A. Answer the following questions:

1. (1 mark) Evaluate the exact integral value $I(f)$ of the function $f(x) = \frac{1}{x(lnx)^2}$, which is continuous on the interval $[e, e+1]$.
2. (6 marks) Consider the function $f(x) = 3x^2 + 25x + 0.2$. Now, compute the numerical intergral by using Trapezium rule (Newton-Cotes formula with $n = 1$) over the interval $[0, 2]$ and also find the relative error in percentage.
3. (6 marks) Consider the function $f(x) = sin(x) + e^{0.5x}$ which is to be integrated on the interval $[0, 2]$. Now, compute the numerical integral by using Newton-Cotes formula with $n = 2$ (Simpson's rule) and also find the relative error in percentage.
4. (7 marks) Consider the function $f(x) = x + e^x$ which is continuous on the interval $[1, 3]$. Now, use Composite Newton-Cotes formula to find the numerical integration for $m = 3$ and also find the relative error in percentage.

1.

$$Given,\ f(x) = \frac{1}{x(ln(x))^2}$$

$$I(f) = \int_{e}^{e+1} \frac{1}{x(ln(x))^2} dx$$

$$= 0.23854$$

2.

$$Given,\ f(x) = 3x^2 + 25x + 0.2$$

$$f(a) = f(0) = 3(0)^2 + 25(0) + 0.2 = 0.2$$

$$f(b) = f(2) = 3(2)^2 + 25(2) + 0.2 = 62.2$$

$$I(f) = \int_0^2 \left(3x^2 + 25x + 0.2\right) dx = 58.4 \qquad [Actual\ Integration\ ]$$

$$I_1(f) = \frac{b-a}{2}[f(a) + f(b)] \qquad [Approximate\ Integration]$$

$$\Rightarrow I_1(f) = \frac{2-0}{2}[0.2 + 62.2] = 62.4$$

$$relative\ error\ (in\ percentage) = \frac{|I_1 - I|}{|I|} \times 100 = \frac{|62.4-58.4|}{|58.4|} \times 100 = 6.85\%$$

3.

$Given,\ f(x) = sin(x) + e^{0.5x}$

$$f(a) = f(0) = sin(0) + e^{0.5\times0} = 1$$

$$f(b) = f(2) = sin(2) + e^{0.5\times2} = 3.63$$

$$f(m) = f\left(\frac{a+b}{2}\right) = f\left(\frac{0+2}{2}\right) = sin(1) + e^{0.5\times1} = 2.49$$

$$I(f) = \int_0^2 sin(x) + e^{0.5x}dx = 4.85 \qquad [Actual\ Integration\ ]$$

$$I_2(f) = \frac{b-a}{6}\ [f(a) + 4\,f(m) + f(b)] \qquad [Approximate\ Integration]$$

$$\Rightarrow I_2(f) = \frac{2-0}{6}\ [1 + (4 \times 2.49) + 3.63]$$

$$\Rightarrow I_2(f) = \frac{1}{3}\ [14.59] = 4.86$$

$$relative\ error\ (in\ percentage) = \frac{\left|I_2 - I\right|}{|I|} \times 100 = \frac{|4.86-4.85|}{|4.85|} \times 100 = 0.206\%$$

4.

$Given,\ f(x) = x + e^{x}$

$$Actual\ result,\ I(f) = \int_1^3 x + e^{x}dx = 21.37$$

$$h = \frac{b-a}{m} = \frac{3-1}{3} = \frac{2}{3}$$

$If\ m = 3,\ then\ need\ to\ find\ x_0\ to\ x_3\ nodes$

$$x_0 = a = 1$$

$$x_1 = x_0 + h = 1 + \frac{2}{3} = \frac{5}{3}$$

$$x_2 = x_1 + h = \frac{5}{3} + \frac{2}{3} = \frac{7}{3}$$

$$x_3 = x_2 + h = \frac{7}{3} + \frac{2}{3} = \frac{9}{3} = 3$$

$Composite\ Newton\ Cotes\ formulae \rightarrow$

$$C_{1,m}(f) = \frac{h}{2}\left[f\left(x_0\right) + 2f\left(x_1\right) + 2f\left(x_2\right) + 2f\left(x_3\right) + \ldots\ldots + 2f\left(x_{m-1}\right) + f\left(x_m\right)\right]$$

$$So,\ C_{1,3}(f) = \frac{\frac{2}{3}}{2}\left[f\left(x_0\right) + 2f\left(x_1\right) + 2f\left(x_2\right) + f\left(x_3\right)\right]$$

$$= \frac{\frac{2}{3}}{2}\left[f(1) + 2f\left(\frac{5}{3}\right) + 2f\left(\frac{7}{3}\right) + f(3)\right]$$

$$= \frac{1}{3}\left[\left(1 + e^{1}\right) + 2\left(\frac{5}{3} + e^{\frac{5}{3}}\right) + 2\left(\frac{7}{3} + e^{\frac{7}{3}}\right) + \left(3 + e^{3}\right)\right]$$

$$= \frac{1}{3} \times 66.0173 = 22.01$$

$$relative\ error\ (in\ percentage) = \frac{|22.01-21.37|}{|21.37|} \times 100 = 2.99\%$$