MCQ: Choose Only One Answer.

1. How many operations are required to solve a linear system where $Lx = b$? Here, L is a lower triangular 4×4 matrix.

   A. 16. **B.** 14. **C.** 9. **D.** 2.

   1. **A**

2. A 6×6 square matrix, A, is changed to an upper triangular form by row operations in Gaussian elimination method. After the completion of the 3^{rd} row operation, how many matrix elements of A have been changed to zero by the row operations?

   A. 18. **B.** 12. **C.** 6. **D.** 3.

   2. **B**

3. Which of the following statement(s) is/are correct about the matrices in LU-decomposition method?

   A. $\det A = \det U$. **B.** $\det L = 1$. **C.** Both of these. **D.** None of these.

   3. **C**

4. A linear system has 6 variables and 6 equations. To solve is by LU-decomposition method, how many Frobenious matrices will be needed?

   A. 2. **B.** 3. **C.** 4. **D.** 5.

   4. **D**

5. **Problem Solving**: Answer the following:

(a) (3 marks) A linear system is described by the following linear equations

$$\begin{aligned} 2x - y + z &= 0\ , \\ x + 2y - z &= 1\ , \\ \text{and} \quad x - y + 2z &= 2. \end{aligned}$$

Find $\det U$ for the above linear system.

Solution: We know that, for a linear system, the matrix A, can be expressed as $A = LU$ by using LU-decomposition method. Therefore,

$$\det(A) = \det(LU) = \det(L)\det(U) = \det(U)\ .$$

Since L is unit triangular matrix, we have, $\det(L) = 1$. Hence, from the given matrix, we get,

$$\det(A) = \begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & -1 \\ 1 & -1 & 2 \end{vmatrix} = 6\ .$$

Therefore, $\det(U) = \det(A) = 6$. ✓

(b) (3 marks) Show that the number of row multipliers, m_{ij}'s, needed to solve a $n \times n$ linear system is given by

$$\frac{1}{2}n(n-1)\ .$$

Solution: There are two ways we can solve it.

First method: The number of row multipliers is equal to the number of elements in the lower triangular part of the square matrix. For an $n \times n$ matrix, there are n^2 number of elements. There are n-number of diagonal elements. So, the sum of lower and upper triangular elements of the matrix is $n^2 - n$. Therefore, the number of lower triangular elements is $(1/2)(n^2 - n)$. Hence, the number of row multipliers need is $\frac{1}{2}n(n-1$.

Second method: For an $n \times n$ matrix, we need $(n-1)$-number of row operations. The first row operation need $(n-1)$ row multipliers, second row operation need $(n-2)$ row multipliers, and so on. Finally, the second last row operation need 2 row multipliers, and the last row operation need only one row multipliers. Therefore, the total row multipliers needed is $(n-1) + (n-2) + \cdots + 2 + 1 = \frac{1}{2}(n-1)(n-1+1) = \frac{1}{2}n(n+1)$. ✓