MCQ: Choose Only One Answer

1. (a) Which of the following is the correct expression for the Forward Difference method?

   A. $\frac{f(x+h)-f(x)}{2h}$ **B.** $\frac{f(x+h)-f(x-h)}{h}$ **C.** $\frac{f(x)-f(x-h)}{h}$ **D.** $\frac{f(x+h)-f(x)}{h}$

   (a) **D**

   (b) If Central Difference $f'(1.1)$ is -0.18558 where $f'(x=1.1) = -0.14304$. Then what is the value of relative error?

   A. 0.29739 **B.** 2.29739 **C.** 0.04254 **D.** 1.29739

   (b) **A**

   (c) Determine the upper bound of truncation error for Backward difference method where $f^2(0.9) = 1.9441$ and $f^2(1) = 1.1791$ in the interval $[x-h, x] = [0.9, 1]$.

   A. 0.097205 **B.** 0.058955 **C.** 0.58955 **D.** 1.17910

   (c) **A**

   (d) Which of the following is the correct expression for the Backward Difference method?

   A. $\frac{f(x)-f(x-h)}{2h}$ **B.** $\frac{f(x+h)-f(x-h)}{h}$ **C.** $\frac{f(x)-f(x-h)}{h}$ **D.** $\frac{f(x+h)-f(x)}{h}$

   (d) **C**

   (e) If Forward Difference $f'(1.1)$ is 0.18558 where $f'(x=1.1) = 0.14304$. Then what is the value of relative error?

   A. 0.29739 **B.** 2.29739 **C.** 0.04254 **D.** 1.29739

   (e) **A**

   (f) Determine the upper bound of truncation error for central difference method where $f^3(0.9) = -9.7204$ and $f^3(1.1) = -3.5759$ in the interval $[x-h, x+h] = [0.9, 1.1]$.

   A. -5.95983×10^{-3} **B.** -595.983×10^{-3} **C.** -23.83933×10^{-3} **D.** -16.20067×10^{-3}

   (f) **A**

   (g) Which of the following is the correct expression for the Central Difference method?

   A. $\frac{f(x+h)-f(x-h)}{2h}$ **B.** $\frac{f(x+h)+f(x-h)}{2h}$ **C.** $\frac{f(x)-f(x-h)}{2h}$ **D.** $\frac{f(x+h)-f(x)}{2h}$

   (g) **A**

   (h) If Backward Difference $f'(1.1)$ is -0.18558 where $f'(x=1.1) = -0.14304$. Then what is the value of relative error?

   A. 0.29739 **B.** 2.29739 **C.** 0.04254 **D.** 1.29739

   (h) **A**

   (i) Determine the upper bound of truncation error for Forward difference method where $f^2(1) = 1.9441$ and $f^2(1.1) = 1.1791$ in the interval $[x, x+h] = [1, 1.1]$.

   A. 0.097205 **B.** 0.058955 **C.** 0.58955 **D.** 1.17910

   (i) **A**

Problems: Marks are as indicated

2. (7 marks) Derive an expression for $D_h^{(1)}$ from D_h by replacing h with $\frac{4h}{3}$ using Richardson extrapolation method.

$D_h = \frac{f(x+h)-f(x-h)}{2h}$

<u>From Taylor series-</u>

$f\,(x) = f\left(x_0\right) + f^1\left(x_0\right)\left(x - x_0\right) + \frac{f^2\left(x_0\right)}{2!}\left(x - x_0\right)^2 + \frac{f^3\left(x_0\right)}{3!}\left(x - x_0\right)^3 + \ldots\ldots\ldots$

$Now,$

$f\,(x + h) = f(x) + f^1(x)(h) + \frac{f^2(x)}{2!}(h)^2 + \frac{f^3(x)}{3!}(h)^3 + \frac{f^4(x)}{4!}(h)^4 + \frac{f^5(x)}{5!}(h)^5 + O\left(h^6\right)$

$f\,(x - h) = f(x) - f^1(x)(h) + \frac{f^2(x)}{2!}(h)^2 - \frac{f^3(x)}{3!}(h)^3 + \frac{f^4(x)}{4!}(h)^4 - \frac{f^5(x)}{5!}(h)^5 + O\left(h^6\right)$

$So,$ $\left[5^{th}\ order\ approximation\right]$

$D_h = \frac{f(x+h)-f(x-h)}{2h}$

$\Rightarrow D_h = \frac{1}{2h}\left(2f^1(x)(h) + 2\frac{f^3(x)}{3!}(h)^3 + 2\frac{f^5(x)}{5!}(h)^5 + O\left(h^7\right)\right)$

$D_h = f^1(x) + \frac{f^3(x)}{3!}(h)^2 + \frac{f^5(x)}{5!}(h)^4 + O\left(h^6\right) \qquad - (1)$

$D_{\frac{4h}{3}} = f^1(x) + \frac{f^3(x)}{3!}\left(\frac{4h}{3}\right)^2 + \frac{f^5(x)}{5!}\left(\frac{4h}{3}\right)^4 + O\left(h^6\right) \qquad - (2)$

$Now,\ \left(\frac{3}{4}\right)^2 \times (2) - (1) \rightarrow$

$$\left(\frac{3}{4}\right)^2 \times D_{\frac{4h}{3}} - D_h = \left(\frac{3}{4}\right)^2 \times \left(f^1(x) + \frac{f^3(x)}{3!}\left(\frac{4h}{3}\right)^2 + \frac{f^5(x)}{5!}\left(\frac{4h}{3}\right)^4 + O\left(h^6\right)\right)$$

$$- \left(f^1(x) + \frac{f^3(x)}{3!}(h)^2 + \frac{f^5(x)}{5!}(h)^4 + O\left(h^6\right)\right)$$

$$= \left(\left(\frac{3}{4}\right)^2 - 1\right)f^1(x) + 0 + \left(\left(\frac{4}{3}\right)^2 - 1\right)f^1\ \frac{f^5(x)}{5!}(h)^4 + O\left(h^6\right)$$

$$\frac{\left(\frac{3}{4}\right)^2 \times D_{\frac{4h}{3}} - D_h}{\left(\left(\frac{3}{4}\right)^2 - 1\right)} = f^1(x) + \left(\frac{\left(\left(\frac{4}{3}\right)^2 - 1\right)}{\left(\left(\frac{3}{4}\right)^2 - 1\right)}\right)\frac{f^5(x)}{5!}(h)^4 + O\left(h^6\right)$$

$$\frac{\left(\frac{3}{4}\right)^2 \times D_{\frac{4h}{3}} - D_h}{\left(\left(\frac{3}{4}\right)^2 - 1\right)} = f^1(x) - \frac{16}{9}\,\frac{f^5(x)}{5!}(h)^4 + O\left(h^6\right)$$

$$\therefore D_h^{(1)} = f^1(x) - \frac{16}{9}\,\frac{f^5(x)}{5!}(h)^4 + O\left(h^6\right)$$