Solution: Assignment-5

#1 Here: $Aug(A) \equiv (A \mid b) = \left(\begin{array}{ccc|c} 2 & 1 & -1 & 1 \\ 1 & 2 & 1 & 0 \\ -1 & -1 & 2 & 2 \end{array}\right)$.

1st row operation: $m_{21} = \frac{a_{21}}{a_{11}} = \frac{1}{2} = 0.5$ & $m_{31} = \frac{a_{31}}{a_{11}} = -0.5$

$\therefore r_1 \to r_1' = r_1 = (2 \quad 1 \quad -1 \quad 1)$

$r_2 \to r_1' = r_2 - m_{21} r_1 = (1 \quad 2 \quad 1 \quad 0) - 0.5(2 \quad 1 \quad -1 \quad 1)$

$= (0 \quad 1.5 \quad 1.5 \quad -0.5)$

$r_3 \to r_3' = r_3 - m_{31} r_1 = (-1 \quad -1 \quad 2 \quad 2) - (-0.5)(2 \quad 1 \quad -1 \quad 1)$

$= (0 \quad -0.5 \quad 1.5 \quad 2.5)$

$\therefore$ After 1st row operation: $Aug(A) \to \begin{pmatrix} 2 & 1 & -1 & 1 \\ 0 & 1.5 & 1.5 & -0.5 \\ 0 & -0.5 & 1.5 & 2.5 \end{pmatrix}$

Now for 2nd row operation: $m_{32} = \frac{a_{32}}{a_{22}} = \frac{-0.5}{1.5} = -\frac{1}{3}$

Therefore:

$r_1 \to r_1' = r_1 = (2 \quad 1 \quad -1 \quad 1)$

$r_2 \to r_2' = r_2 = (0 \quad 1.5 \quad 1.5 \quad -0.5)$

and $r_3 \to r_3' = r_3 - m_{32} r_2 = (0 \quad -0.5 \quad 1.5 \quad 2.5) + \frac{1}{3}(0 \quad 1.5 \quad 1.5 \quad -0.5)$

$= (0 \quad 0 \quad 2 \quad 7/3)$

Hence, $Aug(A) \to \begin{pmatrix} 2 & 1 & -1 & 1 \\ 0 & 1.5 & 1.5 & -0.5 \\ 0 & 0 & 2 & 7/3 \end{pmatrix}$

Therefore, we find, $U = \begin{pmatrix} 2 & 1 & -1 \\ 0 & 1.5 & 1.5 \\ 0 & 0 & 2 \end{pmatrix}$ and $b' = \begin{pmatrix} 1 \\ -0.5 \\ 7/3 \end{pmatrix}$.

#2 Now, using $Ux = b'$, we find by back substitution method.

$2z = 7/3 \Rightarrow \boxed{z = 7/6}$ ✓

$1.5y + 1.5z = -0.5 \Rightarrow y = \dfrac{-0.5 - 1.5(7/6)}{1.5} \Rightarrow \boxed{y = -\frac{3}{2} = -1.5}$ ✓

and $2x + y - z = 1$

$\Rightarrow x = \dfrac{1 + 1.5 + 7/6}{2} \Rightarrow \boxed{x = 11/6}$ ✓

#3 Here: $P^{(1)} = \begin{pmatrix} 1 & 0 & 0 \\ -m_{21} & 1 & 0 \\ -m_{31} & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -0.5 & 1 & 0 \\ 0.5 & 0 & 1 \end{pmatrix}$ ✓ & $P^{(2)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -m_{32} & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1/3 & 1 \end{pmatrix}$ ✓

#4 $L = \begin{pmatrix} 1 & 0 & 0 \\ m_{21} & 1 & 0 \\ m_{31} & m_{32} & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0.5 & 1 & 0 \\ -0.5 & -1/3 & 1 \end{pmatrix}$. ✓

#5 $Ly = b$ & $Ux = b$ from $Ax = b$ by LU method.

For $Ly = b \Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ .5 & 1 & 0 \\ -0.5 & -1/3 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} \Rightarrow \begin{cases} y_1 = 1 \\ y_2 = -0.5 \\ y_3 = 7/3. \end{cases}$

And $Ux = y \Rightarrow \begin{pmatrix} 2 & 1 & -1 \\ 0 & 1.5 & 1.5 \\ 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -0.5 \\ 7/3 \end{pmatrix} \Rightarrow \begin{cases} x = 11/6 \\ y = -1.5 \\ z = 7/6 \end{cases}$ ✓.

(Same as #2 above).