MCQ: Choose Only One Answer

1. (a) How many coefficients $P_{13}(x)$ has?
   A. n **B.** 12 **C.** 13 **D.** 14

   (a) **D**

   (b) Consider the vector space of polynomials spanned by the basis $\{1, x, x^2, x^3\}$. What is the maximum degree of a polynomial that can be expressed as a linear combination of these basis elements?
   A. 2 **B.** 3 **C.** 4 **D.** 5

   (b) **B**

   (c) Vandermonde matrix is generated 7×7. What is the degree of the polynomial?
   A. 5 **B.** 6 **C.** 7 **D.** 8

   (c) **B**

   (d) Suppose you are using Newton's Divided/Difference method to find interpolating polynomial. If $f(x) = \frac{1}{x}$ then $f[x_0, x_1]$ is
   A. -1 **B.** $-\frac{1}{x_0 x_1}$ **C.** $-\frac{1}{x_0^2 x_1^2}$ **D.** $-\frac{1}{x_0^3 x_1^3}$

   (d) **B**

   (e) Which of the following statements is/are not true?
   i. We cannot add a new node easily after computation in Newton's method.
   ii. We need to do the calculation of $l_k(x)$ newly again if we added a new node.
   iii. Suppose you are given n nodes, the polynomial will be of degree $n + 1$.
   A. (i) only. **B.** (iii) only. **C.** (i, iii) only. **D.** (ii, iii) only. **E.** All of these.

   (e) **C**

   (f) Which of the following statements is/are true?
   i. If $P_n(x) \approx P_\infty(x), P(x)$ will act as $f(x)$.
   ii. In Newton's Polynomial, $n_n(x) = (x - x_0)(x - x_1)(x - x_2).....(x - x_{n-1})$
   iii. A system is solved using a 3×3 Vandermonde matrix. Now if the system is solved by Lagrange method, then 3 Lagrange basis elements are needed.
   A. (i, ii) only. **B.** (i, iii) only. **C.** (ii, iii) only. **D.** All of these.

   (f) **D**

   (g) How many coefficients $P_{11}(x)$ has?
   A. n **B.** 10 **C.** 11 **D.** 12

   (g) **D**

   (h) Consider the vector space of polynomials spanned by the basis $\{x^0, x^1, x^2, x^3\}$. What is the maximum degree of a polynomial that can be expressed as a linear combination of these basis elements?
   A. 2 **B.** 3 **C.** 4 **D.** 5

   (h) **B**

   (i) Vandermonde matrix is generated 5×5. What is the degree of the polynomial?
   A. 4 **B.** 5 **C.** 6 **D.** None of these

   (i) **A**

   (j) Which of the following statements is/are not true?
   i. We can add a new node easily after computation in Newton's method.
   ii. We don't need to do the calculation of $l_k(x)$ newly again if we added a new node.
   iii. Suppose you are given n nodes, the polynomial will be of degree $n + 1$.
   A. (ii) only. **B.** (iii) only. **C.** (i, iii) only. **D.** (ii, iii) only. **E.** All of these.

   (j) **D**

   (k) Which of the following statements is/are true?
   i. If $P_n(x) \approx P_\infty(x), P(x)$ will act as $f(x)$.
   ii. In Newton's Polynomial, $n_3(x) = (x - x_0)(x - x_1)(x - x_2)(x - x_3)$
   iii. A system is solved using a 3×3 Vandermonde matrix. Now if the system is solved by Lagrange method, then 3 Lagrange basis elements are needed.
   A. (i, ii) only. **B.** (i, iii) only. **C.** (ii, iii) only. **D.** All of these.

   (k) **B**

(l) How many coefficients $P_7(x)$ has?
A. 1 **B.** 6 **C.** 7 **D.** 8

(l) **D**

(m) Consider the vector space of polynomials spanned by the basis $\{1, x, x^2, x^3, x^4\}$. What is the maximum degree of a polynomial that can be expressed as a linear combination of these basis elements?
A. 3 **B.** 4 **C.** 5 **D.** 6

(m) **B**

(n) Vandermonde matrix is generated 11×11. What is the degree of the polynomial?
A. 10 **B.** 11 **C.** 12 **D.** None of these

(n) **A**

(o) Which of the following statements is/are not true?
i. We can add a new node easily after computation in Newton's method.
ii. We need to do the calculation of $l_k(x)$ newly again if we added a new node.
iii. Suppose you are given n nodes, the polynomial will be of degree $n + 1$.
A. (i) only. **B.** (iii) only. **C.** (i, iii) only. **D.** (ii, iii) only. **E.** None of these.

(o) **B**

(p) Which of the following statements is/are true?
i. If $P_n(x) \approx P_\infty(x), P(x)$ will act as $f(x)$.
ii. In Newton's Polynomial, $n_n(x) = (x - x_0)(x - x_1)(x - x_2).....(x - x_n)$
iii. A system is solved using a 9×9 Vandermonde matrix. Now if the system is solved by Lagrange method, then 9 Lagrange basis elements are needed.
A. (i, ii) only. **B.** (i, iii) only. **C.** (ii, iii) only. **D.** All of these.

(p) **B**

Problems: Marks are as indicated

2. (4 marks) $f(x) = e^{-x}$ using the nodes $x_0 = 0, x_1 = 1, x_2 = 2$ in the interval [-0.5, 2.5]. Working to 3 significant figures, compute the upper bound of the estimated error if f(x) is interpolated by a degree two polynomial.

$There\ have\ 3\ nodes \rightarrow P_2(x)$

$$\left|f(x) - P_2(x)\right| = \left|\left(\frac{f^{(3)}(\xi)}{3!}\right)\right| \times |(x-0)(x-1)(x-2)|$$

$$= \left(\frac{f^{(3)}(\xi)}{6}\right)(x-0)(x-1)(x-2) \quad \left[f(x) = e^{-\xi}\right]$$

$$= \left(\frac{f^{(3)}(\xi)}{6}\right)(x-0)(x-1)(x-2) \quad \left[f^{(3)}(x) = -e^{-\xi}\right]$$

$$\rightarrow \left(e^{-\xi}\right).\ w(x) \quad [w(x) = (x-0)(x-1)(x-2)]$$

$For\ \left(e^{-\xi}\right),\ maximum\ value\ of\ e^{-\xi}\ in\ interval\ [-0.5, 2.5]\ is\ 1.65$

$For\ w(x),$

$$w(x) = (x-0)(x-1)(x-2)$$

$$= x^3 - 3x^2 + 2x$$

$$w'(x) = 0 \Rightarrow 3x^2 - 6x + 2 = 0$$

$$\Rightarrow x = 1 \pm \frac{1}{\sqrt{3}}$$

$So,\ we\ have\ total\ four\ x\ values\ in\ interval\ [-0.5, 2.5].$

$Now,$

$For\ w(x),\ find\ maximum\ value\ of\ w(x) \rightarrow$

x	$\lvert w(x)\rvert = \lvert(x-0)(x-1)(x-2)\rvert$
$x = 1 + \frac{1}{\sqrt{3}}$	0.385
$x = 1 - \frac{1}{\sqrt{3}}$	0.385
$x = -0.5$	1.875
$x = 2.5$	1.875

$$\therefore \left|f(x) - P_2(x)\right| = \left|\left(\frac{f^{(3)}(\xi)}{3!}\right)\right| \times |(x-0)(x-1)(x-2)|$$

$$Max\ error = \left|\left(\frac{1}{6} \times 1.65\right)\right| \times |1.875|$$

$$\approx 0.516$$