Binomial

A **binomial distribution** can be thought of as simply the probability of only two outcomes that is in this case a SUCCESS or FAILURE outcome in an experiment or survey that is repeated multiple times.

1. **The number of observations or trials is fixed.** In other words, you can only figure out the probability of something happening if you do it a certain number of times. This is common sense—if you toss a coin once, your probability of getting a tails is 50%. If you toss a coin a 20 times, your probability of getting a tails is very, very close to 100%.
2. **Each observation or trial is independent**. In other words, none of your trials have an effect on the probability of the next trial.
3. The **probability of success** (tails, heads, fail or pass) is **exactly the same** from one trial to another

The binomial distribution formula is: $B(x, n, p) = {}_nC_x * p^x * (1 - p)^{n-x}$

$$= \binom{n}{x} * p^x * (1-p)^{n-x}$$

$$= \frac{n!}{(n-x)!x!} * p^x * (1-p)^{n-x}$$

- b = binomial probability
- x = total number of "successes" (pass or fail, heads or tails etc.)
- p = probability of a success on an individual trial
- n = number of trials

Example 1

A coin is tossed 8 times.

A) What is the probability of getting exactly 6 heads?

n = 8

x = number of heads which is 6

$p = \frac{1}{2}$

$P(X = 6) = {}_8C_6 * (\frac{1}{2})^6 * (1 - \frac{1}{2})^{8-6}$

$= \binom{8}{6} * (\frac{1}{2})^6 * (\frac{1}{2})^2$

$$= \frac{8!}{(8-6)!6!} * \left(\frac{1}{2}\right)^6 * \left(\frac{1}{2}\right)^2$$

B) What is the probability of getting less than 4 heads?

$P(X < 4) = P(X = 0) \quad + P(X = 1) \quad + P(X = 2) \quad + P(X = 3)$

$$= \binom{8}{0}.\left(\frac{1}{2}\right)^0.\left(\frac{1}{2}\right)^8 + \binom{8}{1}.\left(\frac{1}{2}\right)^1.\left(\frac{1}{2}\right)^7 + \binom{8}{2}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^6 + \binom{8}{3}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^5$$

C) What is the probability of getting more than 5 heads?

$P(X > 5) = P(X = 6) \quad + P(X = 7) \quad + P(X = 8)$

$$= \binom{8}{6}.\left(\frac{1}{2}\right)^6.\left(\frac{1}{2}\right)^2 + \binom{8}{7}.\left(\frac{1}{2}\right)^7.\left(\frac{1}{2}\right)^1 + \binom{8}{8}\left(\frac{1}{2}\right)^8\left(\frac{1}{2}\right)^0$$

Example 2

Paper clips are produced in variety of colors. The proportion of red paper clips is 0.20. Determine the probability that in a random sample of 50 colored paper clips the number of red clips is

(i)fewer than 3

$P(X < 3) = P(X = 0) \quad + P(X = 1) \quad + P(X = 2)$

$$= \binom{50}{0}.(0.20)^0.(1-0.20)^{50-0} + \binom{50}{1}.(0.20)^1.(1-0.20)^{50-1}$$

$$+ \binom{50}{2}.(0.20)^2.(1-0.20)^{50-2}$$

$$= 0.00001427248 + 0.00017840596 + 0.00109273651 = 0.00128541495$$

(ii)at least 8 but at most 12

$P(8 \leq X \leq 12) = P(X = 8) + P(X = 9) + P(X = 10) + P(X = 11) + P(X = 12)$

$= 0.11692181617 + 0.13640878554 + 0.13981900517 + 0.12710818652 + 0.10327540155$

$= 0.623533195$

Practice Problems

1. The probability of a bolt being faulty is 0.3. Find the probability that in a random sample of 20 bolts there are
   a) Exactly 2 faulty bolts
   b) More than 3 bolts

2. A factory produces components of which 1% are defective. The components are packed in boxes of 10. A box is selected at random.

(a) Find the probability that the box contains exactly one defective component.
(b) Find the probability that there are at least 2 defective components in the box

3. A manufacturer supplies DVD players to retailers in batches of 20. It has 5% of the players returned because they are faulty.
(a) Find the probability that the batch contains no faulty DVD players.
(b) Find the probability that the batch contains more than 4 faulty DVD players

The mean and variance of the binomial distribution

if $X \sim B(n, p)$, then the expected value/mean of X is given by $E(x) = np$

the variance value of X is given by $V(x) = npq$

Example

A random variable X is binomially distributed with mean 6 and variance 4.2. Find $P(X \leq 6)$.

Mean $= np = 6$
Variance $= npq = 4.2$
Substituting 1 in 2 we get Variance $= 6q = 4.2$
$q = 0.7$
hence $p = 1 - q = 1 - 0.70 = 0.30$
therefore mean $= 6 = 0.30\,n$
$6 / 0.30 = n = 20$

$$P(X \leq 6) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5) + P(X = 6)$$
$$= 0.6080$$