2   Assignment 2: Syntax of Propositional Logic (100 points)

2.1   Which of the following are well formed formulas? (20 points)

List the indices of all well-formed formulas by its formal definition, not by convention.

1. $\lor pq$

2. $(p \to \neg q)$

3. $(\neg(p \to (q \land p)))$

4. $(\neg(p \to (q = p)))$

5. $((p \land (\neg q)) \lor (q \to r))$

6. $p\neg r$

7. $()$

8. (p)

9. q

10. $(\neg(\neg p))$

2.2   Parse Tree (20 points)

Build the simplified parse tree of the following formulas. Note that brackets in the following formulas may be omitted by convention.

- $\neg p \land q \to r$

- $p \lor q \to \neg p \land r$

- $(p \to q) \land \neg(r \lor p \to q)$

- $(\neg((\neg(p \land q)) \lor (\neg r)))$

2.3   Properties of well-formed formulas (40 points)

(1) Let U, V, W be non-empty expressions in $\mathscr{L}^P$. Prove that UV and VW cannot both be formulas in $\mathscr{L}^P$.

(2) Let $A \in Form(\mathcal{L}^P)$. Assume that s is the total number of symbols (atoms, logical operators, and parentheses) in A, m is the number of atoms, and n is the total number of logical operators. Proof that

$$s = 3n + m.$$

2.4 Formalization (20 points)

Formalize the following sentences from natural language to propositional formulas. You should first formalize the atomic propositions in the sentence using propositional symbols, and then formalize the entire sentence using logical connective symbols. You may ignore brackets by convention.

1. Prozac relieves depression and either Allegra combats allergies or Zocor lowers cholesterol.

2. If it rains tomorrow and the temperature drops below freezing, then the roads will be icy and the traffic will be heavy.

3. The team will win the match only if both the striker and the goalkeeper perform well.

4. The computer will turn off if and only if the battery is dead and the charger is unplugged.

5. If the car is not parked properly, then either the ticket inspector will issue a fine or the car will be towed.

6. The concert will happen provided that the weather is clear, but if the weather is stormy, the event will be canceled.

7. The movie screening will be held if and only if the projector works and the film is available, but not if the theater is under renovation.

8. The new phone will launch if both the design and functionality meet the company's standards.

9. The system will alert the user if the internet connection is slow, unless the software update is in progress.

10. A sufficient condition for the project's approval is that both the budget and timeline are approved by the committee, but the approval of the marketing strategy is not essential.