Ejercicio #3 Guía

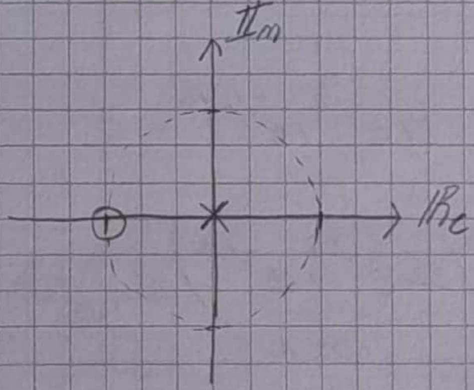
a) $h_1(k) = (1,1)$

$Y(k) = (1,1)$, cuando $X(k) = (1,0,0,0,\dots)$

$Y(k) = X(k) + X(k-1)$

$Y(z) = X(z) + z^{-1} \cdot X(z)$; $Y(z) = X(z)\left(1+z^{-1}\right)$

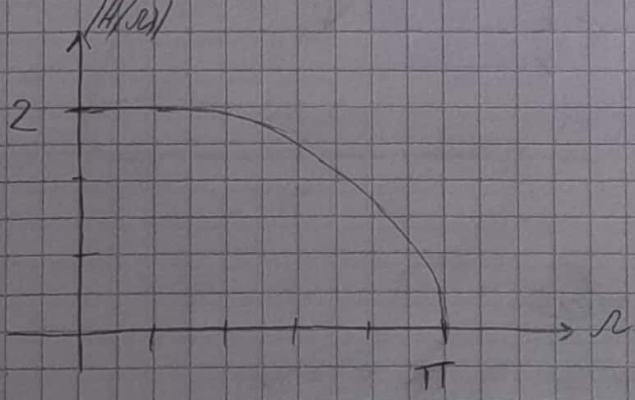
$$H(z) = \frac{Y(z)}{X(z)} = 1 + z^{-1} = \frac{z+1}{z} \; ; \; \boxed{H(z) = \frac{z+1}{z}}$$

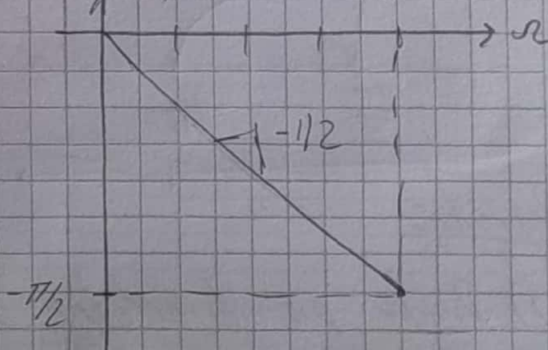


$$H(\Omega) = H(z)\Big|_{z=e^{j\Omega}} = \frac{e^{j\Omega}+1}{e^{j\Omega}} = \frac{e^{j\Omega/2}}{e^{j\Omega}} \cdot \left(e^{j\Omega/2} + e^{-j\Omega/2}\right)$$

$$H(\Omega) = \frac{e^{j\Omega/2}}{e^{j\Omega}}\, 2\cos\Omega \quad ; \quad H(\Omega) = e^{-j\Omega/2} \cdot 2\cos(\Omega/2)$$

$$\therefore \boxed{|H(\Omega)| = 2\cos\left(\frac{\Omega}{2}\right)} \quad ; \quad \boxed{\theta(\Omega) = -\frac{\Omega}{2}}$$





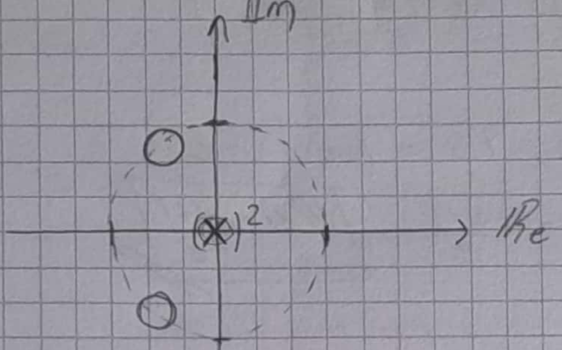
$h_2(k) = (1,1,1)$

$Y(k) = X(k) + X(k-1) + X(k-2)$; $Y(z) = X(z) + z^{-1}X(z) + z^{-2}X(z)$

$Y(z) = X(z)(1 + z^{-1} + z^{-2})$; $H(z) = \frac{Y(z)}{X(z)} = 1 + z^{-1} + z^{-2} = \frac{z^2+z+1}{z^2}$

$$\boxed{H(z) = \frac{z^2+z+1}{z^2}}$$

Ceros $\begin{cases} 1\, e^{j2\pi/3} \\ 1\, e^{-j2\pi/3} \end{cases}$

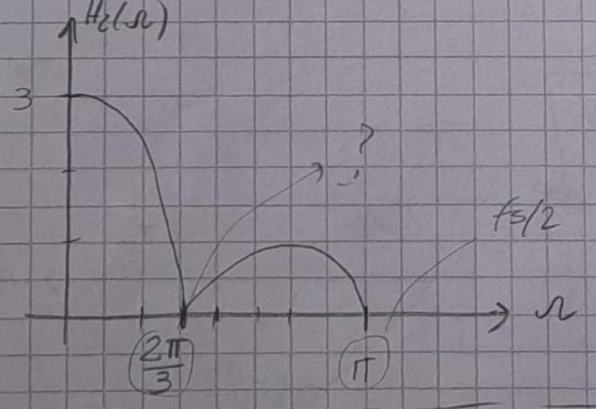


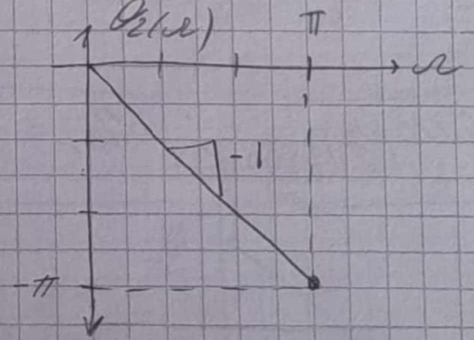
$H_2(\Omega) = H_2(z)\big|_{z=e^{j\Omega}} = \frac{e^{j2\Omega} + e^{j\Omega} + 1}{e^{j2\Omega}} = 1 + e^{-j\Omega} + e^{-j2\Omega}$

$H_2(\Omega) = e^{-j\Omega}\left(e^{+j\Omega} + 1 + e^{-j\Omega}\right) = e^{-j\Omega}\left(1 + \underbrace{e^{j\Omega} + e^{-j\Omega}}_{2\cos\Omega}\right)$

$$\boxed{H_2(\Omega) = e^{-j\Omega}(1 + 2\cos\Omega)}$$

$\boxed{|H_2(\Omega)| = 1 + 2\cos(\Omega)}$, $\boxed{\theta_2(\Omega) = e^{-j\Omega}}$





Preguntas

1) Dividir por la cantidad de muestras

2) $\frac{fs}{2} \longrightarrow \pi$

$X \longleftarrow \frac{2\pi}{3}$ $\Rightarrow X = \frac{\frac{2\pi}{3}\cdot\frac{fs}{2}}{\pi}$; $X = \frac{fs}{3}$; $X = 50\ Hz$

NOTA

$\Rightarrow \boxed{fs = 150\ Hz}$

HOJA Nº

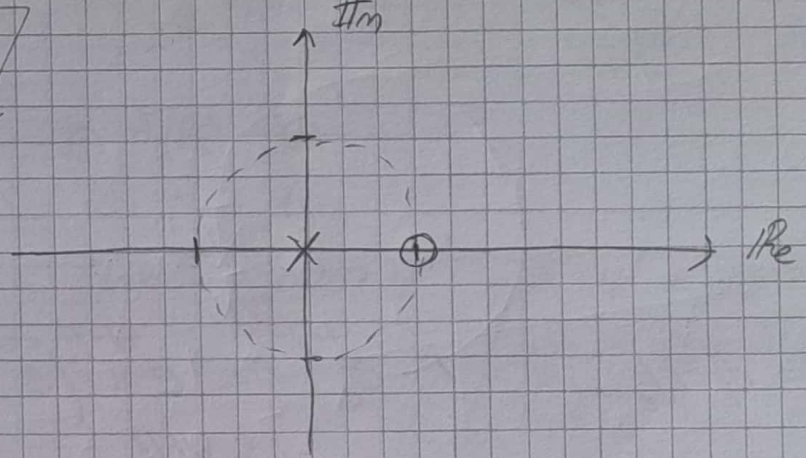
FECHA

5) Filtro Diferenciador

$h_1(k) = (1, -1)$

$y(k) = x(k) - x(k-1)$; $Y(z) = X(z) - z^{-1}X(z) = X(z)(1 - z^{-1})$

$$\boxed{H_1(z) = \frac{Y(z)}{X(z)} = \frac{z-1}{z}}$$

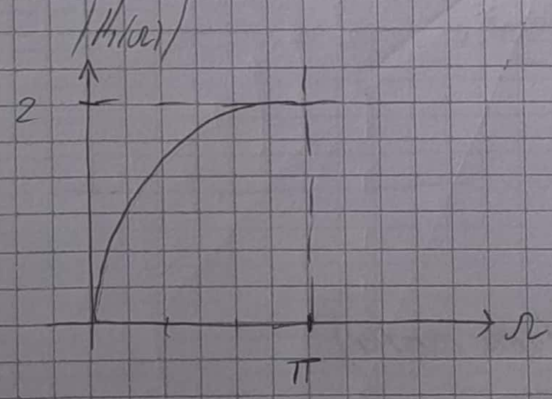


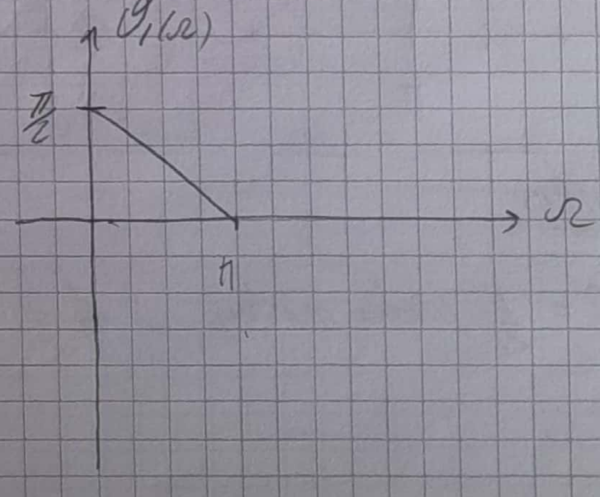
$H(\Omega) = H(z)\Big|_{z=e^{j\Omega}} = 1 - e^{-j\Omega} = e^{-j\Omega/2}\left(e^{j\Omega/2} - e^{-j\Omega/2}\right)$

$H(\Omega) = e^{-j\Omega/2} \cdot 2j \cdot \operatorname{sen}\left(\frac{\Omega}{2}\right) = e^{-j\Omega/2} \cdot 2 \cdot e^{j\pi/2} \cdot \operatorname{sen} \Omega/2$

$H(\Omega) = e^{j(\pi/2 - \Omega/2)} \cdot 2 \operatorname{sen}(\Omega/2)$

$$\begin{cases} \boxed{|H_1(\Omega)| = 2 \operatorname{sen}(\Omega/2)} \\ \boxed{\theta_1(\Omega) = \frac{\pi}{2} - \frac{\Omega}{2}} \end{cases}$$



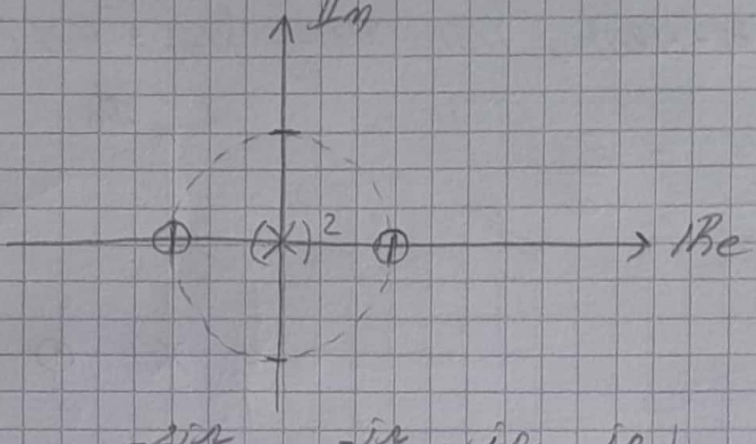


NOTA

$h_2(k) = (1, 0, -1)$

$Y(k) = X(k) - X(k-2)$; $Y(z) = X(z) - z^{-2} X(z)$; $H(z) = \frac{Y(z)}{X(z)} = 1 - z^{-2}$

$$\boxed{H(z) = \frac{z^2 - 1}{z^2}}$$

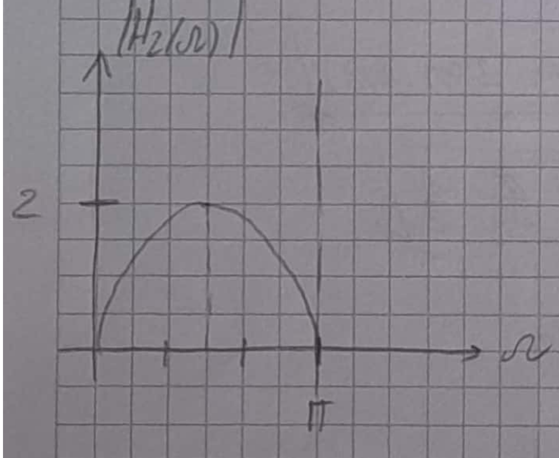


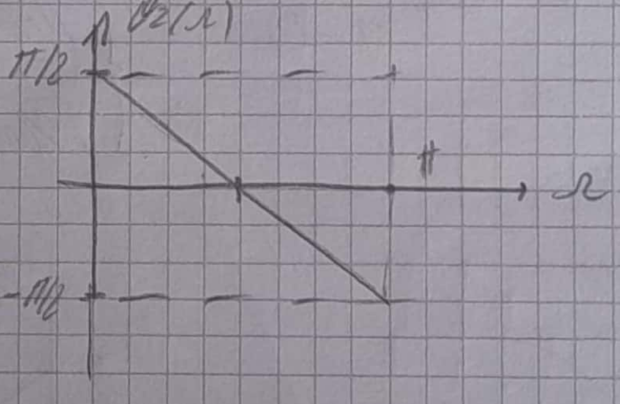
$H_2(\Omega) = H_2(z)\Big|_{z=e^{j\Omega}} = 1 - e^{-j2\Omega} = e^{-j\Omega}\left(e^{j\Omega} - e^{-j\Omega}\right)$

$H_2(\Omega) = e^{-j\Omega} \cdot 2j \cdot \operatorname{sen}\Omega = e^{-j\Omega} \cdot e^{j\pi/2} \cdot 2 \operatorname{sen}\Omega$

$H_2(\Omega) = e^{j(\pi/2 - \Omega)} \cdot 2 \operatorname{sen}\Omega$

$$\begin{cases} \boxed{|H_2(\Omega)| = 2 \operatorname{sen}\Omega} \\ \boxed{\theta_2(\Omega) = \frac{\pi}{2} - \Omega} \end{cases}$$





Preguntas

1) $\tau_1 = \frac{1}{2}$ (media muestra) ; $\tau_2 = 1$ (una muestra)

NOTA

HOJA Nº

FECHA

2) Diferenciador de Primer Orden

$$\mathcal{E}_{\%} = \frac{\Omega - |H_1(\Omega)|}{\Omega} = \frac{\Omega - 2\,\text{sen}(\Omega/2)}{\Omega} \quad ; \quad \mathcal{E}_{\%} = 1 - \frac{2\,\text{sen}(\Omega/2)}{\Omega}$$

$$\mathcal{E}_{\%} = 0{,}05 \Rightarrow 1 - \frac{2\,\text{sen}\left(\frac{\Omega}{2}\right)}{\Omega} = 0{,}05 \quad ; \quad 0{,}95 = \frac{2\,\text{sen}\left(\frac{\Omega}{2}\right)}{\Omega} \quad ; \quad \frac{\text{sen}\left(\frac{\Omega}{2}\right)}{\Omega} = 0{,}475$$

$$\Rightarrow \underline{\Omega = 1{,}104}$$

$$\pi \longrightarrow f_s/2$$

$$1{,}104 \longrightarrow 0{,}1757 \cdot f_s$$

$\Rightarrow$ $|H_1(\Omega)|$ se comporta como un derivador ideal hasta el $\approx$ 17,6 % de f_s

Diferenciador de Segundo Orden

$$\mathcal{E}_{\%} = \frac{\Omega - |H_2(\Omega)|}{\Omega} = 1 - \frac{H_2(\Omega)}{\Omega} = 1 - \frac{2\,\text{sen}\,\Omega}{\Omega}$$

$$\mathcal{E}_{\%} = 0{,}05 \Rightarrow 1 - \frac{2\,\text{sen}\,\Omega}{\Omega} = 0{,}05 \quad ; \quad \frac{2\,\text{sen}\,\Omega}{\Omega} = 0{,}95 \quad ; \quad \frac{\text{sen}\,\Omega}{\Omega} = 0{,}475$$

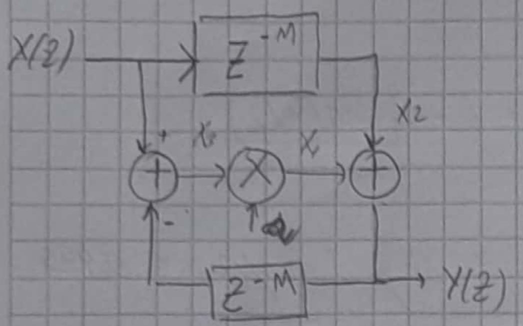
$$\Rightarrow \Omega = 1{,}953 \qquad \pi \rightarrow f_s/2$$

$$1{,}953 \longrightarrow 0{,}3108 \cdot f_s$$

$\Rightarrow$ $|H_2(\Omega)|$ se comporta como un derivador ideal hasta el $\approx$ 31,1 % de f_s

NOTA

Ejercicio #2

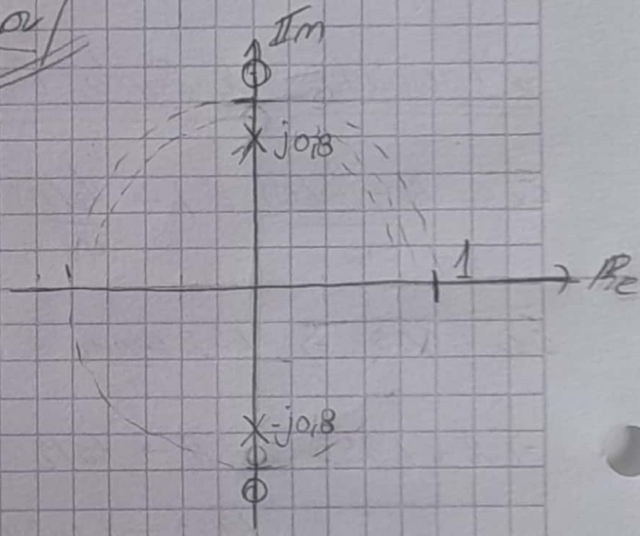


a) $Y(z) = X_1 + X_2 = a X_3 + z^{-M} \cdot X(z) = a \cdot \left(X(z) - z^{-M} \cdot Y(z)\right) + z^{-M} \cdot X(z)$

$Y(z) = a \cdot X(z) - a \cdot z^{-M} \cdot Y(z) + z^{-M} \cdot X(z) \;;\; Y(z)\left(1 + a \cdot z^{-M}\right) = X(z)\left(a + z^{-M}\right)$

$H(z) = \dfrac{Y(z)}{X(z)} = \dfrac{a + z^{-M}}{1 + a z^{-M}} \;;\; \boxed{H(z) = \dfrac{a \cdot z^{M} + 1}{z^{M} + a}}$

$M = 2 \;;\; a = 0{,}8 \Rightarrow \boxed{H(z) = \dfrac{0{,}8 \cdot z^{2} + 1}{z^{2} + 0{,}8}}$



Polos $= j \pm 0{,}8$

Ceros $\cong 1{,}2 \cdot e^{j\pi/2} \;;\; 1{,}2 \cdot e^{-j\pi/2}$

$H(\Omega) = H(z)\big|_{z = e^{j\Omega}} = \dfrac{0{,}8 \cdot e^{j2\Omega} + 1}{e^{j2\Omega} + 0{,}8}$

$H(0) = 1 \cdot e^{j0}$

$H(\pi/4) = 1 \cdot e^{-j \cdot 0{,}22}$

$H(\pi/2) = 1 \cdot e^{j\pi}$

$H(3\pi/4) = 1 \cdot e^{j0{,}22}$

$H(2\pi) = 1 \cdot e^{j0}$

$|H(z)| = 0$

NOTA