5 Assignment 5 (100 points)

5.1 Soundness and Completeness of Natural Deduction (20 points)

Prove or disprove the statement (α, β, γ are wff in propositional logic):

$$\text{If } \{\alpha, \beta\} \vdash_{ND} \gamma, \text{ then } \varnothing \vDash (\alpha \wedge \beta) \rightarrow \gamma$$

Hint: ND is sound.

5.2 CNF and Resolution (20 points)

You are in charge of scheduling meetings in a conference room. You have three potential meetings: A, B, and C, with the following constraints:

- If Meeting A is scheduled, then Meeting B must not be scheduled, and Meeting C must be scheduled.

- If Meeting C is not scheduled, then either Meeting A or Meeting B must be scheduled, but not both.

- If Meeting A is not scheduled, then Meeting C cannot be scheduled.

1. Formalize the problem using propositional logic. Then, convert your formalization into CNF (not principal CNF) using logical equivalence (show your steps).

2. Can you find a meeting schedule that satisfies all constraints? If yes, list all possible meeting schedules by converting the formula to principal CNF; If no, use Resolution to derive $\perp$.

5.3 Syntax (30 points)

Define the following symbols:

- Constants: a, b

- Functions: f^1, g^2

- Predicates: P^1, R^2, Q^3

Which of the following expressions are well-formed FOL formulas? In this question, a well-formed formula with parenthesis omitted by our convention can also be considered as a well-formed formula.

1. $Q(a)$

2. $P(y)$

3. $P(g(a, b))$

4. $R(\neg x, a)$

5. $Q(x, P(a), b)$

6. $P(g(f(a)), g(x, f(x))))$

7. $R(a, R(a, a))$

8. $R(a, g(a, x))$

9. $g(a, g(x, y))$

10. $\forall x(\neg P(x))$

11. $\exists R(f(a), x)$

12. $\exists a R(a, a)$

13. $\exists x Q(x, f(x), b) \rightarrow \forall x R(a, x)$

14. $\exists x \forall y R(x, y)$

15. $R(\forall x, a)$

5.4 Formalization (30 points)

Use FOL to formalize the following statements.

Note: If you use $S(x)$, you need to specify the meaning of it (e.g., "x is a student"); or, you could directly use a self-explanatory predicate name "Student(x)" without specifying the meaning.

1. All Students are smart.

2. Every course has at least one prerequisite course.

3. Some students registered for all courses.

4. No student is both a TA and a professor.

5. Only professors can access the restricted section of the library.

6. There is a professor who has never taught any course.

7. Every student loves some student.

8. Every student loves some other student.

9. There is a student who is loved by every other student.

10. Some students love only themselves.

11. There is at least one student.

12. There is only one student.

13. There are at least two students.

14. There are more than two students.

15. Exactly two students failed Geometry.