3 Answers to Assignment 3: PL semantics

3.1 15 points

1. tautology 2. tautology 3. neither 4. contradiction 5. neither

3.2 15 points

1. correct 2. not correct 3. correct 4. not correct 5. not correct

3.3 15 points

1. correct 2. not correct 3. correct 4. correct 5. not correct

3.4 15 points

Examples of truth valuation v such that all premises are true and the conclusion is false. Listing only one example is enough.

- $A^v = 0, B^v = 0, C^v = 1$
- $A^v = 0, B^v = 1, C^v = 0$
- $A^v = 0, B^v = 1, C^v = 1$

3.5 30 points

P1 (10 points):
Fragment 1:

$$\begin{aligned}
&((i \vee (\neg u)) \wedge (\neg(u \wedge q))) \\
&\equiv ((i \vee (\neg u)) \wedge (\neg u \vee \neg q)) && \text{(De Morgan's Law)} \\
&\equiv \neg u \vee (i \wedge \neg q) && \text{(Distributive Law)}
\end{aligned}$$

Fragment 2:

$$\begin{aligned}
&(\neg(i \wedge u \wedge q)) \wedge (\neg((\neg i) \wedge u)) \\
&\equiv (\neg i \vee \neg u \vee \neg q) \wedge (i \vee \neg u) && \text{(De Morgan's Law)} \\
&\equiv \neg u \vee ((\neg i \vee \neg q) \wedge i) && \text{(Distributive Law)} \\
&\equiv \neg u \vee ((\neg i \wedge i) \vee (\neg q \wedge i)) && \text{(Distributive Law)} \\
&\equiv \neg u \vee (\text{False} \vee (\neg q \wedge i)) && \text{(Domination Laws)} \\
&\equiv \neg u \vee (\neg q \wedge i) && \text{(Identity Laws)} \\
&\equiv \neg u \vee (i \wedge \neg q) && \text{(Commutative Laws)}
\end{aligned}$$

P2 (10 points):

$$
\begin{aligned}
&(((i \vee (\neg u)) \wedge (\neg(\neg(u \wedge q)))) \wedge (u \wedge (\neg q))) \\
&\equiv (((i \vee (\neg u)) \wedge (u \wedge q)) \wedge (u \wedge (\neg q))) && \text{(Double Negation)} \\
&\equiv ((i \vee (\neg u)) \wedge ((u \wedge q) \wedge (u \wedge (\neg q)))) && \text{(Associativity)} \\
&\equiv ((i \vee (\neg u)) \wedge ((u \wedge u) \wedge (q \wedge (\neg q)))) && \text{(Commutative Law)} \\
&\equiv ((i \vee (\neg u)) \wedge (u \wedge \text{False})) && \text{(Idempotent Law, Negation Law)} \\
&\equiv ((i \vee (\neg u)) \wedge \text{False}) && \text{(Domination Law)} \\
&\equiv \text{False} && \text{(Domination Law)}
\end{aligned}
$$

P3 (10 points):

$$
\begin{aligned}
&((i \vee (\neg u)) \wedge (\neg(\neg(u \wedge q)))) \wedge (\neg(u \wedge (\neg q))) \\
&\equiv ((i \vee (\neg u)) \wedge (u \wedge q)) \wedge (\neg(u \wedge (\neg q))) && \text{(Double Negation)} \\
&\equiv ((i \vee (\neg u)) \wedge (u \wedge q)) \wedge (\neg u \vee q)) && \text{(De Morgan's Law)} \\
&\equiv (i \vee (\neg u)) \wedge ((u \wedge q) \wedge (\neg u \vee q))) && \text{(Associativity Law)} \\
&\equiv (i \vee (\neg u)) \wedge (u \wedge (q \wedge (\neg u \vee q))) && \text{(Associativity Law)} \\
&\equiv (i \vee (\neg u)) \wedge (u \wedge q) && \text{(Absorption Law)} \\
&\equiv ((i \vee (\neg u) \wedge u) \wedge q && \text{(Associativity Law)} \\
&\equiv ((i \wedge u) \vee (\neg u \wedge u)) \wedge q && \text{(Distributive Law)} \\
&\equiv ((i \wedge u) \vee False) \wedge q && \text{(Domination Law)} \\
&\equiv (i \wedge u) \wedge q && \text{(Identity Law)}
\end{aligned}
$$

3.6 10 points

Proof: By Lemma 5.2, $\{\neg, \wedge\}$ is an adequate set. So it suffices to show that $\neg$ and $\wedge$ are definable in terms of $\uparrow$.

Consider any well-formed formula $(\neg\alpha)$ and $(\alpha \wedge \beta)$, where α and β are well-formed.

$$
\begin{aligned}
(\neg\alpha) &\equiv (\neg(\alpha \wedge \alpha)) && \text{(Idempotent laws)} \\
&\equiv (\alpha \uparrow \alpha) && \text{(By definition of } \uparrow\text{)}
\end{aligned}
$$

$$
\begin{aligned}
(\alpha \wedge \beta) &\equiv (\neg(\neg(\alpha \wedge \beta))) && \text{(Double negation)} \\
&\equiv (\neg(\alpha \uparrow \beta)) && \text{(By definition of } \uparrow\text{)} \\
&\equiv ((\alpha \uparrow \beta) \uparrow (\alpha \uparrow \beta)) && \text{(By previous proof)}
\end{aligned}
$$

Hence, $\{\uparrow\}$ is adequate.