

## 4 Assignment 4: Formal Proof Systems of PL (100 points)

**Important:** Each step in your proof must be numbered and accompanied by the rule used, just as we demonstrated in class. Points will be deducted for incorrect formatting.

You can use the theorems and derived rules that we've covered in class.

You CANNOT use truth tables or logical equivalence ( $\equiv$ ) for the proofs, which should be purely syntactic.

### 4.1 The Hilbert-style proof system (40 points)

Use the Hilbert-style proof system to prove the following :

1.  $\vdash (\neg A \rightarrow A) \rightarrow A$
2.  $\vdash (\neg A \rightarrow \text{false}) \rightarrow A$  (You can't use Reductio ad absurdum directly)
3.  $\vdash (A \rightarrow B) \rightarrow ((\neg A \rightarrow B) \rightarrow B)$  (you may use the previous answers directly as Theorems)
4.  $\{\neg A\} \vdash (\neg B \rightarrow A) \rightarrow B$

### 4.2 ND proof system (50 points)

Use the Natural Deduction proof system to prove the following. **Subproofs must be boxed; otherwise, they will be considered as incorrect proofs.**

1.  $\neg(\neg p \vee q) \vdash p$
2.  $(p \vee q) \vee r \vdash p \vee (q \vee r)$
3.  $p \rightarrow (q \rightarrow r) \vdash (p \rightarrow q) \rightarrow (p \rightarrow r)$
4.  $\neg(p \vee q) \vdash \neg p \wedge \neg q$
5.  $\neg p \wedge \neg q \vdash \neg(p \vee q)$

### 4.3 Formalization (10 points)

If the train arrives late and there are no taxis at the station, then John is late for his meeting. John is not late for his meeting. The train did arrive late. Therefore, there were taxis at the station.

- Formalize this problem into a set of premises and a conclusion.
- Prove the validity of this argument using the ND proof system.