

Subject: Date: 19/8/23

Summary Measurer and Decisions

The mean and standard deviation are a good way to measure central tendency.

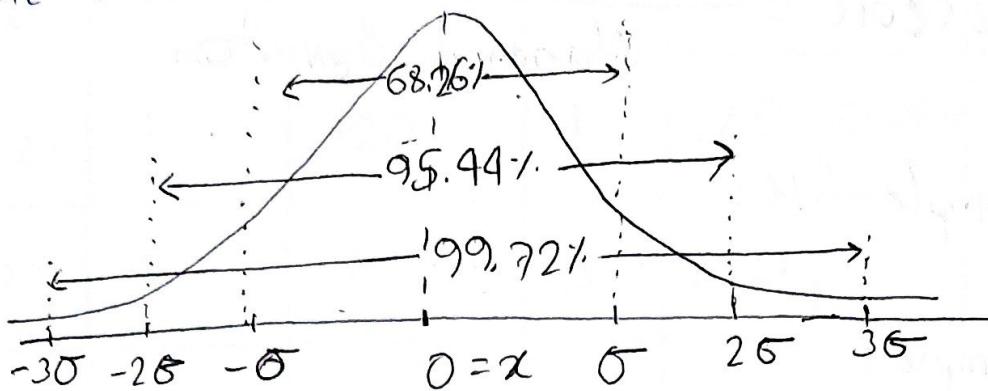
For any mound shape, nearly symmetric distribution data or data;

→ The interval $\bar{x} \pm s$ contains approximately 68% of data points.

→ The interval $\bar{x} \pm 2s$ contains approx. 95% of data points.

→ Interval $\bar{x} \pm 3s$ contains approx. all the data points.

Standard normal deviation: $M=0$ $S.d.=1$



Example-1.15 pg-39 (book)

~~One of the~~

[mid point also called class terms]

[\bar{x}/μ can be used for mean. μ means whole population.

Sample of mean we use \bar{x}/s]

Standardized values and z-scores:

We can calculate any data point if we know

the mean and standard deviation.

z-score doesn't have a unit so it is easier to compare data point of two different datas.

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$$Z\text{ score} = \frac{\text{measurement} - \text{mean}}{\text{standard deviation}}$$

Example - 1.16 :

Example :

138	164	150	132	144	125	199	157
146	158	190	147	136	148	152	144
168	126	138	176	170	163	153	154
146	173	142	147	135	119	140	135
161	145	135	192	150	156	145	128

largest weight = 176 lb

smallest weight = 119 lb

$$\text{Range} = 176 - 119 = 57 \text{ lb}$$

class interval = 5

$$\text{No. of classes} = \frac{57}{5} \approx 12$$

(146.875)

Subject: $\Sigma f = 40$ Date: 5875

Class Interval	Frequency	Midpoint x_i	Cumulative Freq. c.f.	$f x_i$	$x - \bar{x}$	$(x - \bar{x})^2$	$f(x - \bar{x})^2$
118-122	1	120	1	120	-26.875	722.265	722.265
123-127	2	125	3	250			
128-132	2	130	5	260			
133-137	4	135	9	540			
138-142	6	140	15	840			
✓ 143-147	8	145	23	1160			
✓ 148-152	5	150	28	750			
153-157	4	155	32	620			
158-162	2	160	34	320			
✓ 163-167	3	165	37	915			
168-172	1	170	38	170			
173-177	2	175	40	350			

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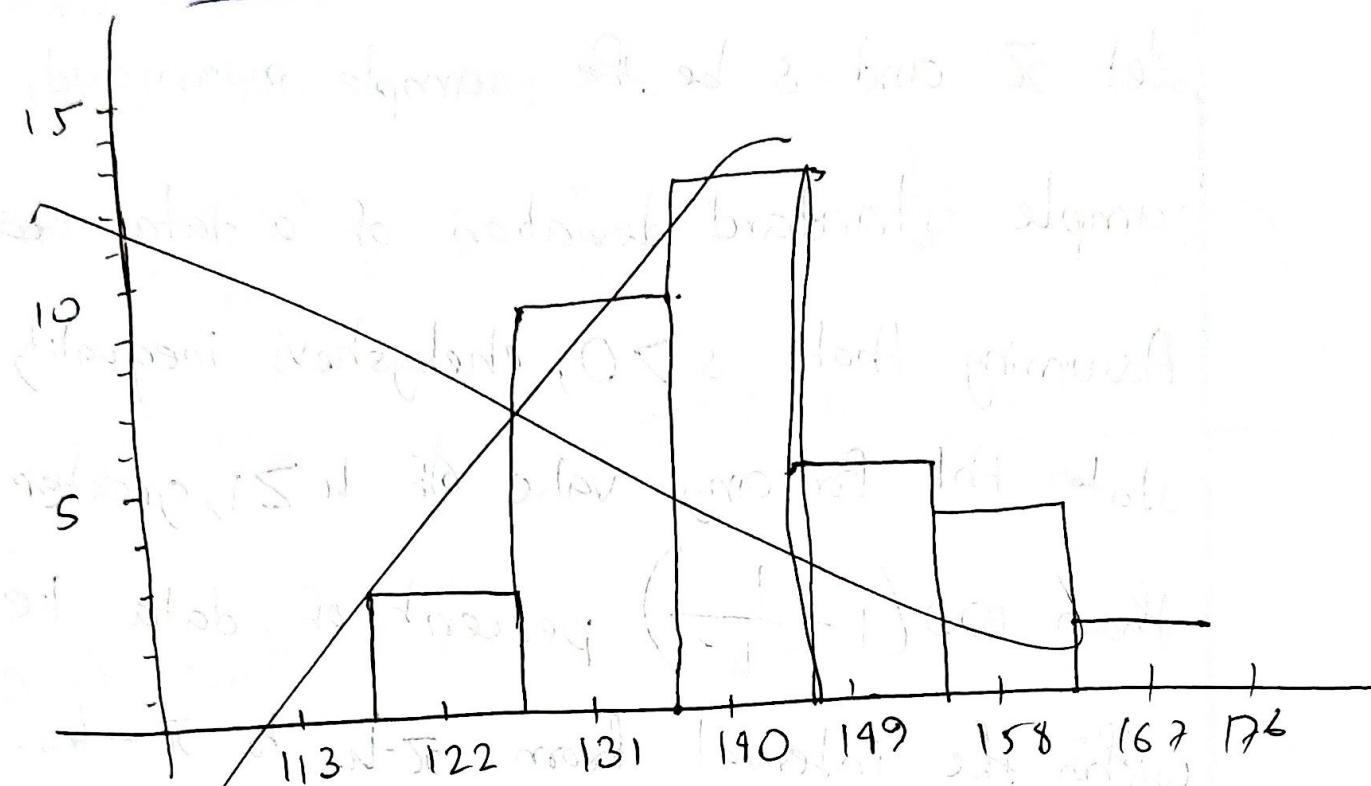
	frequency	midpoint
118-126	3	122
127-135	5	131
136-144	9	140
145-153	12	149.5
154-162	5	158
163-171	4	167
172-180	2	176
	$\sum f_i = 40$	

$$\text{Mean}, \bar{x} = \frac{\sum f_i m_i}{n} = \frac{5875}{40} = 146.875$$

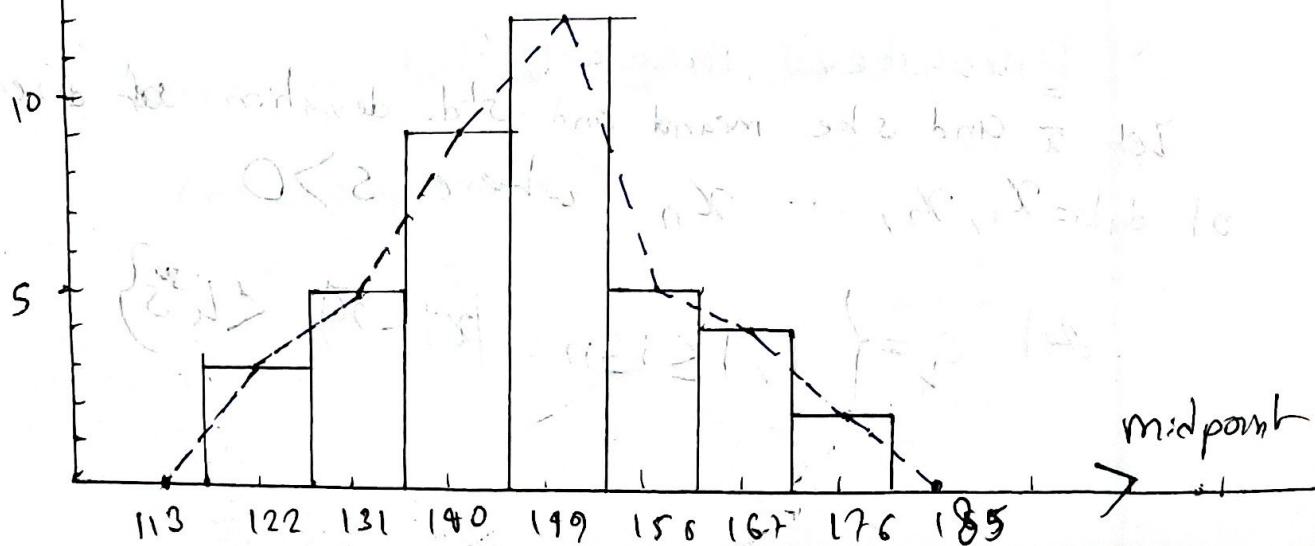
$$\text{Mode} = 143 + \frac{8-6}{(8-6)+(8-5)} \times 5 = 145$$

$$\text{Median} = 143 + \frac{20-15}{8} \times 5 = 146.125$$

Histogram and Frequency Polygon



frequency polygon ← midpoint
frequency →



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Chebychev's theorem inequality : [more generalized than empirical rule]

Let \bar{x} and s be the sample mean and sample standard deviation of a data set.

Assuming that $s > 0$, chebychev's inequality states that for any value of $k \geq 1$, greater than $100\left(1 - \frac{1}{k^2}\right)$ percent of data lie within the interval from $\bar{x} - ks$ to $\bar{x} + ks$.

Suppose, $k = \frac{2}{3}$; $100\left(1 - \frac{1}{\left(\frac{2}{3}\right)^2}\right) = 100 \times \frac{5}{9} \approx 55.56\%$.

of data lie in $(\bar{x} - 1.5s, \bar{x} + 1.5s)$

Chebychev's inequality :

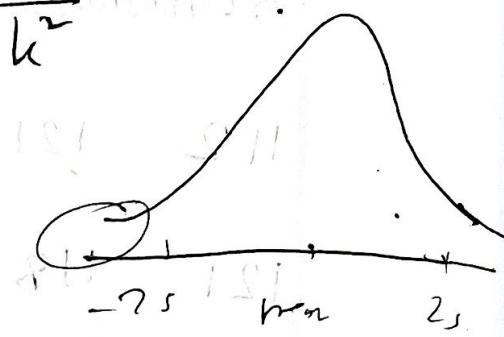
Let \bar{x} and s be mean and std. deviation of set consisting of data x_1, x_2, \dots, x_n where $s > 0$

Let $s_k = \left\{ i ; 1 \leq i \leq n : |x_i - \bar{x}| \leq ks \right\}$

and let $N(s_u)$ be the number of elements in set, s_u . Then for any $k \geq 1$ and s_u

$$\frac{N(s_u)}{n} \geq 1 - \frac{n-1}{nk^2} > 1 - \frac{1}{k^2}$$

$$|x_i - \bar{x}| < k_s \quad \bar{x} - k_s < x_i < \bar{x} + k_s$$



$$N(u) = \text{number of } i : |x_i - \bar{x}| \geq k_s$$

$$\frac{N(u)}{n} \leq \frac{1}{k^2}$$

Next class: 14. data point set, mean, median, mode, what % of data belongs to 1.5s. Compare

Chebyshev's inequality and empirical rule

values not able to remember formula

classmate: $\sigma = \sqrt{\sum (x_i - \bar{x})^2 / n}$

Example: The following data represent the lifetimes (in hours) of a sample of 90 transistors.

112	121	126	108	141	104	130	134
121	118	143	118	108	122	127	190
113	117	126	130	134	120	131	133
118	125	151	147	137	190	132	119
110	124	132	152	135	130	136	128

smallest data = 104
largest data = 190

- (a) Determine the sample mean, median & mode.
- (b) Draw cumulative relative frequency plot.
- (c) Are the data approximately normal?
- (d) What percentage of data fall within $\bar{x} \pm 1.5$ sec?

No need to round up for human count

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(e) Compare the result in (d) to that given by empirical rule?

(f) Is it bounded by Chebychev's inequality?

Chebychev's inequality?

$$\frac{1}{n} \sum (x_i - \bar{x})^2 \leq \frac{\sigma^2}{n} \Rightarrow \frac{1}{n} \sum (x_i - \bar{x})^2 \leq \frac{\sigma^2}{n}$$

Mean = 127.425 Median = 127.5

Mode = 108 Standard deviation = 11.873

Range = 152 - 109 = 43

Range	f	relative frequency	cumulative frequency	cumulative relative frequency
104-116	7	7/14	7	0.175
117-128	14	14/14	21	0.525
129-140	14	14/14	35	0.875
141-152	5	5/14	40	1

$$\sum d = 1$$

Chebychev's inequality yields that: $100 \left(1 - \frac{1}{(\frac{3}{2})^2}\right) = 100 \left(1 - \frac{4}{9}\right) = 55.55\%$

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Stem and Leaf plot

stem → 6

leaf → 5, 9, 2

data: 65, 69, 62

data: 605, 609, 602

6 | 2, 5, 9

60 | 2, 5, 9

[most data uses single digit as stem]

Stem and leaf plot of da prev. data :

10	4, 8, 8	(3)
11	0, 2, 3, 6, 7, 8, 8	(7)
12	0, 1, 1, 2, 4, 5, 6, 6, 7, 8	(10)
13	0, 0, 1, 2, 2, 3, 4, 4, 5, 6, 6, 7	(12)
14	0, 0, 1, 3, 7	(5)
15	1, 2	(2)

Fairly normal distribution

Big Mode, ~~Highest frequency~~ = Multi mode!

Median: avg or 20^{th} and 21^{st} value

cummulative frequency plot \rightarrow ogive

$$\text{1st quartile, } \frac{10+11}{2} = \text{avg}$$

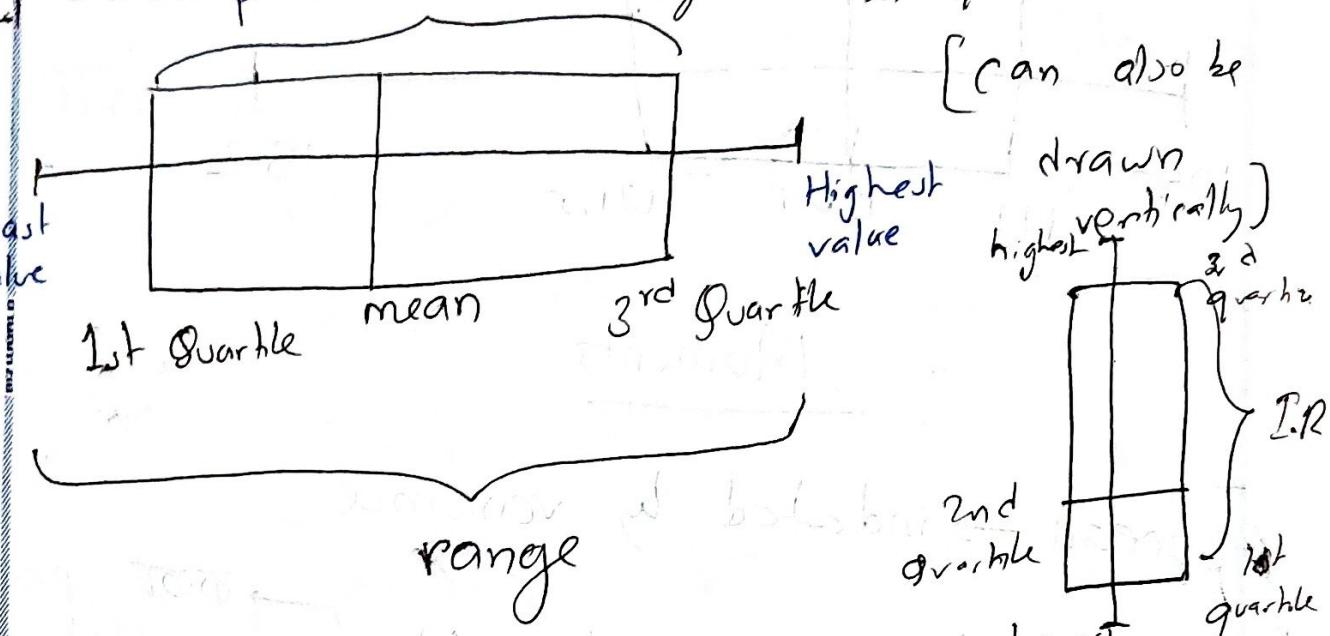
$$\text{2nd Quartile, } \frac{20+21}{2} = \text{avg}$$

$$\text{3rd Quartile, } \frac{30+37}{2} = \text{avg}$$

$$\text{4th Quartile, } \frac{40+47}{2} = \text{avg}$$

[If data point is odd/cannot be divided into 8 equal by 4 then we take avg of two nearest value]

② Box plot interquartile range = 3rd quartile - 1st quartile



[sometimes some data points are very far from typical range of data, and can drastically effect mean but not variance. These data are **Outliers**. They are included in range]

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after dividing in percentile then data shows percentage of data in 100.

$$1^{\text{st}} \text{ quartile} = 111$$

$$2^{\text{nd}} \text{ quartile} = 120.5$$

$$3^{\text{rd}} \text{ quartile} = 131.5$$

