Online Final Examination : CSE330. All Sections.
Department of Computer Science & Engineering
BRAC University
Fall 2023 Semester
Date : December 19, 2023
Exam Time : One hour 30 minutes
Scanning and Uploading time : 15 minutes.

Faculty Name (Initial) : _________ Student ID# : ___________________________ Section#: ______

Instructions:

- Answer as instructed below. Total marks 60.
- Answer questions sequentially. **DO NOT** mix part of one question with another question. Write legibly so that we can follow your thoughts.
- Number your pages, and scan them sequentially when done, and prepare a single pdf file with the Top Sheet as the first page. Before submission rename your pdf file as 'ID#_FirstName_Section#.pdf'. As for example: '12345678_Examinee_18.pdf'.
- Use pencil for your answers (preferable). No break for bathroom/freshroom is allowed. **Must use your own calculator**. Cell phones must be turned off (Not in vibration mode). We assume that you know how to use scientific calculator of model CASIO fx-991 ES or equivalent.
- All students MUST follow the final exam policy as given already.
- All examinees must abide by the 'Regulations of Students Conduct' of Brac university.
- Wait and stay in front of the camera till the end of exam. **DO NOT** leave before that.
- **NO** Email submission. The scanned pdf file of your answer script **MUST** be submitted through the Google Form Submission Link provided in the Discord server for CSE330.

Read carefully the questions below and answer properly:

1. (6 marks) **CO-1:** Answer any one from Questions-(1a-1b). Marks break down are as stated.
   (a) (6 marks) A linear system is described by the matrix equation $Ax = b$ where A is a square $n \times n$ matrix, x and b are $n \times 1$ column matrices. If the matrix A is decomposed into a product of unit lower triangular matrix L and an upper triangular matrix U, **show that** $\det A = \det U$.
   (b) i. (2 marks) **Define** an over-determined system.
   ii. (2 marks) **Name the methods** that you learned to solve an over-determined system.
   iii. (2 marks) Let the over-determined system is described by $Ax = b$ where the matrix A is a 4×2 matrix. Since $\det A$ does not exist, **state** the condition to find a solution of the over-determined system.

2. (12 marks) **CO-2:** Answer any one from Questions-(2a-2b). Marks break down are as stated.
   (a) In the interval $[-4, 4]$, the function, $f(x) = x^3 - x^2 - 3x + 2$, has three roots at 2.000, 0.6180 and -1.618; and two turning points at $x = -0.721$ and $x = 1.387$.
   i. (3 marks) **Explain** why it might not be possible to find the root of the given function and interval using the interval bisection method.
   ii. (3 marks) **Write down** the intervals, including the root it contains, such that the problem in the previous part can be avoided.
   iii. (6 marks) **Find** the minimum number of iterations required to find the root if the error bound is 1.0×10^{-5} in the interval $[-4, -0.5]$.
   (b) Answer the following:
   i. (2+4 marks) Consider the fixed point function, $g(x) = \sqrt{2x + 3}$ which has been derived from the function $f(x) = x^2 - 2x - 3$ with roots -1 and 3. **State** the domain of the fixed point function $g(x)$ and **explain** to which root the iteration $x_{k+1} = g(x_k)$ will converge to based on your understanding of the 'Contraction Mapping Theorem' if you start the iteration from $x_0 = -1.5$.

ii. (6 marks) **Verify** that matrix below for a linear system

$$A = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & \sqrt{2} \end{pmatrix}$$

consists of orthonormal vectors.

3. (18 marks) **CO-3:** Answer any two from Questions-(3a-3c). Marks break down are as stated.

(a) Consider the function, $f(x) = x^3 - 4x^2 + 6$.

i. (3 marks) **Compute** any two fixed point functions, $g_1(x)$ and $g_2(x)$, from the given function $f(x)$.

ii. (3 marks) For $g_1(x)$ **evaluate** the convergence rate and **determine** to which root it is converging to or divergent.

iii. (3 marks) For $g_2(x)$ **evaluate** the convergence rate and **determine** to which root it is converging to or divergent.

(b) Consider a set of four data points: $f(0) = 3$, $f(4) = -2$, $f(-1) = 2$, $f(1) = 1$. In the following, you are asked to find the best fit polynomial of degree 2 by using the Discrete-square approximation method as follows:

i. (3 marks) From the given data, **write down** the matrices A, b and x.

ii. (3 marks) **Evaluate** $A^{\mathrm{T}}A$ and $\det(A^{\mathrm{T}}A)$.

iii. (3 marks) **Compute** the best-fit polynomial of degree 2.

(c) A function is given by $f(x) = e^{0.5x} + \sin x$ which is to be integrated on the interval $[0, 2]$.

i. (2 marks) **Evaluate** the exact integral $I(f)$.

ii. (3 marks) **Compute** the numerical integral by using the Newton-Cotes formula with $n = 2$.

iii. (4 marks) Also **evaluate** the numerical integral $C_{1,4}$ by using the Composite Newton-Cotes formula.

4. (24 marks) **CO-4:** Answer any two from Questions-(4a-4c). Marks break down are as stated.

(a) Consider a cubic function, $f(x) = 2x^3 - 2x - 5$, which has only one real root.

i. (5 marks) Use the Bisection Method to **compute** the approximate root of the given function for the interval $I = [1.2, 1.8]$ after five iterations and keeping 4 decimal places.

ii. (3 marks) **Compute** the fixed point function $g(x)$ for the given function $f(x)$ using Newton's method.

iii. (4 marks) **Compute** four iterations to find an approximate root of the given function using Newton's method with the initial point, $x_0 = 1.9$. Consider five significant figures.

(b) A linear system is described by the following equations

$$\begin{aligned} x_1 - 2x_2 + 4x_3 &= 6 \\ 2x_1 - 7x_2 + x_3 &= 8 \\ 3x_1 + 3x_2 + 4x_3 &= 2 \end{aligned}$$

and you are asked to obtain the solution by using the LU-decomposition method as follows:

i. (3 marks) **Evaluate** the row multipliers m_{21} and m_{31}, the Frobenius matrix $F^{(1)}$ and the matrix $A^{(2)}$.

ii. (2 marks) **Evaluate** the row multiplier m_{32}, the Frobenius matrix $F^{(2)}$, and the matrix $A^{(3)}$.

iii. (2 marks) **Compute** the lower unit triangular matrix L using the row multipliers, and the upper triangular matrix U.

iv. (5 marks) **Evaluate** the solution of the system by using the matrices L and U.

(c) Consider the coordinates: $(x, f(x)) = (0, 1), (0.5, 1.4), (1, 1.7), (1.5, 2)$. In the following, you are asked to contruct the best-fit linear polynomial by using the QR-decomposing method as follows:

i. (2 marks) **Construct** the matrices A, b and x.

ii. (4 marks) **Evaluate** the orthonornmal vectors q_1 and q_2, and **construct** the matrix Q.

iii. (3 marks) **Compute** the matrix R.

iv. (3 marks) Using Q and R, **evaluate** the matrix x, and hence **compute** the best-fit linear polynomial.