MCQ: Choose Only One Answer.

1. (a) Which of the following is/are true about Chebyshev points/nodes?
   i. The Chebysheb points are equally spaced.
   ii. The Chebysheb nodes are more densed near the end of the interval.
   iii. The Chebyshev nodes resolve the Runge problem.
   A. (i, ii) only. **B.** (i, iii) only. **C.** (ii, iii) only. **D.** All of these.

   (a) **C**

   (b) A set of data values are given as $x = 3.0, 3.1, 3.2, 3.3$ and $f(x) = 2.5, 2.8, 3.2, 3.4$ respectively. The first derivative of $f(x)$ at $x = 3.2$ according to the backward difference formula is
   A. 4.0. **B.** 3.0. **C.** 2.0. **D.** 1.0.

   (b) **A**

   (c) In addition to the first derivative condition, if we impose the second derivatives as interpolation condition to obtain the interpolation polynomial, *i.e.*,

$$f(x_k) = p_n(x_k), \quad f'(x_k) = p_n'(x_k) \quad \text{and} \quad f''(x_k) = p_n''(x_k) \ ,$$

   where $k = 0, 1, 2, \cdots, n$, then the degree of the interpolation polynomial will be
   A. n. **B.** $2n+1$. **C.** $3n+2$. **D.** $4n+3$.

   (c) **C**

Problems: Marks are as indicated

2. Let $f(x) = x^3 \ln(x)$, $x_0 = 2$ and $h = 0.12$. Answer the following:

   (a) (4 marks) Compute the upperbound of the truncation error at x_0 for the central difference method.
   Solution: Here we have,

$$\begin{aligned}
\text{Upper Bound} &= \left|\frac{h^2}{3!} f'''(\xi)\right|_{\max, \xi \in (x_0 - h, x_0 + h)} , \\
&= \frac{(0.12)^2}{6} \left|\frac{d^3\xi}{d\xi^3}\left(\xi^3 \ln \xi\right)\right|_{\max, \xi \in (1.88, 2.12)} , \\
&= \frac{(0.12)^2}{6} \left|11 + 6 \ln \xi\right|_{\max, \xi \in (1.88, 2.12)} , \\
&= \frac{(0.12)^2}{6} \left|11 + 6 \ln(2.12)\right| , \\
&= 0.03722 \ . \quad \checkmark
\end{aligned}$$

   (b) (3 marks) Compute the first order Richardson extrapolation, $D_h^{(1)}$, using $h \to h/3$ replacement.
   Solution: As has been derived in the class, for $h \to h/n$ (where n is a positive number greater than 1), the first order Richardson extrapolation is defined as

$$D_h^{(1)} = \frac{n^2 D_{h/n} - D_h}{n^2 - 1} \ .$$

   Using $n = 3$ and $h = 0.12$, we get,

$$D_h^{(1)} = \frac{3^2 D_{h/3} - D_h}{3^2 - 1} = \frac{9 D_{0.04} - D_{0.12}}{8} \ .$$

   Now, we calculate from $D_h = \dfrac{f(x_0 + h) - f(x_0 - h)}{2h}$,

$$\begin{aligned}
D_{0.04} &= \frac{f(2 + 0.04) - f(2 - 0.04)}{2 \times 0.04} = \frac{f(2.04) - f(1.96)}{0.08} = \frac{(2.04)^3 \ln(2.04) - (1.96)^3 \ln(1.96)}{0.08} = 12.3213 \ . \\
D_{0.12} &= \frac{f(2 + 0.12) - f(2 - 0.12)}{2 \times 0.12} = \frac{f(2.12) - f(1.88)}{0.24} = \frac{(2.12)^3 \ln(2.12) - (1.88)^3 \ln(1.88)}{0.24} = 12.3525 \ . \\
D_{0.12}^{(1)} &= \frac{9 D_{0.04} - D_{0.12}}{8} = \frac{9 \times 12.3213 - 12.3525}{8} = 12.3174 \ . \quad \checkmark
\end{aligned}$$