**Final Examination : CSE330. All Sections. Set # 1.**

Department of Computer Science & Engineering
BRAC University
Summer Semester 2023
Date : September 03, 2023
Time : One hour 40 minutes

Faculty Name (Initial) : _____          Student ID# : _____          Section#: _____

**Instructions**:

- There are four question. Answer **all** questions. Total marks 60.

- Use preferably pencil to write down the answers. No break for bathroom/freshroom is allowed. **Must use your own calculator**. Cell phones must be turned off (Not in vibration mode). We assume that you know how to use scientific calculator of model CASIO fx-991 ES or equivalent.

- Return this question paper along with your answer script. Write the Question Set Number on the top of your answer script.

- All examinees must abide by the 'Regulations of Students Conduct' of Brac university.

**Read carefully the questions below and answer properly**:

1. Consider the function, $f(x) = x^3 - 6x^2 + 11x - 6$.
   (a) (3 marks) **[CO-1] Find out** the actual roots of the function $f(x)$.
   (b) (2+2+2 marks) **[CO-3] Compute** any two fixed point function $g(x)$ (except what is given in the Part-(c)) from the given function. For each $g(x)$, **determine** whether the $g(x)$'s are convergent or divergent.
   (c) (4+1+1 marks) **[CO-4]** Starting with $x_0 = 0.8$, and the fixed point function $g(x) = \dfrac{6}{x^2 - 6x + 11}$, **evaluate** up to six iterations, $x_6$, keeping up to 5 significant figures, by copying and filling up the following table in your answer script:

   | $k$ | $x_k$ | $x_{k+1} = g(x_k)$ |
   |-----|-------|--------------------|
   | 0   | 0.8   |                    |
   | ⋮   | ⋮     | ⋮                  |
   | 5   |       |                    |

   **Determine** which root it is converging to. Also **evaluate** the percent error between $x_6$ and the root it is converging to.

2. A linear system is described by the following equations:

$$\begin{aligned} x_1 + 2x_2 + 4x_3 &= 11 \\ 2x_1 + 5x_2 + 2x_3 &= 3 \\ 4x_1 - x_2 + x_3 &= 8. \end{aligned}$$

   Based on these equations, answer the questions below.
   (a) (2 marks) **[CO-2] Identify** the matrix $A$ and **examine** if the matrix $A$ has any pivoting problem? Explain why or why not?
   (b) (2+1+2+2 marks) **[CO-3] Construct** the Frobenius matrices $F^{(1)}$ and $F^{(2)}$ from this system. Also **compute** the unit lower triangular matrix $L$ and the upper triangular matrix $U$.
   (c) (3+3 marks) **[CO-4]** Now **evaluate** the solution of the linear system using LU decomposition method by using the triangular matrices $L$ and $U$ obtained in the previous part.

3. A business analyst has a set of data of accident involving three types of cars: Luxury, Intermediate, and Economy. The following table shows the average weight in Metric tons and the percentage of accident occurrence of three car types. The analyst is trying to find a best-fit straight line that gives the least error for these data. From the best-fit line, the analyst wants to know the percentage of accident occurrences if the car type has an average weight of 8.5 Metric tons. You need to solve this over-determined system using the QR Decomposition Method

| Car Class | Average Weight (Metric Tons) $x$ | Percentage of Accident Occurrence $f(x)$ |
|---|---|---|
| Luxury | 10 | 3 |
| Intermediate | 7 | 4 |
| Economy | 6 | 5 |

by answering the following step-by-step:

(a) (2 marks) **[CO1]** From the given data, consider the values $x$ as the average weights and $f(x)$ as the percentage of accident occurrences for three vehicles types. Now, **write down** the matrix $A$, $x$ and $b$ from these data values.

(b) (2 marks) **[CO1]** Also, **identify** the linearly independent column vectors $u_1$ and $u_2$ from the matrix A.

(c) (3 marks) **[CO3]** Use the Gram-Schmidt process to **compute** the orthonormal column matrices $q_1$ and $q_2$ from the linearly independent column vectors $u_1$ and $u_2$, and **write down** the $Q$ matrix.

(d) (3 marks) **[CO3]** Now **calculate** the upper triangular matrix $R$.

(e) (3 marks) **[CO4] Evaluate** $Q^T b$, and finally **solve** the system by evaluating $x$.

(f) (2 marks) **[CO2] Write down** the best-fit straight line $p_1(x)$ for the given data, and **find** out the approximate percentage of accident occurrences if the car type has an average weight of 8.5 Metric tons?

4. Consider the following functions $f(x) = x^3 + 2x^2 + 3$ and $g(x) = x^4 + 2x^2 + 3$, which are continuous on the interval $[0, 2]$. Answer the following questions:

(a) (2 marks) **[CO1] Find out** the exact integral values $I(f)$ and $I(g)$.

(b) (3 marks) **[CO3]** Use Composite Newton-cotes formula to **evaluate** the numerical integration $C_{1,4}$ for the function $f(x)$ using 4 segments.

(c) (2 marks) **[CO2] Compute** the relative error in percentage. **Explain** how this error can be decreased?

(d) (4+4 marks) **[CO4]** Use Simpson's rule to **evaluate** $I_2(f)$ and $I_2(g)$, and **verify** that the Simpson's rule gives exact results for integration of $f(x)$, but not for $g(x)$.