1 Assignment 1: Preliminaries (100 points)

1.1 Set operations (10 points)

In a party, the total number of people with white or red shirts or black shoes is 21. In particular,

- 10 people have white shirts

- 8 people have red shirts

- 4 people have black shoes and white shirts

- 3 people have black shoes and red shirts

How many people have black shoes? Please model the problem using set and set operations, and write out the full problem-solving process instead of just providing the final answer.

1.2 Equivalence relation (20 points)

Define a relation $\sim$ on the set Z by:

$$x \sim y \Leftrightarrow x^2 - y^2 = 4k$$

for some $k \in Z$. Please:

- Prove $\sim$ is an equivalence relation.

- Describe its equivalence classes and explain why.

1.3 Equivalence class (10 points)

If R is defined on the set $\{1, 5, 12, 22, 35, 41, 55\}$, such that:

$$aRb \Leftrightarrow a \equiv b \pmod{7}$$

Please find all the equivalence classes.

1.4 Partial order and total order relation (30 points)

Whether R is a partial order/total order relation? If yes, show your proof; if no, explain why.
a) R is a relation on the set of positive integers $\mathbb{Z}^+$, defined by

$$R = \{\, (x, y) \mid \exists\, z \in \mathbb{Z}^+,\ x = y^z \,\}.$$

b) R is a relation defined on the set of natural numbers $\mathbb{N}$:

$$aRb \Leftrightarrow \frac{b}{a} = 2^k, k = \{0, 1, 2...\}$$

1.5 One-to-one and onto functions (20 points)

Determine whether the following function is one-to-one/onto? Explain why.

a) The function $f : \mathbb{Z} \to \mathbb{Z}$ is defined by

$$f(x) = \begin{cases} 2x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$$

b) The function $f : \mathbb{Z} \to \mathbb{Z}$ is defined by:

$$f(x) = x^3 + x$$

1.6 Proof by induction (10 points)

Prove by induction that $1^2 + 2^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}$ for every positive integer n.