

Exercise sheet 13

Question 13.1

1. for every edge is checked for once at most.

$$\Rightarrow O(|E|)$$

2. apart from checking every edge for once at most, we need to initialize an array `inDegree` to store the in-degree for every vertex.

$$\Rightarrow O(|E| + |V|)$$

3. check the column of vertex v in matrix

$$\Rightarrow O(|V|)$$

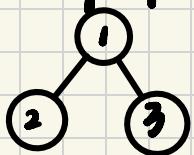
4. check every element of the matrix

$$\Rightarrow O(|V|^2)$$

Question 13.2

The strategy is not good, for it will lead to large waste.
It doesn't always produce an optimal solution.

example :



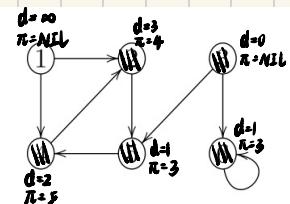
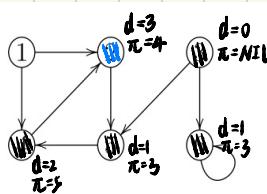
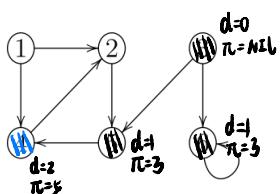
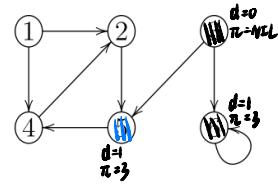
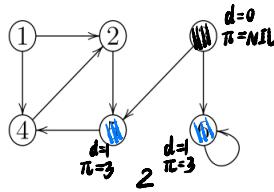
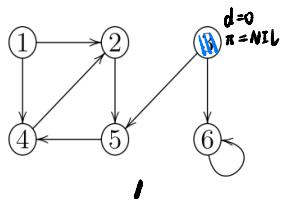
from the strategy given: put in 1.2;
however, the optimal choice is to put in 1).

The strategy doesn't come close, for it ignores the overlap where a vertex can monitor multiple edges.

Optimal strategy: Start with all edges marked as unmonitored, select the vertex covering the maximum number of unmonitored edges, put camera there and mark the roads covered as monitored. Repeat the last step until all roads are monitored.

Question 13.3

blue \rightarrow grey



Question 13.4

The function stays the same and the algorithm can still run normally.

Question 13.5

in BFS's while loop, every time after DEQUEUE, we scan every vertex u to find whether v to u has a edge it take $O(|V|)$ for every vertex. And before while we still do $O(|V|)$
 So the total time: $O(|V| \times |V| + |V|) = O(|V|^2)$