

# Rules of Natural Deduction

## Propositional Logic

<i>Connective or Quantifier</i>	<i>Introduction Rule(s)</i>	<i>Elimination Rule(s)</i>
$\wedge$	$\frac{\alpha \quad \beta}{(\alpha \wedge \beta)} \wedge i$	$\frac{(\alpha \wedge \beta)}{\alpha} \wedge e \quad \frac{(\alpha \wedge \beta)}{\beta} \wedge e$
$\vee$	$\frac{\alpha}{(\alpha \vee \beta)} \vee i \quad \frac{\alpha}{(\beta \vee \alpha)} \vee i$	$\frac{(\alpha \vee \beta) \quad \boxed{\begin{smallmatrix} \alpha \\ \vdots \\ \gamma \end{smallmatrix}} \quad \boxed{\begin{smallmatrix} \beta \\ \vdots \\ \gamma \end{smallmatrix}}}{\gamma} \vee e$
$\rightarrow$	$\frac{\boxed{\begin{smallmatrix} \alpha \\ \vdots \\ \beta \end{smallmatrix}}}{(\alpha \rightarrow \beta)} \rightarrow i$	$\frac{\alpha \quad (\alpha \rightarrow \beta)}{\beta} \rightarrow e$
$\neg$	$\frac{\boxed{\begin{smallmatrix} \alpha \\ \vdots \\ \perp \end{smallmatrix}}}{(\neg \alpha)} \neg i$	(same as $\perp i$ )
$\perp$	$\frac{\alpha \quad (\neg \alpha)}{\perp} \perp i$	$\frac{\perp}{\alpha} \perp e$
$\neg\neg$	(derived)	$\frac{(\neg(\neg \alpha))}{\alpha} \neg\neg e$

## Derived Rules

$\frac{(\alpha \rightarrow \beta) \quad (\neg \beta)}{(\neg \alpha)} \text{ MT}$	$\frac{\alpha}{(\neg(\neg \alpha))} \neg\neg i$
$\frac{\boxed{\begin{smallmatrix} (\neg \alpha) \\ \vdots \\ \perp \end{smallmatrix}}}{\alpha} \text{ PBC}$	$\frac{}{(\alpha \vee (\neg \alpha))} \text{ LEM}$