MCQ: Choose Only One Answer

1. (a) Rate of convergence of the Newton Method is generally-
A. Quadratic **B.** Linear **C.** Super Linear **D.** Cubic

(a) **C**

(b) What is the formula of the 2^{nd} iteration of finding a root using Quasi-Newton's method?
A. $x_2 = x_1 - \frac{f(x_0)-(x_0-x_1)}{f(x_0)-f(x_1)}$ **B.** $x_2 = x_1 - \frac{f(x_0)-(x_0-x_{-1})}{f(x_0)-f(x_{-1})}$
C. $x_2 = x_1 - \frac{f(x_1)-(x_1-x_0)}{f(x_1)-f(x_0)}$ **D.** None of these

(b) **D**

(c) An equation for $f(x)$ is given as $x^2 - 4$. If the initial approximation is $x = 6$, then what will be the value of the 2^{nd} iteration using Newton's method?
A. 2.2665 **B.** 3.3333 **C.** 2.0157 **D.** None of these

Given $x_0 = 6$ and $f(x) = x^2 - 4$. So $f'(x) = 2x$
Now, $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 6 - \frac{6^2-4}{2\times6} = 3.3333$
$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 3.3333 - \frac{3.3333^2-4}{2\times3.3333} = 2.2665$

(c) **A**

(d) An equation for $g(x)$ is given as $x^2 + 3x$. To be super-linearly convergent, what will the root be?
A. -1.5 **B.** $3i$ **C.** 0 **D.** None of these

Given $g(x) = x^2 + 3x$. Now find value of x from $|g'(x)| = 0$

(d) **A**

(e) What is the formula of the 1^{st} iteration of finding a root using Quasi-Newton's method?
A. $x_1 = x_0 - \frac{f(x_0)-(x_0-x_1)}{f(x_0)-f(x_1)}$ **B.** $x_1 = x_0 - \frac{f(x_0)-(x_0-x_{-1})}{f(x_0)-f(x_{-1})}$
C. $x_1 = x_1 - \frac{f(x_1)-(x_1-x_0)}{f(x_1)-f(x_0)}$ **D.** $x_1 = x_0 - \frac{f(x_1)-(x_0-x_1)}{f(x_1)-f(x_0)}$

(e) **None of these**

(f) An equation for $f(x)$ is given as $x^2 - 5$. If the initial approximation is $x = 6$, then what will be the value of the 2^{nd} iteration using Newton's method?
A. 2.2665 **B.** 3.3333 **C.** 2.0157 **D.** None of these

Given $x_0 = 6$ and $f(x) = x^2 - 5$. So $f'(x) = 2x$
Now, $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 6 - \frac{6^2-5}{2\times6} = 3.4161$
$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 3.4161 - \frac{3.4161^2-5}{2\times3.4161} = 2.4400$

(f) **D**

(g) An equation for $g(x)$ is given as $x^2 + 5x$. To be super-linearly convergent, what will the root be?
A. -1.5 **B.** -2.5 **C.** $3i$ **D.** 0

Given $g(x) = x^2 + 5x$. Now find value of x from $|g'(x)| = 0$

(g) **B**

(h) An equation for $f(x)$ is given as $x^2 + 4$. If the initial approximation is $x = 6$, then what will be the value of the 2^{nd} iteration using Newton's method?
A. 2.2665 **B.** 3.3333 **C.** 2.0157 **D.** None of these

Given $x_0 = 6$ and $f(x) = x^2 + 4$. So $f'(x) = 2x$
Now, $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 6 - \frac{6^2+4}{2\times6} = 2.6667$
$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 2.6667 - \frac{2.6667^2-5}{2\times2.6667} = 0.5833$

(h) **D**

Problems: Marks are as indicated

2. (2+4 marks) For $f(x) = x^2 - 5x + 4$, where $x_0 = 1$, construct two $g(x)$ from the given function $f(x)$ and determine they are convergent or divergent.

Given that,

$$f(x) = x^2 - 5x + 4$$

Actual roots are,

$$x^2 - 5x + 4 = 0$$
$$\Rightarrow (x - 1)(x - 4) = 0$$
$$Roots\ are\ at\ x = 1,\ 4$$

Construct two $g(x) = x$

$$(1)\ x^2 - 5x + 4 = 0$$
$$\Rightarrow x^2 = 5x - 4$$
$$\Rightarrow x = \sqrt{5x - 4}\ \left[g_1(x) = \sqrt{5x - 4}\right]$$

$$(2)\ x^2 - 5x + 4 = 0$$
$$\Rightarrow x^2 - x - 4x + 4 = 0$$
$$\Rightarrow x = x^2 - 4x + 4\ \left[g_2(x) = x^2 - 4x + 4\right]$$

$$(1)\ g_1(x) = \sqrt{5x - 4} = (5x - 4)^{\frac{1}{2}}$$

$$g_1'(x) = (5x - 4)^{-\frac{1}{2}} = \frac{1}{\sqrt{5x-4}}$$
$$\Rightarrow \lambda = \left|g'\left(x_*\right)\right| = |g'(1)| = 1\ \ (not\ < 1)$$
$$\Rightarrow \lambda = \left|g'\left(x_*\right)\right| = |g'(4)| = \frac{1}{4}\ \ (< 1)$$
$$\therefore x_0 = 1\ converges\ to\ the\ root,\ x_* = 4$$

$$(2)\ g_2(x) = x^2 - 4x + 4$$

$$g_2'(x) = 2x - 4$$
$$\Rightarrow \lambda = \left|g'\left(x_*\right)\right| = |g'(1)| = |-2| = 2\ \ (not\ < 1)$$
$$\Rightarrow \lambda = \left|g'\left(x_*\right)\right| = |g'(4)| = 4\ \ (not < 1)$$
$$\therefore x_0 = 1\ does\ not\ converge\ to\ any\ root.\ It\ will\ all\ diverge.$$