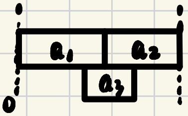
Exercise sheet 12
12310520 芮堤涵

Question 12.1

1. No




the optimal solution is $\{a_1, a_2\}$
but the strategy choose $\{a_3\}$

2. Yes. proof: Consider any nonempty subproblem $S_k$.

Let $A_k$ be a maximum-size subset of mutually compatible activities in $S_k$, and let $a_m$ be an activity in $S_k$ that overlaps with fewest remaining activities. $a_m$ is the greedy choice. And let $a_j$ be the activity that overlaps with fewest remaining activities in $A_k$

so: $a_m$ has no more overlapping activities than $a_j$

And we try to prove. there is a maximum-size compatible subset that includes $a_m$.

If $A_k$ includes the greedy choice $a_m$ ($a_j = a_m$) we're done

Otherwise, if $a_m$ and $a_j$ have same numbers of overlapping activities, we need to proceed with its subproblem. (As what we do in assumption)

if $a_m$ has fewer overlapping activities than $a_j$, then the subproblem of choosing $a_m$ substracts fewer overlapping activities than the subproblem of choosing $a_j$.

Thus, the greedy choice strategy holds.

3. Yes, proof: consider any nonempty subproblem $S_k$, and let $a_m$ be an activity in $S_k$ with the latest start time.

$a_m$ is the greedy choice. And let $a_j$ be the last-starting activity selected in $A_k$, so $S_m \geqslant S_j$.

To prove: there is a maximum-size compatible subset that include $a_m$.

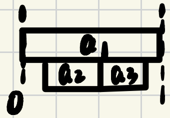
if $A_k$ includes the greedy choice $a_m$ ($a_j = a_m$), we're done.

Otherwise, let's swap $a_j$ for greedy choice $a_m$:

$A_k' = A_k \setminus \{a_j\} \cup \{a_m\}$, since $S_m \geqslant S_j$ and $a_j$ is last-starting, no incompatibilities are created.

Since all activities in $A_k$ were compatible, they are compatible in $A_k'$. As $|A_k'| = |A_k|$, $A_k'$ is a maximum-size subset of compatible activities.

4. No,




the optimal solution is $\{a_2, a_3\}$
but the strategy choose $\{a_1\}$

Question 12.2

n terms, the ith item is worth $v_i$ Yuan and weighs $w_i$ grams (all integers), can carry at most W grams in his knapsack.

Strategy: calculate $a_i = \frac{v_i}{w_i}$, and add $w_i$ number of $a_i$ in the list L; And sort the L in descending order, pick the first W numbers of the elements in L; and if the number of elements of L is smaller than W, pick all elements in L. And the number of $a_i$ picked means that you pick how many grams of ith-item.

Proof: Consider any nonempty subproblem $S_k$, let $a_m$ be the largest element in L left. $a_m$ is the greedy choice. Let $A_k$ be a maximum-value subset that can be picked out, $a_j$ be the maximum element in $A_k$. so $a_j \leq a_m$.

If $A_k$ include all $a_m$ ( $a_j = a_m$ ) we're done

Otherwise, swap $a_j$ for $a_m$ : $A_k' = A_k \setminus \{a_j\} \cup \{a_m\}$

$\Rightarrow$ the value in $A_k'$ is no smaller that $A_k$

$\Rightarrow$ the greedy choice strategy holds.

Question 12.3

(a) Start from $s_1$ with feed.

At $s_i$ if supplies is not enough for $s_i$ to $s_{i+1}$, then feed at $s_i$ ($i = 2, 3, \cdots n-1$)

Repeat the process until reaching $s_n$.

(b) Get supplies only when it can't get to next station. Get supply in advance won't reduce the number of stops, and it will make the next supply in advance.

And we can divide into subproblems,

suppose the strategy has determined the minimal stops to $s_i$, then determing the minimal stops from $s_i$ to $s_n$ only depends on the supply left in $s_i$.

With the greedy choice, we ensure largest supply left in $s_i$ on condition of minimal stops before.

(Otherwise we need to stop once more or get less supply)