4.9.2023

Poisson RV may be used to approximate a binomial RV when the binomial parameter n is large and p is small.

Binomial RV $\longrightarrow$ $P(i) = \binom{n}{i} (p)^i (1-p)^{n-i}$

let, X be a binomial RV with parameter n & p such that,

$$\lambda = np$$

$$P\{X = i\} = \frac{n!}{(n-i)!\, i!} p^i (1-p)^{n-i}$$

$$\frac{n(n-1)(n-2)\cdots \cancel{(n-i+1)(n-i)\cdots 3\cdot 2\cdot 1}}{\cancel{\{(n-i)\cdots 3\cdot 2\cdot 1\}}\cdot i!}$$

$$= \underbrace{\frac{n(n-1)(n-2)\cdots(n-i+1)}{\cancel{i!}\; n^i}}_{\to 1} \left(\frac{\lambda^i}{i!}\right) \underbrace{\left(1 - \frac{\lambda}{n}\right)^{n-i}}_{\frac{\left(1-\frac{\lambda}{n}\right)^n}{\left(1-\frac{\lambda}{n}\right)^i}} \quad \text{--- (1)}$$

$$\frac{(\text{very big no.})}{(\text{very big no.})} \approx 1$$

$$e^{\lambda} = 1 + \frac{\lambda}{1!} + \frac{x^2}{2!} + \cdots$$

$$e^{-\lambda} = 1 - \frac{\lambda}{1!} + \frac{\lambda^2}{2!} - \cdots$$

$$(x+y)^n = \sum_{r=0}^{n} {}^nc_r (x)^{n-r} y^r$$

$$\left(1 - \frac{\lambda}{n}\right)^n = \left\{1 + \left(-\frac{\lambda}{n}\right)\right\}^n$$

$$= {}^nc_0 (1)^{n-0} \cdot \left(-\frac{\lambda}{n}\right)^0 + {}^nc_1 (1)^{n-1} \left(-\frac{\lambda}{n}\right)^1 + {}^nc_2 (1)^{n-2} \cdot \left(-\frac{\lambda}{n}\right)^2 + \cdots$$

$$= 1 + \cancel{n} \cdot \left(-\frac{\lambda}{\cancel{n}}\right) + \frac{\lambda^2}{n^2} \cdot \frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}\; 2!}$$

$$= 1 - \frac{\lambda}{1!} + \frac{\lambda^2}{2!} - \cdots \quad ?$$

$$= e^{-\lambda}$$

Assignment $\longrightarrow$ ross $\longrightarrow$ pg 27
Do this derivation in detail

$$\frac{\left(1-\frac{\lambda}{n}\right)^n}{\left(1-\frac{\lambda}{n}\right)^i} \quad \begin{matrix} \longrightarrow e^{-\lambda} \\ \longrightarrow 1 \end{matrix}$$

[n is a really big no. ; so, $\frac{1}{n}$ is really small]

so, $\left(1-\frac{\lambda}{n}\right)^i \approx 1^i = 1$

From (1),

$$P\{X=i\} = 1 \times \left(\frac{\lambda^i}{i!}\right) \times \left(1-\frac{\lambda}{n}\right)^{n-i}$$

$$= 1\left(\frac{\lambda^i}{i!}\right) \cdot e^{-\lambda}$$

PDF $\longrightarrow$
distribution $\longrightarrow$ sum is 1.

5.9.23

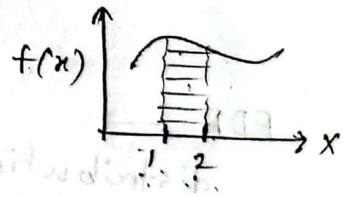
$$S_T = \{ t \mid -5 \le t \le +5 \}$$ → chances বেশি to be correct

$$= \{ t \mid -2 \le t \le +2 \}$$

The amount of probability in an interval gets smaller and smaller as the interval shrinks.

A random variable X is called continuous if there exists a non-negative function $f(x)$, which is called Probability Density Function (PDF), such that

$$\underset{\substack{\uparrow \\ \text{random var.}}}{P(X} \in \underset{\substack{\uparrow \\ \text{range}}}{B)} = \int_B f(x) \cdot dx$$ for every set of B in the real line



Note : CDF $\underset{\text{Int.}}{\overset{\text{Diff.}}{\rightleftarrows}}$ PDF

CDF → area of an interval

PDF → denotes a curve

- $$\boxed{P\{X \in (-\alpha, \alpha)\} \text{ or } P\{-\alpha \le x \le \alpha\} = \int_{-\alpha}^{\alpha} f(x) \cdot dx}$$

↳ generic form of continuous R.V.

$a \le x \le b \xrightarrow{\text{CDF}} \int_a^b f(x)\cdot dx$

$a < x \le b$

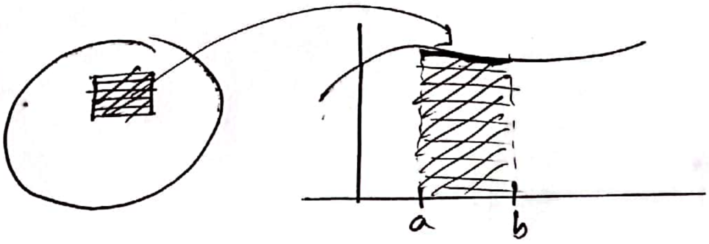
$a \le x < b$

$a < x < b$

default

→ lower limit $-\infty$

→ upper " $+\infty$



Total area will be 1

a to b is → CDF

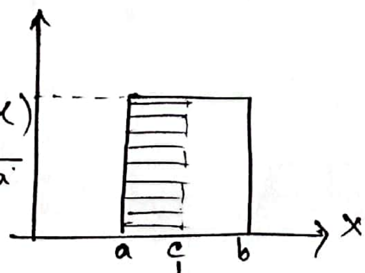
CDF

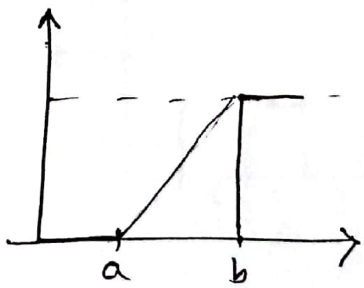
$$F_X(x) = P(X \le x) = \begin{cases} \sum_{k \le x} P(k) & \text{if } x \text{ is } \underline{\text{discrete}} \\ \int_{-\infty}^{\infty} f(t)\cdot dt & \text{" " " } \underline{\text{continuous}} \end{cases}$$

X → name of var.

x → value X can take

read note } $f(x) = \frac{1}{b-a}$





↳ CDF

$\text{area} = (c-a)\cdot\frac{1}{(b-a)}$

$= \frac{c-a}{b-a}$

Note :

$\underbrace{f(x)\cdot(b-a)}_{\text{area}} = 1$

$\Rightarrow f(x) = \frac{1}{b-a}$

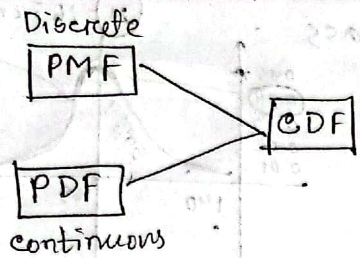
Upper limit = lower limit

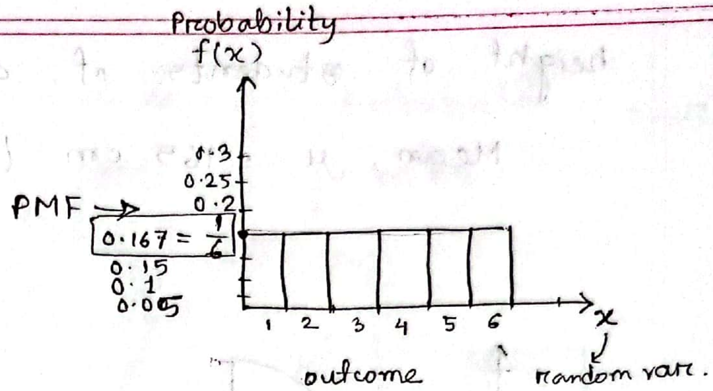
$\mapsto$ area 0

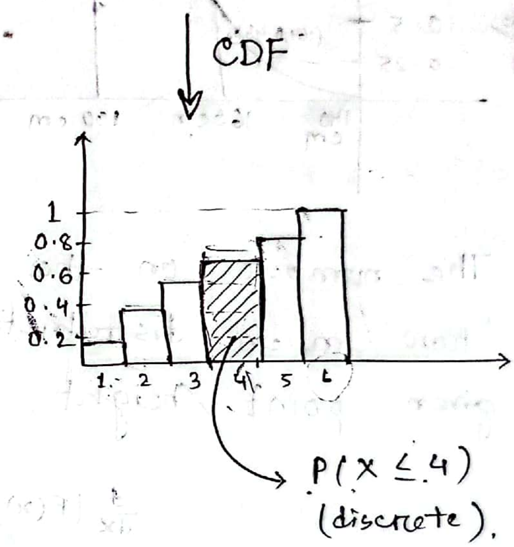
$\longrightarrow$ a single line

$\longrightarrow$ turns into Discrete R.V.

12.09.2023

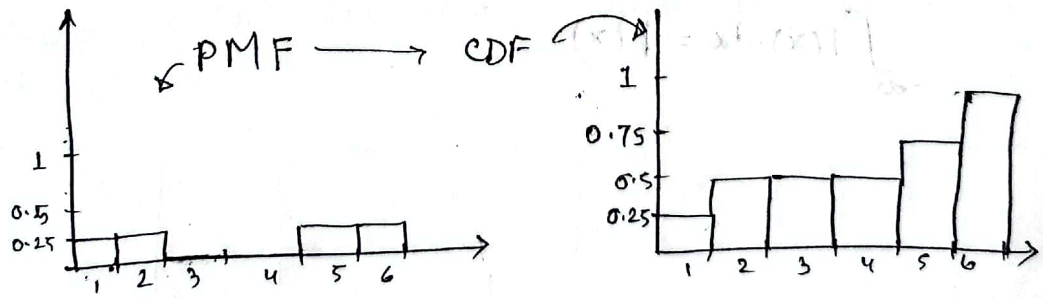






Imp property of CDF :

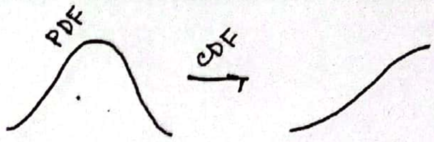
The final bar needs to be equal to 1.



* 3, 4 উঠবে না

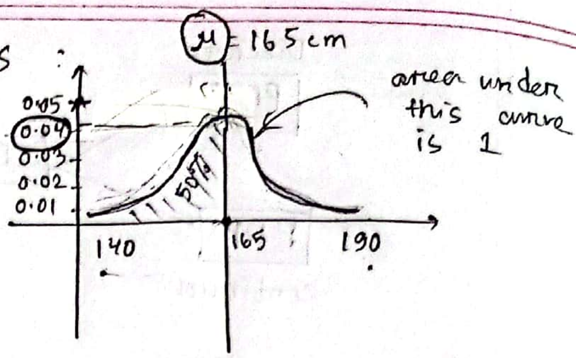
$$P(X \le 4) = P(X \le 2)$$
$$= P(X=1) + P(X=2) + \underbrace{P(X=3)}_{0} + \underbrace{P(X=4)}_{0}$$
$$= P(X \le 2)$$

The flatness in CDF indicates that there's no 'mass' around 3 and 4.



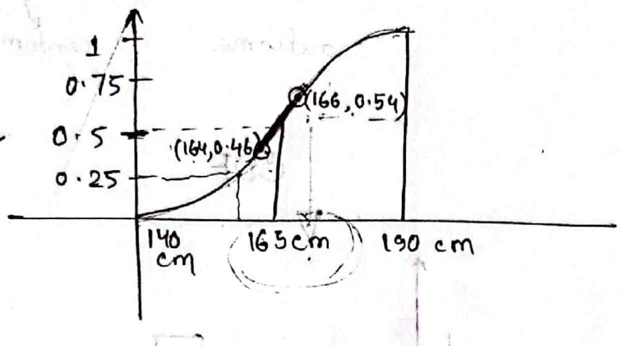
height of students of a class :

Mean, μ = 165 cm (let)



PDF

$\frac{rise}{run} = \frac{0.54-0.46}{166-164} = \frac{0.08}{2} = \boxed{0.04}$



Gradient $= \frac{0.08}{2} = 0.04$

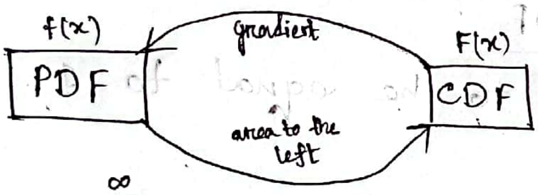
CDF -এর grad = PDF এর y axis এর value.

The number on the Y-axis actually telling us 'how much distribution is on the left of the given point / height.

PDF $\rightarrow f(x)$

CDF $\rightarrow F(x)$

$$\frac{d}{dx}(F(x)) = f(x)$$

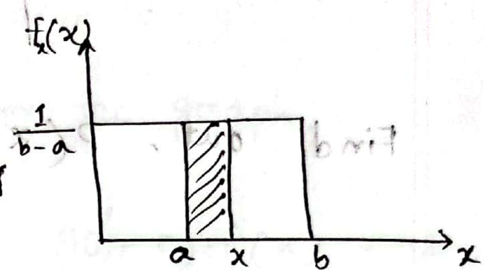


$$\int_{-\infty}^{\infty} f(x) \cdot dx = F(x)$$

Uniform RV :

A R.V. is said to be uniformly distributed over interval $(0, 1)$ if it's PDF is given by,

$$f(x) = \begin{cases} 1, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$



CDF : $\int_{-\infty}^{\infty} \underbrace{f(x)}_{\text{PDF}}\, dx \xrightarrow[\text{R.V}]{\text{for Uniform}} \int_0^1 1 \cdot dx = 1$

$A = B \times H$

$= (b-a) \times f(x)$

$\Rightarrow 1 = (b-a) \times f(x)$

$\Rightarrow \boxed{f(x) = \frac{1}{b-a}}$

$P(a \le x \le b)$

$= \int_a^b f(x) \cdot dx$

$= \int_a^b \frac{1}{b-a} dx = 1$ ~~b-a~~ ?

$$f(x) = \begin{cases} \frac{1}{\beta - \alpha} & \text{if } \alpha < x < \beta \\ 0, & \text{otherwise} \end{cases}$$

$f(x) = \frac{1}{b-a}$; $a<x<b$

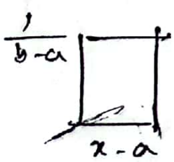
$\mu = \frac{a+b}{2}$

$\sigma = \frac{b-a}{\sqrt{12}}$

$P(a \le x \le b) = \int_a^b \frac{1}{b-a} dx = 1$

Q) Area under the curve upto the point x or to the left of x.

$\rightarrow$



Area $= (x-a) \cdot \frac{1}{b-a}$

$= \frac{x-a}{b-a}$

continuous R.V এর PDF $\longrightarrow$ shape of the curve denote করে

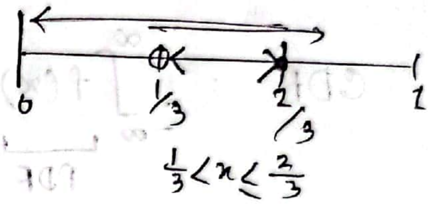
---

⑨ Let x be continuous R.V, with PDF

$$f_x(x) = \begin{cases} 4x^3, & 0 \le x \le 1 \\ 0, & \text{otherwise} \end{cases}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Find out, $P(x \le \frac{2}{3} \mid x > \frac{1}{3})$



$$\rightarrow \frac{P(\frac{1}{3} < x \le \frac{2}{3})}{P(X > \frac{1}{3})}$$

$$= \frac{\int_{1/3}^{2/3} 4x^3\, dx}{\int_{1/3}^{1} 4x^3\, dx}$$

$$= \frac{[x^4]_{1/3}^{2/3}}{[x^4]_{1/3}^{1}}$$

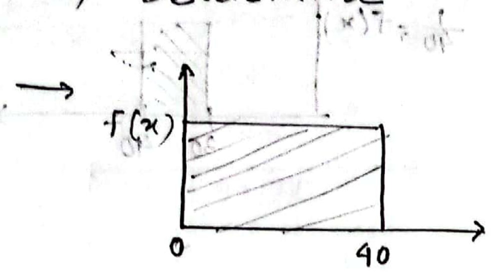
$$= \frac{(\frac{2}{3})^4 - (\frac{1}{3})^4}{1 - (\frac{1}{3})^4}$$

$$= \frac{3}{16}$$

(9) The amount of time a person must wait, for a train to arrive in a certain town is uniformly distributed between 0 to 40 minutes.

a) Determine $f(x)$. b) Draw the graph.

$\rightarrow$



area $= (40-0) \cdot f(x) = 1$

$\Rightarrow f(x) = \frac{1}{40}$

$$f(x) = \begin{cases} \frac{1}{40} & ; \; 0 < x < 40 \\ 0 & ; \; \text{otherwise} \end{cases}$$

you can include 0 and 40 if you want! Doesn't matter!

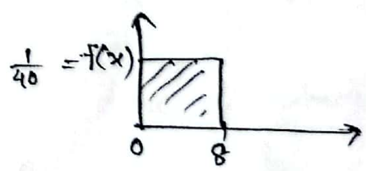
(Ans).

c) What is the probability that a person wait less than 8 minute.

$\rightarrow P(x < 8) = (b-a) \times f(x)$

$= (8-0) \times \frac{1}{40}$

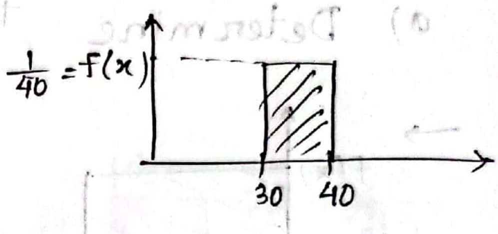
$= \frac{1}{5} = 0.20 = 20\%$



d) What is p that a person must wait more than 30 min ?

$\longrightarrow$ $P(x > 30)$ or $P(30 < x < 40)$

$= (40 - 30) \cdot \frac{1}{40}$



$= \frac{1}{4}$

$= 25\%$

e) Calculate $P(10 < x < 26)$, $P(X = 20)$ and $P(X > 45)$

$(b-a) \cdot f(x)$

$(26 - 10) \cdot \frac{1}{40}$

$= \frac{2}{5} = 40\%$

$P(X = 20)$: 0

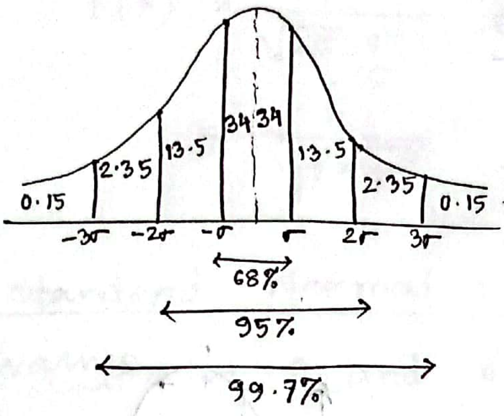
(No area at one point)

$P(X > 45)$: 0

(between 0 to 40)

20. 09. 2023

Normal R.V. : Gaussian R.V. $\longrightarrow$ Bell shaped curve.



* x is a Normal R.V. with parameter μ and σ^2 if the density of x is given by,

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} \cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \quad -\infty < x < \infty$$

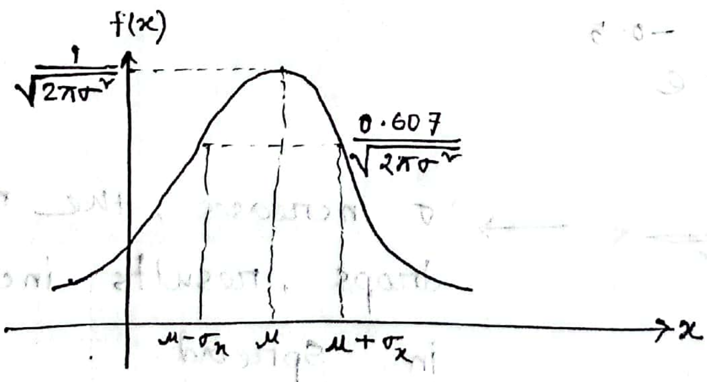
The density function is bell shaped curve that is symmetric around μ.

μ = Mean

σ = std. deviation

σ^2 = variance

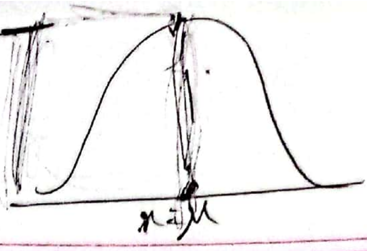
*Notation : $X \sim N(\mu, \sigma^2)$



$\sigma_x^2 \rightarrow$ always a positive number

$\mu \longrightarrow$ can be any no. between $-\infty$ to $+\infty$.

Spread of the distribution depends on σ.



PDF :

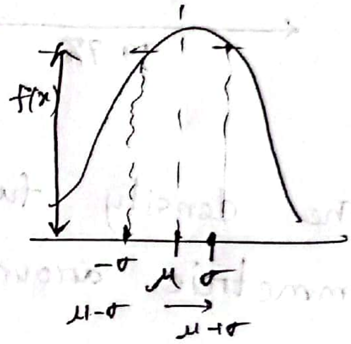
$\boxed{x = \mu}$,

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} \cdot e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

$$= \frac{1}{\sqrt{2\pi}\cdot\sigma} \cdot e^{\frac{-(\mu-\mu)^2}{2\sigma^2}}$$

$$= \frac{1}{\sqrt{2\pi}\cdot\sigma}\, e^{0}$$

$$= \frac{1}{\sqrt{2\pi\sigma^2}}$$



$\boxed{x = \mu - \sigma}$

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} \cdot e^{-\frac{(\cancel{\mu}-\sigma-\cancel{\mu})^2}{2\sigma^2}}$$

$$= \frac{1}{\sqrt{2\pi}\cdot\sigma} \cdot e^{-0.5}$$

$$= \frac{0.607}{\sqrt{2\pi\sigma^2}}$$ $\longrightarrow$ σ increases, the magnitude drops, results increase in "spread.

$$\int_{-\infty}^{\infty} e^{-x^2/2}\, dx = \sqrt{2\pi}$$

area under the curve using integral of $f(x)$.

To make it 1,

$$f(x) = \frac{1}{\sqrt{2\pi}} \cdot e^{-x^2/2}$$

$\boxed{x = \mu + \sigma}$,

$$f(x) = \frac{1}{\sqrt{2\pi}\cdot\sigma} \cdot e^{\frac{-(\mu+\sigma-\mu)^2}{2\sigma^2}}$$

$$= \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-0.5} = \frac{0.607}{\sqrt{2\pi\sigma^2}}$$

Standard Normal : Normal distribution with parameter values $\mu = 0$ and $\sigma^2 = 1$.

$$f(x) = \frac{1}{\sqrt{2\pi}\cdot\sigma} \cdot e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

Notation : $Z \sim N(0,1)$

$$= \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

PDF

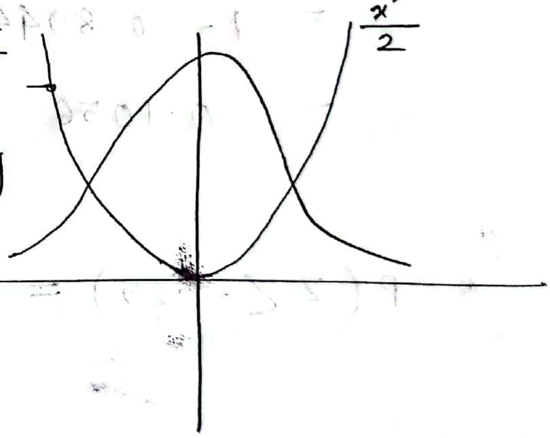
$$f(x) = \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{x^2}{2}}$$

$e^{-\frac{x^2}{2}}$

$= 1$

$[e^0 = 1]$

$\frac{x^2}{2}$



$$P(a \le x \le b) = \int_a^b f(x)\cdot dx$$

it's denoted by Z

$Z \sim N(0,1)$

* Pg 76

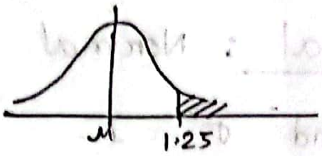
$\phi(1.25)$

CDF :

special notation is used (z)

$$F(z) = \phi(z) = P(Z<z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{x^2}{2}} \, dx$$

* $P(Z \geq 1.25)$



$\rightarrow 1 - P(Z<1.25)$

$= 1 - \phi(1.25)$

$= 1 - 0.8944$ (from table)

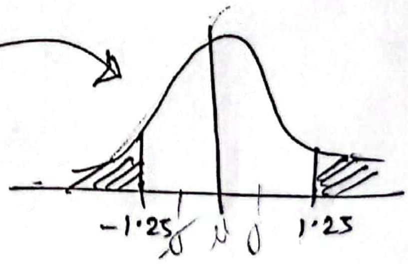
$= 0.1056$

Homework

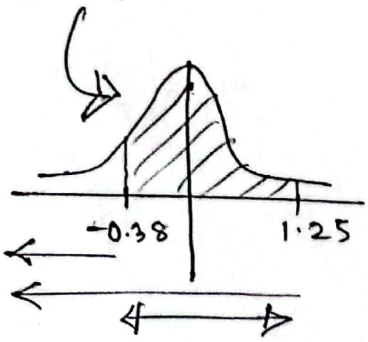
or,

$$\int_{1.25}^{\infty} \underbrace{f(x)}_{\frac{1}{\sqrt{2\pi}} \cdot e^{-x^2/2}} \cdot dx$$

* $P(Z \leq -1.25) = P(Z \geq 1.25)$



* $P(-0.38 \leq Z \leq 1.25) = P(Z < 1.25) - P(Z < -0.38)$

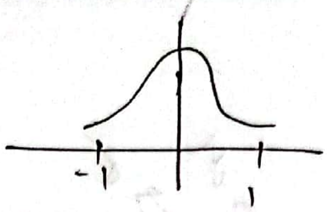


$= \phi(1.25) - \phi(-0.38)$

$= \underbrace{\phi(1.25)}_{0.8944} - 1 + \underbrace{\phi(0.38)}_{0.6480}$

$= 0.5424$

Note : $\phi(-z) = 1 - \phi(z)$



$-1 \le z \le 1 \rightarrow 68\%$

$-2 \le z \le 2 \rightarrow 95\%$

* $P(-1 \le z \le 1)$

$= P(z \le 1) - P(z \le -1)$

$= \varphi(1) - \varphi(-1)$

$= \varphi(1) - 1 + \varphi(1)$

$= (0.8413 \times 2) - 1$

$= 0.6826$

Standard Normal (CDF):

$$\varphi(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-x^2/2} \, dx$$

* If X is a Gaussian (μ, σ) R.V. the CDF of X is,

$$F_X(x) = \varphi\left(\frac{x-\mu}{\sigma}\right)$$

* The probability that X is in interval (a, b) is,

$$P(a \le x \le b) = \varphi\left(\frac{b-x}{\sigma}\right) - \varphi\left(\frac{a-x}{\sigma}\right)$$

Using these formula, we can convert values of Gaussian R.V. X to equivalent values of the standard random variable, Z.

→ a sample value x of the R.V. X, the corresponding value of z is,

$$z = \frac{x-\mu}{\sigma}$$

If X is Gaussian R.V. $(61, 10)$, what is $P(X \le 46)$?

$\mu \quad \sigma$

$\rightarrow \quad F_X(x) = \phi\left(\frac{x-\mu}{\sigma}\right)$

$F_X(46) = \phi\left(\frac{46-61}{10}\right) = \phi\left(-\frac{15}{10}\right) = \phi(-3/2)$

$= \phi(-1.5)$

$\phi(-z) = 1 - \phi(z)$

$\therefore \phi(-1.5) = 1 - \phi(1.5)$

$= 1 - 0.933$

$= 0.067 = 6.7\%$

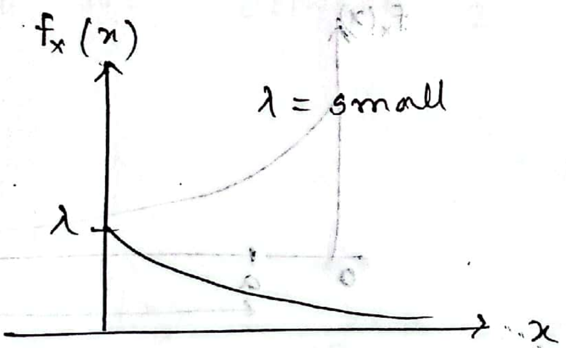
Exponential R.V. :

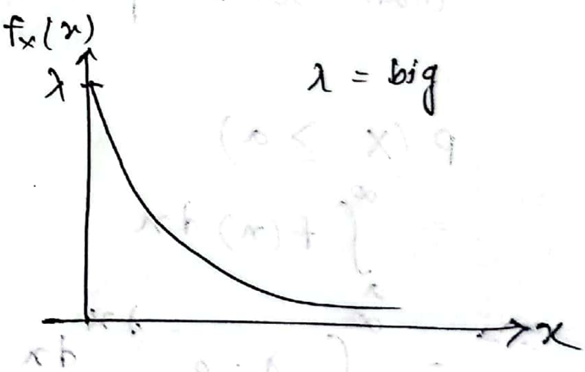
A continuous R.V. is said to be exponential R.V. with parameter λ, for ~~$\lambda <$~~ $\lambda > 0$, if it's PDF is given by.

$$f(x) = \begin{cases} \lambda \cdot e^{-\lambda x}, & \text{if } x \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

$$\boxed{\lambda = \frac{1}{\mu}}$$

* For $-ve$ value of x, PDF, $f(x) = 0$.
  $\hookrightarrow$ It's probability is zero.





$$f(x) = \lambda \cdot e^{-\lambda x} = \lambda e^{-\lambda \cdot 0} = \lambda$$

$$F_x(x) = P(X \leq x)$$

$$F_X(a) = P(X \leq a) = \int_{-\infty}^{a} f_x(x)\, dx$$

$$= \int_{0}^{a} f_x(x)\, dx \leftarrow \text{For exponential R.V.}$$

$$= \int_{0}^{a} \lambda \cdot e^{-\lambda x}\, dx$$

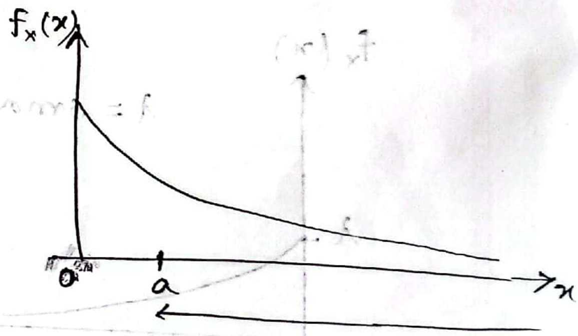
$$= \cancel{\lambda} \cdot \left[\frac{e^{-\lambda x}}{-\cancel{\lambda}} \right]_0^a$$

$$= -\left(e^{-\lambda a} - 1\right)$$

$$= 1 - e^{-\lambda a}$$

$$\boxed{F_X(x) = \int_{-\infty}^{x} f(y) \cdot dy = \begin{cases} 1 - e^{-\lambda x}, & x \geq 0 \\ 0, & \text{otherwise} \end{cases}}$$

Q) calculate the P of x taking a value greater than some point a.



$P(X \geq a)$

$$= \int_a^{\infty} f(x)\, dx$$

$$= \int_a^{\infty} \lambda \cdot e^{-\lambda x}\, dx$$

$$= \cancel{\lambda} \cdot \left[\frac{e^{-\lambda x}}{-\cancel{\lambda}} \right]_a^{\infty}$$

$$= -e^{-\infty} + e^{-\lambda a}$$

$$= e^{-\lambda a}$$

$$\boxed{P(X \geq a) = e^{-\lambda a}}$$

when $a = 0$,
$P(X \geq a) = 1$

1 (Q) If job arrives 15 sec on an average, then $\lambda = 4$ per. min, what is the P of waiting less than a equal to 30 sec? $P(t \leq 0.5)$

2 (Q) The P that a telephone call lasts no more than t min is often modeled as an exponential CDF as,

$$F_T(t) = \begin{cases} 1 - e^{-t/3} & , t \geq 0 \\ 0 & , \end{cases}$$

What is PDF of the duration in min of a telephone conversation? What is P that a conversati will last between 2 to 4 min?

1 (A).

$$f(x) = \begin{cases} \lambda \cdot e^{-\lambda x} & x \geq 0 \\ 0 , & \end{cases}$$

* time is always continuous

$T = 30$ sec $= 0.5$ min

$$P(T \leq 0.5) = \int_0^{0.5} 4 \cdot e^{-4t} \, dt$$

$$= \left[\frac{\cancel{4} \cdot e^{-4t}}{-\cancel{4}} \right]_0^{0.5}$$

$$= -e^{-4 \times 0.5} + e^{-4 \cdot 0}$$

$$= 1 - e^{-2} = 0.86466 \text{ (Ans)}$$

* $1 - e^{-\lambda a}$

$= 1 - e^{-4 \times 0.5}$

$1 - e^{-4 \cdot 0.5}$

$1 - e^{-2}$

2 (A)

$$f_T(t) = \frac{d}{dt} F_T(t)$$

$$= \frac{d}{dt}\left(1 - e^{-t/3}\right)$$

$$= \frac{d}{dt}(1) - \frac{d}{dt}\left(e^{-t/3}\right)$$

$$= -\left(-\frac{1}{3} e^{-t/3}\right) = \frac{1}{3} e^{-t/3}$$

PDF $\rightarrow f_T(t) = \begin{cases} \frac{1}{3} \cdot e^{-t/3}, & t \geq 0 \\ 0, & \text{otherwise} \end{cases}$

$\lambda = \frac{1}{3}$

We know, $f_x(x) = \begin{cases} \lambda e^{-\lambda x}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$

$$P(2 \leq x \leq 4) = \int_2^4 f(t) \cdot dt$$

$$= \int_2^4 \frac{1}{3} \cdot e^{-t/3} \, dt$$

or,

$(1-e^{-\lambda \cdot 4}) - (1-e^{-\lambda \cdot 2})$

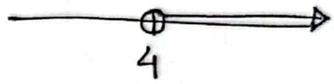
$$P(2 \leq x \leq 4) = F_T(4) - F_T(2)$$

$$= (1 - e^{-\lambda 4}) - (1 - e^{-\lambda 2}).$$

25.09.2023

(19) The time (in hour) required to repair a car is an exponentially distributed R.V. with parameter $\lambda = \frac{1}{2}$. What is the probability that repair time exceeds 4 hr? If it exceeds 4 hr then what is the probability that it will exceed 8 hour?

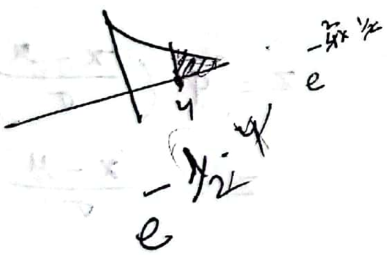
→ $f_T(t) = \lambda \cdot e^{-\lambda t}$



$\lambda = \frac{1}{2}$

$$P(T>4) = \int_4^\infty f(t)\cdot dt = \lambda \int_4^\infty e^{-\lambda t}\, dt$$

$$= \cancel{\lambda}\left[\frac{e^{-\lambda t}}{-\cancel{\lambda}}\right]_4^\infty$$

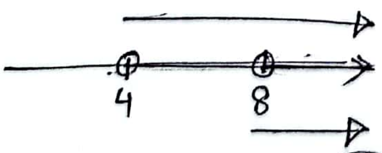


$$= -\left[e^{-\lambda t}\right]_4^\infty$$

$$= -\left(0 - e^{-4\lambda}\right)$$

$$= e^{-\cancel{4}^{2} \times \frac{1}{\cancel{2}}} = e^{-2}$$

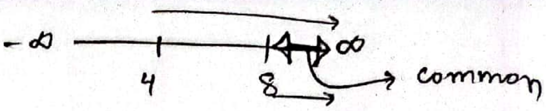
$$= 0.1353352.$$



$P(T>8 \mid T>4)$

$$= \frac{P(T>8)}{P(T>4)} = \frac{e^{-\frac{1}{\cancel{2}}\cdot \cancel{8}^{4}}}{e^{-\frac{1}{\cancel{2}}\cdot \cancel{4}^{2}}}$$

* $P(A|B) = \dfrac{P(AB)}{P(B)}$



* $P(X \geq a) = e^{-\lambda a}$

common part

$$P(T>8 \mid T>4) = \frac{P(T>8)}{P(T>4)}$$

$$= \frac{e^{-\frac{1}{2}\times \cancel{8}^{4}}}{e^{-\frac{1}{2}\times \cancel{4}^{2}}}$$

$$= e^{-4+2} = e^{-2}$$

(29) Give that, $X \sim N(\overset{\mu}{\uparrow}{50}, \overset{\sigma^2}{\uparrow}{10})$; what is the value of mean & standard deviation? what value of X has a Z-score of 1.4? What is the Z-score that corresponds to $X = 30$?

$$Z = \varphi\left(\frac{x-\mu}{\sigma}\right)$$

$$Z = \frac{x-\mu}{\sigma}$$

→ $\mu \to 50$ (Mean)

$\sigma = \sqrt{10}$ (std. d.)

$$Z = \frac{x-\mu}{\sigma}$$

default : σ^2 ধরবো, define করে দিলে ওভাবে ধরবো, ⇒ $\boxed{x = z\sigma + \mu}$

$$= 50 + (1.4 \times \sqrt{10})$$

$$Z = \frac{x-\mu}{\sigma} = \frac{30-50}{\sqrt{10}}$$

(39) The continuous R.V. X has PDF $f(x)$ which is given by,

$$f(x) = \begin{cases} k(x^2 - 2x + 2) & ; 0 < x \leq 3 \\ 3k & ; 3 < x \leq 4 \\ 0 & , \text{otherwise} \end{cases}$$

1) Find out value of k :

$$\text{Total}, P = 1 = \int_{-\infty}^{\infty} f(x)\, dx$$

$$\int_0^3 k(x^2 - 2x + 2)\, dx$$

$$= k\left[\frac{x^3}{3} - \cancel{2}\cdot\frac{x^2}{\cancel{2}} + 2x\right]_0^3$$

$$= k\cdot(9 - 9 + 6) = 6k$$

$$\int_3^4 3k\, dx$$

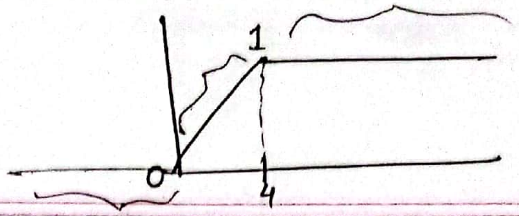
$$= 3k\cdot[x]_3^4$$

$$= 3k(4-3)$$

$$= 3k$$

$$\therefore 6k + 3k = 1$$

$$\Rightarrow \boxed{k = 1/9}$$



2) Find out the CDF ?

$$F(x) = \int \frac{1}{9}(x^2 - 2x + 2)\, dx$$

$$= \frac{1}{9}\left(\frac{x^3}{3} - 2 \cdot \frac{x^2}{2} + 2x + c\right)$$

So, $F(0) = 0$

$$\boxed{F(x) = \frac{1}{9}\left(\frac{x^3}{3} - x^2 + 2x\right)}$$

$k = 1/9$

$$F(x) = \int 3k\, dx = \int \frac{1}{3}\, dx = \frac{1}{3}x + D$$

$$F(4) = \frac{1}{3} \times 4 + D = 1$$

$$\Rightarrow D = -1/3$$

$$\boxed{F(x) = \frac{1}{3}x - 1/3}$$

$$F(x) = \begin{cases} 0, & x \leq 0 \\ \frac{1}{9}\left(\frac{1}{3}x^3 - x^2 + 2x\right), & 0 < x \leq 3 \\ \frac{1}{3}x - 1/3, & 3 < x \leq 4 \\ 1, & x > 4 \end{cases}$$

variance বের করতে বললে Expectation এর formula ব
করতে হবে।

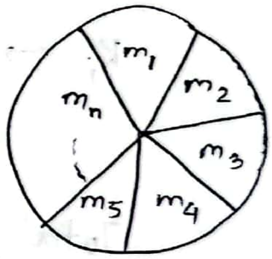
3) Find the mean of X or E[X]

<u>Expectation of X :</u>

→ Weighted Avg of the possible values of x.

→ Expected value of a R.V. is the theoretical Mean of the R.V.

→ In Simple term, expected value or mean is a measure of central tendency. of a probability distribution.



Total money received

$= m_1 K_1 + m_2 K_2 + m_3 K_3 + \cdots + m_i K_i$

$$avg = \frac{m_1 K_1 + m_2 K_2 + \cdots + m_i K_i}{K}$$

↑

K times roll

$K_1 \to m_1$

$K_2 \to m_2$

$$P_i = \frac{K_i}{K}$$

$= m_1 P_1 + m_2 P_2 + m_3 P_3 + \cdots$

27.9.2023

Expectation of R.V :

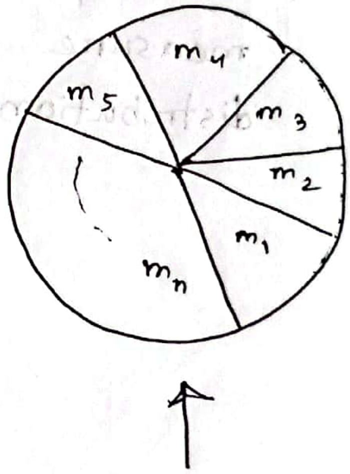
→ Expectation of x → weighted avg

→ Theoretical mean of x

→ A measure of central tendency

→ It represents that the avg value that a R.V is likely to take on.

K → number of times the wheel is spinned



K_i → is the number of time that the o/p is m_i

Total received amount (Per spin)

$$= \frac{m_1K_1 + m_2K_2 + m_3K_3 + \dots + m_nK_n}{K}$$

m_i আসবে K_i times

$$P_i = \frac{K_i}{K}$$

$$= m_1 \cdot P_1 + m_2P_2 + m_3P_3 + \dots + m_nP_n$$

$$\boxed{E[X] = \sum_x x \cdot P_{X}(x)}$$

output o/p এর Prob.

$x \to$ value
$X \to$ R.V.

X	P
1	$\frac{1}{2}$
2	$\frac{1}{2}$

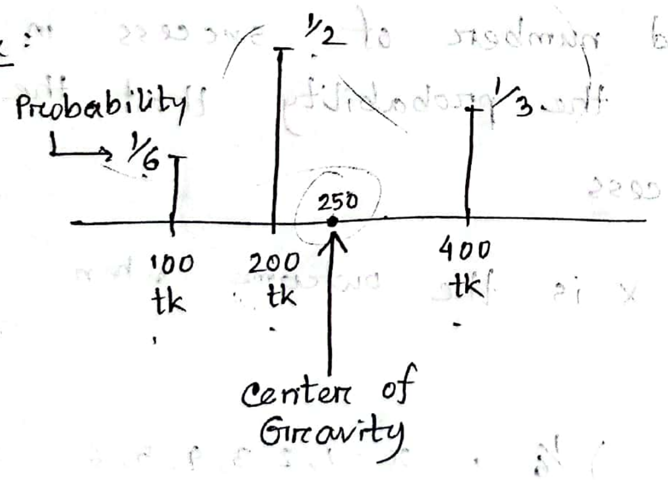
$$E[X] = 1 \cdot \frac{1}{2} + 2 \cdot \frac{1}{2}$$

$$= \frac{3}{2} \longrightarrow$$ Expectation can be more than 1.

$$\boxed{E[X] = \sum_{x : p(x) > 0} x \cdot P(x)}$$

Expected value of X is a weighted avg of the possible values that X can take on, each value being weighted by it's probability that X assumes that value.

Ex :



$$100 \cdot \frac{1}{6} + 200 \cdot \frac{1}{2} + 400 \cdot \frac{1}{3}$$

$$= 250 \longrightarrow E[X]$$

Expectation of Bernoulli R.V :

$$P_X(x) = \begin{cases} P & , \text{ if } x = 1 \\ 1-P & , \text{ if } x = 0 \end{cases} \leftarrow \text{PMF for bernoulli R.V.}$$

$P(0) = 1-P \rightarrow$ failure

$P(1) = P \rightarrow$ success

$$E[X] = \sum_x x \cdot P(x) = 0 \cdot (1-P) + 1 \cdot P = P$$

$\boxed{\text{For Bernoulli R.V, } E[X] = P}$

Show that, the expected number of success in a single trial is just the probability that the trial will be a success.

(Q) Find out $E[X]$ where X is the outcome when fair die is tossed?

$\rightarrow$ The PMF is, $P_X(x) = \begin{cases} 1/6 & , x = 1, 2, 3, 4, 5, 6 \\ 0 & , \text{otherwise} \end{cases}$

$$E[X] = \sum_{x=1}^{6} x \cdot P(x) = \left(1 \times \frac{1}{6}\right) + \left(2 \times \frac{1}{6}\right) + \left(3 \times \frac{1}{6}\right) + \left(4 \times \frac{1}{6}\right) + \left(5 \times \frac{1}{6}\right) + \left(6 \times \frac{1}{6}\right)$$

$$= \frac{7}{2}$$ (Ans)

$\frac{21}{6}$

$E[X] = np$

12

Expectation of binomial R.V :

$$P(i) = \binom{n}{i} \cdot p^{i} \cdot (1-p)^{n-i} \quad \text{when } i = 0, 1, 2, 3 \ldots, n$$

$$\binom{n}{i} = \frac{n!}{(n-i)! \cdot i!}$$

$$E[X] = \sum_{i=1}^{n} i \cdot P(i) = \sum_{i=1}^{n} i \cdot \binom{n}{i} \cdot p^{i} \cdot (1-p)^{n-i}$$

$$= \sum_{i=1}^{n} i \cdot \frac{n!}{(n-i)!\, i!} \cdot p^{i} \cdot (1-p)^{n-i}$$

$$= \sum_{i=1}^{n} \cancel{i} \frac{n!}{(n-i)!\,(i-1)!\,\cancel{i}} \cdot p^{i} \cdot (1-p)^{n-i}$$

$$= \sum_{i=1}^{n} \frac{n!}{(n-i)!\,(i-1)!} \cdot p^{i} \cdot (1-p)^{n-i}$$

$$= \sum_{i=1}^{n} \frac{n \cdot (n-1)!}{(n-i)!\,(i-1)!} \cdot p\,(p)^{i-1} \cdot (1-p)^{n-i}$$

$$= np \cdot \sum_{i=1}^{n} \frac{(n-1)!}{(n-i)!\,(i-1)!} \cdot p^{i-1} \cdot (1-p)^{n-i}$$

$$(n-1)-(i-1)$$

$$= np \sum_{i=1}^{n} \frac{(n-1)!}{\{(n-1)-(i-1)\}!\,(i-1)!}\, p^{i-1} \cdot (1-p)^{n-i}$$

let, $K = i - 1$,

So, when, $i = 1$, $K = 0$

$i = n$, $K = n-1$

$$E[X] = np \sum_{K=0}^{n-1} \frac{(n-1)!}{\{(n-1)-K\}!\, K!} \cdot p^K (1-p)^{n-i}$$

$$= np \cdot \sum_{K=0}^{n-1} \frac{(n-1)!}{\{(n-1)-K\}!\, K!} \cdot p^K \cdot (1-p)^{n-i+1-1}$$

$$= np \cdot \sum_{K=0}^{n-1} \frac{(n-1)!}{\{(n-1)-K\}!\, K!} \cdot p^K \cdot (1-p)^{(n-1)-K}$$

$$\boxed{E[X] = np \cdot \sum_{K=0}^{n-1} \binom{n-1}{K} \cdot p^K \cdot (1-p)^{(n-1)-K}}$$

$$\sum_{i=0}^{\infty} P(i) = 1 = (p + (1-p))^n$$

$$\sum_{i=0}^{n} \binom{n}{i} p^i (1-p)^{n-i} = (\cancel{p} + (1-\cancel{p}))^n = 1$$

$$E[X] = np(p + (1-p))^{(n-1)}$$

$$= np \cdot (1)^{n-1} = np \rightarrow E[X]$$

trials success

Prove that, The expected number of success in n independent trials is n multiplied by the Prob. that the trial result in success.

Expectation of geometric R.V :

From def^n of geometric R.V,

$$P_X(n) = P\{X = n\} = (1-p)^{n-1} \cdot p^1$$

From definition, $E[X] = \sum_{x=1}^{\infty} x \cdot P(x)$

$$= \sum_{n=1}^{\infty} n \cdot p\,(1-p)^{n-1}$$

For geometric series, $\sum_{n=1} ar^n$

Infinite case : $\sum_{n=1}^{\infty} a r^n$

Finite case : $\sum_{k=0}^{n-1} ar^k$

$$S = ar^0 + \cancel{ar^1} + \cancel{ar^2} + \cdots + \cancel{ar^{n-1}} \quad \text{--- (1)}$$

$$S \cdot r = \cancel{ar^1} + \cancel{ar^2} + ar^3 + \cdots + \cancel{ar^{n-1}} + ar^n \quad \text{--- (2)}$$

(1) $-$ (II)

$$S - Sr = ar^0 - ar^n$$

$$\Rightarrow S(1-r) = a(1-r^n)$$

$$\Rightarrow S = \frac{a(1-r^n)}{1-r}$$

So, for a known range ($n-1$ in this case),

$$\sum_{k=0}^{n-1} ar^k = \frac{a(1-r^n)}{1-r} \quad \text{--- (3)}$$

For infinite case,

$$\sum_{n=0}^{\infty} ar^n = ? \qquad n = \infty$$

$$\sum_{k=0}^{n-1} ar^k = \frac{a(1-r^n)}{1-r}$$

r^∞ will be very very small if r is fraction,

So, $$\sum_{k=0}^{\infty} ar^k = \frac{a}{1-r} \quad \text{———} \quad (IV)$$

$$E[X] = \sum_{n=1}^{\infty} n \cdot P(1-P)^{n-1}$$

let, $\boxed{q = 1-P}$

$$\therefore E[X] = P \sum_{n=1}^{\infty} n \cdot q^{n-1}$$

P & q all are small (fraction)

$$\therefore E[X] = P \sum_{n=1}^{\infty} \frac{d}{dq}(q^n)$$

$$= P \cdot \frac{d}{dq}\left[\left(\sum_{n=0}^{\infty} q^n\right) - q^0\right]$$

$$= P \cdot \frac{d}{dq}\left(\frac{1}{1-q} - 1\right)$$

$$= P \cdot \frac{d}{dq}\left(\frac{\cancel{1} - \cancel{1} + q}{1-q}\right)$$

$$= P \cdot \frac{d}{dq}\left(\frac{q}{1-q}\right)$$

$$\sum_{k=0}^{\infty} ar^k$$

$$\sum_{n=0}^{\infty} 1 \cdot q^n$$

$$= \frac{1}{1-q}$$

a

r

$$= p \cdot \frac{(1-q)\cdot 1 - q\cdot(-1)}{(1-q)^2}$$

$$= p \cdot \frac{1 - \cancel{q} + \cancel{q}}{(1-q)^2}$$

$$= \frac{p}{(1-q)^2} = \frac{p}{p^2} = \frac{1}{p} \longrightarrow E[X]$$

The expected number of independent trials we need to perform until we attain our first success is equal to the reciprocal of the probability that any one trial results in success.

If a continuous R.V having the PDF $f(x)$.

$$E[X] = \int_{-\infty}^{\infty} x \cdot f(x) \cdot dx$$

Expectation of Uniform R.V. :

$$f(x) = \begin{cases} \frac{1}{\beta - \alpha} ; & \text{if } \alpha \le x \le \beta \\ 0 \end{cases}$$

$$E[X] = \int_{\alpha}^{\beta} x \cdot \frac{1}{\beta - \alpha} dx$$

$$= \left[\frac{x^2}{2(\beta - \alpha)} \right]_{\alpha}^{\beta}$$

$$= \frac{\beta^2 - \alpha^2}{2(\beta - \alpha)} = \frac{\beta + \alpha}{2}$$

The expected value of a R.V uniformly distributed over interval (α, β) is just the midpoint of the interval.

E[X] of Exponential R.V. :

If X is a RV which exponentially distributed with parameter λ.

$$f(x) = \begin{cases} \lambda \cdot e^{-\lambda x} & \text{if } x \ge 0 \\ 0 , & \text{otherwise.} \end{cases}$$

$$E[X] = \int_{-\infty}^{\infty} x \cdot f(x) \cdot dx$$

$$= \int_{0}^{\infty} \underbrace{x}_{u} \cdot \underbrace{\lambda \cdot e^{-\lambda x}\, dx}_{dv}$$

$$= x(-e^{-\lambda x})\Big|_{0}^{\infty} - \int -(e^{-\lambda x}) \cdot dx$$

$$= \left[-x \cdot e^{-\lambda x}\right]_{0}^{\infty} + \int_{0}^{\infty} e^{-\lambda x} \cdot dx$$

$$= \underbrace{\left[-\infty \cdot e^{-\lambda \infty} + 0 \cdot e^{-\lambda \cdot 0}\right]}_{0} + \left[\frac{e^{-\lambda x}}{-\lambda}\right]_{0}^{\infty}$$

$$= -\left(\frac{e^{0}}{-\lambda}\right)$$

$$= \frac{1}{\lambda}$$

$\int u\, dv = uv - \int v\, du$

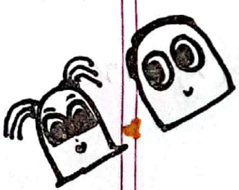
$x = u$

$\Rightarrow \frac{dx}{dx} = \frac{du}{dx}$

$\Rightarrow du = dx$

$v = -e^{-\lambda x}$

$dv = \lambda e^{-\lambda x} dx$



By def, PDF of Normal R.V.

$$f(x) = \frac{1}{\sqrt{2\pi}\cdot\sigma}\cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad (-\infty < x < \infty)$$

$$E[X] = \int_{-\infty}^{\infty} x\cdot f(x)\cdot dx$$

$$= \frac{1}{\sqrt{2\pi}\cdot\sigma}\int_{-\infty}^{\infty} x\cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$$

$x = (x-\mu)+\mu$,

$$E[X] = \frac{1}{\sqrt{2\pi}\cdot\sigma}\int_{-\infty}^{\infty}((x-\mu)+\mu)\, e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$$

$$= \frac{1}{\sqrt{2\pi}\cdot\sigma}\int_{-\infty}^{\infty}(x-\mu)\, e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$$

$$+ \mu\cdot\frac{1}{\sqrt{2\pi}\cdot\sigma}\int_{-\infty}^{\infty} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$$

let, $y = x-\mu$,

$$E[X] = \frac{1}{\sqrt{2\pi}\cdot\sigma}\int_{-\infty}^{\infty} y\cdot e^{\frac{-y^2}{2\sigma^2}} dy + \mu\int_{-\infty}^{\infty} f(x)\cdot dx$$

$$\frac{1}{\sqrt{2\pi}\cdot\sigma}\int_{-\infty}^{\infty} y\cdot e^{-\frac{y^2}{2\sigma^2}} dy$$

$$= \frac{1}{\sqrt{2\pi}\cdot\sigma}\int_{+\infty}^{+\infty} e^{-\frac{x}{2\sigma^2}}\cdot\frac{1}{2}dx$$

$y^2 = x$

$\Rightarrow 2y\cdot dy = dx$

$\Rightarrow y\, dy = \frac{1}{2}dx$

$= 0$

Binomial $\longrightarrow \binom{n}{i} (p^i) (1-p)^{n-i} = P(i)$

$$E[X] = 0 + \mu \underbrace{\int_{-\infty}^{\infty} f(x) \cdot dx}_{1}$$

(area under the curve f(x))

$$= \mu \text{ (Ans).}$$

(Q) consider two independent coin tosses, each with 3/4 probability of a head and let X be the number of heads obtained. This binomial R.V with parameter $n = 2$, and $p = \frac{3}{4}$, $E[X] = ?$

$$P(0) = \binom{2}{0}\left(\frac{3}{4}\right)^0 \cdot \left(1-\frac{3}{4}\right)^{2-0} = \left(\frac{1}{4}\right)^2$$

$$P(1) = \binom{2}{1}\left(\frac{3}{4}\right)^1 \left(1-\frac{3}{4}\right)^{2-1} = 2 \cdot \frac{3}{4} \cdot \frac{1}{4}$$

$$P(2) = \binom{2}{2}\left(\frac{3}{4}\right)^2 \left(1-\frac{3}{4}\right)^{2-2} = \left(\frac{3}{4}\right)^2$$

So, the PMF will be,

$$P_X(k) = \begin{cases} (1/4)^2, & \text{when } k = 0 \\ 2 \cdot \frac{3}{4} \cdot \left(\frac{1}{4}\right) & \text{when } k = 1 \\ (3/4)^2, & \text{when } k = 2 \end{cases}$$

$$E[X] = 0\left(\frac{1}{4}\right)^2 + 1 \cdot \left(2 \cdot 3/4 \cdot \frac{1}{4}\right) + 2\left(\frac{3}{4}\right)^2$$

$$= \frac{3}{2} \text{ (Ans)}$$

CS CamScanner

4. 10. 2023

Suppose X has the following PMF, $P(0) = 0.2$, $P(1) = 0.5$, $P(2) = 0.3$.

Expectation of a function of a R.V.

$E[X] = \sum_x x \cdot P_X(x)$

$E[X] = (0 \times 0.2) + (1 \times 0.5) + (2 \times 0.3) = 1.1$

$Y = E[X^2] \longrightarrow Y$ is a R.V. that can take the values of,

$Y = 0^2, 1^2, 2^2$

$= 0, 1, 4$

$P_Y(0) = 0.2$

$P_Y(1) = 0.5$

$P_Y(4) = 0.3$

$\sum_y y \cdot P_Y(y)$

$E[X^2] = (0 \times 0.2) + (1 \times 0.5) + (4 \times 0.3) = 1.7$

$P(X) = P(X^2) = 0.3$

$\therefore 1.7 = E[X^2] \neq \{E[X]\}^2 = 1.21$

Q (continuous case)

If X is a uniformly distributed over $0, 1$ then calculate $E[X^3] = ?$

Poisson $\longrightarrow$ $\frac{\lambda^{i}}{i!}e^{-\lambda}$ var = λ
$E[X] = \lambda$

$Y = X^3$

$0 \leq a \leq 1$ (Uniform) is true then,

$$F_Y(a) = P\{Y \leq a\}$$
$$= P\{X^3 \leq a\}$$
$$= P\{X \leq a^{1/3}\}$$

$$F_Y(a) \longrightarrow a^{1/3}$$

$$E[X] = \int_{-\infty}^{\infty} x \cdot f(x)\,dx$$

$$f_Y(a) = \frac{d}{da} F_Y(a) = \frac{d}{da}(a^{1/3}) = \frac{1}{3}a^{-2/3}$$

$$E[Y] = E[X^3] \frown \int_{-\infty}^{\infty} a \cdot f_Y(y)\,da$$

$$= \int_0^1 a \cdot \frac{1}{3} \cdot a^{-2/3}\,da$$

$$= \frac{1}{3}\int_0^1 a^{1/3}\,da = \frac{1}{3} \cdot \left[\frac{a^{4/3}}{4/3}\right]_0^1$$

$$= \frac{1}{\cancel{3}} \cdot \frac{\cancel{3}}{4}(1-0)$$

$$= \frac{1}{4}$$

Property 1 : Expectation of function :

If X is a discrete R.V. with PMF $P_X(x)$ for any real valued function or in other words, the derived R.V, $Y = g(x)$, the expected value of Y (or $g(x)$) is given by,

$$E[Y] = E[g(x)] = \sum_{x : P(x) > 0} g(x) \cdot P(x)$$

$$= \sum_{i=1}^{n} g(x) \cdot P(x) \longrightarrow \text{discrete case}$$

$$E[Y] = E[g(x)] = \int_{-\infty}^{\infty} g(x) \cdot f(x) \cdot dx$$

$\longrightarrow$ continuous case

$$* \quad P_X(x) = \begin{cases} 0.2 & \text{when } x = 0 \\ 0.5 & \text{when } x = 1 \\ 0.3 & \text{" } x = 2 \end{cases}$$

let, $g(x) = x^2$

$$\therefore E[X^2] = \sum_{x=0}^{2} g(x) \cdot P(x)$$

$$= (0^2 \times 0.2) + (1^2 \times 0.5) + (2^2 \times 0.3)$$

$$= 1.7$$

For continuous case,

$Y = x^3 = g(x)$ $\longrightarrow$ $\int_0^1 g(x) \cdot dx \underbrace{\int_0 f(x)\, dx}_{1}$

$$E[X^3] = \int_0^1 g(x) \underbrace{f(x)}_{1} dx$$

$$= \int_0^1 x^3\, dx$$

$$= \left[\frac{x^4}{4}\right]_0^1 = \frac{1}{4}.$$

(Q) Given PMF is, $f_X(x) = \begin{cases} 2/3, & x = 1 \\ 1/3, & x = 2 \end{cases}$

calculate $E[x^2 + 1]$

→ Let, $Y = X^2 + 1$

PMF of $Y \Rightarrow f_Y(y) = \begin{cases} 2/3, & y = 2 \\ 1/3, & y = 5 \end{cases}$

$$E[Y] = \left(2 \times \frac{2}{3}\right) + \left(5 \times \frac{1}{3}\right) = 3$$

$$E[X] = \left(1 \cdot \frac{2}{3}\right) + \left(2 \times \frac{1}{3}\right) = \frac{4}{3}$$

$$E[Y] = \int_{-\infty}^{\infty} \underbrace{g(x) \cdot f(x)}_{\#} \cdot dx$$

$$E[X] = \int_{-\infty}^{\infty} x \cdot f(x) \cdot dx$$

remember, $E[Y] = \int_{-\infty}^{\infty} y \cdot f_Y(y) \cdot dy$

Advantage of ① is we do not need to find out the PDF of Y.

<u>Property 2</u> : n^{th} moment of X :

If X is a R.V. then expected value of R.V. is referred to as the mean or the first moment of X.

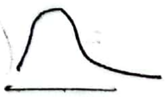
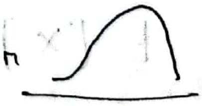
$E[X] = \mu$ = the first moment of X.

$$E[X^n] = \begin{cases} \sum_{x: P(x)>0} x^n \cdot P(x) & \text{if } X \text{ is discrete.} \\ \int_{-\infty}^{\infty} x^n \cdot f(x) \cdot dx & \text{if } X \text{ is continuous.} \end{cases}$$

0^{th} moment $\longrightarrow$ total prob.

1^{st} " $\longrightarrow$ mean

2^{nd} " $\longrightarrow$ variance

3^{rd} " $\longrightarrow$ skewness  or 

4^{th} " $\longrightarrow$ Kurtosis  কিছু পরিমাণ data

$$E[\overset{\text{constant}}{2}] = 2$$

Linearity :

$E[X]$ is linear, if a and b, are constant,

$$E[ax+b] = aE[X] + b$$

Discrete

$$E[g(x)] = \sum_{x:P(x)>0} g(x)\cdot P(x)$$

$$= \sum_{x:P(x)>0} (ax+b)\cdot P(x)$$

$$= a\sum_{x:P(x)>0} x\cdot P(x) + b\underbrace{\sum_{x:P(x)>0} P(x)}_{1}$$

$$= aE[X] + b$$

Continuous :

$$E[g(x)] = \int_{-\infty}^{\infty} g(x)\cdot f(x)\cdot dx$$

$$= \int_{-\infty}^{\infty} (ax+b)f(x)\cdot dx$$

$$= a\underbrace{\int_{-\infty}^{\infty} x\cdot f(x)\cdot dx}_{E[X]} + b\underbrace{\int_{-\infty}^{\infty} f(x)\cdot dx}_{1}$$

$$= aE[X] + b$$

Variance :

$$\boxed{Var[X] = E\{X - E[X]\}^2}$$

$$Var[X] = \int_{-\infty}^{\infty} \{X - E[X]\}^2 f(x) \cdot dx$$

$$= \int_{-\infty}^{\infty} [x^2 - 2x \cdot E[X] + E[X]^2] f(x) \cdot dx$$

$$= \int_{-\infty}^{\infty} x^2 \cdot f(x) dx - 2E[X] \underbrace{\int_{-\infty}^{\infty} x \cdot f(x) dx}_{E[X]}$$

$$+ E[X]^2 \underbrace{\int_{-\infty}^{\infty} f(x) = dx}_{1}$$

$$\boxed{Var[X] = \int_{-\infty}^{\infty} x^2 \cdot f(x) \cdot dx - E[X]^2}$$

Property of independence :

$$E[X + Y] = E[X] + E[Y]$$

$$E[XY] = E[X] \cdot E[Y]$$

- Show that the variance of a R.V. equals the expected value of the square of the R.V minus the square of the expected value of the R.V.

$$Var[X] = E[X^2] - E[X]^2$$

Example: Calculate $Var[X]$ when X represents the outcome when a fair die is rolled?

$P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = \frac{1}{6}$

$E[X] = 1 \cdot \frac{1}{6} + 2 \times \frac{1}{6} + 3 \times \frac{1}{6} + 4 \cdot \frac{1}{6} + 5 \cdot \frac{1}{6} + 6 \cdot \frac{1}{6}$

$= \frac{7}{2}$

$E[X^2] = 1^2 \times \frac{1}{6} + 2^2 \times \frac{1}{6} + 3^2 \times \frac{1}{6} + 4^2 \times \frac{1}{6} + 5^2 \times \frac{1}{6} + 6^2 \cdot \frac{1}{6}$

$= \frac{91}{6}$

$\therefore Var[X] = \frac{91}{6} - \left(\frac{7}{2}\right)^2 = 2 \cdot 91.$

(Ans).

9 1) what will be PMF of R ?

$\longrightarrow$ $P(R=0) = \frac{1}{8} + \frac{1}{8} = 1/4$

$P(R=2) = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{3}{4}$

PMF of R is $P_R(r) = \begin{cases} 1/4, & \text{when } r=0 \\ 3/4, & \text{when } r=2. \end{cases}$

2) what will be expected value, $E[X]$?

$\rightarrow$ $E[R] = \Sigma x \cdot P_x(x)$

$= (0 \times 1/4 + 2 \times 3/4) = 3/2$

3) what will be the expected value of the function $V = g(R) = 4R + 7 = ?$

$\rightarrow Y = 4R + 7$

$E[Y] = E[g(R)] = \sum_{x: P(x)>0} (4R+7) \cdot P(r)$

$= \{(4 \times 0 + 7) \times \frac{1}{4}\} + \{(4 \times 1 + 7) \times \frac{3}{4}\}$

$= 7 \cdot \frac{1}{4} + 15 \cdot \frac{3}{4}$

$= 13$

number of pages X of a FAX

Or,

$$E[ax+b] = a\,E[X]+b$$

$$\Rightarrow E[4R+7] = 4 \times \frac{3}{2} + 7 \quad \leftarrow \begin{array}{l} a=4 \\ b=7 \end{array}$$

$$= 13$$

(Ans)

Example (expected value):

1) $Y = g(x) = \begin{cases} 10.5x - 0.5x^2 \ ; \ 1 \le x \le 5 \\ 50 \ , \ 6 \le x \le 10 \end{cases}$ in cents

Page	1	~~~	10	cent
"	2	"	19	"
"	3	"	27	"
"	4	"	34	"
"	5	"	40	"
"	6	"	50	"
"	7	"	50	"
"	8	"	50	"
"	9	"	50	"
"	10	"	50	

2) $P_X(x) = \begin{cases} 1/4 \ , & x = 1, 2, 3, 4 \\ 0 \ , & \text{otherwise} \end{cases}$

$$P_Y(y) = \begin{cases} 1/4, & y = 10, 19, 27, 34 \\ 0, & \text{otherwise} \end{cases}$$

$$E[Y] = \left(\tfrac{1}{4} \times 10\right) + \left(\tfrac{1}{4} \times 19\right) + \left(\tfrac{1}{4} \times 27\right) + \left(\tfrac{1}{4} \times 34\right)$$

$$= 22.4 \text{ cents}$$

3) The P of the cost of the FAX to be 0.10\$ (when no. of P is 1) = 0.15

$$P_Y(50) = P(6) + P(7) + P(8) = 0.1 + 0.1 + 0.1$$

$$= 0.3$$

$$P_Y(y) = \begin{cases} 0.15, & y = 10, 19, 27, 34 \\ 0.1, & y = 40 \\ 0.3, & y = 50 \\ 0, & \text{otherwise} \end{cases}$$

$$E[Y] = (10 + 19 + 27 \times 34) \times 0.15 + (0.1 \times 40) + (0.3 \times 50)$$

$$= 32.5.$$

decision নিবো → expected value use করব

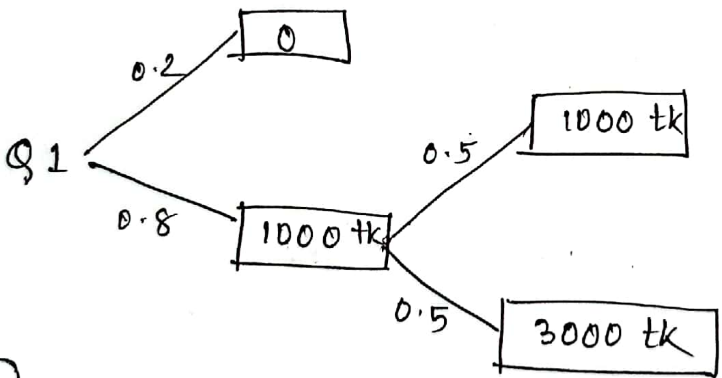
quiz → 2 questions

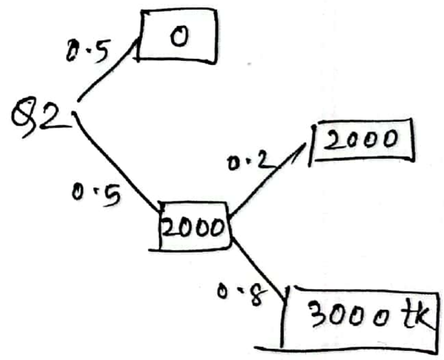
must decide which ques. to answer first.

$Q1 \xrightarrow[\text{answered})]{P(\text{correctly}}$ 0.8 $\xrightarrow{\text{money}}$ 1000 tk

$Q2 \xrightarrow[\text{answered})]{P(\text{correctly}}$ 0.5 $\xrightarrow{\text{"}}$ 2000 tk

If the first question (eith 1 or 2) is wrongly answered the quiz terminates.





Q1

$P_x(0) = 0.2$

$P_x(1000) = 0.8 \times 0.5 = 0.4$

$P_x(3000) = 0.8 \times 0.5 = 0.4$

$E[X] = (0 \times 0.2) + (1000 \times 0.4) + (3000 \times 0.4)$

$= 1600$ tk

Q2

$E[X] = (0 \times 0.5)$

$+ (2000 \times 0.1)$ 0.2×0.5

$+ (3000 \times 0.4)$ 0.5×0.8

$= 1400$ tk.

So, Q1 আগে answer করবো।