MCQ: Choose Only One Answer

1. (a) Which of the following matrix representation of A will have a unique solution?

A. $\begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix}$ **B.** $\begin{bmatrix} 2 & 0 \\ 2 & 0 \end{bmatrix}$ **C.** $\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$ **D.** $\begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$

(a) **D**

(b) How many operations are required to solve a linear system where $Lx = b$? Here, L is a lower triangular 4×4 matrix.

A. 9 **B.** 12 **C.** 14 **D.** 16

(b) **D**

(c) We obtain the lower triangular matrix $\begin{bmatrix} 1 & 0 \\ L_{21} & 1 \end{bmatrix}$ from the matrix $\begin{bmatrix} 2 & 3 \\ 4 & 9 \end{bmatrix}$ by using LU decomposition method. What will be the value of L_{21}?

A. 1 **B.** 2 **C.** 3 **D.** 4

(c) **B**

(d) Which of the following statement is NOT true about the Gaussian elimination method?

A. $det(A)$ does not change.
B. The row operation changes all matrix elements of the matrix A.
C. The lower triangular and upper triangular form gives the same solution.
D. $det(L) = det(U)$, where L and U are the lower and upper triangular forms of A.

(d) **B**

(e) Which of the following matrix representation of A will have a unique solution?

A. $\begin{bmatrix} 0 & 4 \\ 0 & 0 \end{bmatrix}$ **B.** $\begin{bmatrix} 4 & 4 \\ 4 & 0 \end{bmatrix}$ **C.** $\begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix}$ **D.** $\begin{bmatrix} 0 & 4 \\ 0 & 4 \end{bmatrix}$

(e) **B**

(f) How many operations are required to solve a linear system where $Lx = b$? Here, L is a lower triangular 2×2 matrix.

A. 2 **B.** 4 **C.** 8 **D.** 9

(f) **B**

(g) We obtain the lower triangular matrix $\begin{bmatrix} 1 & 0 \\ L_{21} & 1 \end{bmatrix}$ from the matrix $\begin{bmatrix} 3 & 2 \\ 9 & 4 \end{bmatrix}$ by using LU decomposition method. What will be the value of L_{21}?

A. 1 **B.** 2 **C.** 3 **D.** 4

(g) **C**

(h) Which of the following matrix representation of A will have a unique solution?

A. $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ **B.** $\begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix}$ **C.** $\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$ **D.** $\begin{bmatrix} 2 & 4 \\ 2 & 4 \end{bmatrix}$

(h) **C**

(i) How many operations are required to solve a linear system where $Lx = b$? Here, L is a lower triangular 3×3 matrix.

A. 4 **B.** 8 **C.** 9 **D.** 16

(i) **C**

Problems: Marks are as indicated

2. (4+2 marks) Find L and U matrices from $A^{(1)} = \begin{bmatrix} 1 & 2 & 1 \\ 1 & -2 & 2 \\ 2 & 12 & -2 \end{bmatrix}$ using LU decomposition method. (You have to show the Frobenius matrices)

$Solution \rightarrow$

$$A^{(1)} = \begin{bmatrix} 1 & 2 & 1 \\ 1 & -2 & 2 \\ 2 & 12 & -2 \end{bmatrix}$$

$$m_{21} = \frac{a_{21}}{a_{11}} = \frac{1}{1} = 1; \quad m_{31} = \frac{a_{31}}{a_{11}} = \frac{2}{1} = 2$$

$$F^{(1)} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

$$A^{(2)} = F^{(1)} \times A^{(1)}$$

$$A^{(2)} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 2 & 1 \\ 1 & -2 & 2 \\ 2 & 12 & -2 \end{bmatrix}$$

$$A^{(2)} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & -4 & 1 \\ 0 & 8 & -4 \end{bmatrix}$$

$$m_{32} = \frac{a_{32}}{a_{21}} = \frac{8}{-4} =- 2$$

$$F^{(2)} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$$

$$A^{(3)} = F^{(2)} \times A^{(2)}$$

$$A^{(2)} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 2 & 1 \\ 0 & -4 & 1 \\ 0 & 8 & -4 \end{bmatrix}$$

$$A^{(3)} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & -4 & 1 \\ 0 & 0 & -2 \end{bmatrix} = U$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ m_{21} & 1 & 0 \\ m_{31} & m_{32} & 1 \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & -2 & 1 \end{bmatrix}$$

So,

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & -2 & 2 \\ 2 & 12 & -2 \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & -2 & 1 \end{bmatrix}}_{L} \underbrace{\begin{bmatrix} 1 & 2 & 1 \\ 0 & -4 & 1 \\ 0 & 0 & -2 \end{bmatrix}}_{U}$$