

Practice Problems Chapter 7: Numerical Integration

1. $\int_0^{0.8} (0.2 + 25x - 200x^2 + 675x^3 - 900x^4 + 400x^5)dx$

- (a) Find the exact value of the given integral.
- (b) Use the multi-segment Trapezoidal rule with $m = 4$ to approximate the integral.
- (c) Using the previous parts, calculate the relative error.

2. Compute the integration $\int_0^2 f(x)dx$ numerically by using Trapezoidal and Simpson's rules if the function $f(x)$ is given as:

- (a) $f(x) = \sqrt{1 + x^2}$
- (b) $f(x) = \sin(x)$
- (c) $f(x) = e^x$

3. Consider the following function: $f(x) = e^x - x$, which is continuous on the interval $[1, 3]$. Use this function to answer the following:

- (a) Find the actual integral value for this function.
- (b) Use the Composite Newton-Cotes formula to find the numerical integration for 4 segments.
- (c) Compute the error in percentage between the results obtained in the previous two parts. How can we decrease the error more?
- (d) Use the Simpson rule to find the numerical integration.

4. A function is given by $f(x) = 0.2 + 25x + 3x^2$.

Now answer the following based on this function:

- (a) Use the Trapezium rule to numerically integrate over the interval $[0, 2]$
- (b) Compute the exact integrated value of the given function.
- (c) Using the previous parts, calculate the relative error in percentage.

5. Consider a function $f(x) = \frac{1}{x(\ln x)^2}$, which is continuous on the interval $[e, e + 1]$. Now

answer the questions below based on this function:

- (a) Calculate the exact integrated value of the given function.
- (b) Find the numerical integration for $m=4$ using the Composite Newton-cotes formula.
- (c) Calculate the error in percentage from the above two parts.

6. A function is given by $f(x) = 1 + 4x - 2x^2$. Now answer the following based on this function:

- (a) Use the Trapezium rule to integrate over the interval $[1, 3]$ numerically.
- (b) Compute the exact integrated value of the given function.
- (c) Using the previous parts, calculate the relative percentage error.

7. Compute the upper bound of error for the following example:

(a) $n = 1$, $f(x) = \sin(x)$, $[0, \frac{\pi}{2}]$

(b) $n = 2$, $f(x) = \ln(1 + x)$, $[0, 1]$

(c) $n = 2$, $f(x) = \cos(x)$, $[0, \frac{\pi}{4}]$

(d) $n = 1$, $f(x) = x^3$, $[1, 2]$