



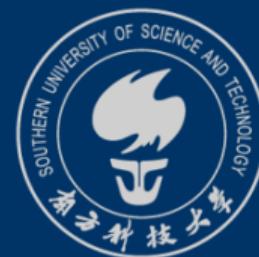
Introduction to Mathematical Logic

For CS Students

CS104

Yida TAO (陶伊达)

2024 年 2 月 20 日



南方科技大学



Course Info

Welcome!

- **Lectures:** Week 1-16
 - Wednesday 3-4, 一教 304 (CS108)
 - Wednesday 5-6, 一教 110 (CS104)
- **No Lab!**
- **Lecturer:** TAO Yida (陶伊达), taoyd@sustech.edu.cn
- **Teaching Assistants**
 - 陈杨 12331247@mail.sustech.edu.cn
 - 黄宇威 12332473@mail.sustech.edu.cn



Course Info

Welcome!

- **Course Site:** Blackboard
- **Course Group:** see announcement on Blackboard course site



Course Evaluation

Welcome!

- Class participation/Quizzes: 20%
- Assignments: 30%
- Exam: 50% (Midterm (negotiable) 20% + Final 30%)



Table of Contents

1 Warm Up

- ▶ Warm Up
- ▶ History
- ▶ Informal Reasoning
- ▶ Applications
- ▶ Course Overview



A Logic Puzzle

1 Warm Up

3 4 5

Zhang San, Li Si, and Wang Wu each have cat(s).

Among them, two people have white cats, and two people have black cats.

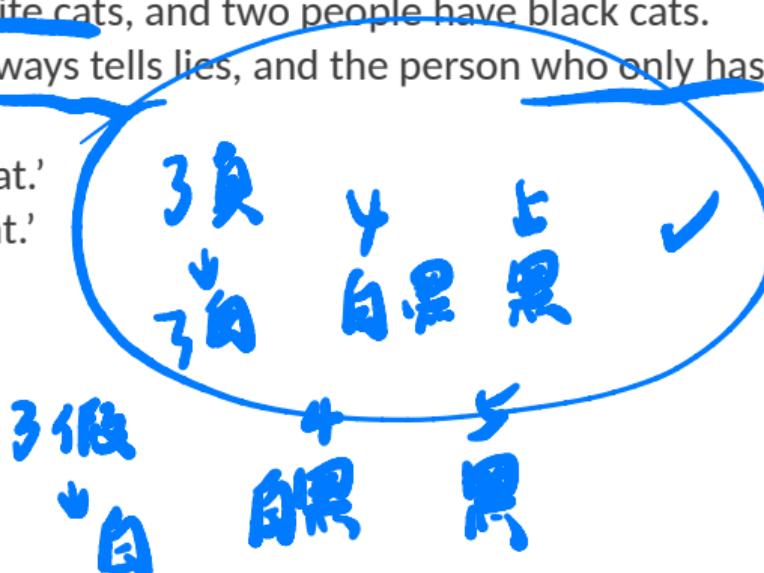
The person who has black cat(s) always tells lies, and the person who only has a white cat may not always tell the truth.

Zhang San says, 'Li Si has a white cat.'

Li Si says, 'Wang Wu has a white cat.'

Question: Who has what cat(s)?

Source: 微信公众号：数理逻辑与哲学逻辑





What is Logic?

1 Warm Up

Have you ever said to someone, "be logical"?

- Whatever your intuition was, that's logic



What is Logic?

1 Warm Up

- Logic is the study of reasoning (推理): the process of using reasons and critical thinking to analyze ideas, find connections, and draw conclusions.
- The basic question studied in logic: what conclusion(s) can be drawn with absolute certainty from a particular set of premises.

前提

从一组特支前提
引出肯定的结论

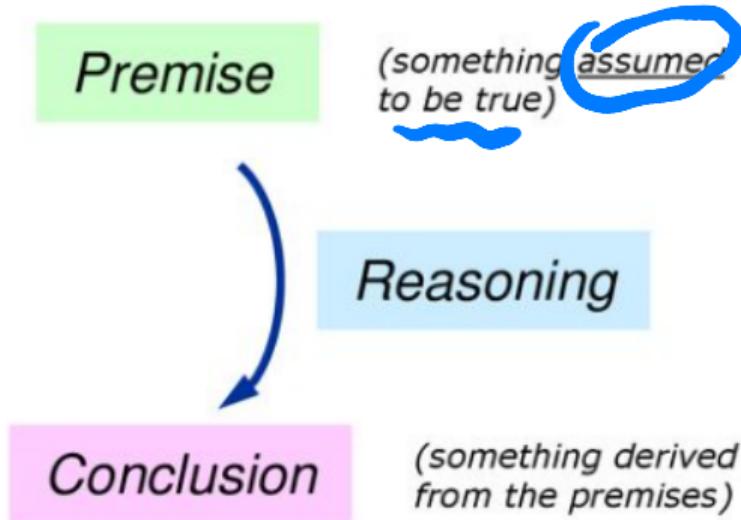


What is Logic?

1 Warm Up

- What entails what?

假設有必要



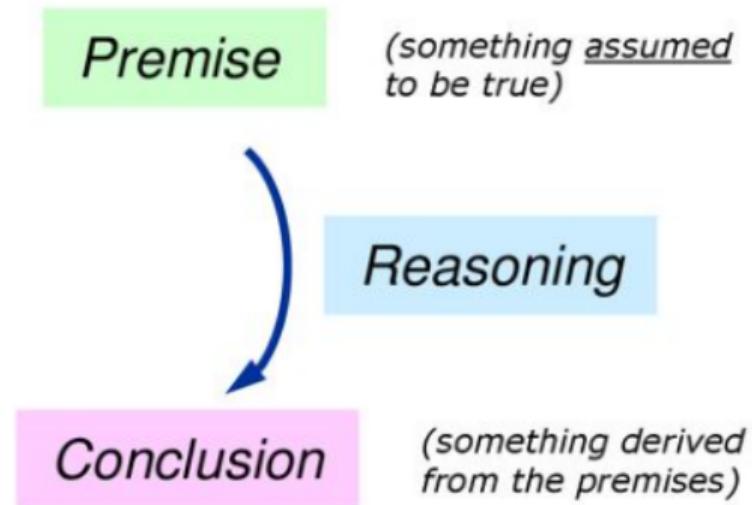
Conclusion/Premise: True/False (T/F)



What is Logic?

1 Warm Up

- What **entails** what?
- What **follows from** what?



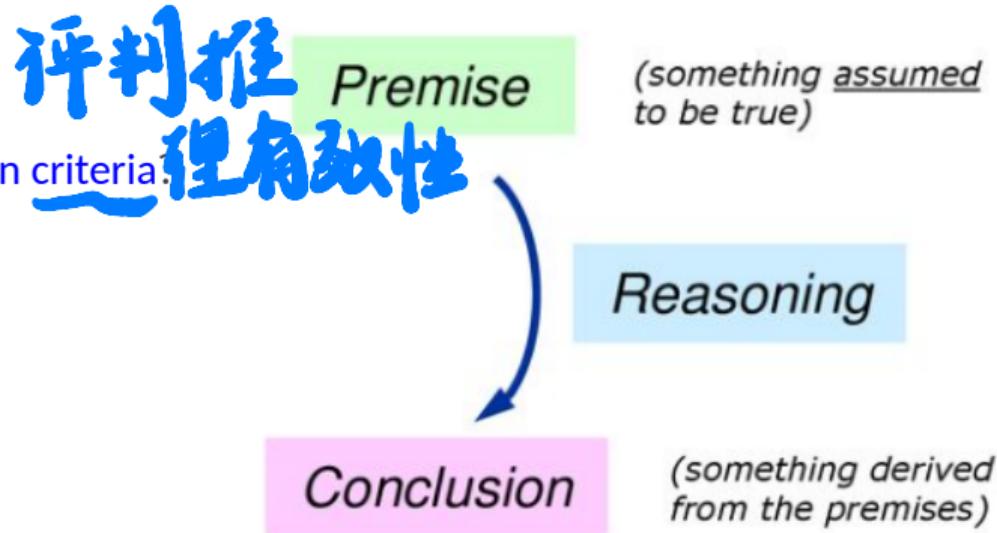
Conclusion/Premise: True/False (T/F)



What is Logic?

1 Warm Up

- What **entails** what?
- What **follows from** what?
- Why? What are the **evaluation criteria**?



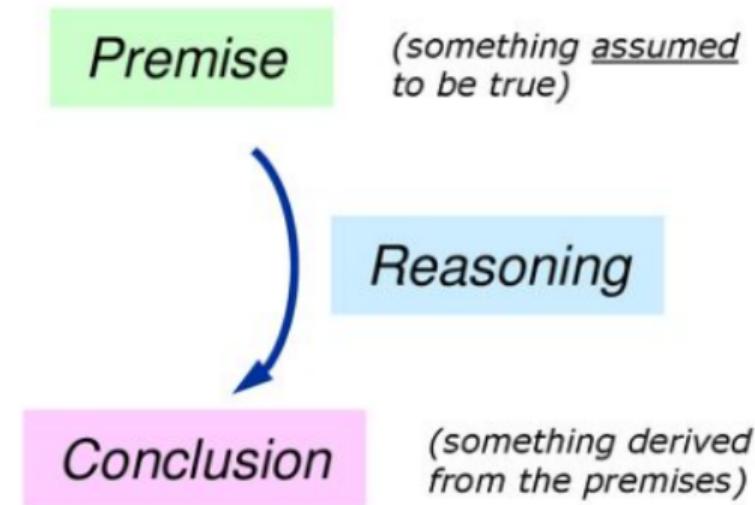
Conclusion/Premise: True/False (T/F)



What is Logic?

1 Warm Up

- What **entails** what?
- What **follows from** what?
- Why? What are the **evaluation criteria**?
- How to **establish/define** the evaluation criteria?



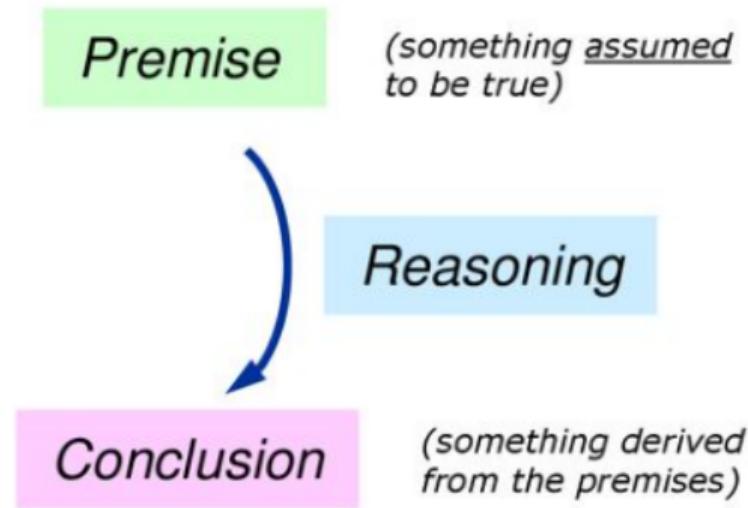
Conclusion/Premise: True/False (T/F)



What is Logic?

1 Warm Up

- What **entails** what?
- What **follows from** what?
- Why? What are the **evaluation criteria**?
- How to **establish/define** the evaluation criteria?
- How to evaluate **arguments/reasoning**?



Conclusion/Premise: True/False (T/F)



What is Logic?

1 Warm Up

In Math term:

- Premise is called Axiom (公理)
- Conclusion is called Theorem (定理), Lemma (引理)
- Reasoning is called Proof (证明)

不用证

证后成立

Premise

(something assumed to be true)

Reasoning

Conclusion

(something derived from the premises)



Conclusion/Premise: True/False (T/F)



Examples of Logic

1 Warm Up

《韩非子 · 难一》

楚人有鬻盾与矛者，誉之曰：“吾盾之坚，物莫能陷也。”又誉其矛曰：“吾矛之利，于物无不陷也。”或曰：“以子之矛陷子之盾，何如？”其人弗能应也。

"In the state of Chu, there was a person selling shields and spears. He praised the shields, saying, 'My shield is sturdy, and nothing can penetrate it.' He also praised his spear, saying, 'My spear is so sharp that it can pierce through anything.' Someone asked, 'What if you use your spear to pierce through your own shield?' The person could not provide an answer." - Translated by ChatGPT



Examples of Logic

1 Warm Up

亚里士多德,《形而上学》

同一个东西同时且在同一方面既属于又不属于同一个东西,这是不可能的。

Reductio ad absurdum (归谬法, Reduction to absurdity) : 如果你从一个主张推出了矛盾, 你就要否定这个主张 (A method of proving the falsity of a statement by assuming its truth and then deriving a contradiction.)。



Examples of Logic

1 Warm Up

"The sum of any number of even numbers is even."

- Empirical Method: Ancient Greek, using pebbles for arithmetic.
- Formal Proof. Assume the proposition (命题) is false, leading to a contradiction. Using Reductio ad absurdum to prove that the original proposition to be true.





Questions Arise.....

1 Warm Up

derive
deduct
infer
reason | 推理

- What is a "contradiction"? Why is a "contradiction" unacceptable?
- What does it mean for a deduction to "derive" a contradiction?
- Besides Reductio ad absurdum, what other rules constrain our reasoning?
- How do we know that a reasoning/deduction is valid?
- ...



What is Mathematical Logic?

1 Warm Up

数学化处理

直觉

- Mathematization/Formalization of the intuition is Mathematical Logic
- Using Mathematical logic, we could formally prove that a reasoning is valid, like we did with Math.
- Using Mathematical logic, even machines could perform reasoning, and validating reasoning.

用数学公式表达出来

就可“计算”命题是否正确

(比如让机器推理)



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Traditional Logic

2 History

- "Logic" originates from ancient greek
- The study of principles of reasoning; a part of philosophy
- Explicit analyses of the principles of reasoning was developed in three cultures:
China, India, and Greece.
- Modern treatment descends from the Greek tradition, particularly Aristotelian Logic,
core of which is the **syllogism** (三段论).



Syllogism (三段论)

2 History

- 大前提 (Major premise): 一般性原则, 描述一个大范围的情况或关系 (A broad statement or generalization)。
- 小前提 (Minor premise): 特殊陈述, 描述一个较小范围的情况或关系 (A specific statement related to or within the scope of the major premise)。
- 结论 (Conclusion): 基于主前提和副前提, 推导出一个逻辑上的结论 (A statement that logically follows from the combination of the major and minor premises)。



Syllogism (三段论)

2 History

Example I

- Major premise: All humans are mortal
- Minor premise: Socrates is a human
- Conclusion: Socrates is mortal

終悟死之的
valid

Example II

- Major premise: Metal conducts electricity.
- Minor premise: Copper is a metal.
- Conclusion: Copper conducts electricity.



The Validity of Reasoning

2 History

Example III

- Major premise: All humans are mortal
- Minor premise: Socrates is mortal
- Conclusion: Socrates is human

This argument has all true premises (and a true conclusion), but it is invalid.

兩前提与结论都对
但无法推断
(违反特设)



The Validity of Reasoning

2 History

Example III

- Major premise: All humans are mortal
- Minor premise: Socrates is mortal
- Conclusion: Socrates is human

Example IV

- Major premise: All humans are mortal
- Minor premise: My friend's dog is mortal.
- Conclusion: My friend's dog is human.



The Validity of Reasoning

2 History

命题

推理

Truth for statements, Validity for arguments/reasoning. They are not the same.

- Valid reasoning does not guarantee a true conclusion.
- Invalid reasoning does not guarantee a false conclusion.
- A false conclusion does not guarantee a invalidity.
- True premises and a true conclusion together do not guarantee a validity.



Ambiguity of Natural Languages

2 History

模棱两可

"natural language 一样"

Example V

- (Major Premise) A student from Class A is the captain of the soccer team .
- (Minor Premise) x knows a student from Class A .
- (Conclusion) x knows the captain of the soccer team.

但不一定指代同一个

不一样自然语言也可以
指同一个

自然语言可能模糊歧义
不准确

数学语言 ↵



Ambiguity of Natural Languages

2 History

- The similarity in natural language forms (a student from Class A) does not guarantee that the logical forms are identical.
- Different natural language forms (Example I and II, Example III and IV) may have the same logical form.



Early Phase

2 History

- 1710 年，莱布尼茨在《神正论》中提出建立一种普遍语言的设想，“这种语言是一种用来代替自然语言的人工语言，它通过字母和符号进行逻辑分析与综合，把一般逻辑推理的规则改变为演算规则，以便更精确更敏捷地进行推理”。
- In the early 18th century, Leibniz outlined his characteristica universalis, an artificial language in which grammatical and logical structure would coincide, allowing reasoning to be reduced to calculation.



Early Phase

2 History

"When there are disputes among persons, we can simply say, 'Let us calculate,' and without further ado, see who is right." —Gottfried Wilhelm Leibniz

Leibniz 将这种语言称为“人类思想的字母表”(alphabet of human thought)，认为在这种语言中，一切理性真理都会被还原为一种演算，所有推理的错误都只成为计算的错误。



Early Phase

2 History

Although Leibniz did not succeed in realizing it, this idea happens to be the essence of mathematical logic. The origins of modern mathematical logic are often attributed to Leibniz.

formal language (与 natural 相反)

- Constructing a "universal language" (通用语言, a universally applicable, precise scientific language).
- Constructing "calculus of reasoning" (推理演算, similar to a computational calculus, capable of proving correctness or incorrectness).



Development

2 History

- George Boole and A. De Morgan presented systematic mathematical treatments of propositional logic

命題邏輯



Development

2 History

- George Boole and A. De Morgan presented systematic mathematical treatments of propositional logic
- Gottlob Frege and B. Russel developed logic with quantifiers





Development

2 History

- George Boole and A. De Morgan presented systematic mathematical treatments of propositional logic
- Gottlob Frege and B. Russel developed logic with quantifiers
- Great Logicians: Bertrand Russell (罗素), David Hilbert(希尔伯特), Gerhard Gentzen(甘岑), Kurt Gödel(哥德尔), and Alfred Tarski(塔斯基)



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Terms

3 Informal Reasoning

1.1 No spear can pierce through this shield. Therefore, this spear cannot pierce through this shield.

1.2 If this spear can pierce through all shields, then it can pierce through this shield.
This spear cannot pierce through this shield. Therefore, it is not the case that this spear can pierce through all shields.

- A **reasoning** consists of (a finite number of) **declarative sentences**, where some sentences **infer** another sentence (indicated by the word "**therefore**").
- **Declarative sentences**: Used for declarations or statements, characterized by **being true or false** (not sentences posing questions, giving commands, or expressing emotions).

陈述句 (只有 T/F) 非问句/祈使句/感叹



Terms

3 Informal Reasoning

1.1 No spear can pierce through this shield. Therefore, this spear cannot pierce through this shield.

1.3 没有矛能刺破这张盾。所以，这支矛不能刺破这张盾。

1.4 All spears cannot pierce through this shield. Therefore, this spear cannot pierce through this shield.

- 1.1 and 1.3 have the same meaning (in different languages).
- 1.1 and 1.4 have the same meaning (different expressions in the same language).



Terms

3 Informal Reasoning

1.1 No spear can pierce through this shield. Therefore, this spear cannot pierce through this shield.

1.3 没有矛能刺破这张盾。所以，这支矛不能刺破这张盾。

1.4 All spears cannot pierce through this shield. Therefore, this spear cannot pierce through this shield.

- Reasoning concerns the abstract meanings of sentences, referred to as propositions(命题).



抽象 *formal meaning*



Terms

3 Informal Reasoning

1.1 No spear can pierce through this shield. Therefore, this spear cannot pierce through this shield.

1.3 没有矛能刺破这张盾。所以，这支矛不能刺破这张盾。

1.4 All spears cannot pierce through this shield. Therefore, this spear cannot pierce through this shield.

- Reasoning concerns the abstract meanings of sentences, referred to as **propositions**(命题).
- 1.1 and 1.3 consist of the same propositions, and the relationships between the propositions are the same, so it is the same reasoning.



Terms

3 Informal Reasoning

1.1 No spear can pierce through this shield. Therefore, this spear cannot pierce through this shield.

1.3 没有矛能刺破这张盾。所以，这支矛不能刺破这张盾。

1.4 All spears cannot pierce through this shield. Therefore, this spear cannot pierce through this shield.

Reasoning is a set of propositions. One of them (called the **conclusion**) is deduced/derived from others (called **premises**). We use **rules** for deduction/derivation.



Rules

3 Informal Reasoning

1.4 All spears cannot pierce through this shield. Therefore, this spear cannot pierce through this shield.

1.5 Everyone reasons. Han Fei is a person. Therefore, Han Fei also reasons.

1.6 Everything undergoes change. A table is something. Therefore, the table undergoes change.

1.7 All Greeks are philosophers. Aristotle is a Greek. Therefore, Aristotle is a philosopher.



Rules

3 Informal Reasoning

1.4 All spears cannot pierce through this shield. Therefore, this spear cannot pierce through this shield.

1.5 Everyone reasons. Han Fei is a person. Therefore, Han Fei also reasons.

1.6 Everything undergoes change. A table is something. Therefore, the table undergoes change.

1.7 All Greeks are philosophers. Aristotle is a Greek. Therefore, Aristotle is a philosopher.

Rule: All X is Y. a is X. Therefore, a is Y.





Rules

3 Informal Reasoning

If this spear can pierce through all shields, then this spear can pierce through this shield.

This spear can pierce through all shields.

Therefore, this spear can pierce through this shield.

Rule: If P then Q; P; Therefore Q



Rules

3 Informal Reasoning

If this spear can pierce through all shields, then this spear can pierce through this shield.

This spear cannot pierce through this shield.

Therefore, it is not the case that this spear can pierce through all shields.

Rule: If P then Q; Not Q; Therefore not P.

道否



Rules

3 Informal Reasoning

P

&

If this spear can pierce through all shields, then this spear can pierce through this shield.

not &

It is not the case that this spear can pierce through all shields.

Therefore, this spear cannot pierce through this shield.

not ♫



Rules

3 Informal Reasoning

If this spear can pierce through all shields, then this spear can pierce through this shield.

It is not the case that this spear can pierce through all shields.

Therefore, this spear cannot pierce through this shield.

Invalid Rule: If P then Q; Not P; Therefore not Q



What We'll Learn in This Course

3 Informal Reasoning

- How do we express ambiguous natural language reasoning into formulas?
- How do we prove that a reasoning is correct?



What We'll Learn in This Course

3 Informal Reasoning

- How do we express ambiguous natural language reasoning into formulas?
 - **Formal languages** (形式语言): a precisely defined set of symbols and syntax rules.
Using symbols and apply syntax rules to form formulas, where formulas represent propositions
- How do we prove that a reasoning is correct?

勾股



Formal Languages

3 Informal Reasoning

1. (Digital sequence understood by computer)

0010101010000010111101000

2. (Programme Language, eg. Java or C)

`s = 1; i = n; while (i > 0) { s *= a; i--; }`

3. (Propositional Logic)

$(\neg((p \vee q) \rightarrow p))$

4. (First-Order Logic)

$\forall \epsilon \exists \delta \forall x (|x - a| < \delta \rightarrow |f(x) - c| < \epsilon)$

5. (Modal Logic)

$\neg(\Diamond p) \leftrightarrow \Box(\neg p)$



What We'll Learn in This Course

3 Informal Reasoning

- How do we express ambiguous natural language reasoning into formulas?
 - **Formal languages** (形式语言): a precisely defined set of symbols and syntax rules. Using symbols and apply syntax rules to form formulas, where formulas represent propositions
- How do we prove that a reasoning is correct?
 - **Formal Deduction System** (形式推演系统): logical truth or falsity of propositions is expressed through deductive calculations based on the formulas and rules.



Example of Deduction - Propositional Logic

3 Informal Reasoning

- | | | |
|-----|--|---------------------------|
| 1. | $((p \rightarrow q) \wedge (\neg r \rightarrow \neg q))$ | Supposition |
| 2. | p | Supposition |
| 3. | $((p \rightarrow q) \wedge (\neg r \rightarrow \neg q))$ | 1 Reiterate |
| 4. | $(p \rightarrow q)$ | 3 Simplification |
| 5. | q | 2, 4 Modus Ponens |
| 6. | $(\neg r \rightarrow \neg q)$ | 3 Simplification |
| 7. | $\neg r$ | Supposition |
| 8. | $(\neg r \rightarrow \neg q)$ | 6 Reiterate |
| 9. | $\neg q$ | 7, 8 Modus Ponens |
| 10. | q | 5 Reiterate |
| 11. | r | 7–10 Reductio ad Absurdum |
| 12. | $p \rightarrow r$ | 2–11 Conditionalization |
| 13. | $((p \rightarrow q) \wedge (\neg r \rightarrow \neg q)) \rightarrow (p \rightarrow r)$ | 1–12 Conditionalization |



Course Part I: Propositional Logic

3 Informal Reasoning

- **Definition:** Propositional logic (命题逻辑) is a branch of mathematical logic that deals with the logical relationships between propositions, which are basic statements that can be either true or false.
- **What we'll learn**
 - Alphabet: Symbols and logical operators (AND, OR, NOT, etc.).
 - Syntax: What makes a valid formula?
 - Semantics: Truth values and meanings for propositions and formulas.
 - Deduction systems: rules for performing deduction.



Example of Deduction - First-Order Logic

3 Informal Reasoning

1	$\forall x \exists y (Px \wedge Qy)$	ass.
2	$\exists y (Pa \wedge Qy)$	1, ($\forall E$)
3	$Pa \wedge Qb$	ass.
4	Pa	3, ($\wedge E$)
5	Pa	3–4, ($\exists E$)
6	$\forall x Px$	5, ($\forall I$)
7	$Pa \wedge Qc$	ass.
8	Qc	7, ($\wedge E$)
9	$\exists x Qx$	8, ($\exists I$)
10	$\exists x Qx$	7–9, ($\exists E$)
11	$\forall x Px \wedge \exists x Qx$	6, 10, ($\wedge I$)
12	$\forall x \exists y (Px \wedge Qy) \rightarrow \forall x Px \wedge \exists x Qx$	1–11, ($\rightarrow I$)



Course Part II: First-Order Logic/Predicate Logic

3 Informal Focusing

两种语言 1 领逻辑 → 有界 symbols

- **Definition:** First-order logic (一阶逻辑) or Predicate logic (一阶谓词逻辑) extends propositional logic to include quantifiers and more complex structures, allowing for a more expressive representation of complex logical relationships.
- **What we'll learn**
 - Alphabet: Symbols, logical operators (AND, OR, NOT, etc.), predicates, constants, variables, and quantifiers.
 - Syntax: What makes a valid formula?
 - Semantics: Truth values and meanings for formulas.
 - Deduction systems: rules for performing deduction.



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Why should we study logic?

4 Applications

悖论

Logic is fun!

The Barber Paradox: a barber who shaves all and only those men in the town who do not shave themselves. Then, who shaves the barber?



Why should we study logic?

4 Applications

Logic is **fun!**

The Liar Paradox: This sentence is false.



Why should we study logic?

4 Applications

Logic is **fun!**

Find the true statement and get a A+ for this course!

The statement in
the right box is
true.

The statement in
the left box is
false.





Why should we study logic?

4 Applications

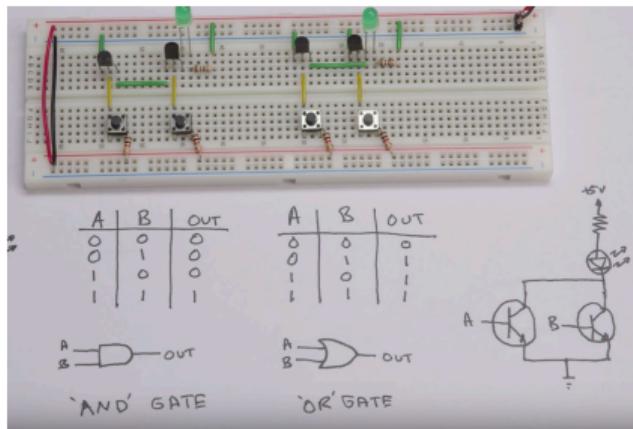
- Logic is fun!
- Logic improves one's ability to think analytically and to communicate precisely
- Logic has many applications in Computer Science.



Circuit Design

4 Applications

- Digital circuits are the basic building blocks of an electronic computer
- We can represent the inputs and outputs of the logic gate with symbols and boolean algebra (propositional logic). Subsequently, we verify the correctness of the circuit through methods such as truth tables.





Databases

4 Applications

Q: ?

- First-order logic is one of the theoretical foundations of [Relational Databases](#).
- A relational database implements a set of predicates to represent relations:
 - $\text{Major}(pid, cs)$: the student with PID pid has declared the major CS.
 - $\text{DeansList}(pid, sem)$: the student with PID pid made the Dean's List in semester sem.
- Properties of the database can be expressed in predicate logic
 - $\exists p(\text{DeansList}(p, 23Spring))$: Some student made the Dean's list in 23Spring.
 - $\forall p(\text{DeansList}(p, 23Spring) \rightarrow \exists m(\text{Major}(p, m)))$: Every student who made the Dean's list in 23Spring has declared some major.



Formal Verification

4 Applications

形式验证
bug-free

- Bugs can be **costly and dangerous** in real life
 - (Hardware bug) Intel's Pentium FDIV bug (1994) cost them half a billion dollars.
 - (Software bug) Cancer patients died due to severe overdose of radiation
- Mathematical logic can be used to prove that a program is **bug-free**.



Software Bug

4 Applications

一行代码导致B站崩溃了三小时。

近日，B站官方发布了一篇文章“2021.07.13 我们是这样崩溃的”，详细回顾了2021.07.13日B站崩溃事件。

B站同学从23: 25到23: 55一直尝试各种方式恢复服务，都未能达到预期效果，最后只能全部重建SLB集群。

从事件发生->初步定位->尝试修复->全部重建->临时解决->逐步恢复，时间从22: 52持续到01: 50，经历了大约三个小时的时间，被称为B站的至暗时刻。

有人就在此算了一笔账，称就是这7行代码，让B站老板一下亏了大约1, 5750, 0000元。



Software Bug

4 Applications

2021.07.13 我们是这样崩的

哔哩哔哩技术 + 关注

```
17 local _gcd
18 _gcd = function (a, b)
19     if b == 0 then
20         return a
21     end
22
23     return _gcd(b, a % b)
24 end
25
```

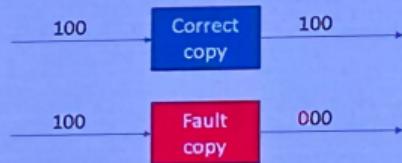
- Lua 是动态类型语言，常用习惯里变量不需要定义类型，只需要为变量赋值即可。
- Lua在对一个数字字符串进行算术操作时，会尝试将这个数字字符串转成一个数字。
- 在 Lua 语言中，如果执行数学运算 $n \leq 0$ ，则结果会变为 nan (Not A Number)。
- `_gcd`函数对入参没有做类型校验，允许参数`b`传入：“0”。同时因为“0” $\neq 0$ ，所以此函数第一次执行后返回是 `_gcd("0",nan)`。如果传入的是int 0，则会触发`[if b == 0]`分支逻辑判断，不会死循环。
- `_gcd("0",nan)`函数再次执行时返回值是 `_gcd(nan,nan)`，然后Nginx worker开始陷入死循环，进程 CPU 100%。



Software Bug

4 Applications

欧洲航天局/罗马大学 (Luca Mottala), 卫星软件容错机制
(2024年4月发射)





Artificial Intelligence

4 Applications

knowledge representation and reasoning

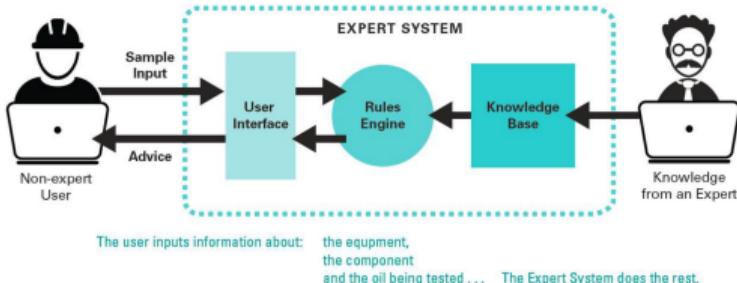
- AI systems (e.g., the expert system) utilize logic-based formalisms such as first-order logic and ontologies to represent and organize knowledge.
- These formal representations enable AI algorithms to reason about the relationships between entities, properties, and events, facilitating intelligent decision-making

A.I.B



Artificial Intelligence

4 Applications



Example: an expert system to assist in diagnosing user illnesses: the system represent the relationships between different symptoms and diseases through a set of symbols and rules based on the knowledge of doctors. This allows reasoning and diagnosis based on the symptoms provided by the patients.



Artificial Intelligence

4 Applications

Automated Discovery in Science

- Reasoning and inference systems have also been used to prove previously unproven results, rather than just simply check proofs produced by a human. One of the most impressive feats of machine reasoning occurred in 1996, when a **first-order logic reasoner was able to prove an open math conjecture that was unsolved for 60 years.**



Artificial Intelligence

4 Applications

Automated Discovery in Science

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- In 2007, a group in Wales and England created a system named Adam (Automated Discovery and Analysis Machine). Adam can automatically form scientific hypotheses, perform experiments to test hypotheses, and record results of experiments. Adam was the first automated system to discover non-trivial scientific information. **It successfully identified the function of some genes in yeast.**



Artificial Intelligence

4 Applications

Inductive Programming

- In inductive programming, the goal is to learn a computer program given a small set of input and output examples.



Artificial Intelligence

4 Applications

Inductive Programming

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- It is estimated that data scientists spend a large portion of their time writing programs for transforming data. Inductive programming methods have been used to learn data cleaning and transformation programs



Artificial Intelligence

4 Applications

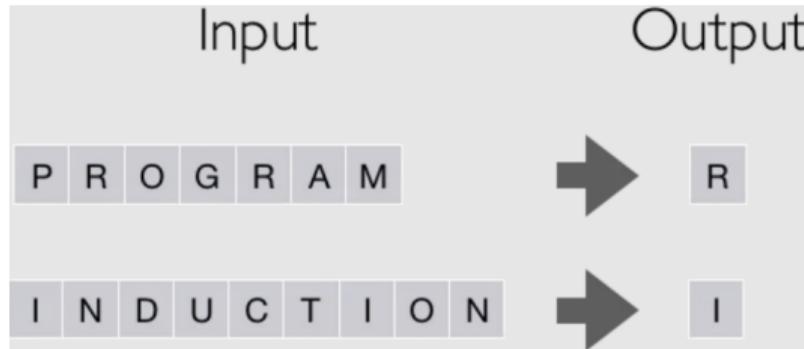
Inductive Programming

- In inductive programming, the goal is to learn a computer program given a small set of input and output examples.
- It is estimated that data scientists spend a large portion of their time writing programs for transforming data. Inductive programming methods have been used to learn data cleaning and transformation programs
- Super helpful for ML and AI that require large amount of high quality training data.



Artificial Intelligence

4 Applications



Goal: to learn a program that finds the first character in an input string that is duplicated.
Tools have been proposed to find an efficient and recursive **Prolog** program that can solve this task.¹

¹<https://medium.com/abacus-ai/an-overview-of-logic-in-ai-and-machine-learning-2f41ccb2a335>



Artificial Intelligence

4 Applications

program in logic

It was only in the early 1970's that the idea emerged to use the formal language of logic as a programming language. An example is PROLOG, which stands for PROgramming in LOGic. A logic program is simply a set of formulas (of a particular form) in the language of predicate logic. The formulas below constitute a logic program for kinship relations. The objects are people and there are two binary predicates 'parent of' (p), and 'grandparent of' (g).

- $A_1: p(\text{art}, \text{bob}).$
- $A_2: p(\text{art}, \text{bud}).$
- $A_3: p(\text{bob}, \text{cap}).$
- $A_4: p(\text{bud}, \text{coe}).$
- $A_5: g(x, z) :- p(x, y), p(y, z).$

syntax is different

$x \text{ is } y \text{ parent} \rightarrow x \text{ is } z \text{ grandparent}$

'art', 'bob', 'bud', 'cap' and 'coe' are individual constants and A_5 stands for $p(x, y) \wedge p(y, z) \rightarrow g(x, z)$.



Artificial Intelligence

4 Applications

Now if we ask the question

?- $g(\text{art}, \text{cap})$

the answer will be ‘yes’, corresponding with the fact that $g(\text{art}, \text{cap})$ can be logically deduced from the premisses or data A_1, \dots, A_5 .

But if we ask the question

?- $g(\text{art}, \text{amy})$

the answer will be ‘no’, corresponding with the fact that $g(\text{art}, \text{amy})$ cannot be logically deduced from A_1, \dots, A_5 . Note that this does not mean that $\neg g(\text{art}, \text{amy})$ logically follows from A_1, \dots, A_5 .



Table of Contents

5 Course Overview

- ▶ Warm Up
- ▶ History
- ▶ Informal Reasoning
- ▶ Applications
- ▶ Course Overview

universal language



Main Topics Covered (Negotiable)

5 Course Overview

- Preliminaries (预备知识)
- Propositional Logic (经典命题逻辑)
- First-order Logic (经典一阶逻辑)
- Program Verification (程序验证)

→ more powerful
application



Expected Output

5 Course Overview

- When encounter any problem, students can identify those logic related elements in the problem by applying related the logic knowledge and can avoid or minimize logical mistakes.
- Students can use classical Mathematical Logics (CML) to formally represent knowledge in empirical fields and construct formal theories for the empirical fields.
- Students can use CML (and automatic reasoning/proof tools, if possible) to solve reasoning/proof problems in empirical fields.
- Students can clearly identify those difficult issues in empirical field applications that are due to the limitations of CML.
- Students can further study various branches of modern logic and their applications based on knowledge and skills acquired this course.



Expected Output

5 Course Overview

- Thinking and communicating precisely
- Problem solving
- Critical thinking
- Creative thinking



Textbooks & References

5 Course Overview

- Text1: 《面向计算机科学的数理逻辑》(科学出版社) 陆钟万
- Text2: 《数理逻辑》(北京大学出版社) 邢滔滔
- Text3: 《数理逻辑证明及其限度》(复旦大学出版社) 郝兆宽等
- Video1: 《数理逻辑》在线课程, 哈尔滨工业大学任世军



Textbooks & References

5 Course Overview

- TextA: Introduction to Logic. 14ed. Copi Cohen. 2011
- TextB: Logic in Computer Science. Huth Ryan. 2004
- TextC: Philosophical and Mathematical Logic. H. Swart. 2018
- TextD: Logic and Proof. Jeremy Avigad, Robert Y. Lewis, and Floris van Doorn
- TextE: Mathematical Logic. Slaman Woodin. 2019.
- TextF: Mathematical Logic for Computer Science. Mordechai Ben-Ari
- TextG: Logic for Computer Scientists. Uwe Schöning
- TextH: A Mathematical Introduction to Logic. Herbert B. Enderton. 2001.
- TextI: A First Course in Logic. Shawn Hedman.

“very large”

different focus
in different
department



Readings

Finished in 2 weeks

- TextA: Chapter 1
- TextF: Chapter 1



Academic Integrity

5 Course Overview

- Although discussion with friends is allowed, assignments and quizzes should be done individually.
- If you get an idea for a solution from others or online resources, you must acknowledge the source in your submission.
- Any dishonest behavior or cheating will be dealt with seriously.



Academic Integrity

5 Course Overview

- If an undergraduate assignment is found to be plagiarized, the first time the score of the assignment will be 0.
- The second time the score of the course will be 0.
- If a student does not sign the Assignment Declaration Form or cheats in the course, including regular assignments, midterms, final exams, etc., in addition to the grade penalty, the student will not be allowed to enroll in the two CS majors through 1+3, and cannot receive any recommendation for postgraduate admission exam exemption and all other academic awards.



Assignment 0

5 Course Overview

Sign this form and upload it to Blackboard (Assignments -> Declaration Form).



南方科技大学
SOUTHERN UNIVERSITY OF SCIENCE AND TECHNOLOGY

| 计算机科学与工程系
Department of Computer Science and Engineering

Undergraduate Students Declaration Form

This is _____ (student ID: _____), who has enrolled in _____ course of the Department of Computer Science and Engineering. I have read and understood the regulations on courses according to "Regulations on Academic Misconduct in courses for Undergraduate Students in the SUSTech Department of Computer Science and Engineering". I promise that I will follow these regulations during the study of this course.



Introduction to Mathematical Logic

*Thank you for listening!
Any questions?*