

Analytic or not :

$$\begin{aligned} 1) f(z) &= e^z \\ &= e^{x+iy} \\ &= e^x \cdot e^{iy} \\ &= e^x \cdot (\cos y + i \sin y) \end{aligned}$$

$$= \underline{e^x \cos y} + i \underline{e^x \sin y}$$

$$\frac{\partial u}{\partial x} = e^x \cos y \quad \leftarrow \text{equal} \quad \frac{\partial v}{\partial x} = e^x \sin y$$

$$\frac{\partial u}{\partial y} = -e^x \sin y \quad \frac{\partial v}{\partial y} = e^x \cos y$$

$$\therefore \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad \text{and} \quad \frac{\partial u}{\partial y} = - \frac{\partial v}{\partial x}$$

So, e^z is analytic (∞) with no singularities.

stages (a) to (c) $\alpha = 5$

exists (v) \vdash nil (c)

$$(u)^\dagger = (v)^\dagger \frac{p(u)}{p(v)}$$

if ϕ is the differentiable at $(x, f(x))$ then:

$$\text{also } \frac{(a)^T \cdot (d+e)^T}{d+e} = \frac{ad+ae}{d+e} = (a)^T$$

$$d + \dots = (di - o) (di + o) *$$

$$2) f(z) = e^{z^2} = e^{(x+iy)^2} = e^{x^2-y^2+2ixy} = e^{x^2-y^2} e^{i2xy}$$

$$= e^{x^2-y^2} (\cos 2xy + i \sin 2xy)$$

$$= e^{x^2-y^2} \cos 2xy + i e^{x^2-y^2} \sin 2xy$$

$$\frac{\partial u}{\partial x} = 2x e^{x^2-y^2} \cos 2xy - 2y e^{x^2-y^2} \sin 2xy$$

$$\frac{\partial u}{\partial y} = -2y \cos 2xy \cdot e^{x^2-y^2} - 2x e^{x^2-y^2} \sin 2xy$$

$$\frac{\partial v}{\partial x} = 2x \sin 2xy \cdot e^{x^2-y^2} + 2y e^{x^2-y^2} \cos 2xy$$

$$\frac{\partial v}{\partial y} = -2y \sin 2xy \cdot e^{x^2-y^2} + 2x e^{x^2-y^2} \cos 2xy$$

$$\therefore \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad \text{and} \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

So, e^{z^2} is analytic.

$$* (a+ib)(a-ib) = a^2 + b^2$$

$$\text{iii) } f(z) = \frac{1}{z}$$

$$= \frac{1}{x+iy}$$

$$= \frac{x-iy}{(x+iy)(x-iy)}$$

$$= \frac{x-iy}{x^2+y^2}$$

$$= \frac{x}{x^2+y^2} + i \left(\frac{-y}{x^2+y^2} \right)$$

$$\frac{\partial u}{\partial x} = \frac{(x^2+y^2) \cdot 1 - x \cdot 2x}{(x^2+y^2)^2}$$

$$\begin{aligned} \frac{\partial u}{\partial y} &= \frac{(x^2+y^2) \cdot 0 - x \cdot 2y}{(x^2+y^2)^2} \\ &= \frac{-2xy}{(x^2+y^2)^2} \end{aligned}$$

$$\frac{\partial v}{\partial x} = \frac{(x^2+y^2) \cdot 0 - (-y) \cdot 2x}{(x^2+y^2)^2}$$

$$\begin{aligned} &= \frac{2xy}{(x^2+y^2)^2} \\ \frac{\partial v}{\partial y} &= \frac{(x^2+y^2) \cdot (-1) - (-y) \cdot 2y}{(x^2+y^2)^2} \\ &= \frac{-x^2 - y^2 + 2y^2}{(x^2+y^2)^2} \\ &= \frac{y^2 - x^2}{(x^2+y^2)^2} \end{aligned}$$

$$\therefore \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad \text{and} \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

So, $\frac{1}{z}$ is analytic.

$$(v) \quad f(z) = \sin 2z \quad (u, v) \text{ at } \frac{1}{c} = u \quad (v)$$

$$= \sin 2(x+iy)$$

$$= \sin(2x + 2iy) \quad \frac{1}{c} = \frac{u^2}{x^2}$$

$$= \sin 2x \cos i \cdot 2y + \sin 2iy \cdot \cos 2x$$

$$= \sin 2x \cdot \cos 2hy + i \cdot \cos 2x \cdot \sin 2hy$$

$$\frac{\partial u}{\partial x} = 2 \cos 2x \cdot \cos 2hy \quad \frac{\partial v}{\partial x} = -2 \sin 2x \sin 2hy$$

$$\frac{\partial u}{\partial y} = \sin 2x \cdot \sin 2hy \quad \frac{\partial v}{\partial y} = 2 \cos 2x \cos 2hy$$

$$\left(\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \right) \text{ and } \left(\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \right)$$

$$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

$$\left[\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \right] \text{ is analytic}$$

$$\left[\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \right]$$

$$(i) \quad f(z) = 3z^2 - 2z + 4$$

$$\begin{aligned} f'(z) &= \lim_{\Delta z \rightarrow 0} \frac{f(z + \Delta z) - f(z)}{\Delta z} \\ &= \lim_{\Delta z \rightarrow 0} \frac{3(z + \Delta z)^2 - 2(z + \Delta z) + 4 - (3z^2 - 2z + 4)}{\Delta z} \\ &= \lim_{\Delta z \rightarrow 0} \frac{\cancel{3z^2} + 6z \cdot \Delta z + 3\Delta z^2 - \cancel{2z} - 2\Delta z + \cancel{4} - \cancel{3z^2} + \cancel{2z} - \cancel{4}}{\Delta z} \\ &= \lim_{\Delta z \rightarrow 0} (6z + 3\Delta z - 2) \end{aligned}$$

$$\boxed{f'(z) = 6z - 2}$$

$$(ii) \quad f(z) = \frac{2z - i}{z + 2i} \quad \text{at } z = -i$$

$$f'(-i) = \lim_{h \rightarrow 0} \frac{f(-i + h) - f(-i)}{h}$$

$$\begin{aligned} f(-i) &= \frac{-2i - i}{-i + 2i} \\ &= \frac{-3i}{i} \\ &= -3 \end{aligned}$$

$$\begin{aligned} f(-i + h) &= \frac{2(-i + h) - i}{-i + h + 2i} \\ &= \frac{-2i + 2h - i}{i + h} \\ &= \frac{2h - 3i}{i + h} \end{aligned}$$

$$f'(-i) = \lim_{h \rightarrow 0} \frac{f(-i+h) - f(-i)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{(2h-3i) - (-3)}{i+h}$$

$$= \lim_{h \rightarrow 0} \frac{2h - 3i + 3}{i+h}$$

$$= \lim_{h \rightarrow 0} \frac{2h - 3i + 3i + 3h}{i+h}$$

$$= \lim_{h \rightarrow 0} \left(\frac{5}{i+h} \right)$$

$$= \frac{5}{i}$$

$$= \frac{5i}{i^2}$$

$$\boxed{f'(-i) = -5i}$$

$$\frac{i - (d+i)}{is + d + i}$$

$$\frac{i - ds + i - i}{d + i}$$

$$\frac{ie - ds}{d + i}$$

$$\frac{i - is}{is + i}$$

$$\frac{is - i}{i}$$

$$s - 1$$

Harmonic or not :

1) $u = x^3 + 6x^2y - 3xy^2 - 2y^3$

$$\frac{\partial^2 u}{\partial x^2} = 6x + 12y - 3y^2 \quad \left| \quad \frac{\partial u}{\partial y} = 6x^2 - 6xy - 6y^2 \right.$$

$$\frac{\partial^2 u}{\partial x^2} = 6x + 12y - 3y^2 \quad \left| \quad \frac{\partial^2 u}{\partial y^2} = -6x - 12y \right.$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 6x + 12y - 6x - 12y = 0 \quad \checkmark \text{ Harmonic}$$

2) $u = x^2 + y^2 - 2xy - 2x + 3y$

$$\frac{\partial u}{\partial x} = 2x - 2y - 2 \quad \left| \quad \frac{\partial u}{\partial y} = -2y - 2x + 3 \right.$$

$$\frac{\partial^2 u}{\partial x^2} = 2 \quad \left| \quad \frac{\partial^2 u}{\partial y^2} = -2 \right.$$

$$\therefore \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 2 - 2 = 0 \quad \checkmark \text{ harmonic}$$

3) $u = x e^x \cos y - y e^x \sin y$

$$\frac{\partial u}{\partial x} = x \cos y \cdot e^x + e^x \cdot \cos y - y e^x \sin y$$

$$\frac{\partial^2 u}{\partial x^2} = x \cos y \cdot e^x + 2e^x \cos y - y e^x \sin y$$

$$\frac{\partial u}{\partial y} = -x e^x \sin y - y e^x \cos y - e^x \sin y$$

$$\frac{\partial^2 u}{\partial y^2} = -x e^x \cdot \cos y + y e^x \sin y - 2e^x \cos y$$

$$\therefore \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \quad \checkmark \text{ harmonic}$$

$$iv) u = e^{-x} (x \sin y - y \cos y)$$

$$= x e^{-x} \sin y - y e^{-x} \cos y$$

$$\frac{\partial u}{\partial x} = -x e^{-x} \sin y + e^{-x} \sin y + y e^{-x} \cos y$$

$$\frac{\partial^2 u}{\partial x^2} = + x e^{-x} \sin y - e^{-x} \sin y - e^{-x} \sin y - y e^{-x} \cos y$$

$$\frac{\partial u}{\partial y} = x e^{-x} \cos y + y e^{-x} \sin y - e^{-x} \cos y$$

$$\frac{\partial^2 u}{\partial y^2} = -x e^{-x} \sin y + e^{-x} \sin y + y e^{-x} \cos y + e^{-x} \sin y$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \quad \checkmark \text{ harmonic}$$

$$u(x, y) = e^{-x} (x \sin y - y \cos y)$$

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$$u(x, y) = e^{-x} (x \sin y - y \cos y)$$

Q) $u = ax^3(-bxy - cxy^2 + dy^3)$ (P, Q) is ()

$$\frac{\partial u}{\partial x} = 3ax^2 - 2bxy - cy^2 \quad \left| \quad \frac{\partial u}{\partial y} = -bx^2 - 2cxy + 3dy^2 \right.$$

$$\frac{\partial^2 u}{\partial x^2} = 6ax - 2by \quad \left| \quad \frac{\partial^2 u}{\partial y^2} = -2cx + 6dy \right.$$

is irrotational $\Rightarrow \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

$$6ax - 2by - 2cx + 6dy = 0$$

$$\Rightarrow (6a - 2c)x + y(-2b + 6d) = 0$$

$$6a - 2c = 0$$

$$\Rightarrow \boxed{c = 3a}$$

$$-2b + 6d = 0$$

$$\Rightarrow \boxed{b = 3d}$$

$$\odot = \frac{u}{x^6} \quad \odot = \frac{u}{y^6}$$

$$u = x^3(-bxy - cxy^2 + dy^3) \quad \frac{u}{x^6} = \frac{u}{y^6}$$

$$x^3(-bxy - cxy^2 + dy^3) = y^3(-bx^2 - 2cxy + 3dy^2)$$

$$-bx^4 - cx^3y + dy^4 = -bx^2y^3 - 2cxy^4 + 3dy^5$$

$$-bx^4 - cx^3y + dy^4 = -bx^2y^3 - 2cxy^4 + 3dy^5$$

Ans Pg 9

$$1) u(x, y) = 2x(1-y) = 2x - 2xy \quad (2)$$

$$\frac{\partial u}{\partial x} = 2 - 2y$$

$$\frac{\partial u}{\partial y} = -2x$$

$$\frac{\partial^2 u}{\partial x^2} = 0$$

$$\frac{\partial^2 u}{\partial y^2} = 0$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \rightarrow \text{harmonic}$$

$$dv = \left(\frac{\partial v}{\partial x} dx \right) + \left(\frac{\partial v}{\partial y} dy \right)$$

$$= \left(-\frac{\partial u}{\partial y} \right) dx + \left(\frac{\partial u}{\partial x} \right) dy$$

$$= \left(\frac{2x}{1} \right) dx + \left(\frac{2-2y}{1} \right) dy$$

$$\frac{\partial M}{\partial y} = 0, \quad \frac{\partial N}{\partial x} = 0$$

$$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \quad ; \text{ so eqn is exact. }$$

$$\int dv = \int 2x \cdot dx + \int (2-2y) dy$$

$$= 2 \cdot \frac{x^2}{2} + 2y - 2 \cdot \frac{y^2}{2} + c$$

$$v = x^2 - y^2 + 2y + c$$

$$f'(z) = u_1(z, 0) - i u_2(z, 0) \quad \text{--- (iii)}$$

$$u_1(x, y) = \frac{\partial u}{\partial x} = 2(-2y) \quad \bigg| \quad u_2(x, y) = -2x$$

$$\Rightarrow u_1(z, 0) = 2 \quad \bigg| \quad \Rightarrow u_2(z, 0) = -2z$$

$$f'(z) = 2 + i \cdot 2z$$

$$f(z) = 2z + i \cdot 2 \left(\frac{z^2}{2} \right) + \frac{1}{2} \left(\frac{1}{2} - \frac{1}{2} \right) =$$

$$f(z) = 2z + (i\tilde{z} + c) + \text{res}(\text{pole})$$

1000000 loops $\left\{ \begin{array}{l} \mu_{200}^{ns} = \frac{140}{100} \\ \mu_{200}^{ns} = \frac{140}{200} \end{array} \right.$

$$v(b(p_{200}^n - p_{100}^n)) + v(p_{100}^n) = v(b) \therefore$$

$$\int_0^1 y \, dx = V$$

$$(0, \Sigma) \rightarrow (0, \Sigma), u = (\Sigma)^7$$

$$S = \{0, 1\}^{\mathbb{N}} \quad (S, \tau)$$

$$0 = \lim_{\epsilon \rightarrow 0} \frac{1}{\epsilon} = (0, \infty) \cup \infty$$

$$(v) \quad u = \frac{1}{2} \ln(x^2 + y^2)$$

$$\frac{\partial u}{\partial x} = \frac{1}{2} \cdot \frac{2x}{(x^2 + y^2)} = \frac{x}{x^2 + y^2}$$

$$\frac{\partial u}{\partial y} = \frac{1}{2} \cdot \frac{2y}{(x^2 + y^2)} = \frac{y}{x^2 + y^2}$$

$$dv = \frac{\partial v}{\partial x} dx + \frac{\partial v}{\partial y} dy$$

$$= \left(-\frac{\partial u}{\partial y}\right) dx + \left(-\frac{\partial u}{\partial x}\right) dy$$

$$= \underbrace{\left(\frac{-y}{x^2 + y^2}\right)}_M dx + \underbrace{\left(\frac{-x}{x^2 + y^2}\right)}_N dy$$

$$\int dv = - \int \left(\frac{y}{x^2 + y^2}\right) dx + \int \left(\frac{-x}{x^2 + y^2}\right) dy$$

$$= -y \int \frac{1}{x^2 + y^2} dx + \int 0 dy = \frac{v_2}{\mu_2}$$

$$= -y \cdot \frac{1}{y} \tan^{-1} \frac{x}{y} + C \quad \left[\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + C \right]$$

$$\boxed{v = -\tan^{-1} \left(\frac{x}{y} \right) + C}$$

$$(viii) \quad u = e^x \cos y$$

$$\frac{\partial u}{\partial x} = e^x \cos y \quad \left| \quad \frac{\partial u}{\partial y} = -e^x \sin y \right.$$

$$\frac{\partial^2 u}{\partial x^2} = e^x \cos y \quad \left| \quad \frac{\partial^2 u}{\partial y^2} = -e^x \cos y \right.$$

$$\therefore \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \quad \rightarrow \text{harmonic}$$

$$dv = \frac{\partial v}{\partial x} dx + \frac{\partial v}{\partial y} dy$$

$$= \left(-\frac{\partial u}{\partial y}\right) dx + \left(\frac{\partial u}{\partial x}\right) dy$$

$$= \underbrace{(e^x \sin y)}_M dx + \underbrace{(e^x \cos y)}_N dy$$

$$\frac{\partial M}{\partial y} = e^x \cos y$$

$$\frac{\partial N}{\partial x} = e^x \cos y \quad \left. \begin{array}{l} \frac{\partial M}{\partial y} \\ \frac{\partial N}{\partial x} \end{array} \right\} \text{equal so exact.}$$

$$\therefore \int dv = \int e^x \sin y dx + \int (e^x \cos y - e^x \cos y) dy$$

$$\boxed{v = e^x \sin y + c}$$

$$f'(z) = u_1(z, 0) - i u_2(z, 0)$$

$$\therefore u_1(z, 0) = e^z \cdot \cos 0 = e^z$$

$$u_2(z, 0) = -e^z \cdot \sin 0 = 0$$

$$\begin{aligned}\therefore f'(z) &= e^z - i \cdot 0 \\ &= e^z\end{aligned}$$

$$\Rightarrow \boxed{f(z) = e^z + c}$$