MATH

STATISTICS

Numbers of possible outcomes = sample space

$$\text{probability} = \frac{\text{favourable outcome (event)}}{\text{possible outcome}} = \frac{n(E)}{n(S)}$$

(*) $0 \leq P(A) \leq 1$

(*) $P(A^c) = 1 - P(A)$

Decks of Cards:

52 Cards
- → Suits
  - → Spade → 13
  - → Heart → 13
  - → Diamond → 13
  - → Clubs → 13
- → face cards (12)
  - → King (K)
  - → Queen (Q)
  - → Jack (J)
- → Color (red/black)
  - → 26
  - → 26

$P(\text{not getting a face card}) = P(F^c) = 1 - P(F) = 1 - \frac{12}{52}$

We toss a coin 5 times.
P(at least 1 head) =

$$= 1 - \left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right)$$

Addition Rule

(events are connected or, non-disjoint)

* $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

* $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

If there are no connections

$P(A \cup B \cup C \ldots \cup Z) = P(A) + P(B) + P(C) + \ldots + P(Z)$.

Multiplication Rule

* not connected

$P(E) = P(A) . P(B) \ldots P(Z)$

Ex: 3 hats, 4 shirts, 5 shoes all together.

$P(1 \text{ hat}, 1 \text{ shoe}) = \frac{3}{12} \cdot \frac{5}{11}$

(multiple events happening at same time)

* Connected: (Conditional probability)

$P(A|B)$ = probability of event A given event B has happened

$$= \frac{P(A \cap B)}{P(B)}$$

- What's the probability of number less than 4 or an even number appearing on the dice

Ans: $A = \text{less than } 4 = \{1, 2, 3, 4, 5\} = \frac{3}{6} = \frac{1}{2}$

$B = \text{even number} = \{1, 2, 3, 4, 5, 6\}$

$= \frac{3}{6} = \frac{1}{2}$

$\therefore$ Probability $= \frac{1}{2} + \frac{1}{2} - \frac{1}{6} = \frac{5}{6}$.

- There's a deck of card.

(a) $P(\text{Jack} \cup \text{King} \cup \text{Queen} \cup \text{Ace})$

(b) neither king nor Queen

(c) a club or a spade

→ we are taking out just 1 card.

Sol^n:

(a): $P(\text{Jack} \cup \text{King} \cup \text{Queen} \cup \text{Ace})$

$= P(J) + P(K) + P(Q) + P(A)$

$= \frac{4}{52} + \frac{4}{52} + \frac{4}{52} + \frac{4}{52} = \frac{4 \times 4}{52} = \frac{4}{13}$

(b): P(neither king nor queen)

$= 1 - P(\text{king} \cup \text{Queen})$

$= 1 - \frac{4}{52} - \frac{4}{52}$

$= \frac{11}{13}$

(c): P(a club or a spade)

$= \frac{13}{52} + \frac{13}{52}$

$= \frac{1}{2}$

MATH

Multiplication rule:

(*) Without replacement:

We take 2 heart cards.

$$P(E) = \frac{13}{52} \times \frac{12}{51}$$

(*) With replacement.

We take 2 heart cards

$$P(E) = \frac{13}{52} \times \frac{13}{52}.$$

Without replacement:

$$\left.\begin{aligned} P(\text{heart}, \text{spade}) &= \frac{13}{52} \cdot \frac{13}{51}. \\ P(\text{spade}, \text{heart}) &= \frac{13}{52} \cdot \frac{13}{51} \end{aligned}\right] \text{has orders}$$

P(1 spade & 1 will be heart)

$$= P(\text{heart}, \text{spade}) + P(\text{spade}, \text{heart})$$

$$= \frac{13}{52} \cdot \frac{13}{51} + \frac{13}{52} \cdot \frac{13}{51}$$

(*) $P(\text{heart, jack}) = \frac{13}{52}.$

here, the heart could be a jack

*

C, H, I, C, K, E, N

P(<u>1 will be c</u> & other will not be c)

$$= \left[\frac{2}{7}\cdot\left(1-\frac{1}{6}\right)\right] + \left[\left(1-\frac{2}{7}\right)\cdot\frac{2}{6}\right]$$

$$= \left(\frac{2}{7}\cdot\frac{5}{6}\right) + \left(\frac{5}{7}\cdot\frac{2}{6}\right)$$

$$P(2 \text{ not } c) = \frac{5}{7}\cdot\frac{4}{6}$$

Conditional Probability

What is the probability of getting a jack given first card is heart.

$$P(\text{Jack} \mid \text{heart}) = \frac{\frac{1}{52}}{\frac{13}{52}} = \frac{1}{13}$$

$$P(\text{heart} \mid \text{Jack}) = \frac{\frac{1}{52}}{\frac{4}{52}} = \frac{1}{4}$$

① In a town, it's rainy $\frac{1}{3}$ of days.

Given it's rainy, there will be heavy traffic, $P = \frac{1}{2}$

" " not " " " " " " $P = \frac{1}{4}$

If it is rainy & heavy traffic, I'm late, $P = \frac{1}{2}$

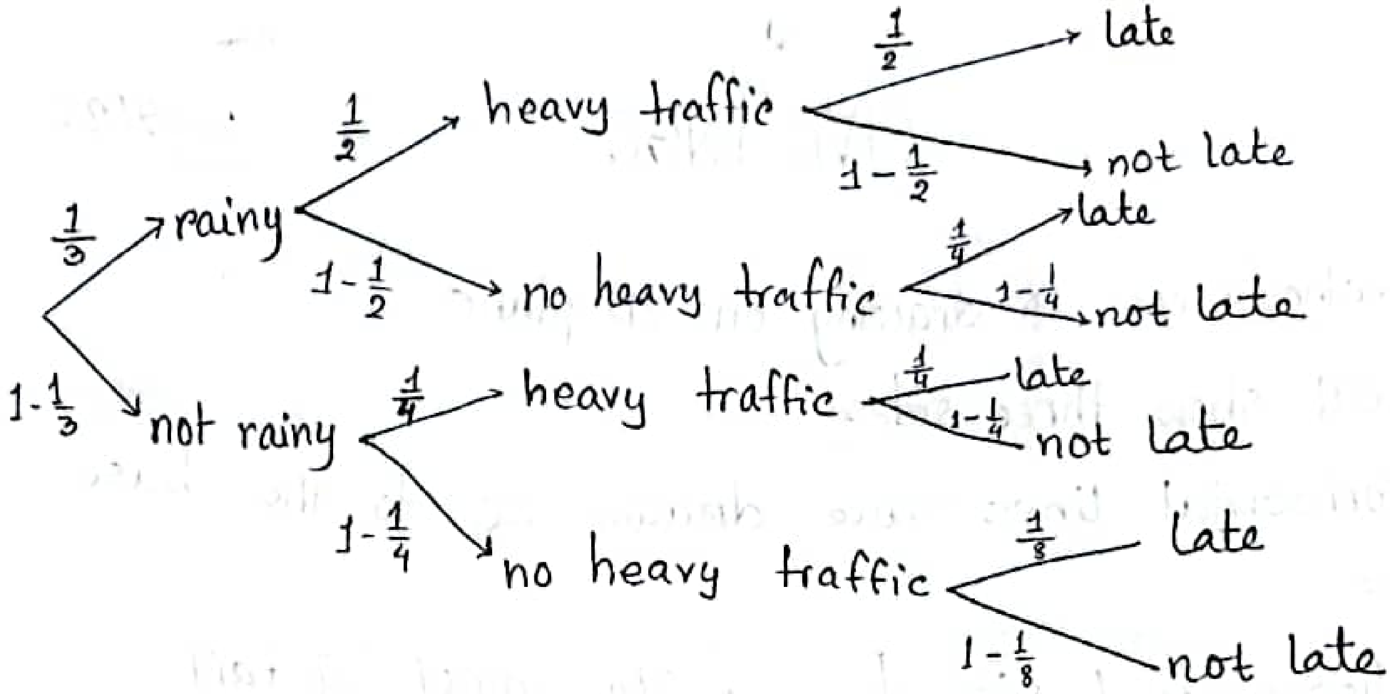
" " " not " & no " " " " $P = \frac{1}{8}$

In other situation, $P(\text{late}) = \frac{1}{4}$

$P(\text{late}) = ?$

$P(\text{rainy} | \text{late}) = ?$

Ans: For complex problems like these, we draw probability tree.



(*) $P(late) = \left(\frac{1}{3}\cdot\frac{1}{2}\cdot\frac{1}{2}\right) + \left(\frac{1}{3}\cdot\frac{1}{2}\cdot\frac{1}{4}\right)$

$+ \left(\frac{2}{3}\cdot\frac{1}{4}\cdot\frac{1}{4}\right) + \left(\frac{2}{3}\cdot\cancel{\frac{1}{4}}\,\frac{3}{4}\cdot\frac{1}{8}\right)$

$= \frac{11}{48}$

(*) $$P(rainy \mid late) = \frac{P(rainy \cap late)}{P(late)}$$

$$= \frac{\left(\frac{1}{3}\cdot\frac{1}{2}\cdot\frac{1}{2}\right) + \left(\frac{1}{3}\cdot\frac{1}{2}\cdot\frac{1}{4}\right)}{\frac{11}{48}}$$

$$= \frac{6}{11}$$

Probability distribution

X	1	2	3	4	5	6
$P(X)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

variable : when we roll a dice, the faces we get.

- Discrete variables
  - Probability Mass Function (PMF) → exactly the number I'm looking for
  - Cumulative Density Function (CDF) → range of numbers.
- ~~Concrete~~ Continuous variables
  - Probability density function (PDF)
  - Cumulative Density Function (CDF)

* PMF of $(X=6) = \frac{1}{6}$

$$CDF(x<6) = P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5)$$

$$= \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{5}{6}.$$

Discrete:

(1) Probability Distribution table for the sum of two dice when rolled.

x		2	3	4	5	6	7	8	9	10	11	12
$P(x)$		$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

$\longrightarrow$ 1st dice

↓ 2nd dice	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

→ sample space = 36

$$PMF\,(x = 9) = \frac{4}{36}$$

(2) Raffle problem :

1000 raffle ticket sold for \$1 each
Each has equal chance of winning

1st price = \$ 300
2nd price = \$ 200
3rd price = \$ 100

Let, X = net gain from the ticket.

* Now, when we buy a ticket, we have to buy a ticket first & then get the price.
So, net gain = 300 − 1 = 299.

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X	\$299	\$199	\$99	\$ −1
$P(X)$	$\frac{1}{1000}$	$\frac{1}{1000}$	$\frac{1}{1000}$	

$\rightarrow \left(1 - \frac{1}{1000} - \frac{1}{1000} - \frac{1}{1000}\right)$

$= \frac{997}{1000}$

since each has equal chance of winning

Discrete

Mean | Variance

Mean $= \sum x . P(X = x)$

x	0	1	2	3
$P(x)$	0.95	0.02	0.02	0.01

mean of probability distribution, μ

$= (0 \times 0.95) + (1 \times 0.02) + (2 \times 0.02) + (3 \times 0.01)$

Variance $= \sum (x - \mu)^2 P(x)$

4. Given, PMF $f(x) = bx^3$ for $x = 1, 2, 3$.
Find the value of b.

<u>Soln</u>: Here,
$$\sum_{1}^{3} bx^3 = 1$$

or, $b(1)^3 + b(2)^3 + b(3)^3 = 1$

or, $b + 8b + 27b = 1$

or, $b = \frac{1}{36}$

<u>(Ans)</u>

<u>variance</u>: how much scattered our values are from the mean.

Continuous Probability Distribution

- PDF → $f(x)$
- CDF → $F(x)$

$$P(X) = \int_a^b f(x)\, dx$$

* The area between density curve and horizontal axis $= \displaystyle\int_{-\infty}^{\infty} f(x)\, dx = 1.$

Let, X be a random variable with PDF given by:

$$f(x) = \begin{cases} cx^2, & |x| \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

(a) Find c.

$$\int_{-\infty}^{\infty} f(x)\, dx = 1$$

$$\Rightarrow \int_{-1}^{1} cx^2 \, dx = 1$$

$$\Rightarrow c.\left[\frac{x^3}{3}\right]_{-1}^{1} = 1$$

$$\Rightarrow \frac{c}{3}\left((1)^3 - (-1)^3\right) = 1$$

$$\Rightarrow \frac{c}{3}(1+1) = 1$$

$$\Rightarrow \frac{2c}{3} = 1$$

$$\Rightarrow c = \frac{3}{2}.$$

⊞ $$f(x) = \begin{cases} 4x^3, & 0 < x \leq 1 \\ 0, & \text{otherwise} \end{cases}$$ 0 নাও থাকতে পারে।

$$P\left(x \leq \frac{2}{3} \,\middle|\, x > \frac{1}{3}\right) = \frac{P\left(\frac{1}{3} < x \leq \frac{2}{3}\right)}{P\left(x > \frac{1}{3}\right)}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{\int_{\frac{1}{3}}^{\frac{2}{3}} 4x^3 \, dx}{\int_{\frac{1}{3}}^{\cancel{\infty}\,1} 4x^3 \, dx}$$

$$= \frac{[x^4]_{\frac{1}{3}}^{\frac{2}{3}}}{[x^4]_{\frac{1}{3}}^{1}}$$

$$= \frac{\left(\left(\frac{2}{3}\right)^4 - \left(\frac{1}{3}\right)^4\right)}{\left((1)^4 - \left(\frac{1}{3}\right)^4\right)}$$

$$= \frac{3}{16}$$

$\boxed{4}$ $f(x) = \begin{cases} \frac{2}{21} x; & 0 \leq x \leq k \\ \frac{2}{15}(6-x); & k \leq x \leq 6 \\ 0; & \text{Otherwise} \end{cases}$

(a) Find $P\left(x < \frac{1}{3}k \mid x < k\right)$

<u>Ans</u>: We know,

$$\int_0^k \frac{2}{21} x \, dx + \int_k^6 \frac{2}{15}(6-x)\, dx + 0 = 1$$

$$\frac{2}{21 \times 2}[x^2]_0^k + \frac{2}{15}\left[6x - \frac{x^2}{2}\right]_k^6 = 1$$

or, $\frac{2}{21 \times 2} [x^2]_0^k + \frac{2}{15} \left[\left(36 - \frac{36}{2}\right) - \left(6k - \frac{k^2}{2}\right)\right] = 1$

or, $\frac{\cancel{2}}{\cancel{21} \times \cancel{2}} \frac{1}{21}(k^2) + \frac{2}{15}\left[\left(36 - \frac{36}{2}\right) - \left(6k - \frac{k^2}{2}\right)\right] = 1$

or, $\frac{k^2}{21} + \frac{2}{15}\left(18 - 6k + \frac{k^2}{2}\right) = 1.$

or, $\frac{k^2}{21} + \frac{12}{5} - \frac{4}{5}k + \frac{1}{15}k^2 = 1$

or, $\frac{4}{35}k^2 - \frac{4}{5}k + \frac{7}{5} = 0$

$k = \frac{7}{2}.$

$P\left(x < \underset{1.166}{\frac{7}{6}} \,\middle|\, x < \underset{3.5}{\frac{7}{2}}\right) = \frac{P\left(x < \frac{7}{6}\right)}{P\left(x < \frac{7}{2}\right)}$ } when there is no intersection

numerator comes from $\frac{\text{probability of event}}{\text{probability of total income}}$

denominator comes from $\frac{P(A \cap B)}{P(B)}$

$$P\left(x < \frac{7}{6} \,\middle|\, x < \frac{7}{2}\right) = \frac{\int_0^{\frac{7}{6}} \frac{2}{21} x \, dx}{\int_0^{\frac{7}{2}} \frac{2}{21} x \, dx}$$

$$= \frac{\frac{1}{21}\left[x^2\right]_0^{\frac{7}{6}}}{\frac{1}{21}\left[x^2\right]_0^{\frac{7}{2}}} = \frac{\frac{1}{21}\left\{\left(\frac{7}{6}\right)^2\right\}}{\frac{1}{21}\left\{\left(\frac{7}{2}\right)^2\right\}} = \frac{7^2}{6^2} \times \frac{2^2}{7^2} = \frac{2^2}{6^2}$$

$$= \frac{4}{36} = \frac{1}{9}$$

MATH - 215 10/10/22

Probability & Statistics

- variable
  - Discreet
    - Prob. distribution
      - PMF
      - CDF
    - types
      - Binomial
  - Continuous
    - Prob. distribution
      - PDF
      - CDF
    - types

Binomial properties:

(I) number of outcomes = 2 (success / fail
pass / fail
Head / Tail)

(II) Number of trials is fixed and independant
(example: coins are tossed <u>20 times</u>)

(III) P = probability of success / fail / points etc.

PMF $\Rightarrow P(X=x) = \frac{n!}{x!\,(n-x)!} p^x (1-p)^{n-x}$

Where, n = no. of trials

CDF : $P(X \leq x)$

Mean : np

Variance: $np(1-p) = npq$

Example:

① Tossing a coin 6 times. X = number of heads

(a) $P(X=2)$ [2 বার Head পাবো]

(b) At least 4 heads

(c) At most 1 head

(d) mean, variance & standard deviation.

(a): $n=6$, $p=\frac{1}{2}$, $x=2$

$\therefore P(X=2) = \frac{2!}{2!\,(6-2)!} \left(\frac{1}{2}\right)^2 \left[1-\left(\frac{1}{2}\right)\right]^{6-2}$

$= 0.234375$

(b): $P(x \geq 4) = P(X=4) + P(X=5) + P(X=6)$

$= 0.3437$

(c): $\underbrace{P(X \leq 1) = P(X=0) + P(X=1)}$

$= 0.109375.$

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(d): Mean $= 6 \times \frac{1}{2} = 3$

variance $= np(1-p)$

$= 6 \cdot \frac{1}{2}\left(1-\frac{1}{2}\right)$

$= 3\left(\frac{1}{2}\right) = \frac{3}{2}$

Standard deviation $= \sqrt{\text{variance}}$

$= \sqrt{\frac{3}{2}}$

④ A random variable X is binomially distributed with mean = 6, variance = 4·2. Find $P(X \leq 6)$.

Ans: $P(X \leq 6) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5) + P(X=6)$

mean = 6

$np = 6$

$\Rightarrow n = \frac{6}{p} = \frac{6}{0.3}$

$= 20$

$np(1-p) = 4.2$

$6(1-p) = 4.2$

$p = 0.3$

Math

Nafisa Mam

Syllabus

(I) Probability & conditional probability
(II) Probability distribution < Discreet / Continuous

Discrete distribution

→ Binomial coefficients
→ Poisson's ~~ratio~~ distribution

Poisson's distribution

An event occuring a certain numbers of times within a given interval of time.

$$PMF : P(x=k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

no. in a given time (1000 people in 1 min. $\therefore \lambda = 1000$)

~~lamba~~ lamda

example

accident : 2 per week

$P(X=0)$ in 1 week

$$= \frac{\lambda^k e^{-\lambda}}{k!} \quad \Big| \quad \lambda = 2$$

$$= \frac{2^0 e^{-2}}{0!}$$

$P(X=0)$ in 2 week

$$= \frac{\lambda^k e^{-\lambda}}{k!}$$

$$= \frac{4^0 e^{-4}}{0!}$$

A person receives average 3 emails per hour. Probability of receiving 5 email in 2 hr?

(a):

$$(X=5) = \frac{6^5 e^{-6}}{5!} \qquad \Big|\ \lambda = 6$$

(b): $P(x>2)$ over period of 2 hours.

$$P(x>2) = P(X=3) + P(X=4) + \dots + \infty$$

$$= 1 - P(X\leq 2)$$

$$= 1 - [P(X=0) + P(X=1) + P(X=2)]$$

$$P(X=0) = \frac{\lambda^k e^{-\lambda}}{k!}$$

$$= \frac{6^0 e^{-6}}{0!} = e^{-6}$$

$$P(X=1) = \frac{6^1 e^{-6}}{1!} = 6e^{-6}$$

$$P(X=2) = \frac{6^2 e^{-6}}{2!} = 18e^{-6}$$

In poisson's distribution:

mean $= \lambda$

variance $= \lambda$

* understand which distribution applies where *

Geometric Distribution

- doing something until we get the event I'm looking for.
- No. of trial is not fixed.
- Conduct as many trials as necessary until first success.

PMF: $P(X=x) = P(1-P)^{x-1}$ ← because we already have found success

P = probability of getting my success

x = no. of trials needed to get my success

CDF:

(1) $P(X \leq x) = 1 - (1-p)^x$

(2) $P(X > x) = 1 - P(X \leq x)$

(3) $P(X \geq x) = (1-p)^{x-1}$

(4) $P(X < x) = 1 - P(X \geq x)$