## **Computer Vision**

CS308
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SUSTech CS Vision Intelligence and Perception
Week 2





- Geometric primitives and transformations
- Projections
- Photometric image formation
- The digital camera



#### **Image Formation**



3D geometric primitives to 2D geometric primitives



# Components of the Image Formation Process

• Image formation process: 3D (real-world) to 2D (matrix)

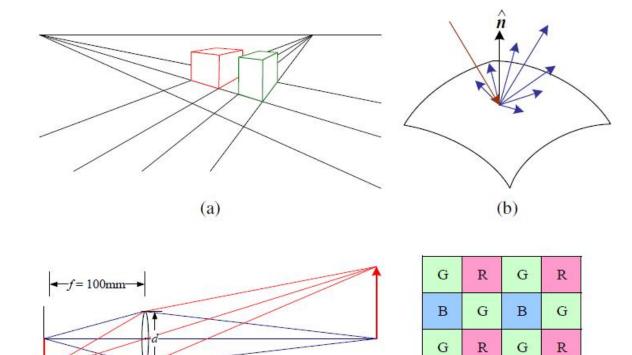
 $-z_i = 102 \text{mm} -$ 

(c)

#### > (a) Dananactive

这张图片展示了**图像形成过程的组成部分**,其中讲述了如何将三维世界(3D)转化为二维矩阵(2D)。具体的内容如下:

- 1. 透视投影 (Perspective projection) (a):
  - 这是将三维场景映射到二维平面上的一种方法。图中显示了一个立方体,利用 透视投影将其从不同角度投射到二维图像上。透视投影会模拟物体在远离观察 者时看起来变小的效果。
- 2. 光线在表面上的散射 (Light scattering when hitting a surface) (b):
  - 这一部分描述了光线与表面相互作用时的散射现象。当光线碰到表面时,它会在不同的方向上散射,影响最终图像中的亮度和颜色。图中的箭头表示光线的不同散射方向。
- 3. 镜头光学 (Lens optics) (c):
  - 这部分展示了镜头的光学原理,光线通过镜头时如何聚焦到成像平面上。图示中,红色和蓝色的光线分别表示进入镜头的不同光束,展示了聚焦的过程,以及镜头的焦距和物距。
- 4. 拜尔色彩滤波阵列 (Bayer color filter array) (d):
  - 这一部分描述了相机传感器的拜尔色彩滤波阵列。拜尔阵列是常用于数字图像传感器中的颜色滤波模式,它通过红色、绿色和蓝色的滤光片来捕捉图像中的不同颜色。图中展示了一个典型的拜尔阵列的布局,其中包含红色(R)、绿色(G)和蓝色(B)的滤光片。 (→)



В

G

(d)

G

# Geometric primitives and transformations



#### Geometric Primitives

2D points

$$x=(x,y)\in \mathcal{R}^2$$

 $x = \begin{bmatrix} x \\ y \end{bmatrix}$ 

> Homogeneous coordinates

齐次坐标是计算机图形学中常用的一种坐标系统,它通过引入额外的坐标来表示点,以便 能够处理仿射变换和透视变换。

$$ilde{m{x}} = ( ilde{x}, ilde{y}, ilde{w}) \in \mathcal{P}^2$$

w^~是缩放因子。

> Augmented vector

$$\bar{x} = (x, y, 1)$$

增强向量:在原始向量基础上,末尾增添一个额外的分

量;变成齐次坐标方便做集合变换

> Relationship

$$\tilde{x} = (\tilde{x}, \tilde{y}, \tilde{w}) = \tilde{w}(x, y, 1) = \tilde{w}\bar{x},$$

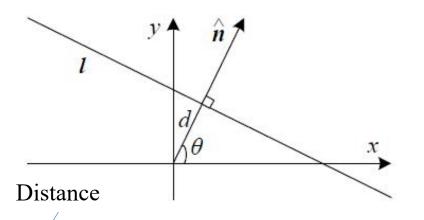
增强向量的一个关键优势是它使得仿射变换能够通过矩阵乘法来统一表示

#### Geometric Primitives

x^-是点(x,y,1)

• 2D lines

$$\bar{x} \cdot \tilde{l} = \underline{ax + by + c} = 0$$
 $\tilde{l} = (a, b, c)$  Direction



- > Polar coordinates
- $l = (\hat{n}_x, \hat{n}_y, d) = (\hat{n}, d)$ 
  - √ The direction (normal vector) is a function of a rotation angle
- Advantageous

- $\hat{n} = (\hat{n}_x, \hat{n}_y) = (\cos \theta, \sin \theta)$
- > Intersection of two lines
- Line joining two points

$$ilde{oldsymbol{x}} = ilde{oldsymbol{l}}_1 imes ilde{oldsymbol{l}}_2 \qquad ilde{oldsymbol{l}} = ilde{oldsymbol{x}}_1 imes ilde{oldsymbol{x}}_2$$

$$ilde{l} = ilde{x}_1 imes ilde{x}_2$$



#### Geometric Primitives

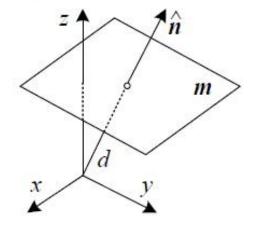
• 3D points

$$oldsymbol{x} = (x,y,z) \in \mathcal{R}^3 \quad ilde{oldsymbol{x}} = ( ilde{x}, ilde{y}, ilde{z}, ilde{w}) \in \mathcal{P}^3$$

$$\bar{x} = (x, y, z, 1)$$
  $\tilde{x} = \tilde{w}\bar{x}$ 

• 3D planes

$$\bar{x} \cdot \tilde{m} = ax + by + cz + d = 0$$
$$m = (\hat{n}_x, \hat{n}_y, \hat{n}_z, d) = (\hat{n}, d)$$

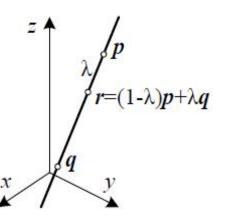


> The direction (normal vector) is a function of two rotation angles

$$\hat{n} = (\cos\theta\cos\phi, \sin\theta\cos\phi, \sin\phi)$$

• 3D lines

$$r = (1 - \lambda)p + \lambda q$$





#### **Transformations**

2D transformations

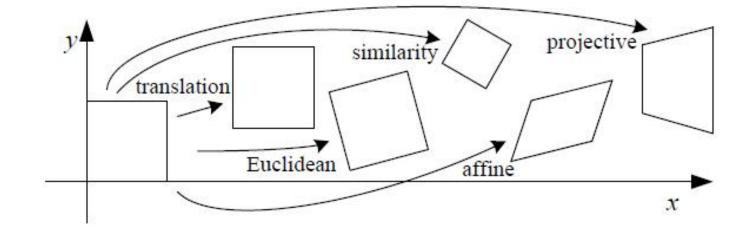
Distributions 
$$ar{x}' = ar{x} + ar{t} = egin{bmatrix} I & t \ \end{bmatrix} ar{x} \qquad ar{x}' = egin{bmatrix} I & t \ 0^T & 1 \end{bmatrix} ar{x}$$

把这个x变成3\*1矩阵,最后一行补上1

Rotation + translation

$$R = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

$$oldsymbol{x}' = oldsymbol{R} oldsymbol{x} + oldsymbol{t} \ = \left[ egin{array}{cc} oldsymbol{R} & oldsymbol{t} \end{array} 
ight] ar{oldsymbol{x}}$$





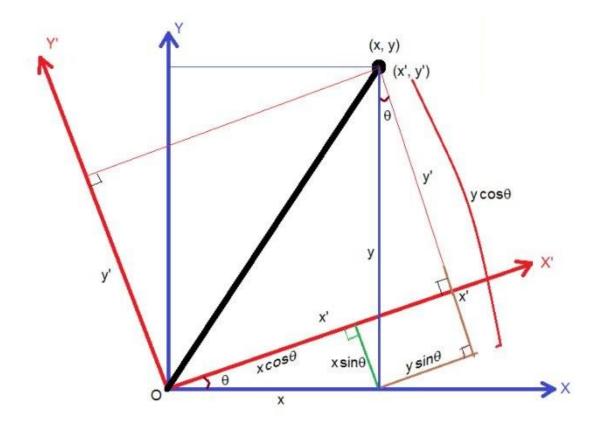
#### **Transformations**

#### Rotation matrix

- After the rectangular coordinate system is rotated by a certain angle
- The relationship between the new and the old coordinate systems

```
x' = x \cos\theta + y \sin\theta

y' = y \cos\theta - x \sin\theta
```





#### Transformations (2D-2D)

• Hierarchy of 2D coordinate transformations

Transformation	Matrix	# DoF	Preserves	Icon
translation	$\left[\begin{array}{c c}I&t\end{array}\right]_{2 imes 3}$	2	orientation	
rigid (Euclidean)	$\left[egin{array}{c c} R & t \end{array} ight]_{2 imes 3}$	3	lengths	$\Diamond$
similarity	$\left[\begin{array}{c c} sR & t\end{array}\right]_{2 imes 3}$	4	angles	$\Diamond$
affine	$\left[egin{array}{c} oldsymbol{A} \end{array} ight]_{2 imes 3}$	6	parallelism	
projective	$\left[egin{array}{c}  ilde{m{H}} \end{array} ight]_{3 imes 3}$	8	straight lines	



## Transformations (3D-3D)

• Hierarchy of 3D coordinate transformations

Transformation	Matrix	# DoF	Preserves	Icon
translation	$\begin{bmatrix} I & t \end{bmatrix}_{3\times4}$	3	orientation	
rigid (Euclidean)	$\left[egin{array}{c c} R & t \end{array} ight]_{3 imes 4}$	6	lengths	$\Diamond$
similarity	$\left[\begin{array}{c c} sR & t\end{array}\right]_{3 imes 4}$	7	angles	$\Diamond$
affine	$\left[\begin{array}{c} A \end{array}\right]_{3 imes4}$	12	parallelism	
projective	$\left[egin{array}{c}  ilde{m{H}} \end{array} ight]_{4 imes 4}$	15	straight lines	

#### Transformations (3D-2D)

- 3D to 2D projections (what information you want to preserved)
  - > Specify how 3D primitives are projected onto the image plane
  - > Use a linear 3D to 2D projection matrix Urthography  $\hat{x}=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1\end{bmatrix}^{3D}$   $\hat{p}$  Scaled orthography
- Orthography

$$x = [I_{2 \times 2}|0] p$$

- ✓ First project the world points onto a local fronto-parallel image plane
- ✓ Then scale this image using regular perspective projection

$$x = [sI_{2\times 2}|0] p$$



#### Transformations (3D-2D)

#### Perspective

- > The most commonly used projection
- Points projected onto the image plane by dividing them by their z component

inhomogeneous 
$$ar{x}=\mathcal{P}_z(p)=egin{bmatrix} x/z \ y/z \ 1 \end{bmatrix}$$

- > A two-step projection
  - ✓ First project 3D points into normalized device coordinates in the range
  - √ Then rescale these coordinates to integer pixel coordinates

the near and far z clipping planes 
$$z_{
m range} = z_{
m far} - z_{
m near}$$
  $ilde{x} = egin{bmatrix} 1 & 0 & 0 & 0 \ 0 & 1 & 0 & 0 \ 0 & 0 & -z_{
m far}/z_{
m range} & z_{
m near}z_{
m far}/z_{
m range} \ 0 & 0 & 1 & 0 \end{bmatrix}^{ ilde{p}} z_{
m near}$ 

# Projections



### The Geometry of Image Formation

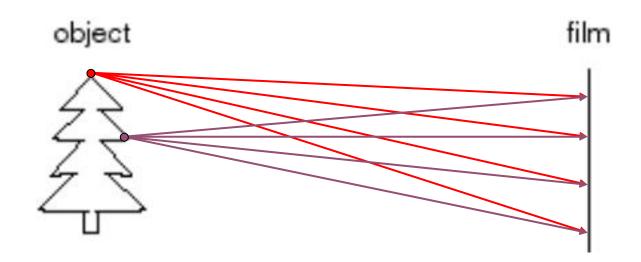
- Mapping between image and world coordinates
  - > Pinhole camera model
  - Projective geometryVanishing points and lines
  - > Projection matrix





#### **Image Formation**

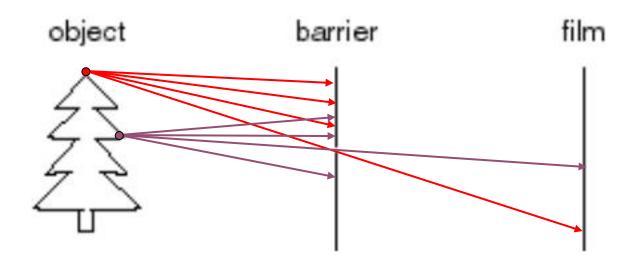
- Let's design a camera
  - > Idea 1: put a piece of film in front of an object
  - > Do we get a reasonable image?





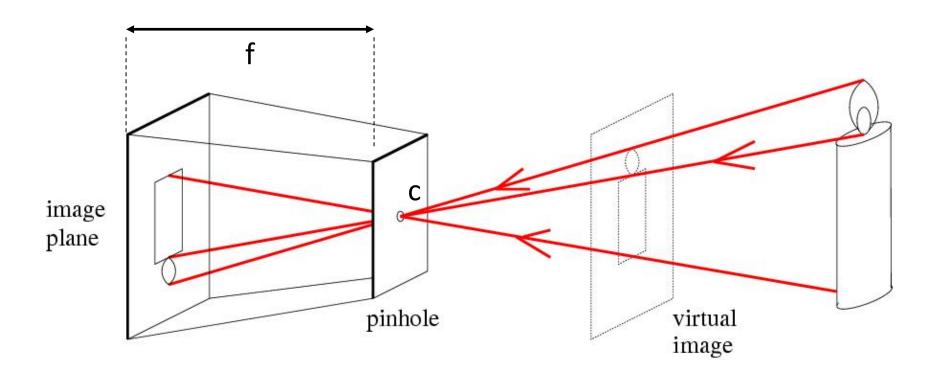
#### Pinhole Camera

- Idea 2: add a barrier to block off most of the rays
  - > This reduces blurring
  - > The opening known as the aperture





#### Pinhole Camera



f = focal length c = center of the camera



#### Camera Obscura: the Pre-Camera

 Known during classical period in China and Greece (e.g. Mo-Ti, China, 470BC to 390BC)

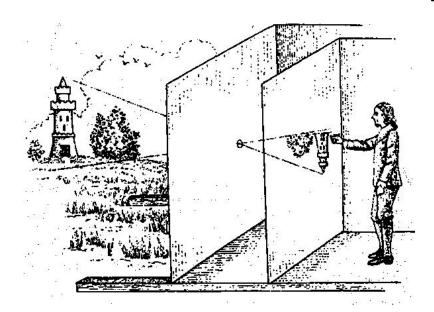


Illustration of Camera Obscura



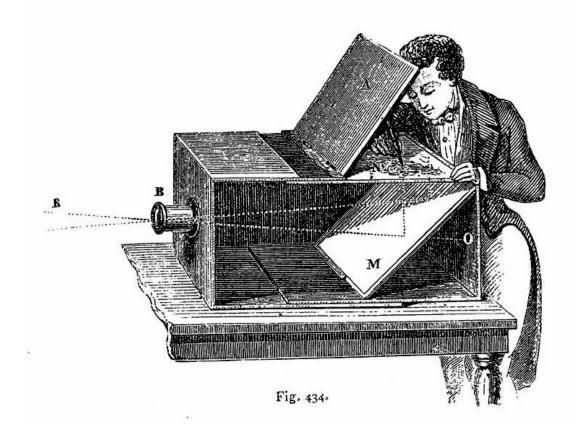
Freestanding camera obscura at UNC Chapel Hill

Photo by Seth Ilys

"景到,在午有端,与景长。说在端。"



## Camera Obscura used for Tracing



Lens Based Camera Obscura, 1568



#### Camera and World Geometry

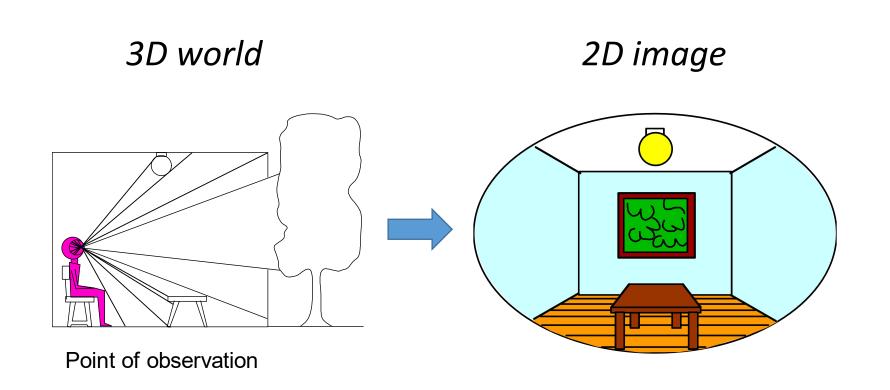
#### • Questions:

- How tall is this woman?
- How high is the camera?
- What is the camera rotation?
- What is the focal length of the camera?
- Which ball is closer?





# Dimensionality Reduction Machine (3D to 2D)





## Projection Can Be Tricky...

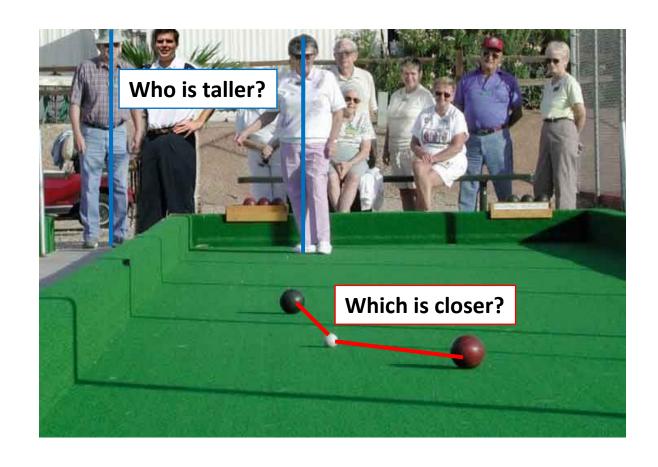




From an another view, it is totally different

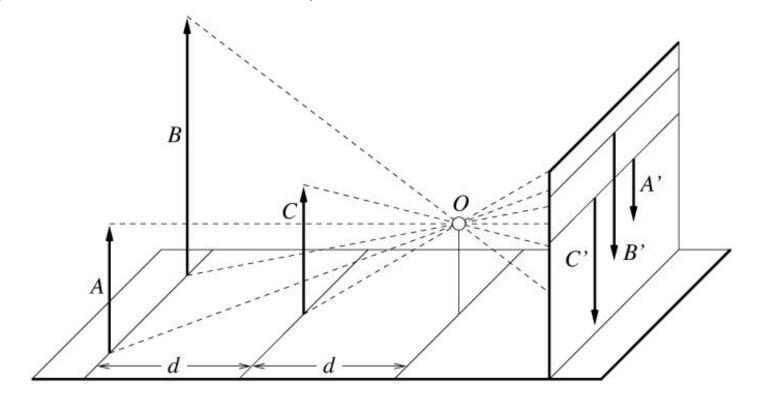


What is lost?Length



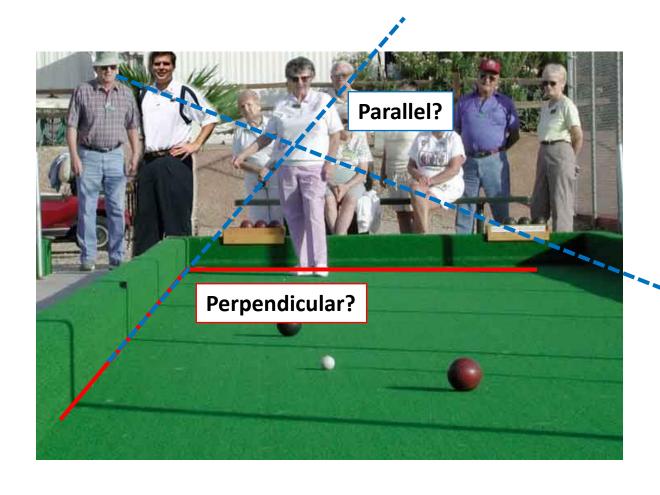


- What is lost?
  - > Length and area are not preserved



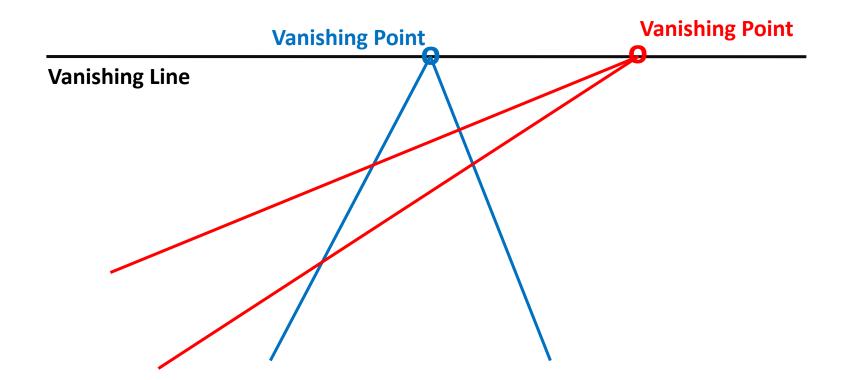


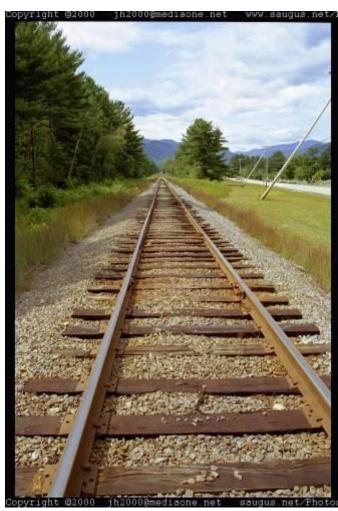
- What is lost?
  - > Length
  - > Angles
- What is preserved?
  - > Straight lines are still straight



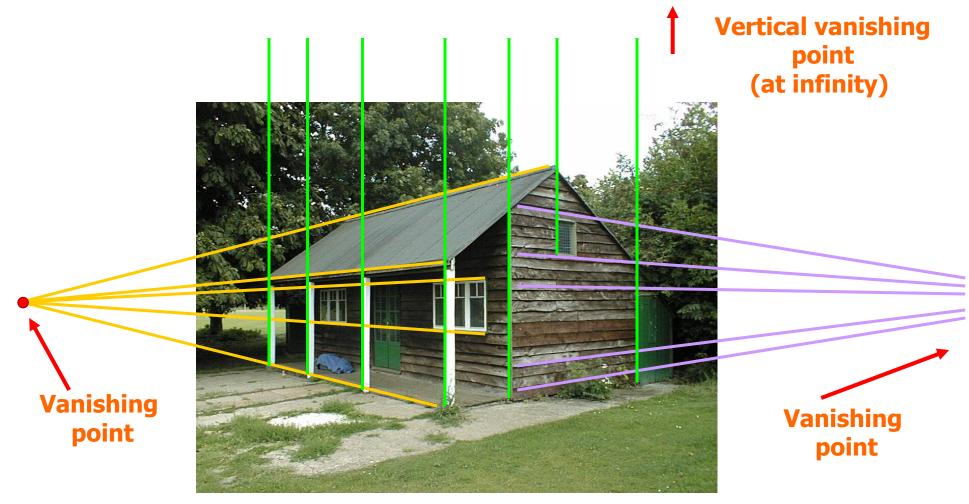


- Vanishing points and lines
  - Parallel lines in the world intersect in the image at a "vanishing point"



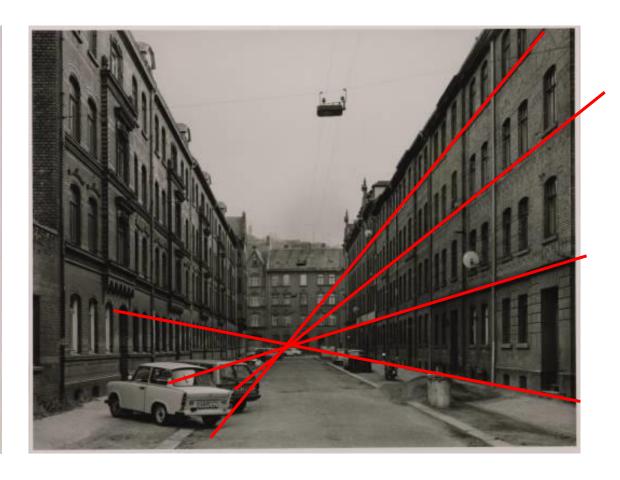










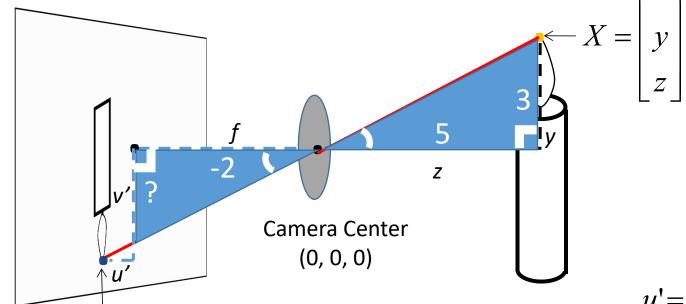


Questions: Why vertical parallel lines haven't have a finite vanishing point?



#### Projection

World coordinates → image coordinates



If X = 2, Y = 3, Z = 5, and f = 2What are U and V?

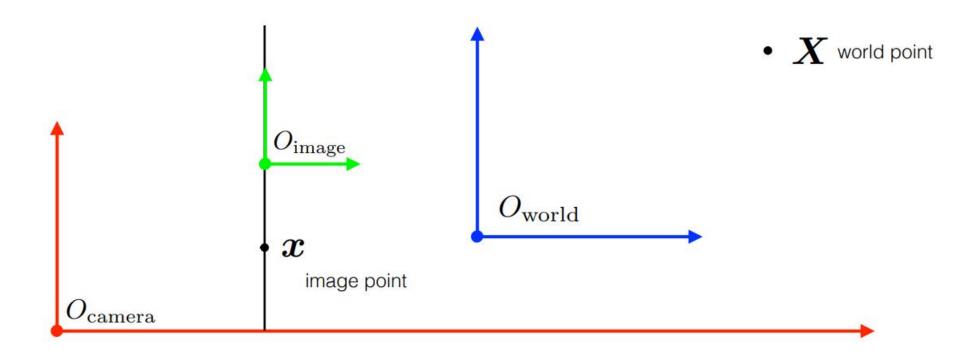
$$\frac{y}{z} \qquad v' = -y * \frac{f}{z}$$

$$u' = -2 * \frac{2}{5}$$
$$v' = -3 * \frac{2}{5}$$



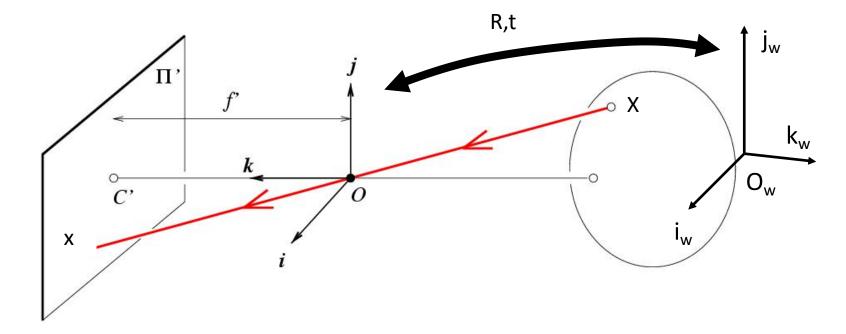
#### Three Different Coordinate Systems

You need the know the transformations between them





#### **Projection Matrix**



$$x = K[R \ t]X$$

x: Image Coordinates: (u,v,1)

**K**: Intrinsic Matrix (3x3)

R: Rotation (3x3)

t: Translation (3x1)

X: World Coordinates: (X,Y,Z,1)



#### **Projection Matrix**

• Inserting photographed objects into images (SIGGRAPH 2007)





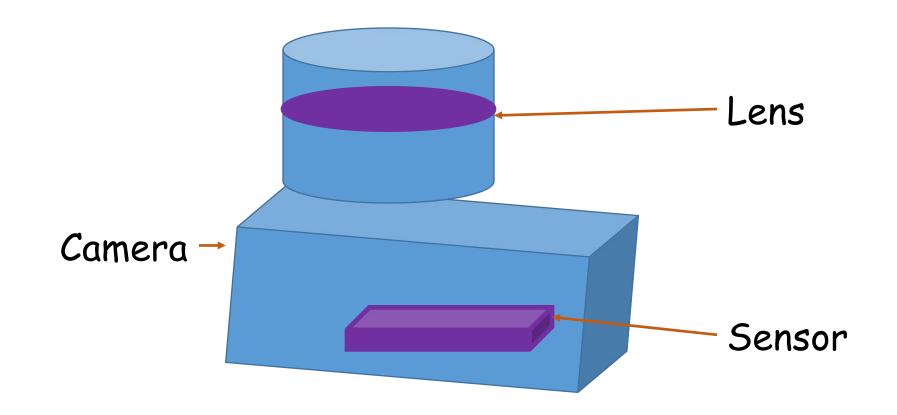
Original

Created



#### Camera Intrinsic

Potential problems caused by the production process



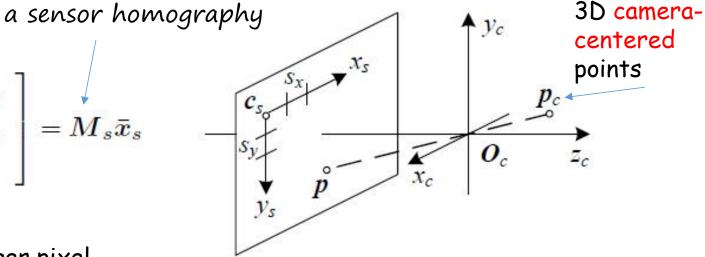


#### Camera Intrinsic

- Pixel values indexed by integer pixel coordinates
- Starting at the upper-left corner of the image
  - √ First scale the pixel values by the pixel spacing
  - √ Then describe the orientation of the sensor array relative to the camera projection center

the sensor planes at location

cation 
$$egin{aligned} \dot{m{p}} = egin{bmatrix} s_s & 0 & 0 \ 0 & s_y & 0 \ 0 & 0 & 0 \ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_s \ y_s \ 1 \end{bmatrix} = m{M}_s ar{x}_s \end{aligned}$$



3D rotation

origin

scale

integer pixel coordinates



#### Camera Intrinsic

- The relationship between the 3D pixel center and the 3D camera-centered point is given by an unknown scaling s
  - > The calibration matrix describes the camera intrinsics

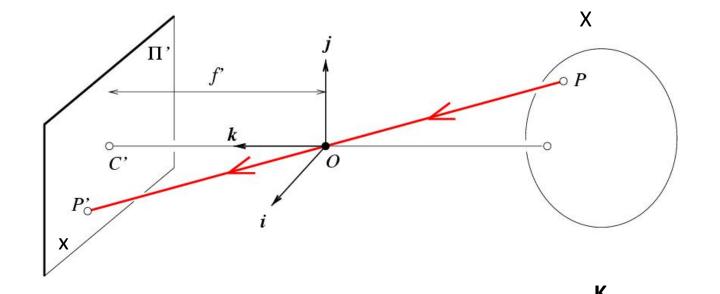
$$oldsymbol{p} = s oldsymbol{p}_c$$
  $oldsymbol{ ilde{x}}_s = s M_s^{-1} oldsymbol{p}_c = K oldsymbol{p}_c$  ansor 3D cameration matrix pixel address calibration matrix

the sensor centered planes at location points

pixel address



- Intrinsic Assumptions
  - > Unit aspect ratio
  - $\triangleright$  Optical center at (0,0)
  - > No skew
- Extrinsic Assumptions
  - No rotation
  - $\triangleright$  Camera at (0,0,0)

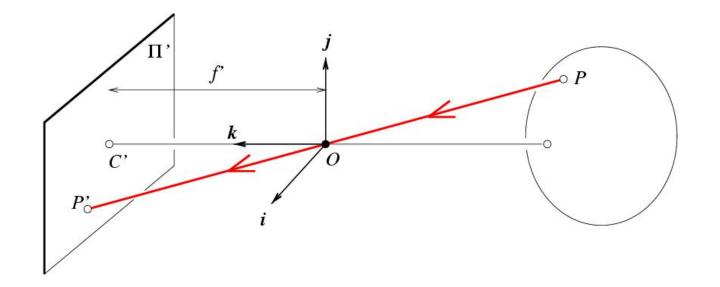


$$\mathbf{X} = \mathbf{K} \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix} \mathbf{X} \implies w \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} y \\ z \\ 1 \end{bmatrix}$$
Perspective



- Intrinsic Assumptions
  - > Unit aspect ratio

  - > No skew
- Extrinsic Assumptions
  - No rotation
  - $\triangleright$  Camera at (0,0,0)

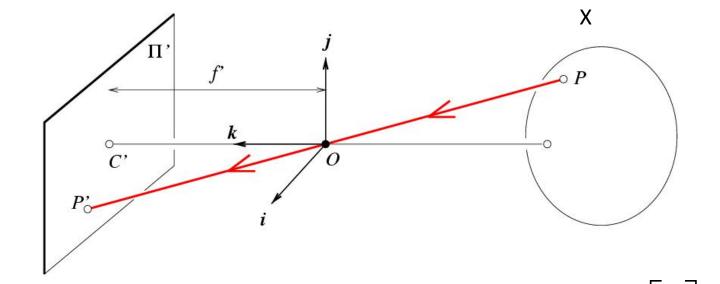


$$\mathbf{X} = \mathbf{K} \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix} \mathbf{X} \implies w \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} J & 0 & u_0 & 0 \\ 0 & f & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} y \\ z \\ 1 \end{bmatrix}$$



- Intrinsic Assumptions

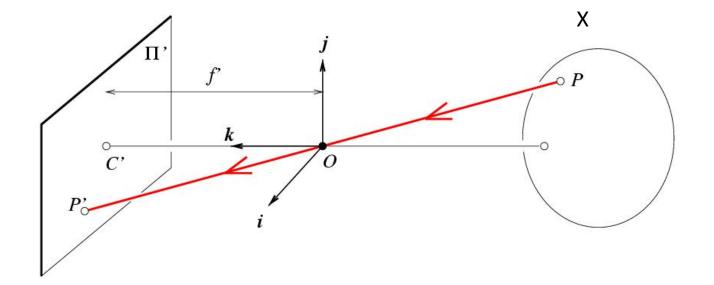
  - > No skew
- Extrinsic Assumptions
  - No rotation
  - $\triangleright$  Camera at (0,0,0)



$$\mathbf{X} = \mathbf{K} \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix} \mathbf{X} \longrightarrow w \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & 0 & u_0 & 0 \\ 0 & \beta & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$



- Intrinsic Assumptions
- Extrinsic Assumptions
  - > No rotation
  - > Camera at (0,0,0)



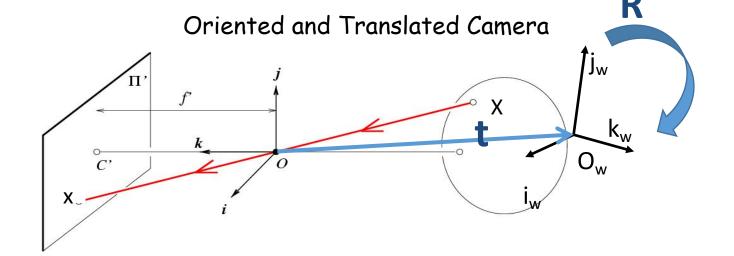
$$\mathbf{X} = \mathbf{K} \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix} \mathbf{X} \Longrightarrow \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & s & u_0 & 0 \\ 0 & \beta & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

**S** encodes any possible skew between the sensor axes due to the sensor not being mounted perpendicular to the optical axis



- Intrinsic Assumptions
- Extrinsic Assumptions
  - No rotation

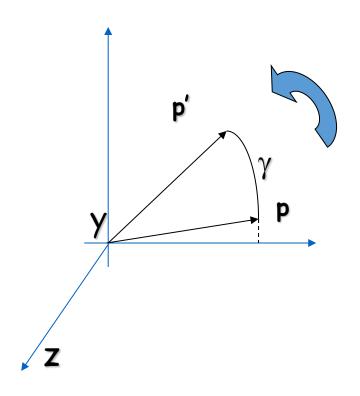




$$\mathbf{X} = \mathbf{K} \begin{bmatrix} \mathbf{I} & \mathbf{t} \end{bmatrix} \mathbf{X} \longrightarrow W \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & \mathbf{s} & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$



- 3D Rotation of Points
  - > Rotation around the coordinate axes, counter-clockwise:



$$R_{x}(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & -\sin \alpha \\ 0 & \sin \alpha & \cos \alpha \end{bmatrix}$$

$$R_{y}(\beta) = \begin{bmatrix} \cos \beta & 0 & \sin \beta \\ 0 & 1 & 0 \\ -\sin \beta & 0 & \cos \beta \end{bmatrix}$$

$$R_{z}(\gamma) = \begin{bmatrix} \cos \gamma & -\sin \gamma & 0 \\ \sin \gamma & \cos \gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



Allow camera rotation

$$\mathbf{x} = \mathbf{K} \begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix} \mathbf{X}$$

$$w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & s & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$



Vanishing point = Projection from infinity

$$\mathbf{p} = \mathbf{K} \begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 0 \end{bmatrix} \Rightarrow \mathbf{p} = \mathbf{K} \mathbf{R} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \Rightarrow \mathbf{p} = \mathbf{K} \begin{bmatrix} x_R \\ y_R \\ z_R \end{bmatrix}$$

$$w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & u_0 \\ 0 & f & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_R \\ y_R \\ z_R \end{bmatrix} \Rightarrow \qquad v = \frac{fx_R}{z_R} + u_0$$

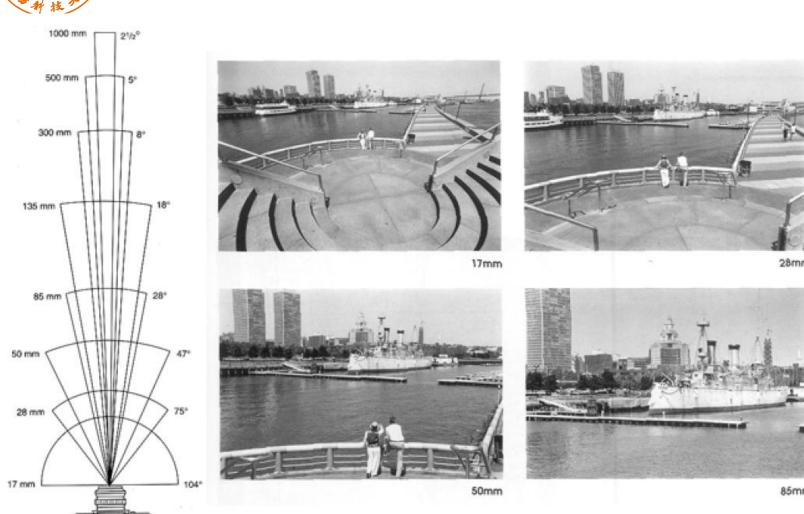
$$v = \frac{fy_R}{z_R} + v_0$$

$$u = \frac{fx_R}{z_R} + u_0$$

$$v = \frac{fy_R}{z_R} + v_0$$



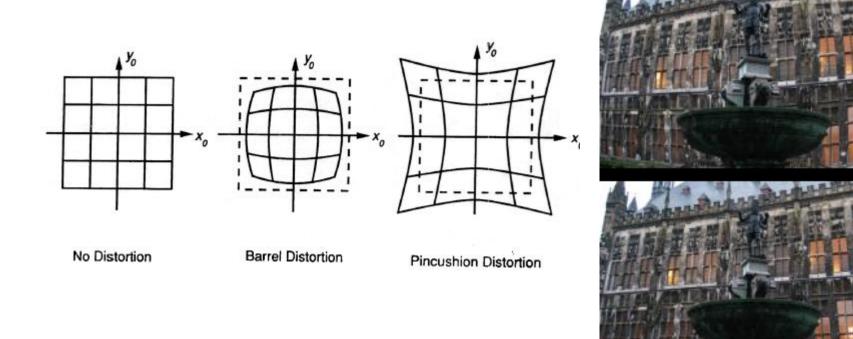
#### Field of View (Zoom, Focal Length)







#### Beyond Pinholes: Radial Distortion

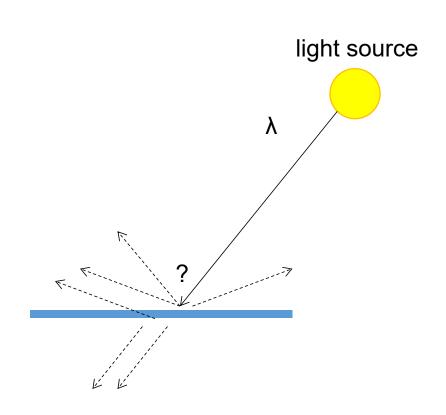


**Corrected Barrel Distortion** 

# Photometric image formation

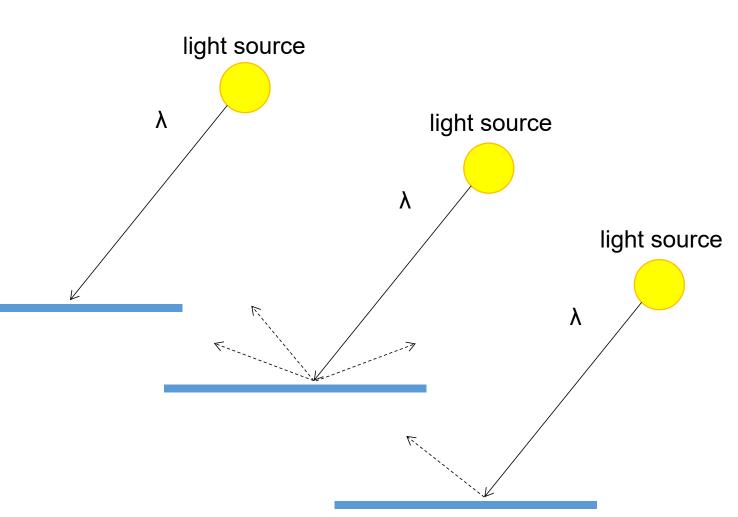


- Absorption 吸收
- Diffusion 漫射
- Reflection反射
- Transparency 透射
- Refraction 折射
- Fluorescence 荧光反应
- Subsurface scattering 次表面散射
- Phosphorescence 磷光
- Interreflection 相互反射



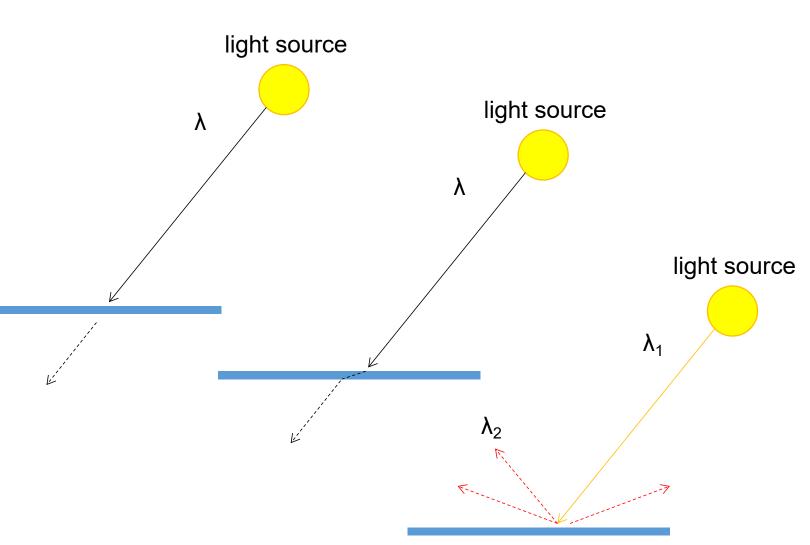


- Absorption
- Diffusion
- Reflection
- Transparency
- Refraction
- Fluorescence
- Subsurface scattering
- Phosphorescence
- Interreflection



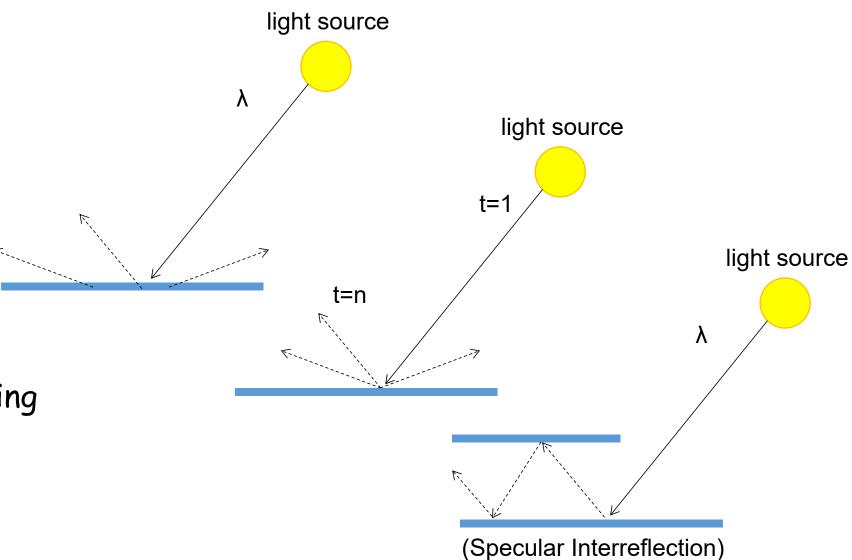


- Absorption
- Diffusion
- Reflection
- Transparency
- Refraction
- Fluorescence
- Subsurface scattering
- Phosphorescence
- Interreflection





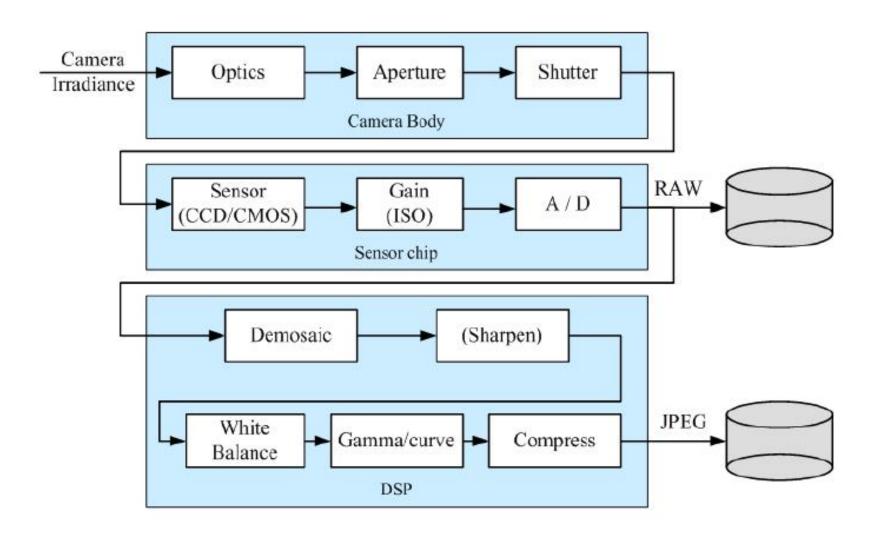
- Absorption
- Diffusion
- Reflection
- Transparency
- Refraction
- Fluorescence
- Subsurface scattering
- Phosphorescence
- Interreflection



## The digital camera



#### Image sensing pipeline





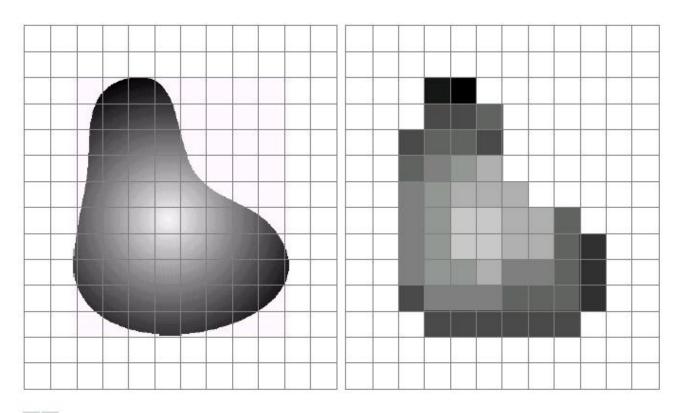
#### Digital Camera

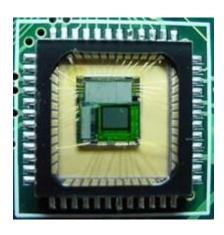
- A digital camera replaces film with a sensor array
  - ➤ Each cell in the array is light-sensitive diode (光敏二极管) that converts photons to electrons
  - > Two common types
    - ✓ Charge Coupled Device (CCD)
    - √ CMOS





#### Sensor Array





**CMOS** sensor

a b

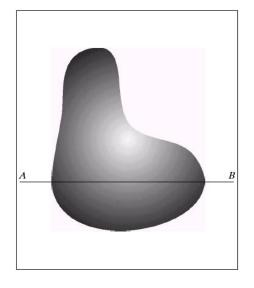
**FIGURE 2.17** (a) Continuos image projected onto a sensor array. (b) Result of image sampling and quantization.

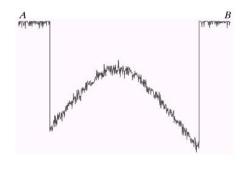


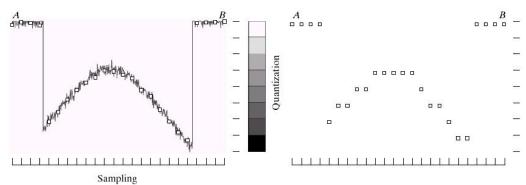
#### Sampling and Quantization

• Shannon's Sampling Theorem

 $f_{
m s} \geq 2 f_{
m max}$ 











Primary and secondary colors

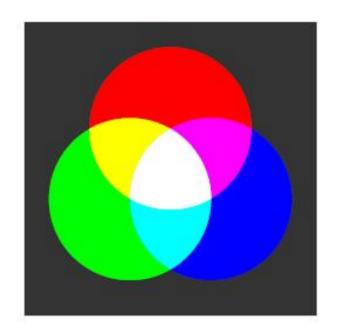
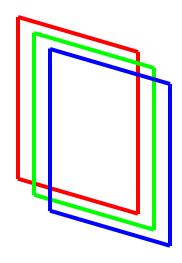




Image: three matrices



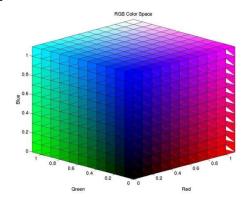


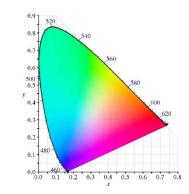
#### Color Spaces

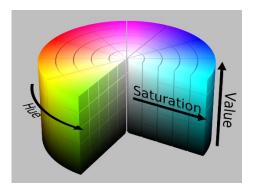
• RGB

• CIE XYZ

- HSV
  - > Hue
  - > Saturation
  - > Value







$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \frac{1}{0.17697} \begin{bmatrix} 0.49 & 0.31 & 0.20 \\ 0.17697 & 0.81240 & 0.01063 \\ 0.00 & 0.01 & 0.99 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

$$x = \frac{X}{X+Y+Z}, \ \ y = \frac{Y}{X+Y+Z}, \ \ z = \frac{Z}{X+Y+Z}$$

$$C = V imes S_{HSV} \ H' = rac{H}{60^{\circ}} \ (R_1, G_1, B_1) = egin{cases} (0,0,0) & ext{if $H$ is undefined} \ (C,X,0) & ext{if $0 \le H' \le 1$} \ (X,C,0) & ext{if $1 < H' \le 2$} \ (0,C,X) & ext{if $2 < H' \le 3$} \ (0,X,C) & ext{if $3 < H' \le 4$} \ (X,0,C) & ext{if $4 < H' \le 5$} \ (C,0,X) & ext{if $5 < H' \le 6$} \end{cases}$$

$$(R,G,B)=(R_1+m,G_1+m,B_1+m)$$



#### Color Filter Arrays

- Color filter array layout
- Interpolated pixel values
  - > The luminance signal is mostly determined by green values
  - > The visual system is much more sensitive to high frequency detail in luminance than in chrominance

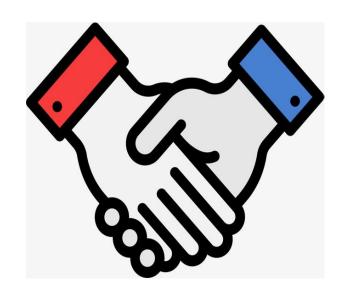
G	R	G	R
В	G	В	G
G	R	G	R
В	G	В	G

rGb	Rgb	rGb	Rgb
rgB	rGb	rgB	rGb
rGb	Rgb	rGb	Rgb
rgB	rGb	rgB	rGb

### Conclusions



- Vanishing points and vanishing lines
- Pinhole camera model and camera projection matrix
- Homogeneous coordinates
- Digital camera



## Thanks



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