



Final Examination : CSE330. All Sections. Set # 2.

Department of Computer Science & Engineering

BRAC University

Spring Semester 2023

Date : May 03, 2023

Time : One hour 40 minutes

Faculty Name (Initial) : _____ Student ID# : _____ Section#: _____

Instructions:

- There are five question. **Answer any four questions.** Total marks 60.
- Use pencil for your answers. No break for bathroom/freshroom is allowed. **Must use your own calculator.** Cell phones must be turned off (Not in vibration mode). We assume that you know how to use scientific calculator of model CASIO fx-991 ES or equivalent.
- Return this question along with your answer script.
- All examinees must abide by the 'Regulations of Students Conduct' of Brac university.

Read carefully the questions below and answer properly:

1. A function $f(x) = e^{-x} [2.2 \sin(x) - 0.5 \cos(x)]$ has one root in the interval $I = [3, 4]$. Answer the following questions considering up to **five significant figures**:
 - (a) (6 marks) [CO-4] Use Bisection Method to **evaluate** an approximate root of the above function for interval I within the error bound of 5×10^{-4} .
 - (b) (3 marks) [CO-3] **Compute** the minimum number of iterations required to find the root if the error bound is 1×10^{-5} for the interval I .
 - (c) (6 marks) Show four iterations to find a root of the given function using Newton's method where the initial point, $x_0 = 3.8$.
2. Consider a function: $f(x) = 2x^2 - 2x - 5$ and one of it's fixed point function $g_1(x) = \frac{5}{2(x-1)}$ such that $f(x) = 0$. Answer the following questions considering up to **five significant figures**:
 - (a) (2 marks) [CO-3] **Calculate** the exact roots of $f(x)$.
 - (b) (4 marks) [CO-4] **Deduce** two more fixed point functions, $g_2(x)$ and $g_3(x)$ [apart from the given one] such that $f(x) = 0$.
 - (c) (2 marks) [CO-3] **Compute** the convergence rate of the given fixed point function $g_1(x)$.
 - (d) (1 mark) [CO-1] In Question-(2c), **state** which root it is converging to or diverging from .
 - (e) (6 marks) [CO-4] Considering the given fixed point function $g_1(x)$ and initial point $x_0 = -1.5$, **estimate** the approximate root by using fixed point iteration method and by applying Aitken acceleration appropriately. [Error bound is 1×10^{-2}].
3. A linear system is described by the following equations:

$$6x_2 + 2x_3 = 10$$

$$3x_1 + 2x_2 + x_3 = 6$$

$$4x_1 + 5x_2 + 2x_3 = 9$$

Based on these equations, answer the questions below:

- (a) (1.5 marks) [CO-1] From the given linear equations, **identify** the matrices A , x and b such that the linear system can be expressed as a matrix equation.
- (b) (1.5 marks) [CO-2] **Explain** if there is any pivoting problem in Question-(3a), and if yes, **remove** it and proceed to answer the remaining parts of the current question.
- (c) (2+3+2 marks) [CO-3] **Construct** the Frobenius matrices, $F^{(1)}$ and $F^{(2)}$, for this system, and then **Compute** the unit lower triangular matrix L .

- (d) (5 marks) [CO-4] Now **evaluate** the solution of the linear system using LU-decomposition method. Use the unit lower triangular matrix found in Question-(3c).

4. Consider the following set of vectors $\mathbf{S} = \{\vec{v}_1, \vec{v}_2, \vec{v}_3\} = \left\{ \begin{bmatrix} 1 & 2 & 0 \end{bmatrix}^T, \begin{bmatrix} 1 & 1 & 0 \end{bmatrix}^T, \begin{bmatrix} 0 & 1 & 2 \end{bmatrix}^T \right\}$.

- (a) (1.5 marks) [CO-1] **Examine** whether the vectors are orthogonal to each other.
- (b) (6 marks) [CO-3] **Construct** an orthonormal set of vectors from the set of vectors \mathbf{S} by using the Gram-Schmidt orthogonalization process.
- (c) (1.5 marks) [CO-2] **Explain** whether the following system of linear equations has an exact solution for x_0 and x_1 ?

$$\begin{bmatrix} 3 & 2 \\ 4 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \\ 1 \end{bmatrix}$$

- (d) (6 marks) **Evaluate** the solution to the above system of linear equations by the Discrete Squares method.
5. Consider the function $f(x) = 3^x - 19x$ which is continuous on the interval $[2, 7]$. Answer the following questions:
- (a) (3 marks) [CO-4] **Calculate** the exact value of integration $I(f)$. You may use the calculator to find the answer, but you need to show detail steps to get full credit.
- (b) (6 marks) **Evaluate** the approximate value of the integration using Composite Newton Cotes formula with 5 segments $C_{1,5}$.
- (c) (2 marks) [CO-2] Also **express** the relative error in percentage using the values found in Questions(5a, 5b).
- (d) (3 marks) [CO-1] **State**, with an explanation, how the error can be reduced further.