Final Examination : CSE330. All Sections. Set # 1 .

Department of Computer Science & Engineering

BRAC University

Fall Semester 2022

Date : December 29, 2022

Time : One hour 40 minutes

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Faculty Name (Initial) : _____   Student ID# : _____   Section#: _____

Instructions:

- There are five question. **Answer any four questions**. Total marks 60.

- Use pencil for your answers. No break for bathroom/freshroom is allowed. **Must use your own calculator**. Cell phones must be turned off (Not in vibration mode). We assume that you know how to use scientific calculator of model CASIO fx-991 ES or equivalent.

- Return this question along with your answer script.

- All examinees must abide by the 'Regulations of Students Conduct' of Brac university.

Read carefully the questions below and answer properly:

1. (a) (5 marks) [**CO-2**]Consider the function $f(x) = x^3 - 2x + 2$ and starting point $x_0 = 0$. **Show** that the sequence x_1, x_2, $\cdots$ using Newton's method fails to approach a root of $f(x)$.

   (b) (10 marks) [**CO-3**] Consider the function $f(x) = \cos(3x) - \sin(x)$. **Compute** the solution of the function, such that $f(x) = 0$, using Newton's method with Aitken's acceleration and starting point, $x_0 = 0$. Consider up to five decimal places.

2. (a) (5 marks) [**CO-3**] **Compute** five iterations using interval bisection method to find the root of $f(x) = x^2 - x - 6$ for the interval $[2.5, 3.15]$. Keep up to three decimal places.

   (b) (4 marks) [**CO-3**] **Calculate** the number of iterations needed to solve the above function for the given interval within the error bound of $\delta = 10^{-2}$.

   (c) (6 marks) [**CO-6**] **Evaluate** a fixed point function for $x^3 - 2x^2 = 0$ such that $f(x) = x$ and show whether it is converging or diverging.

3. A linear system is described by the following equations

$$\begin{aligned} x_1 + 6x_2 + 2x_3 &= 10 \\ 3x_1 + 2x_2 + x_3 &= 6 \\ 4x_1 + 5x_2 + 2x_3 &= 9. \end{aligned}$$

Based on these equations, answer the questions below:

   (a) (3 marks) [**CO-1**] From the given linear equations, **identify** the matrices A, x and b such the linear system can be expressed as a matrix equation.

   (b) (2+3 marks) [**CO-1**] **Construct** the Frobenius matrices $F^{(1)}$ and $F^{(2)}$ from this system.

   (c) (2 marks) [**CO-3**] **Compute** the unit lower triangular matrix L.

   (d) (5 marks) [**CO-6**] Now **Evaluate** the solution of the linear system using LU-decomposition method. Use the unit lower triangular matrix found in the previous question.

4. A student has decided to sell the jerseys of football club as a relaxation after the stressful final exam. There are x_1 number of jerseys of PSG club and x_2 number of jerseys of Barcelona club in his shop. The total number of jersey is 30. On the first day, he sold each jersey of two clubs by 400 tk. and totally he earned 12000 tk. But when Messi left Barcelona club and went to PSG club, each jersey of Barcelona costs 300 tk. and each jersey of PSG costs 500 tk. Then, he earned total 13000 tk. In the following, this overdetermined system will be solved by using the QR Decomposition Method by answering the following step by step:

   (a) (1.5 marks) [**CO-3**] **Write down** the linear equations that relate the variable x_1 and x_2.

   (b) (1.5 marks) [**CO-1**] **Identify** the matrices A, x and b so that the equations in the previous question can be expressed in the standard matrix equation form $Ax = b$.

   (c) (7 marks) [**CO-3**] From matrix A in the previous question, **compute** the matrices Q and R such that $A = QR$, where the symbols have their usual meanings.

   (d) (5 marks) [**CO-5**] **Evaluate** $Q^T b$, and finally solve the system by evaluating x (that is , evaluate x_1 and x_2).

5. Consider the function $f(x) = x^2 - e^{3x}$ which is continuous on the interval $[2, 7]$. Answer the following questions:

   (a) (4 marks) [**CO-3**] **Calculate** the exact value of integration $I(f)$.

   (b) (4+2 marks) [**CO-3**] **Evaluate** the approximate value of the integration using Composite Newton Cotes formula with 4 segments $C_{1,4}$. Then, **calculate** the relative error in percentage using part (a) and (b).

   (c) (5 marks) [**CO-2**] For the Newton-Cotes formula with $n = 2$, **show** that one of weight function/factors is given as $\sigma_0 = (b - a)/6$, where a and b are the lower and upper limits of the integration.