Course Title:
Digital Logic Design

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Course Code: CSE 103
Credit Hr: 3.00
Contact Hr: 3.00

Overview

- **Complements**

Complements

- Used in digital computers
  - for simplifying the subtraction and
  - for logical manipulations (is the process of applying logical operations on a sequence of bits to achieve a required result)
- Two types of complements for base-r system
  - the r’s complement and
  - the (r-1)’s complement

r's Complement-10's Complement

- For a positive number N in base r with an integer part of n digits, the r's complement of N is defined as
  - r^n-N for N≠0 and
  - 0 for N=0
- Example:
  - 10's complement of $(52520)_{10}$ is 10^5-52520=47480
  - Here, N=52520
  - The number of digits in the number is n=5
  - 10's complement of $(0.3267)_{10}$ is 1-0.3267=0.6733
  - No integer part, so $10^n=10^0=1$
  - 10's complement of $(25.639)_{10}$ is 10^2-25.639=74.361

r’s Complement-2’s Complement

- For a positive number N in base r with an integer part of n digits, the r’s complement of N is defined as
  - r^n-N for N$\neq$0 and
  - 0 for N=0
- Example:
  - 2’s complement of $(101100)_2$ is $[(2^6)_{10}-(101100)_2]=[(1000000-101100)_2]=(010100)_2$
  - 2’s complement of $(0.0110)_2$ is $(1-0.0110)_2=0.1010$

Important Notes

- 10’s complement of a decimal number can be formed by
  - leaving all least significant zeros unchanged
  - subtracting the first nonzero least significant digit from 10, and
  - then subtracting all other higher significant digits from 9
- 2’s complement can be formed by
  - leaving all least significant zeros and the first nonzero digit unchanged, and
  - then replacing 1’s by 0’s and 0’s by 1’s in all other higher significant digits.

(r-1)'s Complement

- For a positive number N in base r with an integer part of n digits and a fraction part of m digits, the (r-1)'s complement of N is defined as
  - $(r^n - r^{-m} - N)$ for $N \neq 0$ and
  - 0 for $N=0$
- Example:
  - 9's complement of $(52520)_{10}$ is $(10^5-1-52520)=47479$
  - The number of integer digits in the number is n=5
  - No fraction part, so $10^{-m}=10^0=1$
  - 9's complement of $(0.3267)_{10}$ is $(1-10^{-4}-0.3267)=(0.9999-0.3267)=0.6732$
  - No integer part, so $10^n=10^0=1$
  - 9's complement of $(25.639)_{10}$ is $(10^2-10^{-3}-25.639)=(99.999-25.639)=74.360$

1's Complement

- For a positive number N in base r with an integer part of n digits and a fraction part of m digits, the (r-1)'s complement of N is defined as
  - $r^n - r^{-m} - N$ for $N \neq 0$ and
  - 0 for $N=0$
- Example:
  - 1's complement of $(101100)_2$ is $[(2^6-1)_{10} - (101100)_2] = [(111111-101100)_2] = (010011)_2$
  - 1's complement of $(0.0110)_2$ is $[(1-2^{-4})_{10} - 0.0110)_2] = [0.1111 - 0.0110] = 0.1001$

Important Notes

- 9's complement of a decimal number is formed simply by
  - Subtracting every digit from 9
- 1's complement is even simpler to form
  - The 1's are changed to 0 and the 0's to 1's
- Since (r-1)'s complement is very easily obtained, it is sometimes convenient to use it when the r's complement is desired.
  - r's complement can be obtained from the (r-1)'s complement after the addition of r^{-m} to the least significant digit.
- Example: 2's complement of 10110100 is obtained from the 1's complement 01001011 by adding 1 to give 01001100

Subtraction with r's complement

- Subtraction of two positive numbers (M-N), both of base r, may be done as follows:
- Add the minuend M to the r's complement of the subtrahend N
- Inspect the result obtained in step 1 for an end carry:
  - If an end carry occurs, discard it.
  - If an end carry does not occur, take the r's complement of the number obtained in step 1 and place a negative sign in front

Example:

- Using 10’s complement, subtract 72532-3250.

M= 72532   72532

N = 03250

10’s complement of N=   96750

End carry 1 69282

- Result=69282

Example:

- Using 10's complement, subtract 3250-72532.

M= 03250 03250

N = 72532

10's complement of N=27468

No carry 30718

- Result=-69282 = -(10's complement of 30718)

Example:

- Using 2's complement, subtract 1010100-1000100.

M= 1010100 1010100

N = 1000100

2's complement of N= 0111100

End carry 1 0010000

- Result=10000

Example:

- Using 2's complement, subtract 1000100-1010100.

	M= 1000100	1000100
	N = 1010100	
2's complement of N=		0101100
	No carry	1110000

- Result=-10000 = -(2's complement of 1110000)

Subtraction with (r-1)'s complement

- Subtraction of two positive numbers (M-N), both of base r, may be done as follows:
- Add the minuend M to the (r-1)'s complement of the subtrahend N
- Inspect the result obtained in step 1 for an end carry:
  - If an end carry occurs, add 1 to the least significant digit (end-around carry).
  - If an end carry does not occur, take the (r-1)'s complement of the number obtained in step 1 and place a negative sign in front

Example:

- Using 9's complement, subtract 72532-3250.

M= 72532 72532

N = 03250

9's complement of N= 96749

End carry 1 69281

+1

69282

- Result=69282

Example:

- Using 9's complement, subtract 3250-72532.

M= 03250 03250

N = 72532

9's complement of N= 27467

No carry 30717

- Result=-69282 = -(10's complement of 30717)

Example:

- Using 1's complement, subtract 1010100-1000100.

M= 1010100   1010100

N = 1000100

1's complement of N= 0111011

End-around carry   1   0001111

+1

0010000

- Result=10000

Example:

- Using 1's complement, subtract 1000100-1010100.

	M= 1000100	1000100
	N = 1010100	
1's complement of N=		0101100
	No carry	1101111

- Result=-10000 = -(1's complement of 1110000)

Comparison Between 1’s and 2’s Complement

- 1’s complement has the advantage of being easier to implement by digital components since the only thing that must be done is to change 0’s into 1’s & 1’s into 0’s.
- 2’s complement may be implemented in two ways:
  - by adding 1 to the least significant bit of the 1’s complement, and
  - by leaving all leading 0’s in the least significant positions and the first 1 unchanged, and only then changing all 1’s into 0’s & 0’s into 1’s
- During subtraction of two numbers by complements, the 2’s complement is advantageous in that only one arithmetic addition operation is required.
  - 1’s complement requires two arithmetic additions when an end-around carry occurs.
- 1’s complement has additional disadvantage of processing two arithmetic zeros: one with all 0’s and one with all 1’s.


GOOD NEWS!
THE CLASS IS OVER…
THANK YOU!
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