Midterm Examination : CSE330. All Sections. Set # 1
Department of Computer Science & Engineering
BRAC University
Summer 2023 Semester
Date : July 23, 2023
Time : One hour 30 minutes

Faculty Name (Initial) : _____ Student ID# : _____ Section#: _____

Instructions:

- There are four questions. **Answer any three questions**. Total marks 30.
- Use pencil for your answers. No break for bathroom/freshroom is allowed. **Must use your own calculator**. Cell phones must be turned off (Not in vibration mode). We assume that you know how to use scientific calculator of model CASIO fx-991 ES or equivalent.
- Return this question along with your answer script.
- All examinees must abide by the 'Regulations of Students Conduct' of Brac university.
- **No bathroom break**. You must use the bathroom, if necessary, before the exam starts.

Read carefully the questions below and answer properly:

1. Here the symbols have their usual meanings. Answer the following:
   (a) (2 marks) [**CO-1**] **Explain** why the base β in the floating point representation needs to be an even integer.
   (b) (1 mark) [**CO-2**] Let $\beta = 2$, $m = 4$, $e_{\min} = -2$ and $e_{\max} = 3$. **Find out** the machine epsilon using the standard convention of the floating point representation.
   (c) (2 marks) [**CO-3**] For the floating point system given in Question-(1b), **compute** the maximum and minimum numbers with and without negative support and using the Normalized form of Floating point numbers, and **express** these in decimal format.
   (d) (3+2 marks) [**CO-4**] Consider the quadratic equation, $x^2 - 16x + 1 = 0$. First **compute** where and how the loss of significance occur when you solve the quadratic equation. Then, **evaluate** the correct roots. You need to consider up to 5 significant figures.

2. Consider the following data set:

x	$f(x)$	$f'(x)$
-1	0	1
1	1	0

   (a) (2 marks) [**CO-3**] **Evaluate** the Lagrange bases for the given data.
   (b) (4 marks) [**CO-4**] **Compute** the elements of the Hermite bases.
   (c) (2 marks) [**CO-2**] **Express** the Hermite interpolation polynomial in the natural basis.
   (d) (2 marks) [**CO-1**] **Explain or state** what would be the degree of the Hermite interpolation polynomial if we add the second derivative conditions in addition to first derivative conditions.

3. Consider the function $f(x) = e^{3x} + e^{-3x}$ and the nodes at -1,0,1. Now answer the following using 3 significant figures:
   (a) (1 mark) [**CO-1**] **Write down** the matrices b and V used in Vandermonde matrix method.
   (b) (2 marks) [**CO-2**] **Calculate** the determinant of the Vandermonde matrix V .
   (c) (4 marks) [**CO-3**] Using the results of the previous two parts, **evaluate** the Taylor coefficients a_0, a_1 and a_2 to write the interpolation polynomial.
   (d) (3 marks) [**CO-4**] **Compute** the upper bound of the error for the given function for the interval $[-1.1, 1.1]$ using Cauchy's theorem.

4. Consider the function $f(x) = x\ln(x)$. Now answer the following:
   (a) (2 marks) [**CO-3**] **Evaluate** the numerical derivative of $f(x)$ at $x = 1.0$ with step size $h = 0.1$ using the forward and central difference methods up to 5 significant figures.
   (b) (4 marks) [**CO-4**] **Compute** the upper bound of the truncation error of $f(x)$ using $x_0 = 1.0$ and $h = 0.1$ for the central difference method up to 5 significant figures.
   (c) (4 marks) [**CO-2**] **Deduce** an expression for $D_h^{(1)}$ from D_h by replacing h by $h/3$ using the Richardson extrapolation method.