SOLUTION

1. Suppose that two fair dice are tossed. Let E be an event that the sum of the dice is 8 and F denote the event that the first die is equal to 5. Are these two events are independent or not? Prove your argument. **4(CO 2)**

Let,

E = Sum of the two dice is 8

F = First die is equal to 5

We know that two events (E and F) will be independent if and only if following condition hold good.

$$P(EF) = P(E).P(F)$$

Now,

$P(E) = \frac{5}{36}$ [$\because$ (2, 6), (3, 5), (4, 4), (5, 3), (6, 2)]

$P(F) = \frac{1}{6}$ [$\because$ For the first dice there are total 6 possible outcomes]

$P(EF) = P(E = 8, F = 5) = \frac{1}{36}$

Since we can see here that,

$$P(EF) \neq P(E) \, . \, P(F)$$

Therefore these two events are not independent.

2. A constructive design team, call it C, and an innovative design team, call it N. Both are asked to separately design a new product within a month. From past experience it is known that,

i) The probability that team C is successful is $\frac{2}{3}$

ii) The probability that team N is successful is $\frac{1}{2}$

iii) The probability that at least one team is successful is $\frac{3}{4}$

Assuming that exactly one successful design is produced, what is the probability that it was designed by team N? **5(CO2)**

There are four possible outcomes here, corresponding to the four combinations of success and failure of the two teams:

SS: both succeed

SF: C succeeds, N fails

FF: both fail

FS: C fails, N succeeds

It is given that,

$P(SS) + P(SF) = \frac{2}{3}$, $\quad P(SS) + P(FS) = \frac{1}{2}$, $\quad P(SS) + P(SF) + P(FS) = \frac{3}{4}$,

From these relations, together with the normalization equation,

$$P(SS) + P(SF) + P(FS) + P(FF) = 1,$$

We can find out the probabilities of all outcomes.

$$P(SS) = \frac{5}{12}, \quad P(SF) = \frac{1}{4}, \quad P(FS) = \frac{1}{12}, \quad P(FF) = \frac{1}{4}$$

Therefore the desired probability is,

$$P(FS \mid \{SF, FS\}) = \frac{\frac{1}{12}}{\frac{1}{4} + \frac{1}{12}} = \frac{1}{4} \text{ (Ans)}$$

3. It is known that any item produced by a certain machine will be defective with probability 0.1, independent of any other item. What is the probability that in a sample of 5 items, 2 will be without defect?

4(CO 3)

Let,

X be the number of samples without defect. So here x is a Binomial random variable n=5 and p=0.9 (probability without defect) x=2.

So,

$$P\{x = 2\} = \binom{5}{2}(0.9)^2(0.1)^{5-2} \qquad \text{[Since, } P(X=K) = \binom{n}{k}p^k(1-p)^{n-k}\text{]}$$

$$= \binom{5}{2}(0.9)^2(0.1)^3$$

$$= \frac{5!}{2! \times (5-2)!}(0.9)^2(0.1)^3$$

$$= \frac{5!}{2! \times 3!}(0.9)^2(0.1)^3$$

$$= \frac{5.4.3.2.1}{2.1 \times 3.2.1}(0.9)^2(0.1)^3$$

$$= 10 \times (0.9)^2 . (0.1)^3$$

$$= \frac{81}{10000}$$

$$= 0.0081 \text{ (Ans)}$$

4. Let X be a discrete random variable with following probability mass function,

$$P_X(x) = \begin{cases} 0.1 & \text{for } x = 0.2 \\ 0.2 & \text{for } x = 0.4 \\ 0.2 & \text{for } x = 0.5 \\ 0.3 & \text{for } x = 0.8 \\ 0.2 & \text{for } x = 1 \\ 0 & \text{otherwise} \end{cases}$$

a. Find $P(X \leq 0.6)$
b. Find $P(0.25 < X < 0.75)$
c. Find $P(X = 0.2 \mid X < 0.6)$

2+2+3 (CO 2)

(a)

Event $X \leq 0.6$ can happen only if x is 0.2, 0.4 and 0.5. Thus,

$$P(X \leq 0.4) = P(x = 0.2) + P(x = 0.4) + P(x = 0.5)$$
$$= P_X(0.2) + P_X(0.4) + P_X(0.5)$$
$$= 0.1 + 0.2 + 0.2$$
$$= 0.5 \text{ (Ans)}$$

(b)

$P(0.25 < x < 0.75)$ can happen only if x is 0.4 and 0.5. Thus,

$$P(0.25 < X < 0.75) = P_X(0.4) + P_X(0.5)$$
$$= P_X(0.4) + P_X(0.5)$$
$$= 0.2 + 0.2$$
$$= 0.4 \text{ (Ans)}$$

(c)

This is a conditional probability. We know that conditional probability is given by,

$$P(A|B) = \frac{P(AB)}{P(B)} = \frac{P(A \cap B)}{P(B)}$$

So,

$$P(x = 0.2 \mid x < 0.6) = \frac{P\{(x=0.2) \cap (x<0.6)\}}{P(x<0.6)}$$
$$= \frac{P(x=0.2)}{P(x<0.6)}$$
$$= \frac{P(x=0.2)}{Px(0.2) + Px(0.4) + Px(0.5)}$$
$$= \frac{0.1}{0.1+0.2+0.2}$$
$$= \frac{0.1}{0.5} = 0.2 \text{ (Ans)}$$