MCQ: Choose Only One Answer

1. A contraction mapping is superlinear if the rate is

   A. greater than 1. **B.** zero. **C.** a negative value. **D.** greater than zero, but less than 1.

   1. **B**

2. Consider $f(x) = x^2 - 4$. A fixed point function can be constructed from $f(x) = 0$ which is $g(x) = \dfrac{4+x}{x+1}$. Which of the following is a fixed point?

   A. 4 **B.** 2. **C.** 0. **D.** -1.

   2. **B**

3. For $g(x) = \dfrac{4+x}{x+1}$, $|g'(2)|$ equals

   A. -1. **B.** 1/3 **C.** 3. **D.** 4.

   3. **B**

4. For $f(x) = x^2 - 4 = 0$, which of the following is a superlinear $g(x)$?

   A. $x + \dfrac{x^2-4}{2x}$. **B.** $x - \dfrac{x^2-4}{2x}$ **C.** $\dfrac{x^2-4}{2x}$. **D.** $x + \dfrac{x^2+4}{x}$.

   4. **B**

5. **Problem Solving**: Answer the following:

(a) (3 marks) A fixed-point function $g(x) = \dfrac{4-x}{x-1}$ has the fixed points ± 2. Use $x_{k+1} = g(x_k)$ to calculate six iterations (keep upto 3 decimal places) starting from $x_0 = -3.000$. Which root it seems to be converting to or diverging?

Solution: Here, $x_0 = -3.000$. So, the six iterated values are :

k	$g(x_k)$	$x_{k+1} = \dfrac{4-x_k}{x_k-1}$
0	$g(-3.00)$	-1.750
1	$g(-1.750)$	-2.091
2	$g(-2.091)$	-1.971
3	$g(-1.971)$	-2.010
4	$g(-2.010)$	-1.997
5	$g(-1.997)$	-2.001
6	$g(-2.001)$	-2.000

Hence it seems to be converging to -2.

(b) (3 marks) In Newton's method the fixed point function is expressed as $g(x) = x - \dfrac{f(x)}{f'(x)}$, where $f(x)$ satisfies $f(x_\star) = 0$ (*i.e.*, $x_\star$ is a root of $f(x)$). Verify that the convergence rate λ for the Newton's method is ZERO (*i.e.*, it is superlinear).

Solution: By definition, the convergence rate is given by

$$
\begin{aligned}
\lambda &= \left|\frac{dg(x)}{dx}\right|_{x=x_\star} = \left|\frac{d}{dx}\left(x - \frac{f(x)}{f'(x)}\right)\right|_{x=x_\star}, \\
&= \left|1 - \frac{f'(x)f'(x) - f(x)f''(x)}{[f'(x)]^2}\right|_{x=x_\star}, \\
&= \left|1 - \frac{[f'(x_\star)]^2 - f(x_\star)f''(x_\star)}{[f'(x_\star)]^2}\right|, \\
&= \left|1 - \frac{[f'(x_\star)]^2}{[f'(x_\star)]^2}\right|, \quad \text{(because } f(x_\star) = 0\text{)}, \\
&= |1-1| = 0 \,.\checkmark
\end{aligned}
$$