Midterm Examination : Questions for CSE330. All Sections.
Department of Computer Science & Engineering
BRAC University
Summer Semester
Date : July 24, 2022
Time : One hour 30 minutes

Faculty Name (Initial) : ___________ Student ID# : ________________ Section#: _____

Instructions:

- There are six question. **Answer any four questions**. Total marks 40.
- Use pencil for your answers. No break for bathroom/freshroom is allowed. **Must use your own calculator**. Cell phones must be turned off (Not in vibration mode). We assume that you know how to use scientific calculator of model CASIO fx-991 ES or equivalent.
- Return this question along with your answer script.
- All examinees must abide by the 'Regulations of Students Conduct' of Brac university.

Read carefully the questions below and answer properly (All are CO1 and CO2):

1. For part (a) and (b), assume that the system has the following specification: $\beta = 2$, $m = 3$, $e_{\min} = -1$ and $e_{\max} = 1$.
   (a) (2 marks) Consider two numbers, $x = \frac{5}{8}$, and $y = \frac{6}{8}$. Convert these numbers into floating points $\text{fl}(x)$ and $\text{fl}(y)$, in base 2.
   (b) (4 marks) Compute $x \times y$, then evaluate $\text{fl}(x) \times \text{fl}(y)$, and express the answers in base 10. Compare the two results and state whether the two values match or not. Explain why or why not.

   For part (c) and part (d), assume that the system follows the denormalized convention and has the following specification: $\beta = 2$, 1 bit for sign, 4 bit for exponent and 5 bit for mantissa.
   (c) (1 mark) Find the value for $e_{\min}$ and $e_{\max}$.
   (d) (3 marks) Assuming that $e_{\min}$ and $e_{\max}$ are reserved for representing zero and infinity respectively, calculate the highest possible and the lowest possible non-negative number that can be represented by this system.

2. Consider the Runge function $f(x) = \dfrac{1}{1 + 20x^2}$ which shows Runge phenomenon/divergence behavior when $f(x)$ is interpolated by the equally-spaced nodes over some interval. To overcome this divergence, we may interpolate $f(x)$ with Chebyshev nodes.
   (a) (2 marks) Exactly what property of Chebyshev nodes allows us to overcome the Runge phenomenon?
   (b) (3 marks) Find six Chebyshev nodes over the interval $[-1.5, 1.5]$.
   (c) (5 marks) Find the interpolating polynomial using the Lagrange Form with the first 3 nodes found in part-(b), and simplify the expression of the interpolation polynomial to the standard form.

3. From a function, $f(x)$, we obtain the following data points: $\big(x_i, f(x_i)\big) = \big\{(-0.5, e), (0, 1), (0.5, e^{-1})\big\}$.
   (a) (5 marks) Using the Newton/Divided-Difference method, find the interpolating polynomial for $f(x)$ using the data points above, and using it evaluate the approximate value of $f(0.25)$.
   (b) (3+2 marks) State Cauchy's Theorem (Do not need to prove). Now suppose a function $f(x)$ is five times differentiable, and it is interpolated by a degree five polynomial. Explain or state what would be the interpolation error.

4. Consider the following data set

x	1.1	1.2	1.3	1.4
$f(x)$	30.743	40.918	54.472	72.566

   (a) (2 marks) Using the above data, compute $f'(1.2)$ using the central difference method.

(b) (3 marks) For the interval $[1.1, 1.4]$, compute the upper bound of truncation error if the above data is generated by the function, $f(x) = e^{3x} + 3x^2$ using the method used in Part-(a).

(c) (5 marks) Find an expression for $D_h^{(1)}$ by replacing h by $2h/3$ in D_h in Richardson extrapolation method. Note D_h is the numerical derivative in central difference method.

5. Consider the following function: $f(x) = x^3 + 4x^2 - 10$.

   (a) (5 marks) Find solution of $f(x) = 0$ up to 5 iterations using Newton's method starting with $x_0 = 2$. Keep up to six significant figures.

   (b) (5 marks) Consider the fixed point function, $g(x) = \dfrac{2x+1}{\sqrt{x+1}}$. Show that to be super-linearly convergent, the root must satisfy $x_\star = -\dfrac{3}{2}$.

6. Consider the following function: $f(x) = x^3 + x^2 - 25x - 25$.

   (a) (3 marks) Use interval bisection method to find the root, x_* of $f(x)$, up to 5 iterations, on the interval $[-2, 2]$.

   (b) (2 marks) Compute the minimum number of iterations required to find the root of the given function, where the error bound, $\delta = 10^{-2}$.

   (c) (3 marks) State the exact roots of $f(x)$, and construct two different fixed point functions $g(x)$ such that $f(x) = 0$.

   (d) (2 marks) Compute the convergence rate of each $g(x)$ obtained in the previous part(c) and state if the root is converging linearly, superlinearly or it is diverging.