is difficult is to believe that mathematics is NOT difficult.