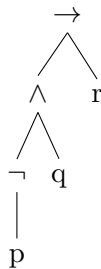# 2 Answers to Assignment 2: PL syntax

2.1 Which are well formed propositional formulas? (20 points)

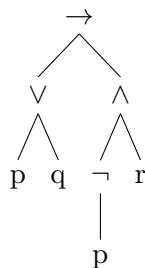3, 5, 9, 10

2.2 Parse Tree (20 points)
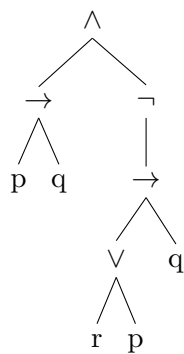
1.

```
          →
         / \
        ∧   r
       / \
      ¬   q
      |
      p
```

2.

```
          →
         / \
        ∨   ∧
       / \ / \
      p  q ¬  r
           |
           p
```

3.

```
          ∧
         / \
        →   ¬
       / \  |
      p  q  →
           / \
          ∨   q
         / \
        r   p
```
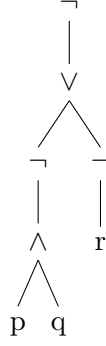
4.

¬
|
∨
/\
¬   ¬
| |
∧   r
/\
p q

2.3 Properties of well-formed formulas (40 points)

(1) Proof by contradition.

Suppose both UV and VW are formulas. V is a proper suffix of the formula UV. Given Lemma 3.3, V has less opening brackets than closing brackets.

V is a proper prefix of the formula VW. Given Lemma 3.3, V has more opening brackets than closing brackets.

Contradiction. UV and VW cannot both be formulas.

(2)

Base case: if α is atomic, then $s = 1$, $n = 0, m = 1$, which satisfies $s = 3n + m$. So the statement holds for the base case.

Inductive hypothesis: We assume α_1 and α_2 satisfy the statement. That is, for α_1

$$s_1 = 3n_1 + m_1,$$

for α_2

$$s_2 = 3n_2 + m_2.$$

Inductive step: If $\alpha = (\neg \alpha_1)$, we have

$$s = s_1 + 3 = 3n_1 + m_1 + 3$$

and

$$3n + m = 3(n_1 + 1) + m_1 = 3n_1 + m_1 + 3.$$

So, in this case $s = 3n + m$.

If $\alpha = (\alpha_1 * \alpha_2)$, we have

$$s = s_1 + s_2 + 3 = 3n_1 + 3n_2 + m_1 + m_2 + 3$$

and

$$3n + m = 3(n_1 + n_2 + 1) + m_1 + m_2 = 3n_1 + 3n_2 + m_1 + m_2 + 3.$$

So, in this case $s = 3n + m$.

By induction, the statement holds.

2.4 Formalization (20 points)

1. P: Prozac relieves depression; A: Allegra combats allergies; Z: Zocor lowers cholesterol

$$P \land (A \lor Z)$$

2. R: it rains tomorrow; T: temperature drops below freezing; I: the roads will be icy; H: the traffic will be heavy

$$R \land T \to I \land H$$

3. W: The team will win the match; S: the striker perform well; G: the goalkeeper perform well.

$$W \to S \land G$$

4. C: The computer will turn off; D: the battery is dead; U: the charger is unplugged.

$$C \leftrightarrow D \land U$$

5. P: the car is parked properly; T: the ticket inspector will issue a fine; C: the car will be towed.

$$\neg P \to T \lor C$$

6. H: The concert will happen; C: the weather is clear; S: the weather is stormy.

$$(C \to H) \land (S \to \neg H)$$

7. M: The movie screening will be held; P: the projector works; F: the film is available; R: the theater is under renovation

$$(M \leftrightarrow P \land F) \land (R \to \neg M)$$

8. L: The new phone will launch; D: design meets the company's standards; F: functionality meets the company's standards

$$D \land F \to L$$

9. A: The system will alert the user; S: the internet connection is slow; U: the software update is in progress.

$$(S \to A) \lor U$$

or

$$\neg U \rightarrow (S \rightarrow A)$$

10. P: the project is approved; B: the budget is approved by the committee; T: the timeline is approved by the committee; M: the marketing strategy approves the project

$$(B \wedge T \wedge (M \vee \neg M)) \rightarrow P$$

or

$$B \wedge T \rightarrow P$$