MCQ: Choose Only One Answer.

1. Let the matrix A be an $m \times n$ matrix that represents a linear system. This system will be considered an over-determined system is

   A. $m > n$. **B.** $m < n$. **C.** $m = n$. **D.** It can not be determined..

   1. **A**

2. You are given a set of five data points and you are asked to find the best-fit quadratic polynomial for the date set. The matrix A that represent this data set must be of order

   A. 5×4. **B.** 5×3. **C.** 4×5. **D.** 3×5.

   2. **B**

3. An over-determined system, can be solved if

   A. $\det A^{\mathrm{T}}A = 0$. **B.** $\det Q^{\mathrm{T}}A \neq 0$. **C.** $\det R^{\mathrm{T}}R = 0$. **D.** $\det A^{\mathrm{T}}A \neq 0$.

   3. **D**

4. In QR-decomposition, which of the following is an identity matrix.

   A. $Q^{\mathrm{T}}Q$. **B.** $R^{\mathrm{T}}R$. **C.** $Q^{\mathrm{T}}R$. **D.** $R^{\mathrm{T}}Q$.

   4. **A**

Problems: Marks are as indicated

5. Consider the data: $f(1) = 1$, $f(2) = 2.2$, $f(3) = 2.8$ and $f(4) = 4$. Find the best-fit straight line for these data by answering the following:

   (a) (1.5 marks) Identify the matrices: A, b and x.
   Solution: Since we want to find the best-fit straight line, we need to find the Taylor coefficients of the linear polynomial. That is: $p_1(x) = a_0 + a_1 x$. Therefore, the using the given data, we find:

$$A = \begin{pmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \end{pmatrix} , \quad b = \begin{pmatrix} 1 \\ 2.2 \\ 2.8 \\ 4 \end{pmatrix} \quad \text{and} \quad x = \begin{pmatrix} a_0 \\ a_1 \end{pmatrix} .$$

   (b) (3 marks) Compute $A^{\mathrm{T}}A$ and $A^{\mathrm{T}}b$.
   Solution: Using the matrices from the previous parts, we get,

$$A^{\mathrm{T}}A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \end{pmatrix} = \begin{pmatrix} 4 & 10 \\ 10 & 30 \end{pmatrix} \quad \text{and} \quad A^{\mathrm{T}}b = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \end{pmatrix} \begin{pmatrix} 1 \\ 2.2 \\ 2.8 \\ 4 \end{pmatrix} = \begin{pmatrix} 10 \\ 29.8 \end{pmatrix} .$$

   (c) (1.5 marks) Evaluate x and write the best-fit straight line.
   Solution: From $A^{\mathrm{T}}Ax = A^{\mathrm{T}}b$, we obtain,

$$\begin{pmatrix} 4 & 10 \\ 10 & 30 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \end{pmatrix} = \begin{pmatrix} 10 \\ 29.8 \end{pmatrix} .$$

   Therefore, we find,

$$\begin{aligned} 4a_0 + 10a_1 &= 10 \implies a_0 + 2.5a_1 = 2.5 , \\ \text{and} \quad 10a_0 + 30a_1 &= 29.8 \implies a_0 + 3a_1 = 2.98 . \end{aligned}$$

   Solving for a_0 and a_1, we obtain, $a_0 = 0.10$ and $a_1 = 0.96$. Hence, the best-fit straight line is:

$$p_1(x) = 0.10 + 0.96x .$$