'Assignment-03'

Amirun Nahin

ID: 23201416

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Ans. to the que. no-01

(a)

Given,

$P(A,B,C) = 0.2$, $P(\neg A,\neg B,C) = 0.35$

$P(A,\neg B,C) = 0.15$, $P(A) = 0.5$

$P(A,\neg B,\neg C) = 0.05$ $P(\neg C) = 0.25$

$P(\neg A,B,C) = 0.05$

$P(\neg A,B,\neg C) = 0.05$

Now,

$$P(A) = P(A,B,C) + P(A,B,\neg C) + P(A,\neg B,C) + P(A,\neg B,\neg C)$$

$$\Rightarrow 0.5 = 0.2 + P(A,B,\neg C) + 0.15 + 0.05$$

$$\Rightarrow P(A,B,\neg C) = 0.5 - (0.2 + 0.15 + 0.05)$$

$$= 0.1$$

$$\therefore P(A,B,\neg C) = 0.1$$

and,

$$P(\neg A) = P(\neg A,B,C) + P(\neg A,B,\neg C) + P(\neg A,\neg B,C) + P(\neg A,\neg B,\neg C)$$

$$\Rightarrow 1 - P(A) = 0.05 + 0.05 + 0.35 + P(\neg A,\neg B,\neg C)$$

$$\Rightarrow 1 - 0.5 = 0.45 + P(\neg A,\neg B,\neg C)$$

$$\Rightarrow P(\neg A,\neg B,\neg C) = 0.05$$

$$\therefore P(\neg A,\neg B,\neg C) = 0.05$$

So, the joint probability distribution table,

	B		¬B	
	C	¬C	C	¬C
A	0.2	0.1	0.15	0.05
¬A	0.05	0.05	0.35	0.05

(b)

$$P(\neg B) = P(A, \neg B, C) + P(\neg A, \neg B, C) + P(A, \neg B, \neg C) + P(\neg A, \neg B, \neg C)$$

$$= 0.15 + 0.35 + 0.05 + 0.05$$

$$= 0.6$$

$$\therefore P(\neg B) = 0.6$$

(c)

$$P(A \mid \neg B, C) = \frac{P(A, \neg B, C)}{P(\neg B, C)}$$

$$= \frac{P(A, \neg B, C)}{P(\neg A, \neg B, C) + P(A, \neg B, C)}$$

$$= \frac{0.15}{0.35 + 0.15}$$

$$= \frac{0.15}{0.5}$$

$$= 0.3$$

$\therefore P(A \mid \neg B, C) = 0.3$

and, $P(\neg A \mid B, \neg C) = \dfrac{P(\neg A, B, \neg C)}{P(B, \neg C)}$

$$= \frac{P(\neg A, B, \neg C)}{P(A, B, \neg C) + P(\neg A, B, \neg C)}$$

$$= \frac{0.05}{0.1 + 0.05}$$

$$= \frac{0.05}{0.15}$$

$$= 0.333$$

$\therefore P(\neg A \mid B, \neg C) = 0.333$ (Ans.)

(d)

$P(A) = 0.5$

$P(B) = 1 - P(\neg B)$
$= 1 - 0.6$
$= 0.4$

and, $P(C) = 1 - P(\neg C)$
$= 1 - 0.25$
$= 0.75$

Now,

$$P(A,B,C) = 0.2$$

and,

$$P(A) \times P(B) \times P(C)$$

$$= 0.5 \times 0.4 \times 0.75$$

$$= 0.15$$

Since, $P(A,B,C) \neq P(A) \times P(B) \times P(C)$.

A, B, C are not independent. (Ans.)

Ans. to the que. no-02

(a)

Let D = Disease

D' = No Disease

T = Tested Positive

T' = Tested Negative

Now, $P(D) = \frac{1}{1000} = 0.001$

$$P(D') = 1 - P(D)$$

$$= 0.999$$

$P(T|D') = \frac{1}{100} = 0.01$

$P(T'|D) = \frac{3}{100} = 0.03$, $P(T|D) = 1 - 0.03$
$= 0.97$

Now, using bayes rule,

$$P(D|T) = \frac{P(T|D) \cdot P(D)}{P(T)}$$

$$= \frac{P(T|D) \cdot P(D)}{P(T|D) \cdot P(D) + P(T|D') \cdot P(D')}$$

$$= \frac{0.97 \times 0.001}{(0.97 \times 0.001) + (0.01 \times 0.999)}$$

$$= 0.0885$$

So, the probability of having that disease given that patient tested positive once is 0.0885.

(Ans.)

(b)

We got $P(D|T) = 0.0885$. We can use this as our new prior,

$$P_n(D) = 0.0885, \quad P_n(D') = 1 - 0.0885 = 0.9115$$

Again using Bayes rule,

$$P(D|T_1, T_2) = \frac{P(T|D) \cdot P_n(D)}{P(T)}$$

$$= \frac{P(T|D) \cdot P_n(D)}{P(T|D) \cdot P_n(D) + P(T|D') \cdot P_n(D')}$$

$$= \frac{(0.97 \times 0.0885)}{(0.97 \times 0.0885) + (0.01 \times 0.9115)}$$

$$= 0.904$$

$\therefore P(D|T_1, T_2) = 0.904$

So, the probability of the patient having that disease given that patient tested positive twice is 0.904.

(Ans.)

Ans. to the que. no-03

(a)

Given,

$$D_1 = \{(8, 1.2), (9, 1.3), (10, 1.5), (12, 1.8), (15, 2.3)\}$$

$$\eta = \frac{100}{100 * 10t}$$

$$\theta_i = i$$

for iteration 1,

$$\eta = 0.1$$

$$\theta_0 = 0$$

$$\theta_1 = 1$$

So,

$$\theta_0 = \theta_0 + \eta \sum_{i=1}^{n} (y^{(i)} - h_\theta(x^{(i)}))$$

$$= 0 + 0.1 \Big[\{(1.2-8)\} + \{(1.3-9)\} + \{(1.5-10)\} + \{(1.8-12)\} + \{(2.3-15)\} \Big]$$

$$= 0.1 \times (-45.9)$$

$$= -4.59$$

$$\theta_1 = \theta_1 + \eta \sum_{i=1}^{n} \left(y^{(i)} - h_\theta(x^{(i)}) \right) x_j^{(i)}$$

$$= 1 + 0.1 \Big[\{(1.2-8)\cdot 8\} + \{(1.3-9)\cdot 9\} + \{(1.5-10)\cdot 10\} + \{(1.8-12)\cdot 12\} + \{(2.3-15)\cdot 15\} \Big]$$

$$= 1 + 0.1 \times (-521.6)$$

$$= -51.16$$

For Iteration 2,

$$\eta = 0.05$$

$$\theta_0 = -4.59$$

$$\theta_1 = -51.16$$

So,

$$\theta_0 = \theta_0 + \eta \sum_{i=1}^{n} \left(y^{(i)} - h_\theta(x^{(i)}) \right)$$

$$= -4.59 + 0.05 \Big[\{1.2 - (-4.59 + (-51.16 \times 8))\} + \{1.3 - (-4.59 + (-51.16 \times 9))\} + \{1.5 - (-4.59 + (-51.16 \times 10))\} + \{1.8 - (-4.59 + (-51.16 \times 12))\} + \{2.3 - (-4.59 + (-51.16 \times 15))\} \Big]$$

$$= -4.59 + 0.05 (415.07 + 466.33 + 517.69 + 620.31 + 774.29)$$

$$= 135.0945$$

$$\theta_1 = \theta_1 + \eta \sum_{i=1}^{n} \left(y^{(i)} - h_\theta(x^{(i)}) \right) x_j^{(i)}$$

$$= -51.16 + 0.05 \Big[\{1.2 - (-4.59 + (-51.16 \times 8))\} \cdot 8 +$$
$$\{1.3 - (-4.59 + (-51.16 \times 9))\} \cdot 9 +$$
$$\{1.5 - (-4.59 + (-51.16 \times 10))\} \cdot 10 +$$
$$\{1.8 - (-4.59 + (-51.16 \times 12))\} \cdot 12 +$$
$$\{2.3 - (-4.59 + (-51.16 \times 15))\} \cdot 15 \Big]$$

$$= -51.16 + 0.05 \left(3320.56 + 4196.97 + 5176.9 + 7443.72 + 11614.35 \right)$$

$$= 1536.465$$

(Ans.)

(b)

Given,

$$D_2 = \{ (10,1), (13,1), (5,0), (15,1), (7,0) \}$$

For iteration 1,

$$\theta_0 = 0$$
$$\theta_1 = 1$$
$$\eta = 0.1$$

$$\theta_0 = \theta_0 + \eta \sum_{i=1}^{n} y^{(i)} - h_\theta(x^{(i)})$$

$$= 0 + 0.1 \left[\{1 - g(10)\} + \{1 - g(13)\} + \{0 - g(5)\} + \{1 - g(15)\} + \{0 - g(7)\} \right]$$

$$= 0.1 \times \left[(4.54 \times 10^{-5}) + (2.26 \times 10^{-6}) + (-0.9933) + (3.06 \times 10^{-7}) + (-0.999) \right]$$

$$= -0.19922$$

$$\theta_1 = \theta_1 + \eta \sum_{i=1}^{n} \{y^{(i)} - h_\theta(x^{(i)})\} x_j^{(i)}$$

$$= 1 + 0.1 \left[\{1 - g(10)\} 10 + \{1 - g(13)\} 13 + \{0 - g(5)\} 5 + \{1 - g(15)\} 15 + \{0 - g(7)\} 7 \right]$$

$$= -0.196$$

For iteration 2,

$$\theta_0 = -0.19922$$

$$\theta_1 = -0.196$$

$$\eta = 0.05$$

$$\theta_0 = \theta_0 + \eta \sum_{i=1}^{n} y^{(i)} - h_\theta(x^{(i)})$$

$$= -0.19922 + 0.05 \times \Big[\{1 - g(-0.19922 - 0.196 \times 10)\} + \{1 - g(-0.19922 - 0.196 \times 13)\} + \{0 - g(-0.19922 - 0.196 \times 5)\} + \{1 - g(-0.19922 - 0.196 \times 15)\} + \{0 - g(-0.19922 - 0.196 \times 7)\} \Big]$$

$$= -0.19922 + 0.1194$$

$$= -0.0798$$

$$\theta_1 = \theta_1 + \eta \sum_{i=1}^{n} \{y^{(i)} - h_\theta(x^{(i)})\} x_j^{(i)}$$

$$= -0.196 + 0.05 \times \Big[\{1 - g(-0.19922 - 0.196 \times 10)\} 10 + \{1 - g(-0.19922 - 0.196 \times 13)\} 13 + \{0 - g(-0.19922 - 0.196 \times 5)\} 5 + \{1 - g(-0.19922 - 0.196 \times 15)\} 15 + \{1 - g(-0.19922 - 0.196 \times 7)\} 7 \Big]$$

$$\{0 - g(-0.19922$$

$$= -0.196 + 0.05 \times (33.179)$$

$$= -0.196 + 1.659$$

$$= 1.463$$

(Ans)

(c)

Let X be the matrix and y be the vector. So our loss function (least squared loss function) that is,

$$J(\theta) = \frac{1}{2} \sum_{i=1}^{n} (\bar{y}^{(i)} - y^{(i)})^2$$

the matrix form for $J(\theta)$,

$$J(\theta) = (X\theta - y)^T (X\theta - y)$$

$$= (\theta^T X^T - y^T)(X\theta - y)$$

$$= (\theta^T X^T X\theta - \theta^T X^T y - y^T X\theta + y^T \cdot y)$$

$$= \theta^T X^T X\theta - 2\theta^T X^T y + y^T \cdot y$$

Now,

$$\frac{\partial}{\partial\theta} J(\theta) = \frac{\partial}{\partial\theta}\left(\theta^T X^T X\theta - 2\theta^T X^T y + y^T y\right)$$

$$= \frac{\partial}{\partial\theta}\left(\theta^T X^T X\theta\right) - 2\frac{\partial}{\partial\theta}\left(\theta^T X^T y\right) + \frac{\partial}{\partial\theta}\left(y^T y\right)$$

$$= 2X^T X\theta - 2X^T y$$

Now, to minimize, the gradient must be 0,

$$2X^TX\theta - 2X^Ty = 0$$

$$\Rightarrow X^TX\theta = X^Ty$$

$$\Rightarrow (X^TX)\theta = X^Ty$$

$$\Rightarrow \theta = (X^TX)^{-1} X^Ty$$

So, the closed-form solution for linear regression,

$$\theta = (X^TX)^{-1} \cdot X^Ty$$

(Ans.)