Midterm Examination : CSE330. All Sections. Set # 1
Department of Computer Science & Engineering
BRAC University
Spring 2023 Semester
Date : March 05, 2023
Time : One hour 10 minutes

Faculty Name (Initial) : _____ Student ID# : _____ Section#: _____

Instructions:

- There are five questions. **Answer any three questions**. Total marks 30.
- Use pencil for your answers. No break for bathroom/freshroom is allowed. **Must use your own calculator**. Cell phones must be turned off (Not in vibration mode). We assume that you know how to use scientific calculator of model CASIO fx-991 ES or equivalent.
- Return this question along with your answer script.
- All examinees must abide by the 'Regulations of Students Conduct' of Brac university.

Read carefully the questions below and answer properly (All are CO1 and CO2):

1. Let $\beta = 2$, $m = 4$, $e_{\min} = -2$ and $e_{\max} = 3$. Here the symbols have their usual meanings. Answer the following:
   (a) (2 marks) [**CO-3**] **Evaluate** the machine epsilon using the standard convention of the floating point representation.
   (b) (2 marks) [**CO-4**] **Evaluate** the maximum and minimum numbers with and without negative support and using Normalized form of Floating point numbers, express these in decimal format.
   (c) (3 marks) [**CO-1**] In all floating point representations (standard, normalized and denormalized), the first digit has to be non-zero. **Explain** why the first digit has to be non-zero.
   (d) (3 marks) [**CO-1**] In all three representations of floating point representation, the first digit in mantissa is set to 1 (one). **State and explain** if it is valid for any number system, like octal, decimal, hexagonal system, etc. or it is specific to binary system. Explain why or why not.
2. Consider the quadratic equation, $2x^2 - 16x + 3 = 0$. Answer the following:
   (a) (5 marks) [**CO-2**] Computing up to 5 significant figures, **show** where and how the loss of significance occur when calculating the roots of the quadratic equation.
   (b) (5 marks) [**CO-4**] **Evaluate** the correct values of the roots up to 5 significant figures, and check that the roots satisfy the fundamental properties of algebra (that is, sum and product of the roots).
3. Consider the function $f(x) = e^x + e^{-x}$ and the nodes are at $-2, 0$ and 2. Now answer the following questions using 3 significant figures.
   (a) Using the Lagrange method,
      i. (3 marks) [**CO-3**] **compute** the interpolating polynomial for the given function, and
      ii. (2 marks) [**CO-2**] **express** the result in the natural basis.
   (b) (5 marks) [**CO-4**] **Evaluate** the upper bound of interpolation error for the given function for the interval $[-1, 1]$.

4. (a) (6 marks) [**CO-2**] In Newton Divided/Difference method, the coefficients, a_k, is defined as

$$a_k = f[x_0, x_1, x_3, \cdots, x_k] \ , \qquad \text{with} \quad k = 0, 1, 2, \cdots, n.$$

Verify that the second Newton coefficient, a_2, can be expressed as

$$a_2 = \frac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0} \ ,$$

where the symbols have their usual meanings.

(b) (4 marks) [**CO-2**] One of the Hermite basis is given as

$$h_k(x) = \Big[1 - 2\big(x - x_k\big) l_k'(x_k)\Big] l_k^2(x) \ .$$

Verify that $h_k(x_j) = \delta_{jk}$, $j, k = 0, 1, 2, \cdots, n$, and the symbols have their usual meanings.

5. Consider the following function $f(x) = x^4 + \ln(x)$. Now answer the following:

(a) (2 marks) [**CO-4**] **Evaluate** the numerical derivative of $f(x)$ at $x = 1.0$ with step size $h = 0.2$ using the central difference method up to 5 significant figures.

(b) (3 marks) [**CO-4**] **Evaluate** the upper bound of truncation error of $f(x)$ using $x_0 = 1.0$ and $h = 0.2$ using the central difference method.

(c) (4+1 marks) [**CO-2**] **Deduce** an expression for $D_h^{(1)}$ from D_h by replacing h with $h/3$ using the Richardson extrapolation method, and calculate the upper bound of error of $D_h^{(1)}$, if $f(x) = \cos x$, with $x_0 = 1$ and $h = 0.1$.