



**Final Examination : Questions for CSE330. All Sections.**

Department of Computer Science & Engineering

BRAC University

Summer Semester

Date : September 06, 2022

Time : One hour 50 minutes

Faculty Name (Initial) : \_\_\_\_\_ Student ID# : \_\_\_\_\_ Section#: \_\_\_\_\_

**Instructions:**

- There are six question. **Answer any four questions.** Total marks 60.
- Use pencil for your answers. No break for bathroom/freshroom is allowed. **Must use your own calculator.** Cell phones must be turned off (Not in vibration mode). We assume that you know how to use scientific calculator of model CASIO fx-991 ES or equivalent.
- Return this question along with your answer script.
- All examinees must abide by the 'Regulations of Students Conduct' of Brac university.

**Read carefully the questions below and answer properly:**

1. [CO-3] A function is given by  $f(x) = x^2 - x - 2$ . Three fixed point functions,  $g_1(x) = x^2 - 2$ ,  $g_2 = \sqrt{x+2}$  and  $g_3(x) = \sqrt{6-x} + 2$  are constructed from  $f(x)$ . Based on these answer the following:
  - (a) (4.5+1.5 marks) **Calculate** the convergence rate (or ratio)  $\lambda$  for these three fixed point functions, and **state** if these are diverging, linear or superlinear. Note that you need to find the roots first to answer this question.
  - (b) (3 marks) **Construct** a fixed point function  $(x)$  that is superlinear, and let's call it  $g_4(x)$ .
  - (c) (3+1+1+1 marks) Use  $g_4(x)$  and  $x_0 = 0$  to **evaluate** the iterated values of  $x_k$  up to  $k = 4$ . Keep up to 6 decimal places. **Decide** which root  $g_4(x)$  is converging to and **explain** why the iteration did not converges to the other root. Also **estimate** the actual error up to six decimal places.
2. [CO-3] A linear system is described by the following equations:

$$\begin{aligned}4x_2 + 2x_3 &= 1 \\2x_1 + 3x_2 + 5x_3 &= 0 \\3x_1 + x_2 + x_3 &= 11\end{aligned}$$

Based on these equations, answer the questions below.

- (a) (3 marks) From the given linear equations, **identify** the matrices  $A$ ,  $x$  and  $b$  such the the linear system can be expressed as a matrix equation.
  - (b) (2 marks) **Examine** if the matrix  $A$  has any pivoting problem? **Explain** why or why not?
  - (c) (6 marks) **Write** down the Augmented matrix,  $\text{Aug}(A)$ , from the given linear system, and **evaluate** the upper triangular matrix  $U$ . Note that you have to show the row multipliers  $m_{ij}$  for each step as necessary.
  - (d) (4 marks) Using the upper triangular matrix found in the previous question, **compute** the solution of the given linear system by Gaussian elimination method.
3. [CO-3] A linear system is described by the following equations:

$$\begin{aligned}2x_1 + 6x_2 - 9x_3 &= 15 \\2x_1 + 4x_2 - 6x_3 &= 10 \\-2x_1 - 3x_2 + 4x_3 &= -6\end{aligned}$$

Based on these equations, answer the questions below.

- (a) (3 marks) From the given linear equations, **identify** the matrices  $A$ ,  $x$  and  $b$  such the the linear system can be expressed as a matrix equation.

- (b) (3 marks) **Compute** the Frobenius matrices  $F^{(1)}$  and  $F^{(2)}$  for this system.
- (c) (3 marks) **Evaluate** the unit lower triangular matrix  $L$ , and the upper triangular matrix  $U$ . **Show** that  $\det A = \det L \times \det U$ .
- (d) (6 marks) Now **compute** the solution of the given linear system using  $LU$ -decomposition method. Use the matrices  $L$  and  $U$  found in the previous question. Show your works.

4. [CO-4] Answer the following questions:

- (a) (6 marks) **Show** that the following set

$$S = \left\{ \frac{1}{\sqrt{5}} (2, -1, 0)^T, \frac{1}{\sqrt{30}} (1, 2, -5)^T, \frac{1}{\sqrt{24}} (2, 4, 2)^T \right\}$$

is an orthonormal set of vectors.

- (b) (2+1 marks) Now, consider the function  $f(x) = \sin x$ , and the data points at  $x_0 = 4$ ,  $x_1 = 9$  and  $x_2 = -6$ . **Identify** the matrices  $V$  and  $b$ . Keep up to 3 decimal places for all evaluated values.
- (c) (6 marks) **Determine** the best fit polynomial of degree one for the data points in Part-(b) using the Discrete Square Approximation method. Keep up to 3 decimal places for all evaluated values.
5. [CO-4] A test has been offered where there are a total of 40 questions and total marks is 100, and there are two sets of question. Set-1 contains  $x_1$  number of 2-marks questions and  $x_2$  number of 4-marks questions. Set-2 contains  $x_1$  number of 3-marks questions and  $x_2$  number of 1-mark questions. In the following, this overdetermined system will be solved by using the  $QR$  Decomposition Method by answering the following step by step:
- (a) (3 marks) **Write** down the linear equations that relate the variable  $x_1$  and  $x_2$ .
- (b) (1+1/2+1/2 marks) **Identify** the matrices  $A$ ,  $x$  and  $b$  so that the equations in the previous question can be expressed in the standard matrix equation form  $Ax = b$ .
- (c) (4+3 marks) From matrix  $A$  in the previous question, **compute** the matrices  $Q$  and  $R$  such that  $A = QR$ , where the symbols have their usual meanings.
- (d) (3 marks) **Evaluate**  $Q^T b$ , and finally **solve** the system by evaluating  $x$  (that is, **evaluate**  $x_1$  and  $x_2$ ).
6. [CO-3] Consider the function  $f(x) = e^{0.5x} + \frac{1}{30}x^2$  which is continuous on the interval  $[-2, 0]$ . Answer the following questions:
- (a) (3 marks) **Calculate** the exact value of integration  $I(f)$ .
- (b) (5+1 marks) **Evaluate** the approximate value of the integration using Composite Newton Cotes formula with 4 segments  $C_{1,4}$ . Then, **calculate** the relative error in percentage using part (a) and (b).
- (c) (3 marks) For the Newton-Cotes formula with  $n = 2$ , **show** that one of weight function/factors is given as  $\sigma_0 = \frac{b-a}{6}$ , where  $a$  and  $b$  are the lower and upper limits of the integral.
- (d) (3 marks) **Evaluate** the approximate value of the integral of  $f(x)$  using Simpson's rule.