**Online Midterm Examination : CSE330. All Sections.**
Department of Computer Science & Engineering
BRAC University
Fall 2023 Semester
Date : November 08, 2023
Exam Time : One hour 15 minutes
Scanning and Uploading time : 15 minutes.

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Faculty Name (Initial) : _____ Student ID# : _____ Section#: _____

**Instructions**:

- Answer as instructed below. Total marks 40.
- Answer questions sequentially. **DO NOT** mix part of one question with another question. Write legibly so that we can follow your thoughts.
- Number your pages, and scan them sequentially when done, and prepare a single pdf file with the Top Sheet as the first page.
- Use pencil for your answers (preferable). No break for bathroom/freshroom is allowed. **Must use your own calculator**. Cell phones must be turned off (Not in vibration mode). We assume that you know how to use scientific calculator of model CASIO fx-991 ES or equivalent.
- All students MUST follow the 'Midterm Exam Policy' published before.
- All examinees must abide by the 'Regulations of Students Conduct' of Brac university.
- Wait and stay in front of the camera till the end of exam. **DO NOT** leave before that.
- **NO** Email submission. The scanned pdf file of your answer script **MUST** be submitted through the Google Form Submission Link provided in the Discord server for CSE330.

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**Read carefully the questions below and answer properly**:

1. (4 marks) **CO-1:** Answer any one from Questions-(1a-1b):
   (a) (4 marks) A function $f(x)$ is interpolated by using Newton's basis at the nodes $\{-1, 0, 1\}$ and the Newton coefficients are found to be $a_2 = 2$, $a_1 = 1$ and $a_0 = 0$. If the interpolation function is computed using Vandermonde method at the same nodes, **what would be** the Taylor coefficients of the interpolating function.
   (b) In Richardson extrapolation, the parameter $h$ is replaced by $h/2$, and hence we define the first order extrapolated derivative as
   $$D_h^{(1)} = \frac{2^2 D_{h/2} - D_h}{3} .$$
      i. (2 marks) **Explain** if it is necessary to replace $h$ by $h/2$.
      ii. (2 marks) **Write down** how we need to define $D_h^{(1)}$ if we replace $h$ by $h/n$ where $n$ is a nonzero positive integer. Note that you do not need to derive the expression.
2. **CO-2:** Answer any one from Questions-(2a-2b):
   (a) You are asked to analyze the climate change data set which contains the average temperature of a day for the last ten years. Answer the following:
      i. (3 marks) **Name** the methods you can use to represent the data set and find the average temperature of a day within the given ten years period.
      ii. (1 mark) If we want to make the data set dynamic, **which method** you should use to keep your results up -to-date.
      iii. (4 marks) Usually equally spaced data set are used to obtain the interpolation polynomial. Suppose you obtained the Taylor coefficients $a_0 = 1$, $a_1 = 0$ and $a_2 = -1$ of an interpolating polynomial using equally spaced data. Now if you use non-equally spaced data, **will** these coefficients have the same values or different values? **Explain** your answer.
   (b) Answer the following:
      i. (2+2 marks) The following Data set is generated by the function $f(x) = x\sin(x) - x + \cos(x)$.

| $x$ | 1.1 | 1.2 | 1.3 |
|---|---|---|---|
| $f(x)$ | 0.3339 | 0.2808 | 0.2201 |

Based on the above data, **compute** $f'(1.2)$ using Central Difference method, and also **calculate** the relative error. Use 4 significant figures.

ii. (4 marks) During the lecture, we derived first order numerical derivative, $D_h^{(1)}$, using Richardson extrapolation method by replacing $h \to h/2$. **Derive** an expression for $D_h^{(1)}$ by replacing $h \to h/4$ using the Richardson extrapolation method.

3. **CO-3:** Answer any two from Questions-(3a-3c):

(a) (6 marks) Consider the nodes at $(1, 10)$, $(2, 15)$, $(3, 20)$. Using Newton's divided difference method, **evaluate** a polynomial that goes through the above nodes.

(b) (6 marks) Consider the function $f(x) = x^2e^x$. **Evaluate** the numerical derivative of $f(x)$ at $x = 2.0$ with step size $h = 0.2$ using the backward and central difference methods up to 5 significant figures.

(c) (6 marks) **Evaluate** the upper bound of truncation error for the function $f(x) = x\sin(x) - x + \cos(x)$ if we compute the numerical derivative at $x = 1.2$ by the Central Difference method and $h = 0.1$.

4. **CO-4:** Answer any two from Questions-(3a-3c):

(a) (4+4 marks) A computing system exists with $\beta = 2$, exponent $e \in [-2, 3]$, and mantissa $m = 3$. **Express** the floating-point representation of the numbers $x = 13.125, y = 11.625$ using the normalized form, and **calculate** $\text{fl}[x + y]$ in binary form and then **convert** it to the decimal form.

(b) An interpolating polynomial, $p_1(x) = 1.648(x - 1)$ is derived for the function $f(x) = x\ln x$ at the nodes ($x_0 = 1$, $x_1 = 3$) using the Lagrange method. Answer the following keeping up to 4 significant figures.

i. (2 marks) **Explain** what you need to do to obtain a degree 3 interpolation polynomial for the same function $f(x)$ and for the same nodal points ($x_0 = 1$, $x_1 = 3$).

ii. (6 marks) **Calculate** the bases of the degree 3 polynomial.

(c) (8 marks) The function $f(x) = e^{3x} - e^{-3x}$ has been interpolated at the nodes at (-1, 0, 1) using Vandermonde matrix method. **Evaluate** the upper bound of the interpolation error for the interval [-1.5, 1.5] using Cauchy's theorem. Keep up to 4 significant figures.