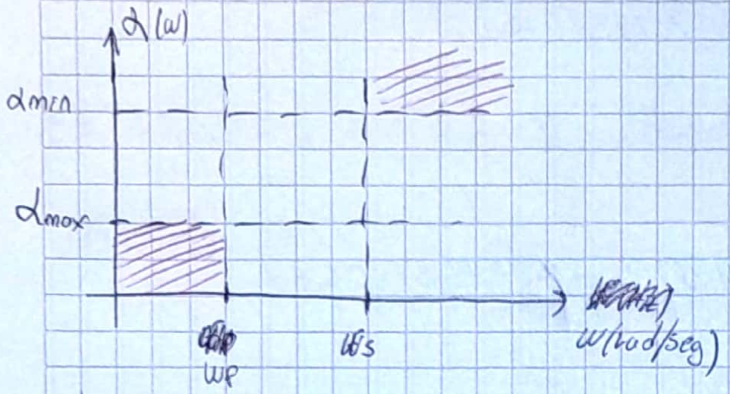
HOJA Nº 1

FECHA

Trabajo Semanal 3



$\alpha_{máx}(dB) = 1$

$\alpha_{min}(dB) = 12$

$f_p(Hz) = 1500$

$f_s(Hz) = 3000$

1) Normalizo con $\Omega_\omega = \omega_p = 2\pi f_p \rightarrow \underline{\omega_p = 1}$ y $\underline{\omega_s = 2}$

$|T(\omega)|^2 = \dfrac{1}{1+\varepsilon^2\omega^{2n}}$; $\alpha(\omega) = \dfrac{1}{|T(\omega)|} \Rightarrow \alpha^2(\omega) = 1+\varepsilon^2\omega^{2n}$

$\alpha(\omega) = \sqrt{1+\varepsilon^2\omega^{2n}} = (1+\varepsilon^2\omega^{2n})^{1/2}$

$\alpha(\omega)(dB) = 20\log(1+\varepsilon^2\omega^{2n})^{1/2}$; $\underline{\alpha(dB) = 10\log(1+\varepsilon^2\omega^{2n})}$

$\alpha_{max}(dB) = \alpha(\omega)\big|_{\omega=\omega_p=1} = 10\log(1+\varepsilon^2)$

$\dfrac{\alpha_{max}(dB)}{10} = \log(1+\varepsilon^2)$; $10^{\frac{\alpha_{max}}{10}} = 1+\varepsilon^2$; $\varepsilon^2 = 10^{\frac{\alpha_{max}(dB)}{10}} - 1$

$\Rightarrow \boxed{\varepsilon^2 = 0,2589}$

$\alpha_{min}(dB) = \alpha(\omega)\big|_{\omega=\omega_s=2} = 10\log(1+\varepsilon^2 . \omega_s^{2n})$

$\Rightarrow 12 = 10\log(1+0,2589 . 2^{2n})$

Iterando,

para $n = 1 \Rightarrow 3,1$

para $n = 2 \Rightarrow 7,1$

para $n = 3 \Rightarrow 12,44 \Rightarrow \boxed{n = 3}$

NOTA

$$|T(\$)|^2 = |T(\omega)|^2\Big|_{\omega = \frac{\$}{j}} = T(\$).T(-\$)$$

$$\therefore \frac{1}{1+\varepsilon^2.\left(\frac{\$}{j}\right)^{2n}} = \frac{1}{1-\varepsilon^2.\$^{2n}} = T(\$).T(-\$)$$

$$\frac{1}{1-\varepsilon^2.\$^6} = \frac{1}{a.\$^3+b.\$^2+c.\$+d} \cdot \frac{1}{-a.\$^3+b.\$^2-c.\$+d}$$

$\$^6$ $\quad -\varepsilon^2 = -a^2 \Rightarrow \boxed{a = \varepsilon}$

$\$^5$ $\quad 0 = a.b - b.a = 0$ ✓

$\$^4$ $\quad 0 = -a.c + b^2 - a.c \Rightarrow b^2 - 2ac = 0$

$\$^3$ $\quad 0 = \cancel{d.a} \cancel{-b.c} + \cancel{c.b} - \cancel{d.a} = 0$ ✓

$\$^2$ $\quad 0 = b.d - c^2 + b.d \Rightarrow c^2 - 2bd = 0$

$\$$ $\quad 0 = \cancel{c.d} - \cancel{c.d} = 0$ ✓

$\quad 1 = d^2 \Rightarrow \boxed{d = 1}$

$\therefore \quad b^2 - 2.\varepsilon.c = 0 \Rightarrow b = \sqrt{2\varepsilon c}$

$\quad c^2 - 2.b.1 = 0 \Rightarrow c^2 - 2\sqrt{2\varepsilon c} = 0 \; ; \; c^2 = 2\sqrt{2\varepsilon c}$

$c^4 = 4.2\varepsilon c \; ; \; c^{\cancel{4}\,3} = 8\varepsilon \cancel{c} \Rightarrow c = \sqrt[3]{8\varepsilon} \Rightarrow c = 2\sqrt[3]{\varepsilon} \Rightarrow \boxed{c = 2\varepsilon^{1/3}}$

$b = \sqrt{2\varepsilon c} = \sqrt{2.\varepsilon.2\varepsilon^{1/3}} = \sqrt{4.\varepsilon^{4/3}} = 2.\varepsilon^{2/3}$

$\Rightarrow \boxed{b = 2\varepsilon^{2/3}}$

$$\Rightarrow T(\$) = \frac{1}{\varepsilon.\$^3 + 2\varepsilon^{2/3}.\$^2 + 2\varepsilon^{1/3}.\$ + 1}$$

$$\boxed{T(\$) = \frac{1/\varepsilon}{\$^3 + 2\varepsilon^{-1/3}.\$^2 + 2\varepsilon^{-2/3}.\$ + 1/\varepsilon}}$$

NOTA

HOJA Nº 2

FECHA

2) $T(\$) = \dfrac{1/\varepsilon}{\$^3 + 2\varepsilon^{-1/3} \cdot \$^2 + 2\varepsilon^{-2/3} \cdot \$ + 1/\varepsilon} = \dfrac{1,96}{\$^3 + 2,5\$^2 + 3,14\$ + 1,96}$

$\varepsilon^2 = 0,2589 \Rightarrow \underline{\varepsilon = 0,5088}$. Usando numpy, obtengo las raíces.

Las raíces están en: $-1,25$

$-0,626 + j \cdot 1,08$

$-0,626 - j \cdot 1,08$

Armo el polinomio de orden 2.

$T_2(\$) = (\$ + 0,626 - j1,08)(\$ + 0,626 + j1,08)$

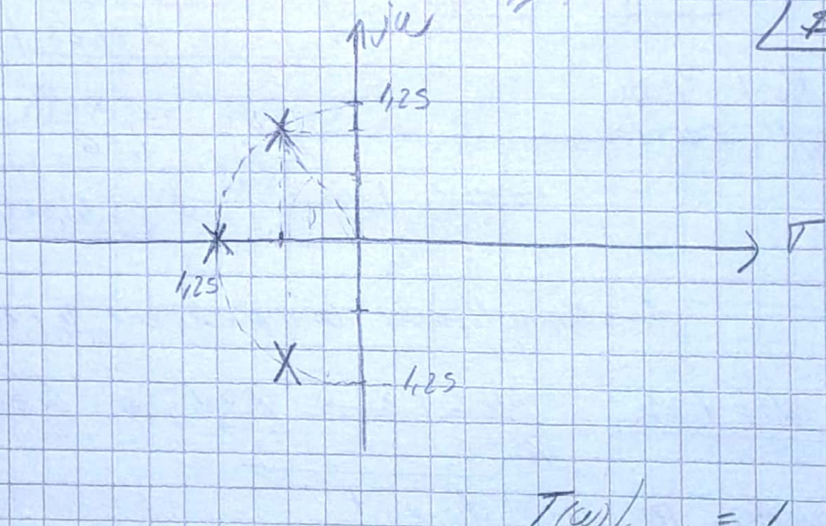
$T_2(\$) = \$^2 + \$0,626 + \cancel{\$j1,08} + \$0,626 + 0,63^2 + \cancel{j0,68} - \cancel{\$.j.1,08} - \cancel{j0,63} + 1,08^2$

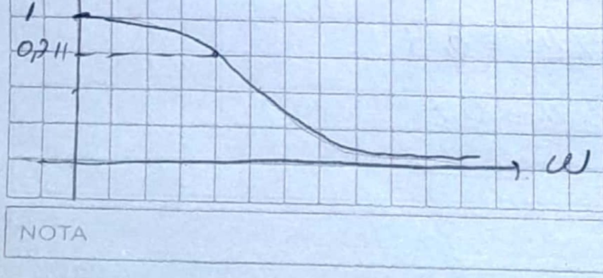
$\underline{T_2(\$) = \$^2 + \$ \cdot 1,25 + 1,56}$

$\Rightarrow \boxed{T(\$) = \dfrac{1,96}{(\$ + 1,25)(\$^2 + \$1,25 + 1,56)}}$

$\omega_0 = 1,25$

$\angle \$$





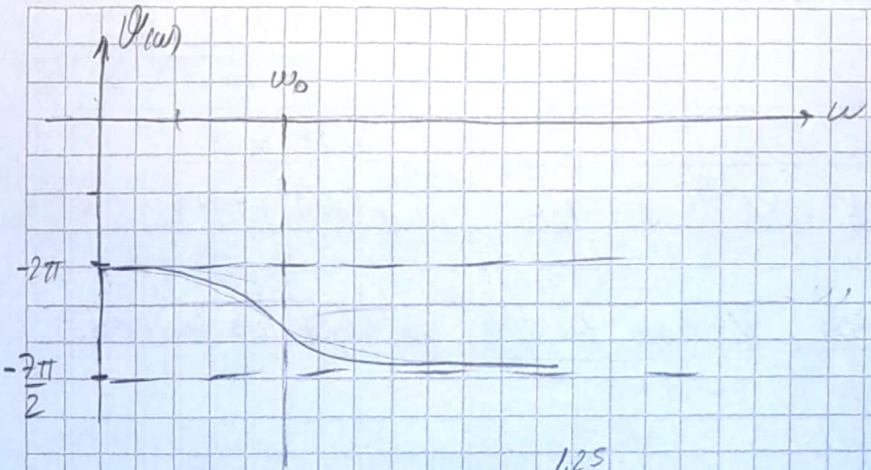
$T(\omega)\big|_{\omega=\varnothing} = 1$

$T(\omega)\big|_{\omega=\omega_0} = 0,711$

$T(\omega)\big|_{\omega\to\infty} = 0$

NOTA

Escaneado con CamScanner



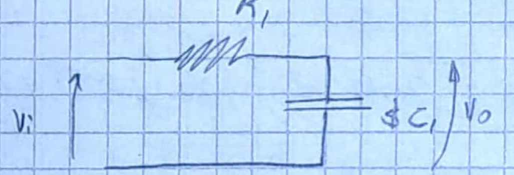
$\theta(\omega)|_{\omega=\phi} = \overbrace{0}^{\text{ceros}} - \overbrace{(2\pi + 0)}^{\text{polos}} = -2\pi$

$\theta(\omega)|_{\omega=\omega_0}$ = mitad de excursión

$\theta(\omega)|_{\omega\Rightarrow\infty} = \tilde{0} - \left(\frac{5\pi}{2} + \frac{\pi}{2} + \frac{\pi}{2}\right) = -\frac{7\pi}{2}$

3) $T(s) = \dfrac{1{,}96 \cdot \frac{1{,}25}{1{,}25}}{(s+1{,}25)(s^2+s1{,}25+1{,}56)} = \underbrace{\dfrac{1{,}25}{s+1{,}25}}_{T_1(s)} \cdot \underbrace{\dfrac{1{,}56}{s^2+s1{,}25+1{,}56}}_{T_2(s)}$

$\underline{\underline{T_1(s)}}$



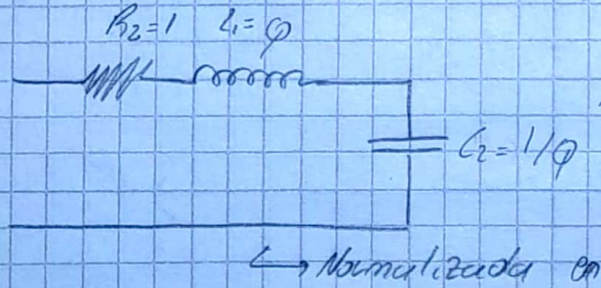
$V_0(s) = V_i(s) \cdot \dfrac{\frac{1}{sC_1}}{\frac{1}{sC_1} + R_1}$

$H(s) = \dfrac{1}{sC_1R_1+1} = \dfrac{1/C_1R_1}{s+1/C_1R_1} = \dfrac{\omega_0}{s+\omega_0}$

$\omega_0 = \dfrac{1}{C_1R_1}$, Normalizando respecto de $R_1 \Rightarrow \omega_0 = \dfrac{1}{C} = 1{,}25 \Rightarrow \underline{C = 0{,}8}$

$\Rightarrow \boxed{R_1 = 1 \;;\; C_1 = 0{,}8} \Rightarrow \boxed{T_1(s) = \dfrac{1{,}25}{s+1{,}25}}$

$\underline{\underline{T_2(s)}}$



$T_2(s) = \dfrac{1{,}56}{s^2+s\,1{,}25+1{,}56} = \dfrac{\omega_0^2}{s^2+s\frac{\omega_0}{\varphi}+\omega_0^2}$

↳ Normalizada en impedancia y frecuencia.

$\omega_0^2 = 1{,}56 \Rightarrow \underline{\omega_0 = 1{,}25}$; $\dfrac{\omega_0}{\varphi} = 1{,}25 \Rightarrow \underline{\varphi = 1}$, $\Rightarrow \underline{L_1 = 1,\ C_2 = 1}$

Desnormalizando en ω,

$R_2 = 1$

$L_1 = 1/\omega_0 = 0{,}8$

$C_2 = 1/\omega_0 = 0{,}8$

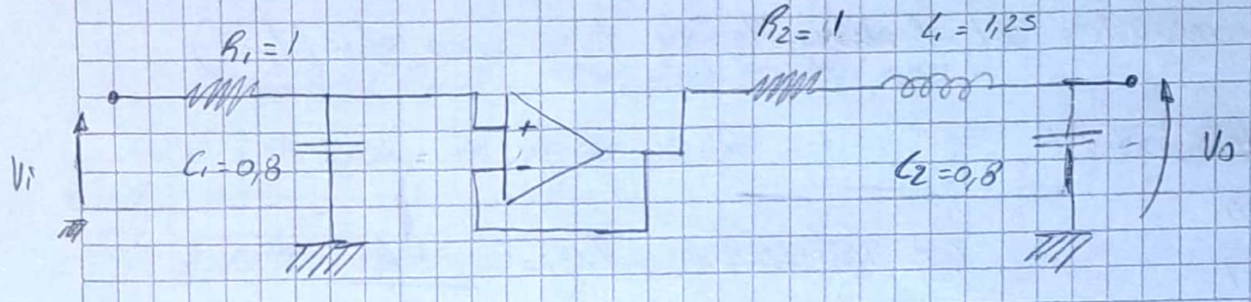
NOTA

$C = \dfrac{1}{j\omega C}$

HOJA Nº 3

FECHA

El circuito completo normalizado quedaría



Con $\Omega_\omega = \omega_p$

4) Necesito una nueva norma de impedancia.

$C_1 = C_2 = 100\,nF. \Rightarrow 100\,nF = \dfrac{0{,}8}{\Omega_\omega \cdot \Omega_Z}$; $\Omega_Z = \dfrac{0{,}8}{2\pi \cdot 1500 \cdot 100 \cdot 10^{-9}}$

$\therefore \underline{\Omega_Z \cong 848{,}83}$

Con eso, desnormalizo en frec. e impedancia el resto de componentes.

$R_1 = R_2 = 1 \cdot \Omega_Z \Rightarrow \underline{R_1 = R_2 \cong 848{,}83}$

$L_1 = 0{,}8 \cdot \dfrac{\Omega_Z}{\Omega_\omega} = 0{,}8 \cdot \dfrac{848{,}83}{2\pi \cdot 1500} \Rightarrow \underline{L_1 = 72{,}05\,mHy}$

$\therefore$ $\boxed{\begin{array}{l} C_1 = C_2 = 100\,nF \\ R_1 = R_2 = 848{,}83\ \Omega \\ L_1 = 72{,}05\ mHy \end{array}}$

NOTA

Escaneado con CamScanner

5) Propongo que $T_2(s)$ sea implementada por un Ackerberg-Mosberg.

$$T_2(s) = \frac{1,56}{s^2 + s\,1,25 + 1,56} = \frac{\omega_0^2}{s^2 + s\frac{\omega_0}{\varphi} + \omega_0^2} \left.\vphantom{\frac{1}{1}}\right\} \begin{array}{l} \omega_0 = 1,25 \\ \varphi = 1 \end{array}$$

Los componentes del Ackerberg-Mosberg tenían como valor (TS2)

$R_1 = 1/k \quad C_2 = 1$

$R_2 = \varphi$

$R_3 = 1$, que implementaban $T(s) = \dfrac{-k}{s^2 + s\cdot\frac{1}{\varphi} + 1}$

$R_4 = 1$

En mi caso, $k=1$, $\varphi = 1$, pero $\omega_0 = 1,25$ por lo que reescalo el capacitor. $C_2 = \dfrac{1}{\omega_0} = 0,8$

Como C_2 sigue teniendo el mismo valor de antes ($\nearrow 0,8$), no hace falta buscar una nueva Ωz, uso la del punto 4.

Desnormalizo los componentes.

$$\boxed{R_1 = R_2 = R_3 = R_4 = 848,83 \quad ; \quad C_2 = 100\,nF}$$

6) Del punto 1, se obtuvieron los valores

$\varepsilon^2 = 0,2589 \quad ; \quad n = 3$

$$|T(j\omega)|^2 = \frac{1}{1+\varepsilon^2\cdot\omega^{2n}} = \frac{1}{1+\left(\varepsilon^{1/n}\cdot\omega\right)^{2n}} = \frac{1}{1+\left(\underbrace{\frac{\omega}{\varepsilon^{-1/n}}}_{\omega'}\right)^{2n}}$$

es decir, $\omega' = \dfrac{\omega}{\varepsilon^{-1/n}} = \dfrac{\omega}{\omega_B}$; $\omega_B = \varepsilon^{-1/3} \longrightarrow \underline{\omega_B \cong 1,25}$

$$\therefore |T(j\omega)|^2 = \frac{1}{1+\omega^{2n}}$$

NOTA

Pero como es un butter, yo ya sé que

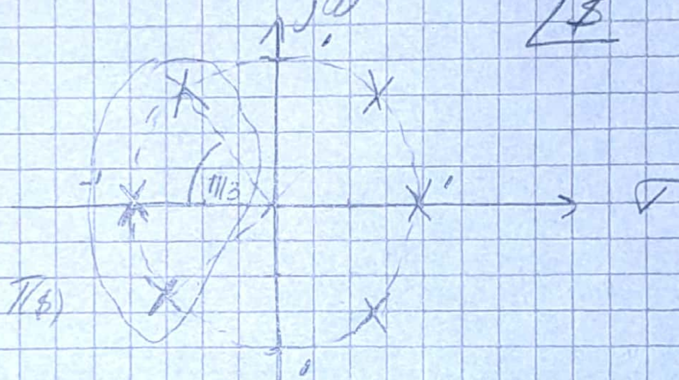
singularidades en circ. unitaria

Separación entre polos de $\frac{\pi}{n}$

Si n es impar, el primer polo es real puro.

Si n es par, el primer polo está en $\frac{\pi}{2n}$.

Tengo 3 polos, separados $\frac{\pi}{3}$ entre sí. Uno es real puro



$$Q = \frac{1}{2\cos\psi} = \frac{1}{2\cos \pi/3} \; ; \; \underline{Q = 1}$$

$$T_{B3}(S) = \frac{1}{(S+1)(S^2 + S\cdot\frac{1}{Q} + 1)} \; , \; Q = 1 \Rightarrow \underline{T_{B3}(S) = \frac{1}{(S+1)(S^2+S+1)}}$$

Ahora, desnormalizo. $S = \frac{s}{\Omega_\omega} = \frac{s}{\omega_B}$

$$\therefore \; T_3(s) = \frac{1}{\left(\frac{s}{1,25} + 1\right)\left(\frac{s^2}{1,25^2} + \frac{s}{1,25} + 1\right)}$$

$$T_3(s) = \frac{1}{\frac{1}{1,25}(s+1,25)\frac{1}{1,25^2}\cdot(s^2 + s\cdot 1,25 + 1,56)}$$

$$\boxed{T_3(s) = \frac{1,95}{(s+1,25)(s^2+s\,1,25+1,56)}}$$

Da lo mismo. En el numerador, en (1) era 1,96, pero es un error de redondeo.

NOTA