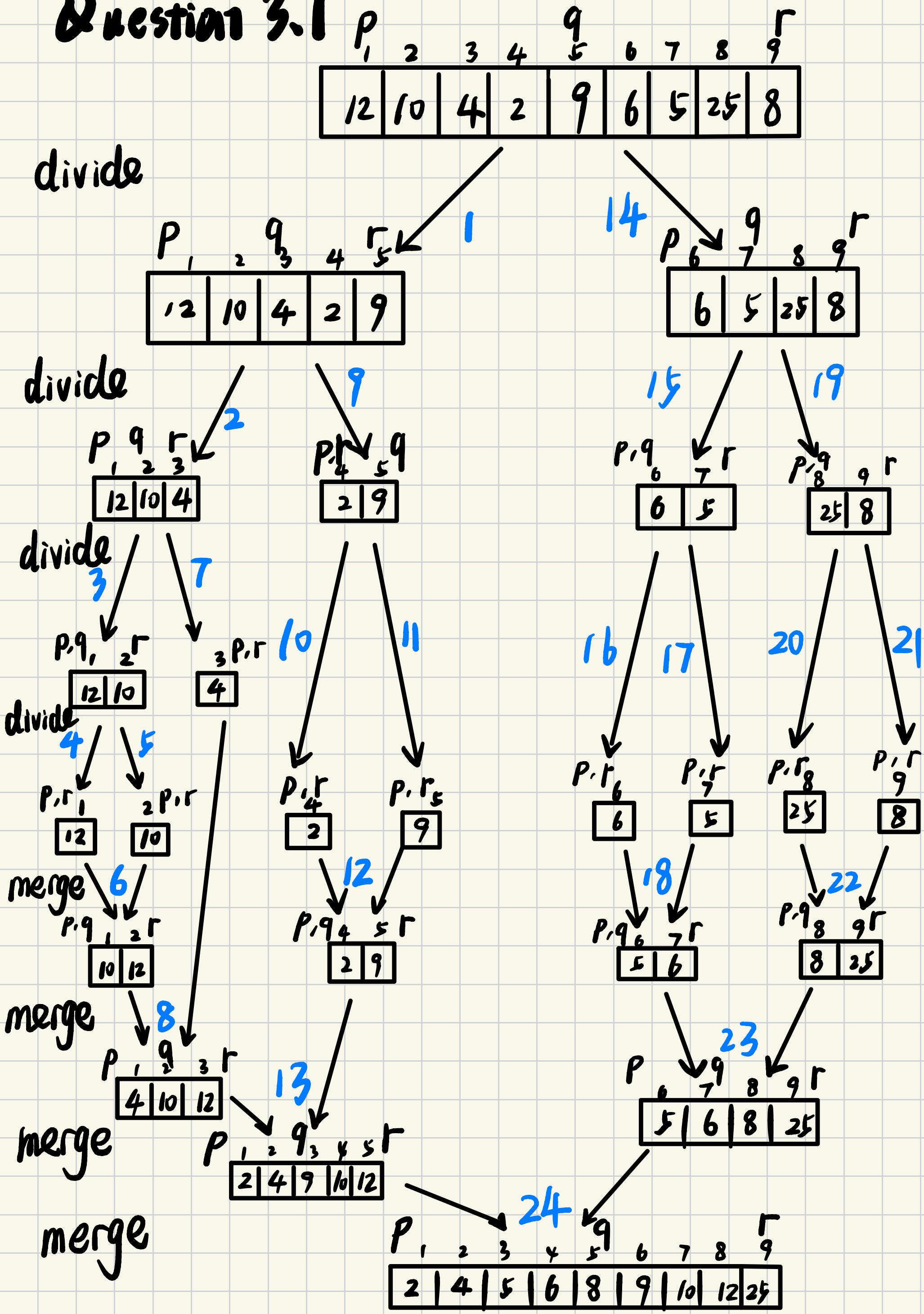
Question 3.1



p q r
1 2 3 4 5 6 7 8 9
12 10 4 2 9 6 5 25 8
divide
1
14
12 10 4 2 9
6 5 25 8
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2
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15
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12 10 4
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divide
4
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merge
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18
22
10 12
2 9
5 6
8 25
merge
8
23
4 10 12
5 6 8 25
merge
13
2 4 9 10 12
merge
24
2 4 5 6 8 9 10 12 25

Question 3.2

Assume that $n = 2^k$, then

$$T(2^k) = \begin{cases} d & \text{if } k=0 \\ 2T(2^{k-1}) + c \cdot 2^k & \text{if } k \geq 1 \end{cases}$$

(1)
Base case: $k=0 \Rightarrow n=1$ $T(n) = d + c\log 1$
$= d$ it holds

(2)
Inductive step: Assume true for $k=a$,
then $T(2^a) = d2^a + c \cdot 2^a \cdot a$,
and calculate $T(2^{a+1}) = 2T(\frac{2^{a+1}}{2}) + c \cdot 2^{a+1}$

$$= 2(d2^a + c \cdot 2^a \cdot a) + c \cdot 2^{a+1}$$
$$= d2^{a+1} + c \cdot 2^{a+1}(a+1)$$

thus, it holds for $k = a+1$
then true for all $k \geq a$

(3) Therefore, for base case and by induction
we can prove for all $k \geq 0$ $T(2^k) = d2^k + c2^k \cdot k$
$n = 2^k \Rightarrow T(n) = dn + cn\log n$

Question 3.3

T1. $a=2$ $b=4$ $f(n)=1$

$$n^{\log_b a} = n^{\frac{1}{2}}$$

$$f(n) = O(n^{\frac{1}{2}-\varepsilon}) \text{ for any } \varepsilon \in (0, \tfrac{1}{2})$$

the watershed function grows polynomially faster than $f(n)$

then $T(n) = \Theta(n^{\frac{1}{2}})$

T2. $a=2$ $b=4$ $f(n)=\sqrt{n}$

$$n^{\log_b a} = n^{\frac{1}{2}}$$

let $k=0$ $f(n) = \Theta(n^{\frac{1}{2}} \lg^k n)$

watershed and driving functions grow asymptotically nearly at the same rate

then $T(n) = \Theta(n^{\frac{1}{2}} \lg n)$

T3. $a=2$ $b=4$ $f(n)=\sqrt{n}\log^2 n$

$n^{\log_b a} = n^{\frac{1}{2}}$

let $k=2$, $f(n) = \Theta(n^{\frac{1}{2}}\lg^k n)$

watershed and driving function grows asymptotically nearly at the same rate

then $T(n) = \Theta(n^{\frac{1}{2}}\lg^3 n)$

T4、$a=2$ $b=4$ $f(n)=n$

$n^{\log_b a} = n^{\frac{1}{2}}$

$f(n) = \Omega(n^{\frac{1}{2}+\varepsilon})$ for any $\varepsilon \in (0, \frac{1}{2})$

and $2f(n/4) \le cf(n)$ for any $c \in [\frac{1}{2}, 1)$

the watershed function grows polynomially slower than $f(n)$

then $T(n) = \Theta(n)$

Question 3.4

```
BINARYSEARCH(A, x, low, high)
    if low > high then                      Θ(1)
        return -1                           Θ(1)
    m = ⌊(low + high)/2⌋                    Θ(1)
    if A[m] = x then                        Θ(1)
        return m                            Θ(1)
    else if A[m] < x then                   Θ(1)
        BINARYSEARCH(A, x, m+1, high)       T(n/2)
    else then                               Θ(1)
        BINARYSEARCH(A, x, low, m-1)        T(n/2)
```

$$T(n) = \begin{cases} \Theta(1) & \text{if } n = 1 \\ T(n/2) + \Theta(1) & \text{if } n > 1 \end{cases}$$

$$\Rightarrow T(n) = T(n/2) + \Theta(1)$$

$a=1 \quad b=2 \quad f(n)=\Theta(1)$

$n^{\log_b a} = n^0 = 1$

let $k=0$ $\quad f(n)=\Theta(n^0 \lg^k n)$

then $\quad T(n)=\Theta(\lg n)$