3 Assignment 3: Semantics of Propositional Logic (100 points)

For 3.1, 3.2, and 3.3, write down the answer and also briefly explain why.

3.1 For each formula, whether it is tautology, contradiction, or neither? (15 points)

- $(p \to r) \to (q \to (p \to r))$
- $(p \land \neg q) \lor (r \to (\neg p \lor q))$
- $(p \lor (q \to r)) \leftrightarrow (p \land (r \lor q))$
- $(p \to q) \land (p \land \neg q)$
- $(\neg (p \leftrightarrow r) \land (q \to s)) \lor (((p \leftrightarrow r) \lor (q \land \neg s)) \to p)$

3.2 Whether the following logical equivalences are correct? (15 points)

- $(p \land \neg p) \to (q \to r) \equiv p \lor \neg p \lor q \lor r$
- $(p \lor q) \land (\neg p \to \neg q) \equiv q$
- $(p \land q) \to r \equiv p \to (q \to r)$
- $\neg p \to \neg q \equiv p \to q$
- $p \land q \land (p \to (q \to r)) \land (p \to (q \to s)) \equiv r \leftrightarrow s$

3.3 Whether the following logical consequences are correct? (15 points)

- $p \to (q \to r) \models q \to (p \to r)$
- $(p \leftrightarrow q) \lor (r \leftrightarrow s) \models p \lor r$
- $(p \to q) \land (r \to q) \models (p \lor r) \to q$
- $p \to (q \to r) \models (q \land \neg r) \to \neg p$
- $(p \to q) \land q \models p$

3.4 Prove (15 points)

- $A \to B \not\models (A \lor C) \to B$
- $\not\models ((A \land B) \to C) \to (B \to C)$
- $A \to (B \to C) \not\models (B \lor C) \to A$

3.5 Logical Equivalence (30 points)

The two code have different syntax, but equivalent semantics.

Listing 1: Your code

```
if (i || !u) {
  if (!(u && q)) {
    P1
  } else if (u && !q) {
    P2
  } else { P3 }
} else { P4 }
```

Listing 2: Your friend's code

```
if ((i && u) && q) {
  P3
} else if (!i && u) {
  P4
} else {
  P1
}
```

To prove that, we have to show that for each code block, the conditions in Fragment 1 and Fragment 2 under which it is executed are logically equivalent.

Block	Fragment 1	Fragment 2
P_1	$(i \vee (\neg u)) \wedge (\neg(u \wedge q))$	$(\neg(i \wedge u \wedge q)) \wedge (\neg((\neg i) \wedge u))$
P_2	$(i \vee (\neg u)) \wedge (\neg(\neg(u \wedge q))) \wedge (u \wedge (\neg q))$	**F**
P_3	$(i \vee (\neg u)) \wedge (\neg(\neg(u \wedge q))) \wedge (\neg(u \wedge (\neg q)))$	$(i \wedge u \wedge q)$
P_4	$(\neg(i \vee (\neg u)))$	$(\neg(i \wedge u \wedge q)) \wedge ((\neg i) \wedge u)$

We have proved P4 in the lecture. Please complete the proof for P1, P2, and P3. Note that, instead of using truth tables, your proof should based on the logical identity laws (e.g., the De Morgan's laws) we've introduced in the lecture.

3.6 Adequate Sets (10 points)

Consider the binary connective, Sheffer stroke operation: $\uparrow$, which means "not both" (also known as the NAND operation). Its truth table is:

p	q	$p \uparrow q$
0	0	1
0	1	1
1	0	1
1	1	0

Proof that the set $\{\uparrow\}$ is adequate. (Hint: you may use the fact that $\{\neg, \wedge\}$ is an adequate set, and prove that $\neg$ and $\wedge$ are definable by $\uparrow$).