

CS310 Natural Language Processing

自然语言处理

Lecture 02 - Word Vectors

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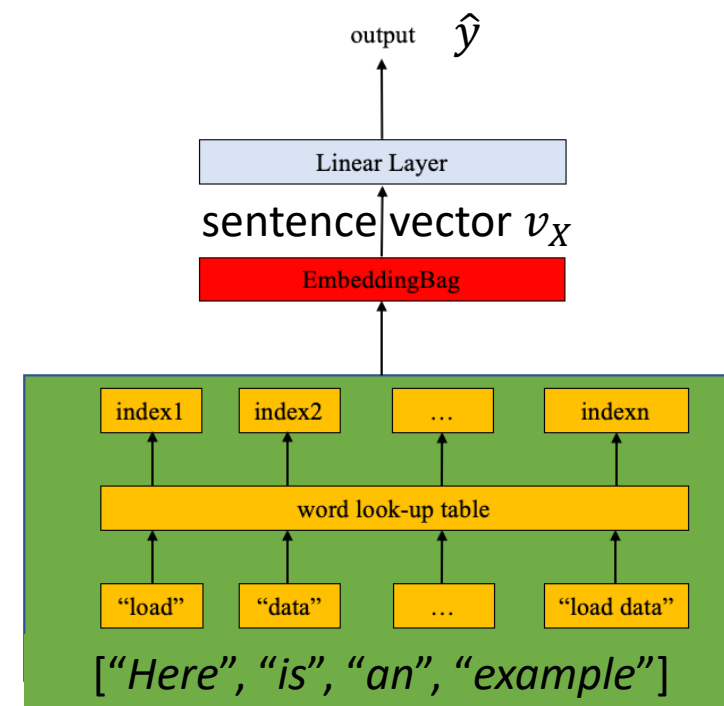
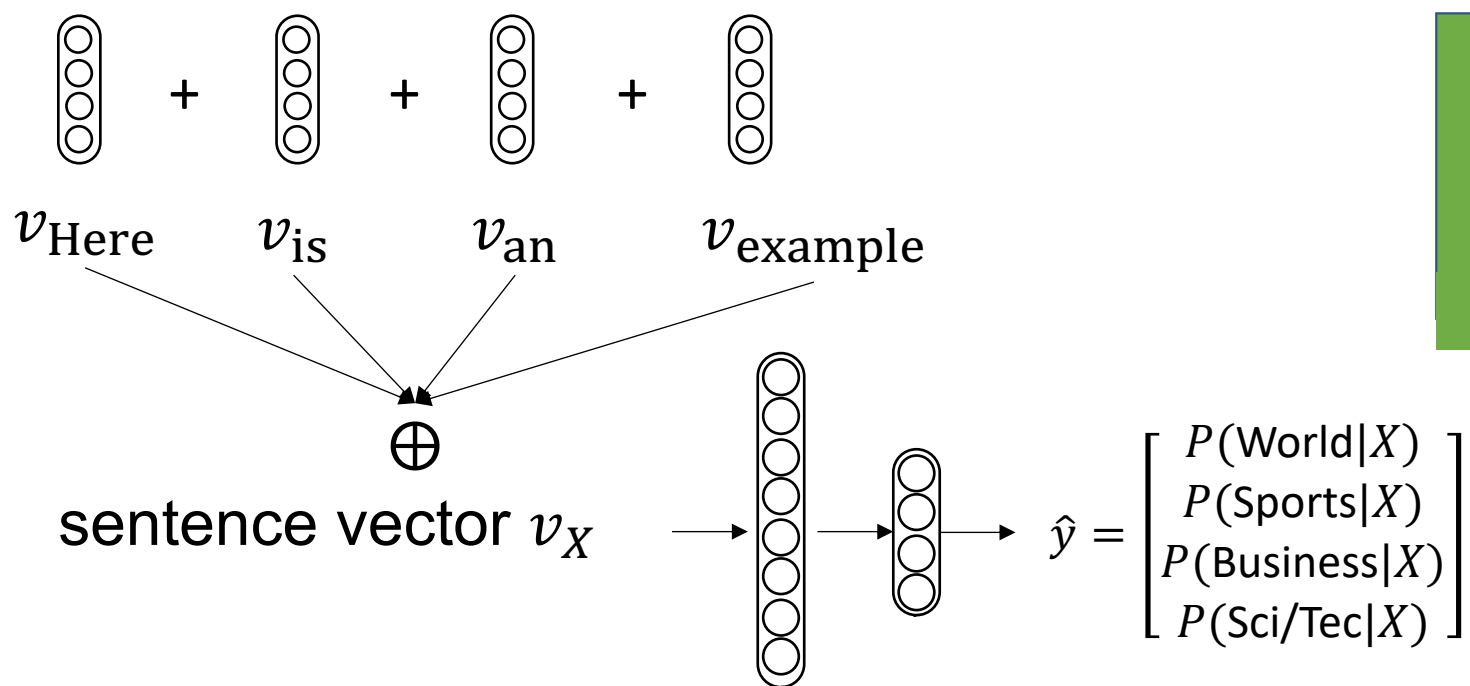
Content

- **Motivation**
- Documents and Counts-based Method
- Neural Network-based Method -- word2vec
- Evaluation and Applications

Recap: Bag-of-Words Neural Networks

Task: News text classification

X : ["Here", "is", "an", "example"]



Naïve method: one-hot vectors

- Words as discrete symbols \Leftrightarrow localist representations
- **One-hot** vectors

Vocabulary (10k) = $\begin{bmatrix} a \\ about \\ all \\ \vdots \\ zoo \end{bmatrix}$

Apple [0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 ... 0]

Orange [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 ... 0]

I would like some **apple** juice

I would like some **orange** _____

Distance between any pair of words is **constant**:

Euclidean distance = $\sqrt{(1-0)^2 + (1-0)^2}$

Cosine distance = 0

One-hot vector is not helpful

Ideally \Rightarrow real-valued word vectors

	Man	Woman	King	Queen	Apple	Orange
Gender	-1	1	-0.98	0.97	0.00	-0.01
Royal	0.01	0.02	0.93	0.98	-0.01	0.00
Age	0.03	0.02	0.72	0.68	0.03	0.02
Food	0.00	0.00	0.01	0.02	0.95	0.97

main difference: gender

main difference: gender

$$e_{Man} = \begin{bmatrix} -1 \\ 0.01 \\ 0.03 \\ 0.0 \end{bmatrix} \quad e_{Woman} = \begin{bmatrix} 1 \\ 0.02 \\ 0.02 \\ 0.0 \end{bmatrix}$$

$$e_{Man} - e_{Woman} = \begin{bmatrix} -2 \\ -0.01 \\ 0.01 \\ 0.00 \end{bmatrix}$$

$$e_{King} - e_{Queen} = \begin{bmatrix} -1.95 \\ -0.05 \\ 0.04 \\ -0.01 \end{bmatrix}$$

With real-valued dense vectors, word similarity can be computed more accurately

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 - **LSA and TF-IDF**
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Documents and Word Counts

- **Goal:** Derive word vectors from a collection of documents
- without annotation -- unsupervised/self-supervised
- **Notations:**
 - x is the collection of C documents
 - x_c is the c th document in the corpus
 - ℓ_c is the length of x_c (in # of tokens)
 - N is the total number of tokens (words), $N = \sum_{c=1}^C \ell_c$

Build Word-Document Matrix (term-document matrix)^[1]

- Build matrix $\mathbf{A} \in \mathbb{R}^{V \times C}$, which contains the count of each word in each document

- Example:**

x_1 : 学 而 时 习 之

x_2 : 学 而 不 思 则 罔

x_3 : 思 而 不 学 则 殆

Entry $\mathbf{A}_{v,c} = \text{count}_{x_c}(v)$, count of word v in the c th document

		\mathcal{C}		
		x_1	x_2	x_3
V	学	1	1	1
	而	1	1	1
	不	0	1	1
	思	0	1	1
	则	0	1	1
	时	1	0	0
	习	1	0	0
	之	1	0	0
	罔	0	1	0
	殆	0	0	1

[1] https://en.wikipedia.org/wiki/Term-document_matrix

Q: Can we directly use this matrix?

• Example

- 《论语》 前十篇内容，8664字，267章

$V = 8664$

$C = 267$

	x_1	x_2	x_3	...
子				...
曰				...
学				...
⋮	⋮	⋮	⋮	

$$v(\text{子}) = [2., 2., 1., 1., 1., 2., 1., 2., \dots, 0., 1.]$$

$$v(\text{曰}) = [1., 1., 1., 1., 1., 1., 2., 1., \dots, 0., 1.]$$

$$v(\text{学}) = [1., 0., 0., 0., 0., 1., 2., 1., \dots, 0., 0.]$$

Most of them are same numbers.
Are they really necessary?

Q: How do we interpret the matrix?

- What is the expected occurrence of word v in document c ?
- Under a simple assumption, the chance of word v to occur at any position is $\frac{\text{count}_x(v)}{N}$, (where $\text{count}_x(v)$ is the count of v over all documents)
- So the expected occurrence of v in a document of length ℓ_c is $\frac{\text{count}_x(v)}{N} \cdot \ell_c$
- Consider the **ratio** of *observed* count of v in document c , $\text{count}_{x_c}(v)$, to the expected count $\frac{\text{count}_x(v)}{N} \cdot \ell_c$

Intuition of surprise in word

	x_1	x_2	x_3
学	1	1	1
而	1	1	1
不	0	1	1
思	0	1	1
则	0	1	1
时	1	0	0
习	1	0	0
之	1	0	0
罔	0	1	0
殆	0	0	1

$$\text{count}_x(\text{学}) = 1 + 1 + 1 = 3$$

Expected count of 学 in x_1 is $\frac{\text{count}_x(\text{学})}{N} \cdot \ell_1 = \frac{3}{17} \cdot 5 \approx 0.88$

The observed count of 学 in x_1 is $\text{count}_{x_1}(\text{学}) = 1$

The **surprise** of seeing 学 in x_1 is:

$$\log \frac{\text{observed}}{\text{expected}} = \log \frac{\text{count}_{x_1}(\text{学})}{\frac{\text{count}_x(\text{学})}{N} \cdot \ell_1} \approx \log \frac{1}{0.88} \approx 0.125$$

Intuition of surprise in word

	x_1	x_2	x_3
学	1	1	1
而	1	1	1
不	0	1	1
思	0	1	1
则	0	1	1
时	1	0	0
习	1	0	0
之	1	0	0
罔	0	1	0
殆	0	0	1

$$\text{count}_x(\text{习}) = 1 + 0 + 0 = 1$$

Expected count of 习 in x_1 is $\frac{\text{count}_x(\text{习})}{N} \cdot \ell_1 = \frac{1}{17} \cdot 5 \approx 0.29$

The observed count of 习 in x_1 is $\text{count}_{x_1}(\text{习}) = 1$

The **surprise** of seeing 习 in x_1 is:

$$\log \frac{\text{observed}}{\text{expected}} = \log \frac{\text{count}_{x_1}(\text{习})}{\frac{\text{count}_x(\text{习})}{N} \cdot \ell_1} \approx \log \frac{1}{0.29} \approx 1.223 > \text{surprise of 学}$$

Pointwise Mutual Information

- From matrix $\mathbf{A} \in \mathbb{R}^{V \times C}$, derive positive **pointwise mutual information**

$$[\mathbf{A}]_{v,c} = \left[\log \frac{\text{count}_{x_c}(v)}{\frac{\text{count}_x(v)}{N} \cdot \ell_c} \right]_+ = \left[\log \frac{N \cdot \text{count}_{x_c}(v)}{\text{count}_x(v) \cdot \ell_c} \right]_+ \quad \text{where } [x]_+ = \max(0, x)$$

More examples:

$$[\mathbf{A}]_{\text{学},2} = \log \frac{17 \cdot 1}{3 \cdot 6} \approx -0.057 \rightarrow 0 \quad \text{rounded to 0 because of max()}$$

$$[\mathbf{A}]_{\text{思},2} = \log \frac{17 \cdot 1}{2 \cdot 6} \approx 0.348$$

Meaning of PMI

Random variable **A** and **B**

$$\begin{aligned}\text{PMI}(a, b) &= \log \frac{p(A = a, B = b)}{p(A = a) \cdot p(B = b)} \\ &= \log \frac{p(A = a \mid B = b)}{p(A = a)} \\ &= \log \frac{p(B = b \mid A = a)}{p(B = b)}\end{aligned}$$

Example:

$$\log \frac{\text{count}_{x_1}(\text{习})}{\frac{\text{count}_x(\text{习})}{N} \cdot \ell_1} \approx \log \frac{1}{0.29} \approx 1.223$$

is high, which means we learn a lot about the global meaning of “习” by reading x_1

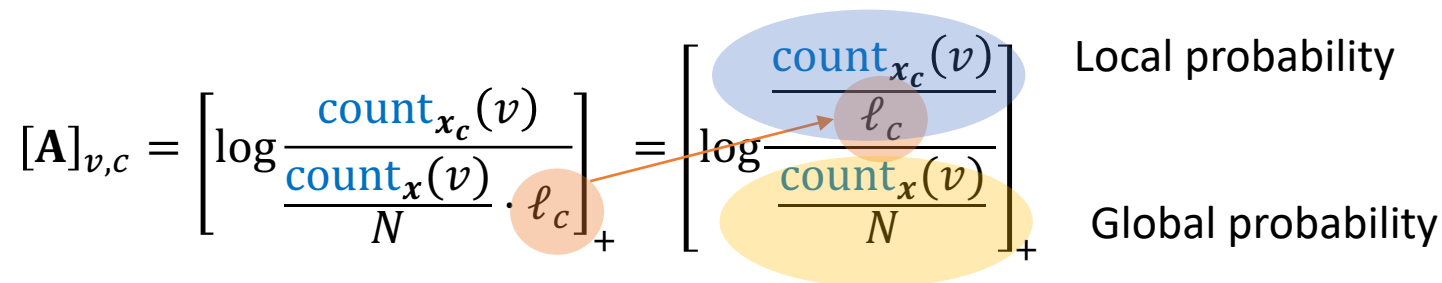
Mutual Information (MI):

The amount of information each r.v. offers about the other.

I.e., how much do we know about **B** by knowing about **A**

$$[\mathbf{A}]_{v,c} = \left[\log \frac{\text{count}_{x_c}(v)}{\frac{\text{count}_x(v)}{N} \cdot \ell_c} \right]_+ = \left[\log \frac{\frac{\text{count}_{x_c}(v)}{\ell_c}}{\frac{\text{count}_x(v)}{N}} \right]_+$$

Local probability
Global probability



How much do we know about the global meaning of v by knowing about its local meaning in document c

Pointwise Mutual Information

$$PMI = [A]_{v,c} = \left[\log \frac{\text{count}_{x_c}(v)}{\frac{\text{count}_x(v)}{N} \cdot \ell_c} \right]_+$$

	x_1	x_2	x_3
学	1	1	1
而	1	1	1
不	0	1	1
思	0	1	1
则	0	1	1
时	1	0	0
习	1	0	0
之	1	0	0
罔	0	1	0
殆	0	0	1

x_1 : 学 而 时 习 之

x_2 : 学 而 不 思 则 罔

x_3 : 思 而 不 学 则 殆

PMIs highlight the most
informative words

	x_1	x_2	x_3
学	0	0	0
而	0	0	0
不	0	0	0
思	0	0	0
则	0	0	0
时	1	0	0
习	1	0	0
之	1	0	0
罔	0	1	0
殆	0	0	1

Properties of PMI

- If a word v has nearly same frequency in every document, then its row $[A]_{v,*}$ will be nearly all zeros
- If a word v only occurs in one document c , then its PMI will be large and positive
- Thus, PMI is sensitive to **rare** words; usually need to smooth the frequencies by filtering rare words

Reflection

- Can we directly use word-document matrix $\mathbf{A} \in \mathbb{R}^{V \times C}$ (or smoothed PMI $[\mathbf{A}]$) to represent word meanings?
- For example, can we use the row vectors as input features for a neural text classifier?
- What are the advantages/disadvantages?

Improvement: Latent Semantic Analysis

(Deerwester et al., 1990)

- LSA seeks to find a more compact (low rank) representation of word-document matrix \mathbf{A}

$$\underset{V \times C}{\mathbf{A}} \approx \underset{V \times d}{\hat{\mathbf{A}}} = \underset{V \times d}{\mathbf{M}} \times \underset{d \times d}{\text{diag}(\mathbf{s})} \times \underset{d \times C}{\mathbf{C}^T}$$

- Can be solved by applying singular value decomposition to \mathbf{A} , and then truncating to d dimensions ($\hat{\mathbf{A}}$)
- \mathbf{M} contains left singular vectors of \mathbf{A}
- \mathbf{C} contains right singular vectors of \mathbf{A}
- \mathbf{s} are singular values of \mathbf{A}

SVD and Truncated SVD

SVD:

$$\begin{array}{c}
 \text{SVD:} \\
 \begin{array}{c}
 \boxed{\mathbf{A}} \\
 \end{array}
 =
 \begin{array}{c}
 V \times V \\
 \boxed{\mathbf{M}} \\
 \end{array}
 \begin{array}{c}
 V \times C \\
 \begin{array}{c}
 \diagdown \\
 \text{diag}(\mathbf{s}) \\
 \diagup
 \end{array}
 \end{array}
 \begin{array}{c}
 C \times C \\
 \boxed{\mathbf{C}^T} \\
 \end{array}
 \end{array}$$

- \mathbf{M} and \mathbf{C} are unitary, i.e., $\mathbf{M}\mathbf{M}^T = \mathbf{I}$ and $\mathbf{C}\mathbf{C}^T = \mathbf{I}$
- $\text{diag}(\mathbf{s})$ only has non-zero elements at diagonal
- \mathbf{M} are eigenvectors of $\mathbf{A}\mathbf{A}^T$
- \mathbf{C} are eigenvectors of $\mathbf{A}^T\mathbf{A}$
- \mathbf{s}^2 are eigenvalues

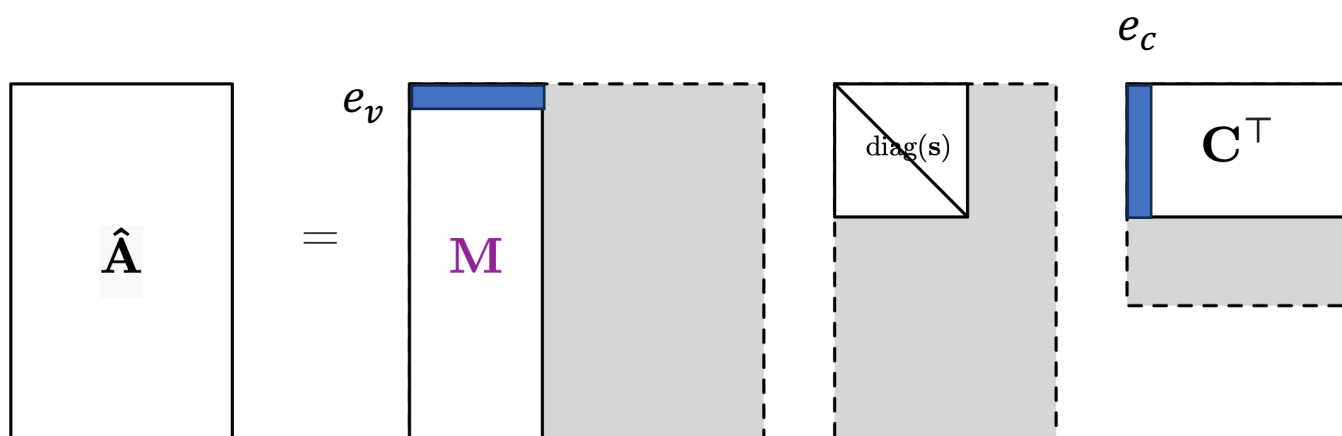
SVD truncated at d dimensions:

$$\begin{array}{c}
 \boxed{\hat{\mathbf{A}}} \\
 \end{array}
 =
 \begin{array}{c}
 \begin{array}{c}
 \boxed{\mathbf{M}} \quad \text{dashed box} \\
 V \times d
 \end{array}
 \begin{array}{c}
 \begin{array}{c}
 \diagdown \\
 \text{diag}(\mathbf{s}) \\
 \diagup
 \end{array} \\
 d \times d
 \end{array}
 \begin{array}{c}
 \begin{array}{c}
 \boxed{\mathbf{C}^T} \\
 \text{dashed box} \\
 d \times C
 \end{array}
 \end{array}
 \end{array}$$

- Truncated: keeping only top d singular values in \mathbf{s}
- corresponding d columns in \mathbf{M} and \mathbf{C}

Truncated SVD => word vectors

$$\mathbf{A} \approx \hat{\mathbf{A}} = \mathbf{M} \times \text{diag}(\mathbf{s}) \times \mathbf{C}^\top$$

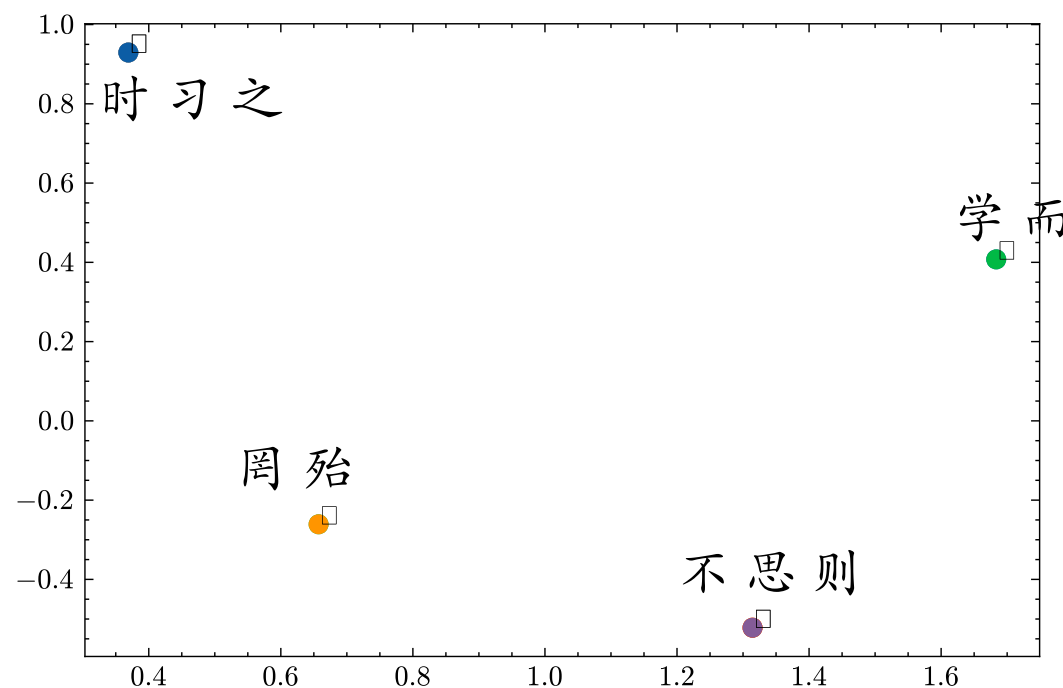


- v th row in \mathbf{M} is the embedding vector for word v
- c th column in \mathbf{C} is the embedding vector for document c

- \mathbf{M} contains useful word vectors (“embeddings”) of d dimensions
- \mathbf{C} contains document vectors

LSA Example $d = 2$

- Word vectors \mathbf{M} plotted
- Note that some words are in the same spot. Why?

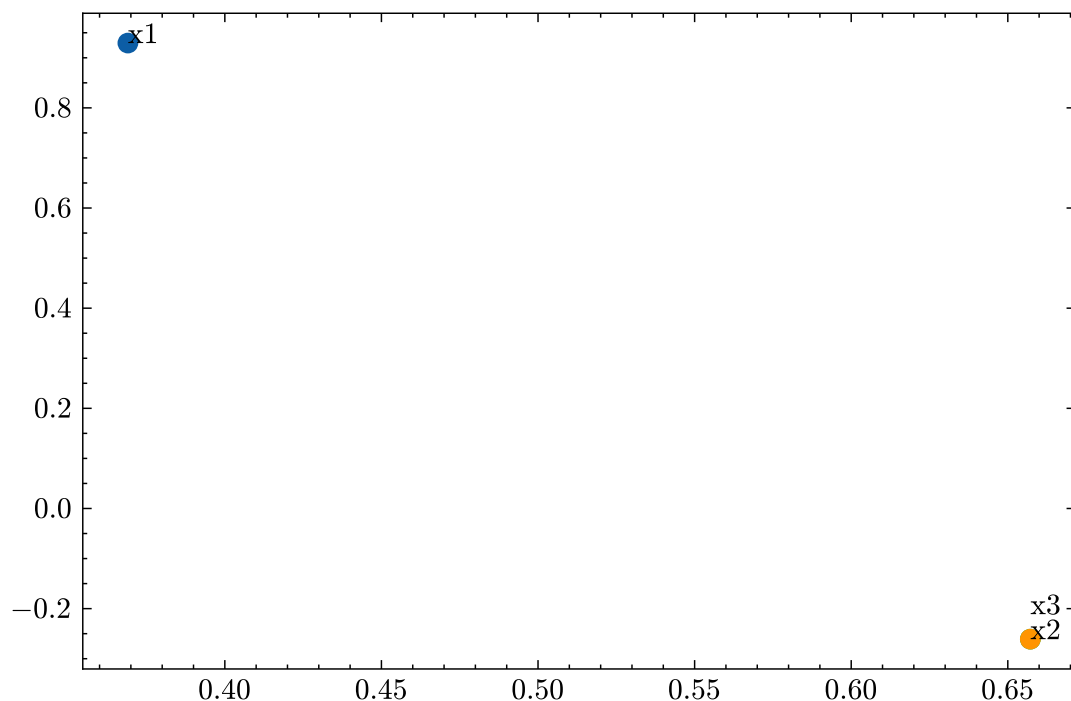


$$\mathbf{A} \approx \hat{\mathbf{A}} = \mathbf{M} \times \text{diag}(\mathbf{s}) \times \mathbf{C}^T \quad \mathbf{A} =$$

	x_1	x_2	x_3
学	1	1	1
而	1	1	1
不	0	1	1
思	0	1	1
则	0	1	1
时	1	0	0
习	1	0	0
之	1	0	0
罔	0	1	0
殆	0	0	1

LSA Example $d = 2$

- Document vectors \mathbf{C} plotted
- Note that documents x_2 and x_3 are in the same spot. Why?



$\mathbf{A} =$

	x_1	x_2	x_3
学	1	1	1
而	1	1	1
不	0	1	1
思	0	1	1
则	0	1	1
时	1	0	0
习	1	0	0
之	1	0	0
罔	0	1	0
殆	0	0	1

LSA Summarized

- It creates a mapping of words and documents into the same low-dimensional space.
- Bag-of-words assumption (Salton et al., 1975):
 - A document is nothing more than the distribution of words it contains.
- Distributional hypothesis (Harris, 1954; J.R. Firth, 1957):
 - Words' meanings are nothing more than the distribution of *contexts* (here, documents) they occur in.
 - Words that occur in similar contexts have similar meanings.
- Word-document matrix \mathbf{A} is sparse and noisy; LSA “fills in” the zeroes and tries to eliminate the noise.
- It finds the best rank- d approximation to \mathbf{A} .

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 - LSA and **TF-IDF**
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TF-IDF

- Background: Find the most relevant document among a collection of documents, using a query

tf: term frequency
idf: inverse document frequency

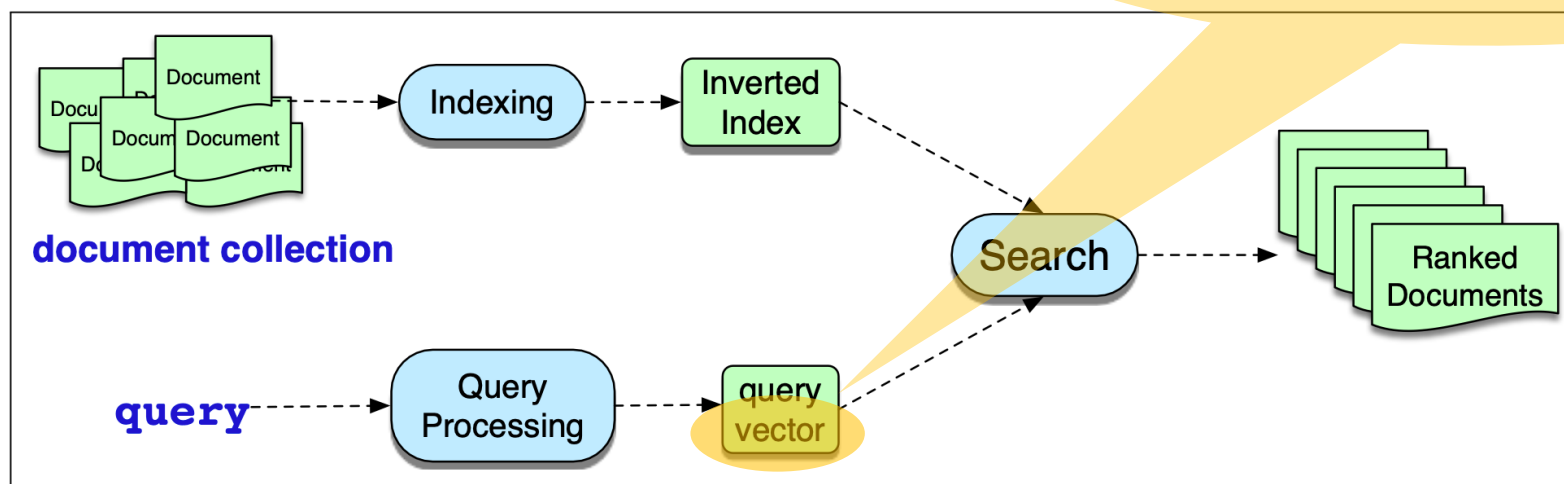


Figure 14.1 The architecture of an ad hoc IR system.

How to match a document a query?

- Compute a term weight for each document term
- **tf**: term frequency
- **idf**: inverse document frequency

term t ; document d

- **tf-idf** \triangleq **tf** \times **idf** (product of the two)

$$\text{tf}_{t,d} = \begin{cases} 1 + \log_{10} \text{count}(t, d) & \text{if } \text{count}(t, d) > 0 \\ 0 & \text{otherwise} \end{cases}$$

- **tf**: words that occur more often in a document are likely to be informative about the document's content
- Use the \log_{10} of word frequency count rather than raw count
- *Why? A word appearing 100 times doesn't make it 100 times more likely*

Tf-idf

$$\text{tf}_{t,d} = \begin{cases} 1 + \log_{10} \text{count}(t,d) & \text{if } \text{count}(t,d) > 0 \\ 0 & \text{otherwise} \end{cases}$$

term t ; document d

term occurs 0 times in document: $\text{tf} = 0$
term occurs 1 times in document: $\text{tf} = 1$
term occurs 10 times in document: $\text{tf} = 2, \dots$

- **document frequency** df_t of a term t is the number of documents it occurs in
- Terms that occur in only **a few** documents are useful for discriminating those documents from the rest of the collection;
- terms that occur across the entire collection aren't as helpful (*the, a, an, ...*)
- **inverse document frequency** or **idf** is defined as:

$$\text{idf}_t = \log_{10} \frac{N}{\text{df}_t}$$

N : total number of documents
The fewer documents in which t occurs, the higher idf_t

Inverse document frequency example

- Some idf values for some words in the corpus of Shakespeare plays

Word	df	idf
Romeo	1	1.57
salad	2	1.27
Falstaff	4	0.967
forest	12	0.489
battle	21	0.246
wit	34	0.037
fool	36	0.012
good	37	0
sweet	37	0

Extremely informative words that occur in only one play like *Romeo*

good or *sweet* are completely non-discriminative since they occur in all 37 plays

Scoring with tf-idf

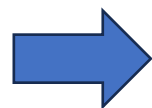
- We can score document d by the cosine of its vector \vec{d} with the query vector \vec{q} :

$$\text{score}(q, d) = \cos(\vec{q}, \vec{d}) = \frac{\vec{q} \cdot \vec{d}}{|\vec{q}| \cdot |\vec{d}|}$$

- in which \vec{q} and \vec{d} are vectors of query length n , whose values are the **tf-idf** values (normalized):

$$\vec{q} = \frac{[\text{tfidf}(t_1, q), \dots, \text{tfidf}(t_n, q)]}{\sqrt{\sum_{t \in q} \text{tfidf}^2(t, q)}}$$

$$\vec{d} = \frac{[\text{tfidf}(t_1, d), \dots, \text{tfidf}(t_n, d)]}{\sqrt{\sum_{t \in d} \text{tfidf}^2(t, d)}}$$



$$\text{score}(q, d) =$$

$$\sum_{t_i \in q} \frac{\text{tfidf}(t_i, q)}{\sqrt{\sum_{t \in q} \text{tfidf}^2(t, q)}} \cdot \frac{\text{tfidf}(t_i, d)}{\sqrt{\sum_{t \in d} \text{tfidf}^2(t, d)}}$$

Tf-idf scoring example

- A collection of 4 nano documents

Query: sweet love

Doc 1: Sweet sweet nurse! Love?

Doc 2: Sweet sorrow

Doc 3: How sweet is love?

Doc 4: Nurse!

Query vector $\vec{q} = (0.383, 0.924)$

word	cnt	tf	df	idf	Query	
					tf-idf	n'lized = tf-idf/ q
sweet	1	1	3	0.125	0.125	0.383
nurse	0	0	2	0.301	0	0
love	1	1	2	0.301	0.301	0.924
how	0	0	1	0.602	0	0
sorrow	0	0	1	0.602	0	0
is	0	0	1	0.602	0	0
$ q = \sqrt{.125^2 + .301^2} = .326$						

Tf-idf scoring example

Query vector $\vec{q} = (0.383, 0.924)$

Document 1					
word	cnt	tf	tf-idf	n'lized	$\times q$
sweet	2	1.301	0.163	0.357	0.137
nurse	1	1.000	0.301	0.661	0
love	1	1.000	0.301	0.661	0.610
how	0	0	0	0	0
sorrow	0	0	0	0	0
is	0	0	0	0	0
$ d_1 = \sqrt{.163^2 + .301^2 + .301^2} = .456$					

$$\vec{d}_1 = (0.357, 0.661)$$

$$\text{score}(\vec{q}, \vec{d}_1) = 0.747$$

Therefore, d_1 is more relevant

Document 2					
word	cnt	tf	tf-idf	n'lized	$\times q$
sweet	1	1.000	0.125	0.203	0.0779
nurse	0	0	0	0	0
love	0	0	0	0	0
how	0	0	0	0	0
sorrow	1	1.000	0.602	0.979	0
is	0	0	0	0	0
$ d_2 = \sqrt{.125^2 + .602^2} = .615$					

$$\vec{d}_2 = (0.203)$$

$$\text{score}(\vec{q}, \vec{d}_1) = 0.0779$$

Query: sweet love

Doc 1: Sweet sweet nurse! Love?

Doc 2: Sweet sorrow

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Motivation: Distributional semantics

- Distributional semantics: A word's meaning is given by the words that frequently appear close-by
- “You shall know a word by the company it keeps” (J. R. Firth 1957: 11)
- When a word w appears in a text, its local **context** is the set of words that co-occur within a fixed-size window

*...government debt problems turning into **banking** crises as happened in 2009...*
*...saying that Europe needs unified **banking** regulation to replace the hodgepodge...*
*...India has just given its **banking** system a shot in the arm...*

The meaning of “banking” is represented by these context words

Example from: <https://web.stanford.edu/class/archive/cs/cs224n/cs224n.1224/>

In What Form of Representation?

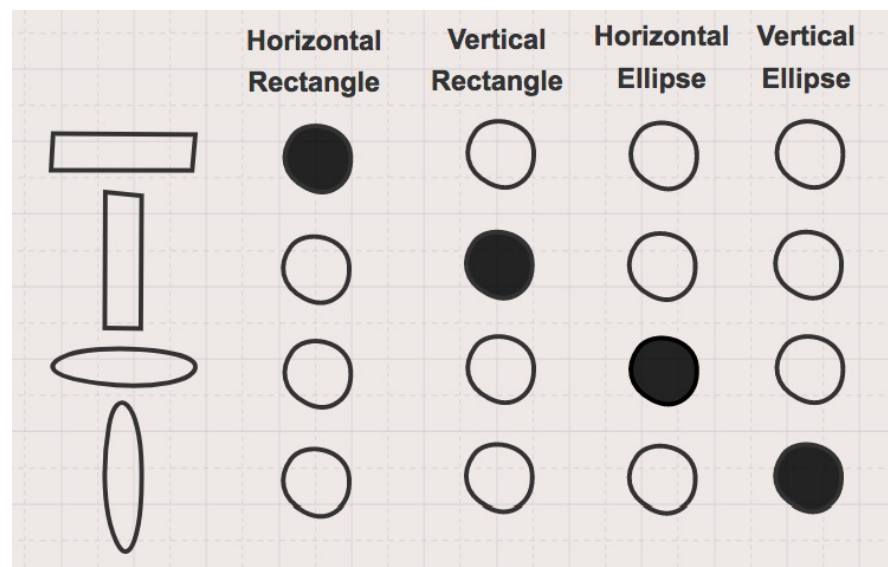
- **Goal:** Obtain a dense vector for each word, so that word sense similarity can be computed via vector distance, such as dot product

$$e_{apple} = \begin{bmatrix} 0.00 \\ -0.01 \\ 0.03 \\ 0.95 \\ \dots \\ 0.21 \end{bmatrix} \quad e_{orange} = \begin{bmatrix} -0.01 \\ 0.00 \\ 0.02 \\ 0.97 \\ \dots \\ 0.22 \end{bmatrix} \quad \left. \vphantom{\begin{bmatrix} 0.00 \\ -0.01 \\ 0.03 \\ 0.95 \\ \dots \\ 0.21 \end{bmatrix}} \right\} \begin{array}{l} \text{Common dimension size:} \\ 100\text{-d}, 200\text{-d}, 300\text{-d}, \dots \end{array}$$

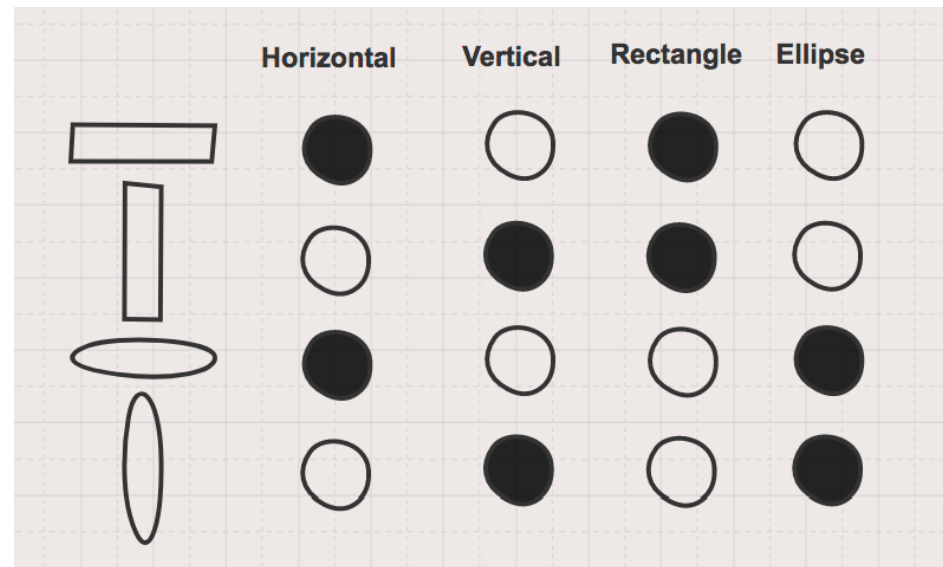
These dense word vectors are also called word **embeddings** (嵌入)
(which implies the idea of placing or mapping words into some continuous vector space)

Intuition: One-hot vs. Distributed repr.

One-hot representation



Distributed representation



The individual dimensions of a word embedding do not have concrete “meanings”

$$E_{Orange} = \begin{bmatrix} -0.01 \\ 0.00 \\ 0.02 \\ 0.97 \\ \dots \\ 0.22 \end{bmatrix}$$

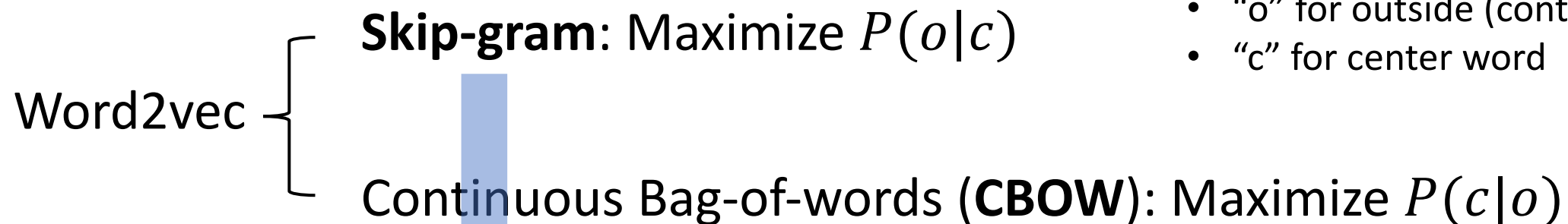
For instance, e_{orange} It does NOT mean
 1st dimension -0.01 is for “animalness”
 4th dimension 0.97 is for “fruitness”
 They are only meaningful when compared to other words

Question: How to obtain word embeddings?

- An effective and efficient method: Word2vec (Mikolov et al. 2013 a&b)
- **Basic Idea:**
- Given a corpus as a list of words
- Go through each position t in the text, which has a center word c and context (“outside”) words o
- Use the **similarity of word vectors** between c and o to **compute the probability** of o given c , i.e., conditional probability $P(o|c)$ (or vice versa)
- **Maximize this probability** by keep adjusting the word vectors

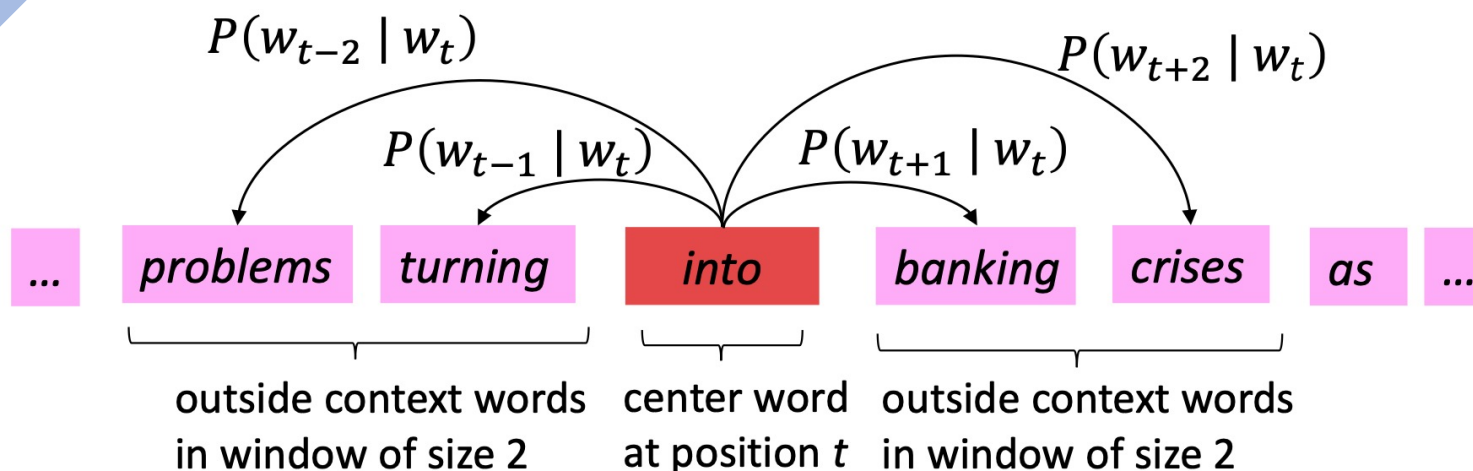
使用中心词 c 与上下文词 o 的词向量相似度来计算条件概率 (l) (或者反过来)

Two architectures of Word2vec



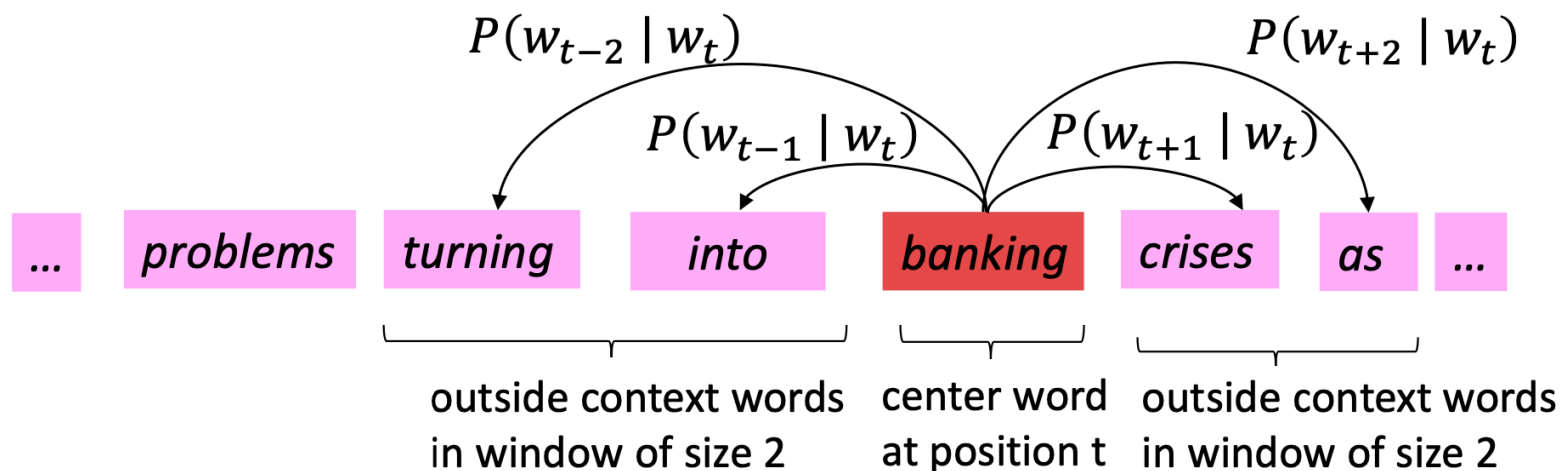
- “o” for outside (context) words
- “c” for center word

Compute probability
 $P(w_{t+j}|w_t)$,
 for $j \in \{-2, -1, 1, 2\}$
 when window size is 2



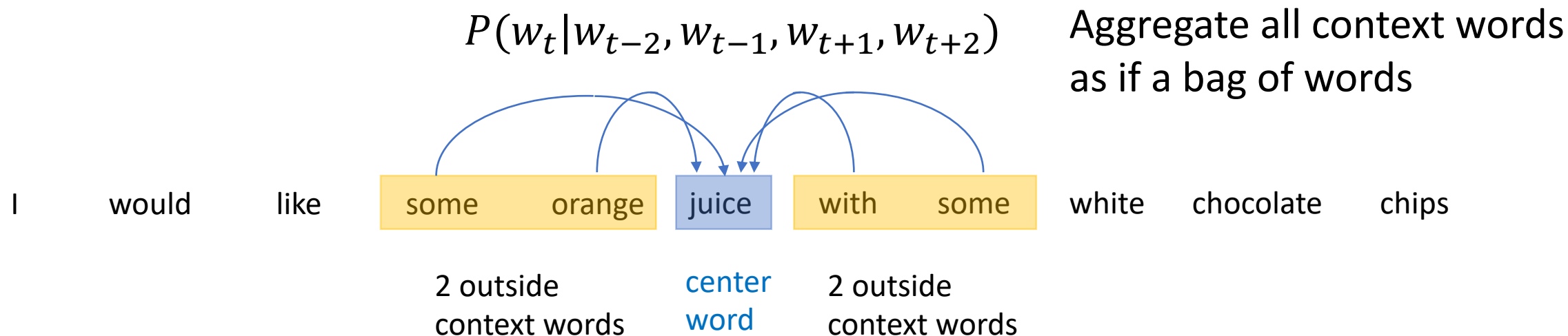
Use A Moving Window $t \leftarrow t + 1$

Skip-gram: Compute probability $P(w_{t+j}|w_t)$, for $j \in \{-2, -1, 1, 2\}$ when window size is 2



Example from: <https://web.stanford.edu/class/archive/cs/cs224n/cs224n.1224/>

Continuous Bag-of-Words (CBOW)



Compute only one probability at position t :
 $P(w_t | w_{t-2}, w_{t-1}, w_{t+1}, w_{t+2})$, for window size 2

Word2vec Objective Function (Skip-gram as example)

- Given a data set of T tokens, for each position $t = 1, \dots, T$, we compute the conditional probability $P(w_{t+j}|w_t)$, for $j \in \{-m, \dots, m\}$, with window size m
- Then the *likelihood* of data is:

$$\mathcal{L}(\theta) = \prod_{t=1}^T \prod_{\substack{-m \leq j \leq m \\ j \neq 0}} P(w_{t+j}|w_t; \theta)$$

θ denotes model parameters, that is, all the word **embeddings** to be learned!

- The objective function (cost/loss) is the negative log-likelihood

$$J(\theta) = -\frac{1}{T} \log \mathcal{L}(\theta) = -\frac{1}{T} \sum_{t=1}^T \sum_{\substack{-m \leq j \leq m \\ j \neq 0}} \log P(w_{t+j}|w_t; \theta)$$

Question: How to compute $P(w_{t+j}|w_t; \theta)$?

- **Solution:** Use *two* vectors per word w
- When w is a center word, its vector is v_w
- When w is a context (outside) word, its vector is u_w
- Then the conditional probability of context word o given center word c can be computed using **softmax** function:

$$P(o|c) = \frac{\exp(u_o^\top v_c)}{\sum_{w \in V} \exp(u_w^\top v_c)}$$

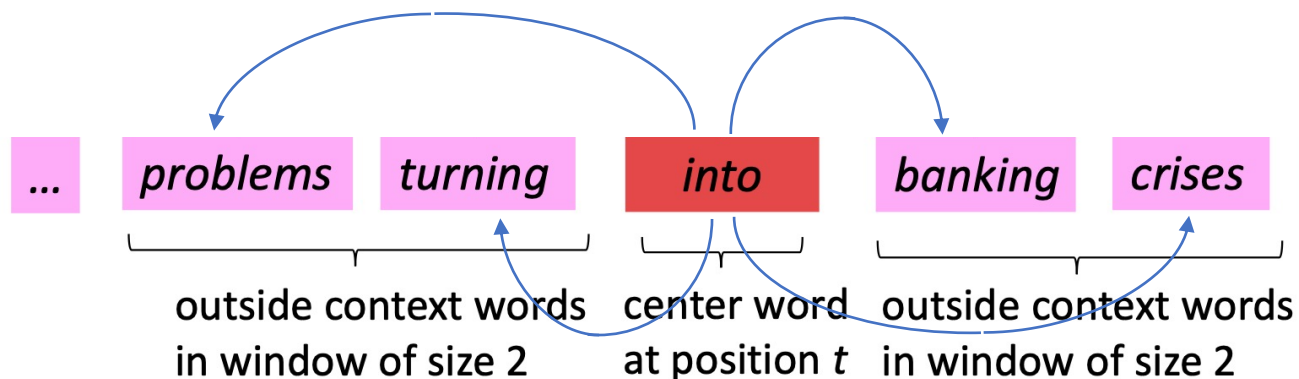
Dot product measures the similarity between o and c

Normalized over the entire vocabulary

Compute probabilities using softmax

$$P(\text{problems}|\text{into}) = \frac{\exp(u_{\text{problems}}^T v_{\text{into}})}{\sum_{w \in V} \exp(u_w^T v_{\text{into}})}$$

$$P(\text{banking}|\text{into}) = \frac{\exp(u_{\text{banking}}^T v_{\text{into}})}{\sum_{w \in V} \exp(u_w^T v_{\text{into}})}$$



$$P(\text{turning}|\text{into}) = \frac{\exp(u_{\text{turning}}^T v_{\text{into}})}{\sum_{w \in V} \exp(u_w^T v_{\text{into}})}$$

$$P(\text{crises}|\text{into}) = \frac{\exp(u_{\text{crises}}^T v_{\text{into}})}{\sum_{w \in V} \exp(u_w^T v_{\text{into}})}$$

Example from: <https://web.stanford.edu/class/archive/cs/cs224n/cs224n.1224/>

Number of Parameters

- Because *two* vectors are used per word w : v_w and u_w
- => Two parameter tables, or, embedding matrices

Usually we keep the target table **V** as the trained embeddings

Total # of params:
 $2 * [\text{vocab-size}] * [\text{emb-size}]$

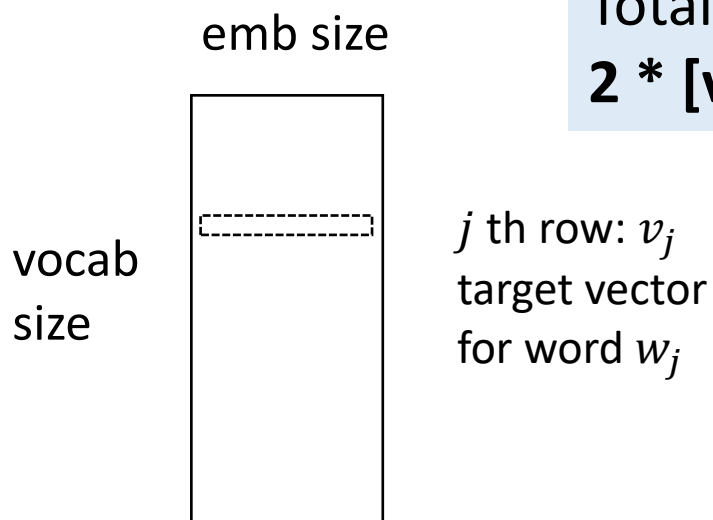


Table **V** contains all parameters for **center** vectors

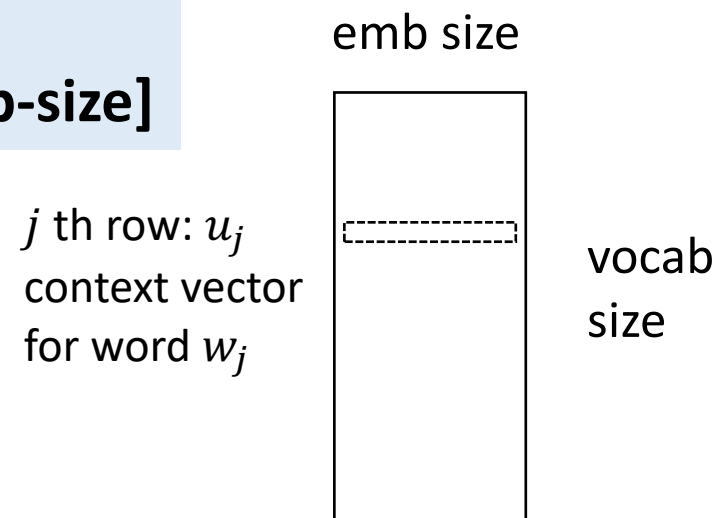
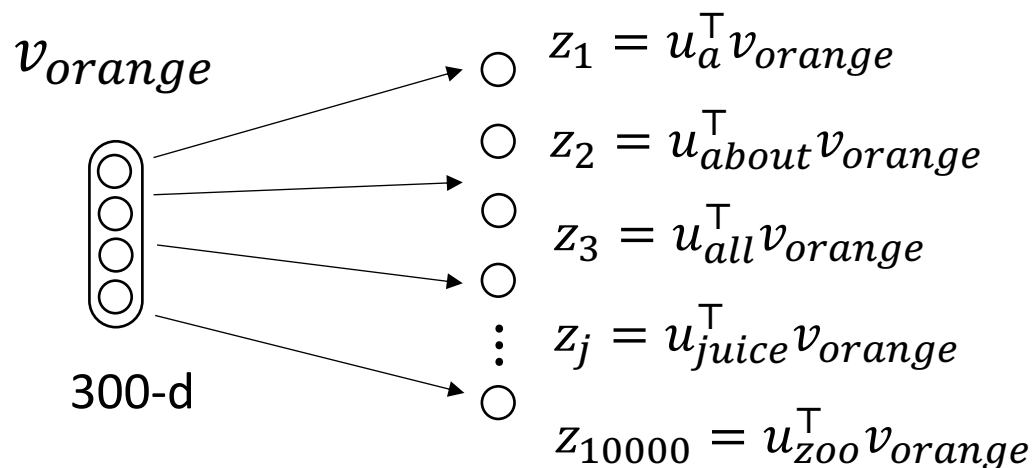


Table **U** contains all parameters for **context** vectors

Problem with Softmax

center context
I would like some orange juice with some white chocolate chips

$$P(\text{juice}|\text{orange}) = \frac{\exp(u_{\text{juice}}^{\top} v_{\text{orange}})}{\sum_{w \in V} \exp(u_w^{\top} v_{\text{orange}})}$$



For a vocabulary of 10,000 words

Needs 10,000 times of dot product to compute the denominator

To Overcome Softmax

Solutions

- 1. Hierarchical softmax
- 2. Negative sampling

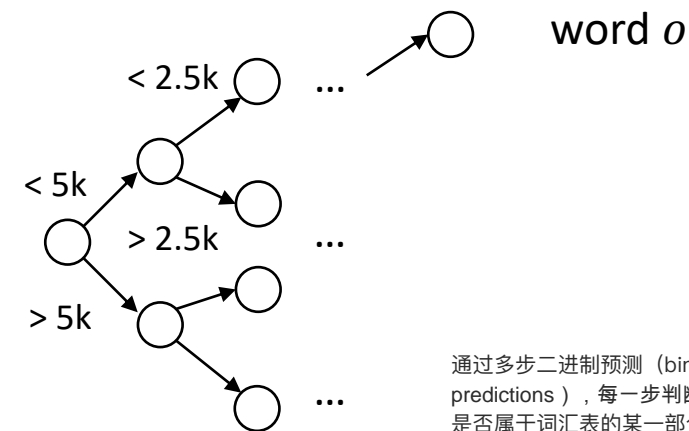


Make binary predictions instead:

$$P\left(o < \frac{|V|}{2} \middle| c\right)$$

The probability of word o belongs to the 1st half of vocabulary

For vocabulary size $|V| = 10k$



Multiple steps of binary predictions until word o is found

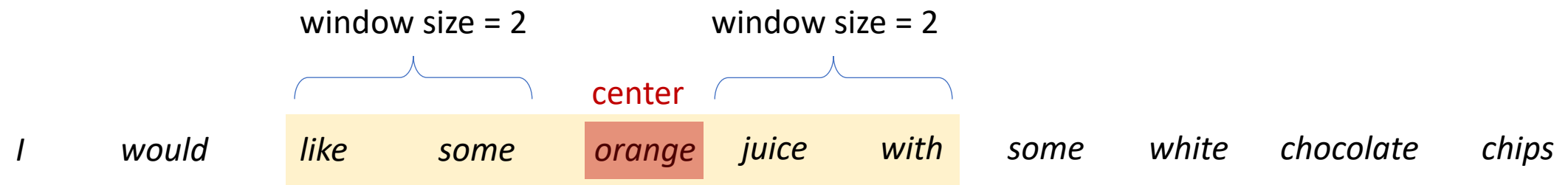
Time complexity $O(\log(|V|))$

$$\text{Then } P(o|c) = P(o < 5k|c) \cdot P(o < 2.5k|c) \cdot P(o < 1.25k|c) \dots$$

product of probabilities along the path

Reference: <http://ruder.io/word-embeddings-softmax/>

Solution 2: Negative sampling intuition



Intuition: Given a center word, predict if a *randomly sampled* word is its context or not (within a fixed window)

Center	Outside Word	Label
orange	juice	1
orange	king	0
orange	the	0
orange	of	0
orange	book	0

x
 y

Step 1: Pick a context word within the window

Positive sample

Step 2: Randomly pick k words from the entire vocabulary that do not appear in the window

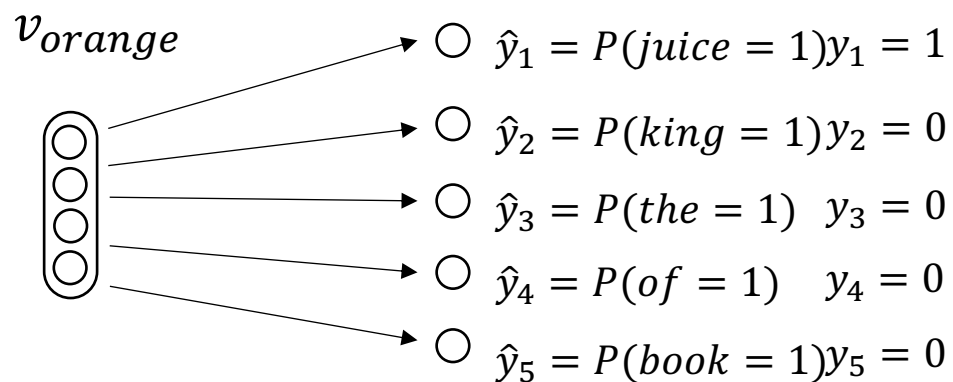
Negative samples

Negative sampling intuition

c	o	y
Center	Outside	Context or not
<i>orange</i>	<i>juice</i>	1
<i>orange</i>	<i>king</i>	0
<i>orange</i>	<i>the</i>	0
<i>orange</i>	<i>of</i>	0
<i>orange</i>	<i>book</i>	0

Instead of using softmax: $P(o|c) = \frac{\exp(u_o \cdot v_c)}{\sum_{j=1 \dots |V|} \exp(u_j \cdot v_c)} = \hat{y}_t$

Use **logistic regression**: $P(y = 1|c, o) = \sigma(u_o \cdot v_c)$



$k+1$ times of logistic regression

Negative Sampling: Objective Function (loss)

- For token at position t , maximize the log-likelihood:

Word o is the positive sample

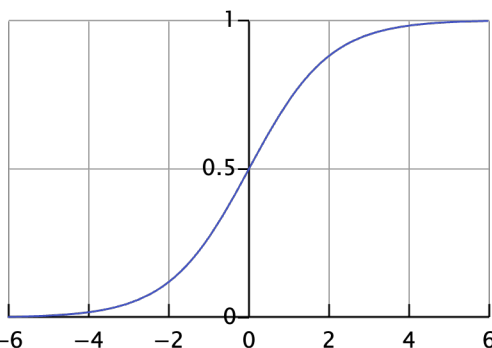
$$J_t(\theta) = \log \sigma(u_o^\top v_c) + \sum_{i=1}^k \mathbb{E}_{w_i \sim P(w)} [\log \sigma(-u_{w_i}^\top v_c)]$$

The k words w_i ($i = 1 \dots k$) are the negative samples

- Sigmoid function $\sigma(u_o^\top v_c)$ outputs the probability of o in the context window of c

$$\sigma(x) = \frac{1}{1 + e^{-x}}$$

a monotone increasing function



Maximizing this term will push the dot product $u_o^\top v_c$ to **larger** values, i.e., making o and c closer in semantic space

Maximizing this term will push the dot product $u_{w_i}^\top v_c$ to **smaller** values, i.e., making w_i and c farther apart in semantic space

Loss \Rightarrow negation of objective

$$\mathcal{L}(\theta) = -J_t(\theta) = - \left[\log \sigma(u_o^\top v_c) + \sum_{i=1}^k \mathbb{E}_{w_i \sim P(w)} [\log \sigma(-u_{w_i}^\top v_c)] \right]$$

$$\mathcal{L}(\theta) = - \left[\log \sigma(u_{\text{pos}} \cdot v) + \sum_{i=1}^k \log \sigma(-u_{\text{neg}_i} \cdot v) \right] \quad \downarrow \text{(Simplify the subscripts)}$$

u_{pos} , u_{neg} , and v are all learnable parameters

Need to derive derivatives: $\frac{\partial \mathcal{L}}{\partial u_{\text{pos}}}$, $\frac{\partial \mathcal{L}}{\partial u_{\text{neg}_i}}$, $\frac{\partial \mathcal{L}}{\partial v}$

Loss: derivatives

$$\mathcal{L}(\theta) = - \left[\log \sigma(u_{\text{pos}} \cdot v) + \sum_{i=1}^k \log \sigma(-u_{\text{neg}_i} \cdot v) \right]$$

Using the knowledge:



$$\frac{\partial \sigma(z)}{\partial z} = \sigma(z)(1 - \sigma(z))$$

$$\frac{\partial \mathcal{L}}{\partial u_{\text{pos}}} = (\sigma(u_{\text{pos}} \cdot v) - 1)v$$

$$\frac{\partial \mathcal{L}}{\partial u_{\text{neg}_i}} = (\sigma(u_{\text{neg}_i} \cdot v))v$$

$$\frac{\partial \mathcal{L}}{\partial v} = (\sigma(u_{\text{pos}} \cdot v) - 1)u_{\text{pos}} + \sum_{i=1}^k \sigma(u_{\text{neg}_i} \cdot v)u_{\text{neg}_i}$$

Gradients update with SGD

- Start with randomly initialized U and V matrices, and do the updates

$$u_{\text{pos}}^{t+1} = u_{\text{pos}}^t - \alpha(\sigma(u_{\text{pos}}^t \cdot v^t) - 1)v^t$$

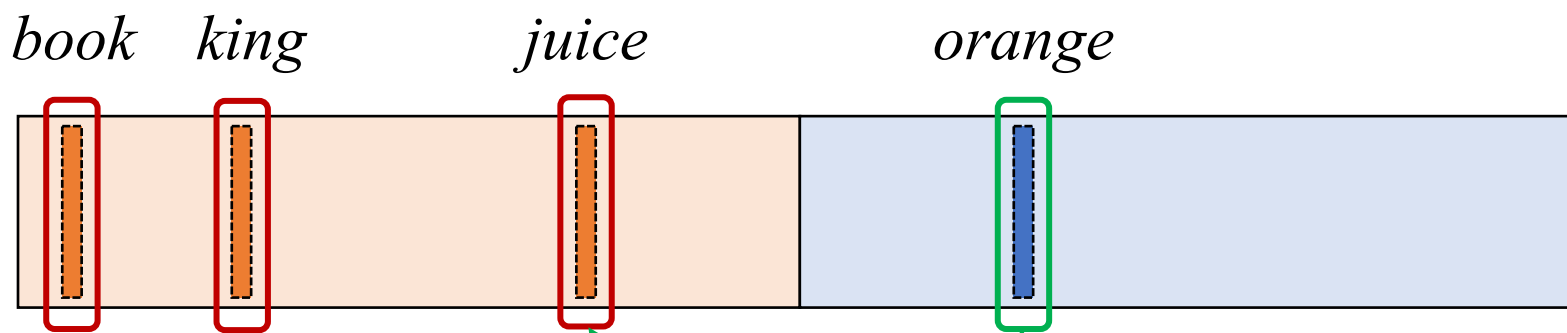
$$u_{\text{neg}_i}^{t+1} = u_{\text{neg}_i}^t - \alpha(\sigma(u_{\text{neg}_i}^t \cdot v^t))v^t$$

$$v^{t+1} = v^t - \alpha \left[\left((\sigma(u_{\text{pos}}^t \cdot v^t) - 1) - 1 \right) u_{\text{pos}}^t + \sum_{i=1}^k \sigma(u_{\text{neg}_i}^t \cdot v^t) u_{\text{neg}_i}^t \right]$$

α is the learning rate

Intuition: One step of gradient descent

Negative samples Target word Center word



Move *orange* and *book* apart

Move *orange* and *king* apart

Move *orange* and *juice* closer
increasing $u_{\text{pos}} \cdot v$

U

V

More details: choosing window size

- **Small windows** ($k = 2$) : nearest words are syntactically similar words in same taxonomy
 - Nearest neighbor of *Hogwarts*: Sunnydale, Evernight, Blandings (Other school names)
- **Large windows** ($k = 5$) : nearest words are related words in same semantic field
 - Nearest neighbor of *Hogwarts*: Dumbledore, half-blood, Malfoy (entities in the HP world)

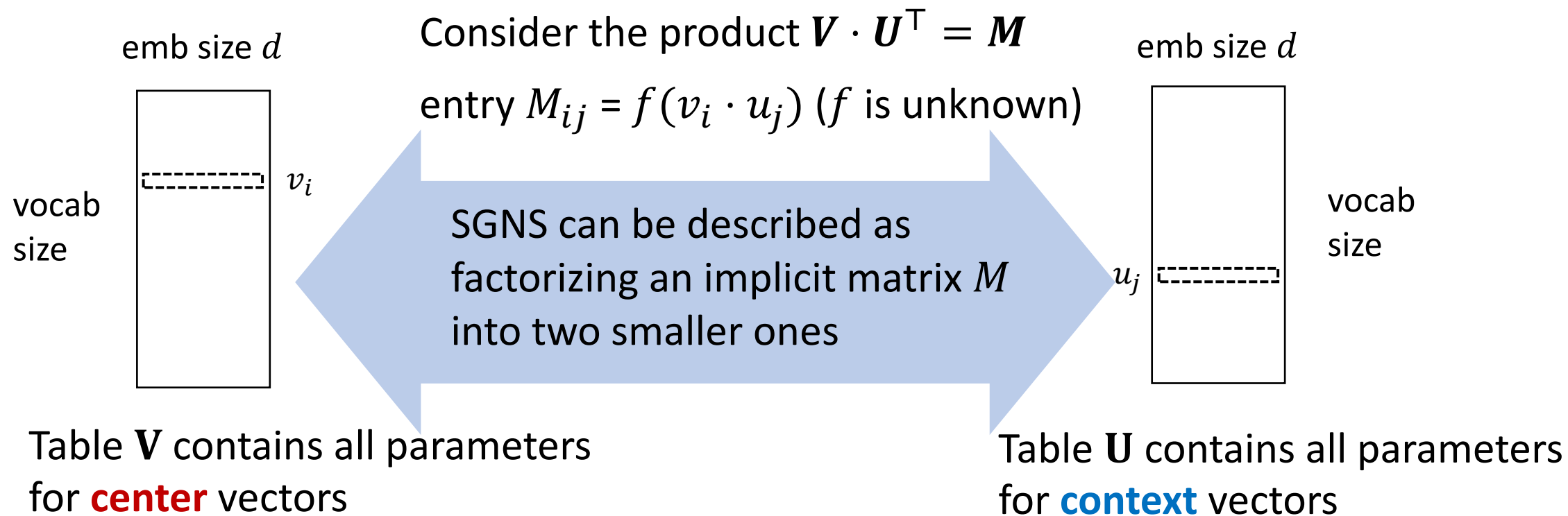
More Details : Sample less frequent words

- Maximize probability that *real* outside word appears;
- Minimize probability that *random* words appear around center word
- Sample from the distribution $P(w) = \frac{U(w)^{\frac{3}{4}}}{Z}$, the unigram frequency distribution $U(w)$ raised to the $\frac{3}{4}$ power (Z is normalization term)
- The power makes less frequent words be sampled more often
- $0.9^{3/4} \approx 0.924 \Rightarrow$ a 2.7% increase in chance being sampled
- $0.1^{3/4} \approx 0.178 \Rightarrow$ a 77.8% increase in chance being sampled

Why word2vec works?

- Levy and Goldberg, 2014, Neural Word Embedding as Implicit Matrix Factorization

Why Skip-gram negative sampling (SGNS) works?



Word2vec as implicit matrix factorization

Equations from Levy and Goldberg, 2014

- How to find the unknown function $M_{ij} = f(v_i \cdot u_j)$?
- Start from the SGNS loss:

$$\begin{aligned}\ell &= \sum_{w \in V_W} \sum_{c \in V_C} \#(w, c) (\log \sigma(\vec{w} \cdot \vec{c})) + \sum_{w \in V_W} \sum_{c \in V_C} \#(w, c) (k \cdot \mathbb{E}_{c_N \sim P_D} [\log \sigma(-\vec{w} \cdot \vec{c}_N)]) \\ &= \sum_{w \in V_W} \sum_{c \in V_C} \#(w, c) (\log \sigma(\vec{w} \cdot \vec{c})) + \sum_{w \in V_W} \#(w) (k \cdot \mathbb{E}_{c_N \sim P_D} [\log \sigma(-\vec{w} \cdot \vec{c}_N)])\end{aligned}$$

This eq from Levy and Goldberg (2014) use different terms:

$w \in V_W \Rightarrow v_w \in V$

$c \in V_C \Rightarrow u_c \in U$

$\#(w, c)$ denotes the number of times
the pair (w, c) appears in D

P_D is the prob. distr.
to sample NSs

Word2vec as implicit matrix factorization

Equations from Levy and Goldberg, 2014

- Explicitly express the expectation term:

$$\begin{aligned}\mathbb{E}_{c_N \sim P_D} [\log \sigma(-\vec{w} \cdot \vec{c}_N)] &= \sum_{c_N \in V_C} \frac{\#(c_N)}{|D|} \log \sigma(-\vec{w} \cdot \vec{c}_N) \\ &= \frac{\#(c)}{|D|} \log \sigma(-\vec{w} \cdot \vec{c}) + \sum_{c_N \in V_C \setminus \{c\}} \frac{\#(c_N)}{|D|} \log \sigma(-\vec{w} \cdot \vec{c}_N)\end{aligned}$$

- Get the local loss for a specific (w, c) pair:

$$\ell(w, c) = \#(w, c) \log \sigma(\vec{w} \cdot \vec{c}) + k \cdot \#(w) \cdot \frac{\#(c)}{|D|} \log \sigma(-\vec{w} \cdot \vec{c})$$

Word2vec as implicit matrix factorization

Equations from Levy and Goldberg, 2014

- Let $x = w \cdot c$, and calculate the partial derivative:

$$\frac{\partial \ell}{\partial x} = \#(w, c) \cdot \sigma(-x) - k \cdot \#(w) \cdot \frac{\#(c)}{|D|} \cdot \sigma(x)$$

- Compare the derivative to zero, with some simplification:

$$e^{2x} - \left(\frac{\#(w, c)}{k \cdot \#(w) \cdot \frac{\#(c)}{|D|}} - 1 \right) e^x - \frac{\#(w, c)}{k \cdot \#(w) \cdot \frac{\#(c)}{|D|}} = 0$$

Word2vec as implicit matrix factorization

Equations from Levy and Goldberg, 2014

- If we let $y = e^x$, it becomes a quadratic equation of y
- and the solution is:

$$y = \frac{\#(w, c)}{k \cdot \#(w) \cdot \frac{\#(c)}{|D|}} = \frac{\#(w, c) \cdot |D|}{\#w \cdot \#(c)} \cdot \frac{1}{k}$$

numerator: among all occurrences of c , the chance of w co-occurs with it

- Substituting y with e^x and x with $w \cdot c$:

$$\vec{w} \cdot \vec{c} = \log \left(\frac{\#(w, c) \cdot |D|}{\#(w) \cdot \#(c)} \cdot \frac{1}{k} \right) = \log \left(\frac{\#(w, c) \cdot |D|}{\#(w) \cdot \#(c)} \right) - \log k$$

$$\log \left(\frac{\#(w, c) / \#(c)}{\#(w) / |D|} \right)$$

denominator: prior (global) probability of w

It is exactly the point-wise mutual information (PMI) between w and c !

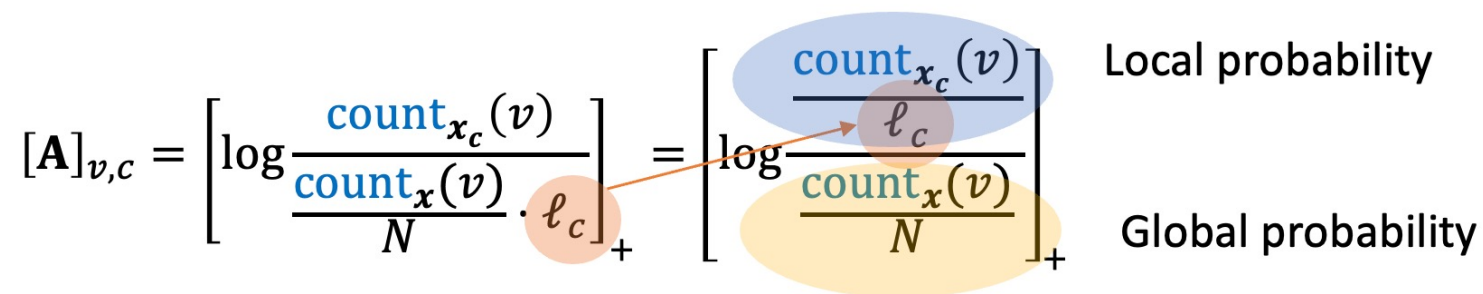
Word2vec SGNS related to PMI

- Recall PMI from LSA

$$[A]_{v,c} = \left[\log \frac{\text{count}_{x_c}(v)}{\frac{\text{count}_x(v)}{N} \cdot \ell_c} \right]_+ = \left[\log \frac{\frac{\text{count}_{x_c}(v)}{\ell_c}}{\frac{\text{count}_x(v)}{N}} \right]_+$$

Local probability

Global probability



- Finally, we can describe the matrix M that SGNS is factorizing:

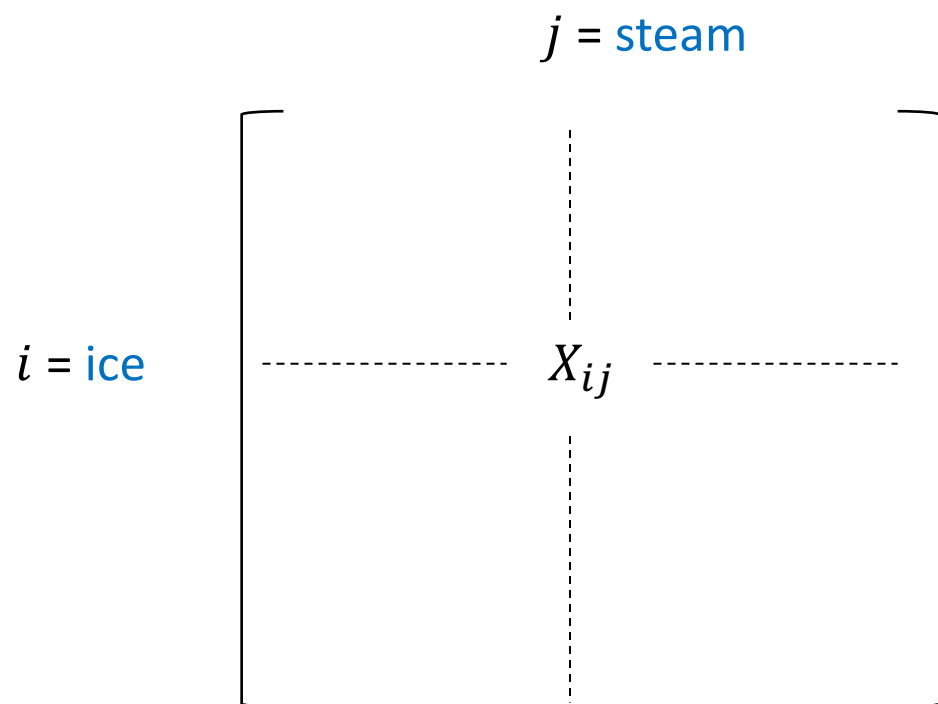
$$M_{ij}^{\text{SGNS}} = W_i \cdot C_j = \vec{w}_i \cdot \vec{c}_j = \text{PMI}(w_i, c_j) - \log k$$

- When $k = 1$, SGNS is factorizing a word-context matrix, in which the unknown association function between w and c is $f(w, c) = \text{PMI}(w, c)$
- When $k > 1$, SGNS is factorizing a *shifted* PMI matrix

GloVe: From a different perspective

Pennington, J., Socher, R., & Manning, C. (2014). Glove: Global vectors for word representation.

- Start from the word-word co-occurrence counts X
 - X_{ij} is the number of times word j occurs in the context of word i



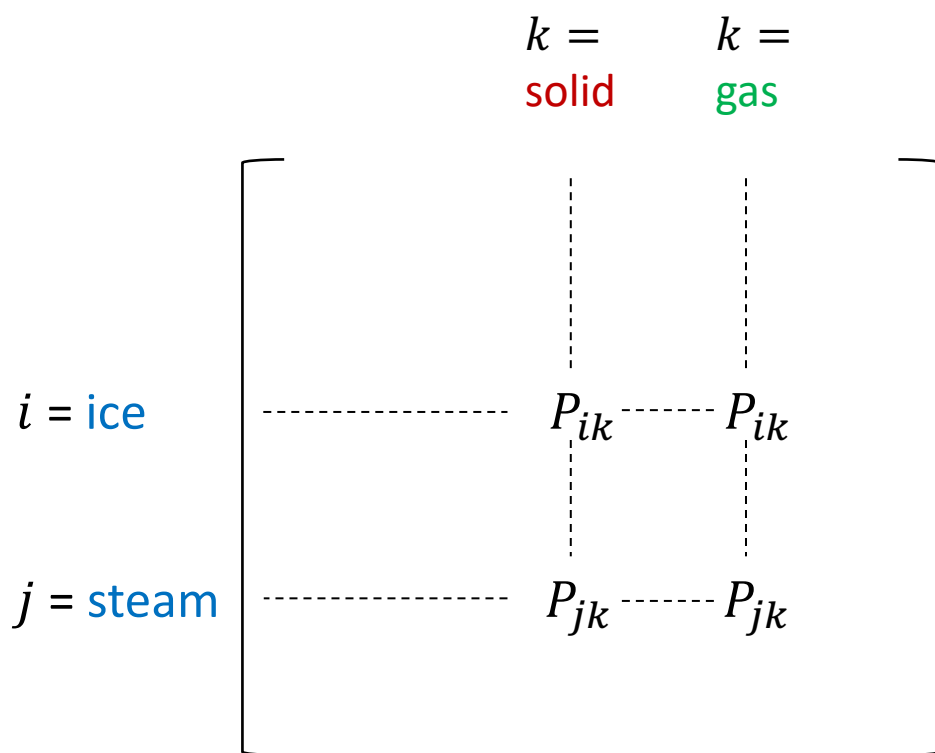
Let $X_i = \sum_k X_{ik}$ be the number of times any word occurs in the context of word i

Let $P_i = P(j|i) = X_{ij}/X_i$ be the probability that word j occurs in the context of word i

GloVe: a showcase

Pennington, J., Socher, R., & Manning, C. (2014)

- Consider $i = \text{ice}$ and $j = \text{steam}$



The relationship between ice and steam can be examined by studying the *ratio of their co-occurrence probabilities* with various **probe** words k

E.g., for $k = \text{solid}$ (related to ice but not steam)

we expect the ratio $\frac{P_{ik}}{P_{jk}}$ to be large, as $P_{ik} \gg P_{jk}$

For $k = \text{gas}$ (related to steam but not ice)

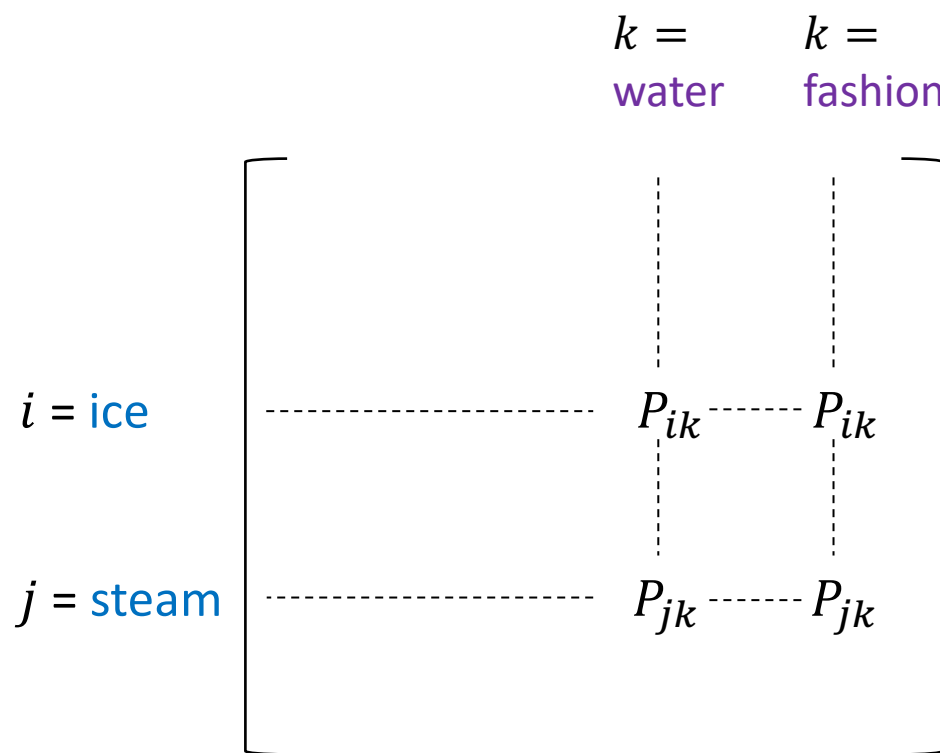
we expect the ratio $\frac{P_{ik}}{P_{jk}}$ to be small, as $P_{ik} \ll P_{jk}$

GloVe: a showcase

Pennington, J., Socher, R., & Manning, C. (2014)

- For words k that are either related to both, say **water**
- or to neither, say **fashion**

The ratio should $\frac{P_{ik}}{P_{jk}}$ be close to one,
as $P_{ik} \approx P_{jk}$



Probability and Ratio	$k = \text{solid}$	$k = \text{gas}$	$k = \text{water}$	$k = \text{fashion}$
$P(k \text{ice})$	1.9×10^{-4}	6.6×10^{-5}	3.0×10^{-3}	1.7×10^{-5}
$P(k \text{steam})$	2.2×10^{-5}	7.8×10^{-4}	2.2×10^{-3}	1.8×10^{-5}
$P(k \text{ice})/P(k \text{steam})$	8.9	8.5×10^{-2}	1.36	0.96

GloVe: introduce the basic idea

- The appropriate starting point for learning word vectors should be the **ratios of co-occurrence probabilities** rather than the probabilities themselves.
- The most general embedding model takes the form:

$$F(w_i, w_j, \tilde{w}_k) = \frac{P_{ik}}{P_{jk}} \quad \Rightarrow \quad F(w_i - w_j, \tilde{w}_k) = \frac{P_{ik}}{P_{jk}}$$

- The final form takes several steps further following a few desiderata

$$F((w_i - w_j)^T \tilde{w}_k) = \frac{P_{ik}}{P_{jk}} \quad \Rightarrow \quad F((w_i - w_j)^T \tilde{w}_k) = \frac{F(w_i^T \tilde{w}_k)}{F(w_j^T \tilde{w}_k)} \quad \Rightarrow \quad F(w_i^T \tilde{w}_k) = P_{ik} = \frac{X_{ik}}{X_i}$$

$$\downarrow$$

$$w_i^T \tilde{w}_k = \log(P_{ik}) = \log(X_{ik}) - \log(X_i)$$

GloVe

Pennington, J., Socher, R., & Manning, C. (2014). Glove: Global vectors for word representation.

Cost function:
$$J = \sum_{i,j=1}^V f(X_{ij})(w_i^\top \tilde{w}_j + b_i + \tilde{b}_j - \log(X_{ij}))^2$$

Dot product of two embeddings Frequency counts of word i and j co-occur (within a fixed window)

Basic idea: words that appear together more often (larger X_{ij}) should have closer meanings (larger dot product)

Advantages: Fast training; scalable to large corpora

GloVe: interesting connection to word2vec

Pennington, J., Socher, R., & Manning, C. (2014). Glove: Global vectors for word representation.

- Choice for the weighting-function $f(\cdot)$

Cost function:
$$J = \sum_{i,j=1}^V f(X_{ij})(w_i^\top \tilde{w}_j + b_i + \tilde{b}_j - \log(X_{ij}))^2$$

- 1. $f(0) = 0$, as $x \rightarrow 0$, it should vanish fast so that $\lim_{x \rightarrow 0} f(x) \log x$ is finite
- 2. $f(x)$ should be non-decreasing so that rare co-occurrences are not overweighted
- 3. $f(x)$ should be small for larger x , so that frequent co-occurrences (“of”, “the”) are not overweighted

GloVe: interesting connection to word2vec

Pennington, J., Socher, R., & Manning, C. (2014). Glove: Global vectors for word representation.

- Choice for the weighting-function $f(\cdot)$ $J = \sum_{i,j=1}^V f(X_{ij})(w_i^\top \tilde{w}_j + b_i + \tilde{b}_j - \log(X_{ij}))^2$

$$f(x) = \begin{cases} (x/x_{\max})^\alpha & \text{if } x < x_{\max} \\ 1 & \text{otherwise} \end{cases}$$

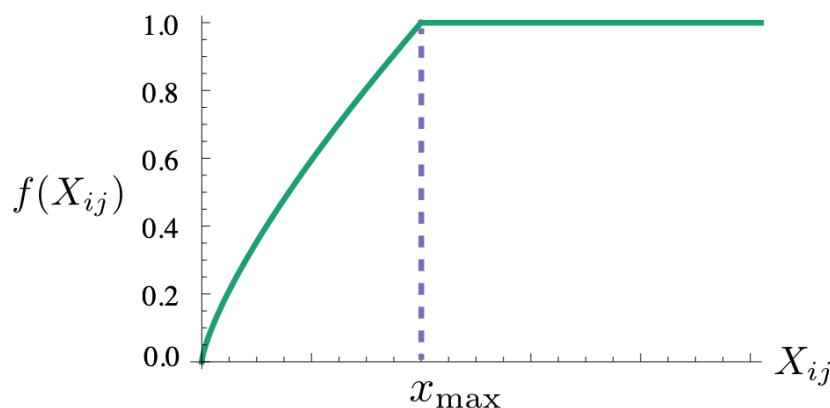


Figure 1: Weighting function f with $\alpha = 3/4$.

It is the similar fractional power scaling used for negative sampling in word2vec

$$P(w) = \frac{U(w)^{\frac{3}{4}}}{Z} \sum_{i=1}^k \mathbb{E}_{w_i \sim P(w)} [\log \sigma(-u_{w_i}^\top v_c)]$$

Content

- Motivation
- Documents and Counts-based Method
- Neural Network-based Method -- word2vec
- **Evaluation and Applications**

General Evaluation in NLP

- Intrinsic (内在的) vs. Extrinsic (外在的)
- Intrinsic:
 - Evaluation on a specific/intermediate subtask
 - Fast to compute
 - Not clear if really helpful unless correlation to real task is found
- Extrinsic:
 - Evaluation on a real task
 - Can take a long time to compute accuracy
 - Unclear if the subsystem is the problem or its interaction with other subsystems

Adapted from: <https://web.stanford.edu/class/archive/cs/cs224n/cs224n.1224/>

Evaluate Word Vectors (Embeddings)

- Intrinsic task: Word semantic similarity task

$$d_1 = \cos(v_{book}, v_{library}), \text{ cosine similarity}$$

Word1	Word2	Human score	Cosine distance
book	library	7.46	d1
bank	money	8.12	d2
wood	forest	7.73	d3
professor	cucumber	0.31	d4
...

Correlation between the two columns are used to evaluate the quality of word embeddings

Evaluate Word Vectors (Embeddings)

- Intrinsic task: Word analogy task

Question: What is to “King” as “woman” to “man”?

$$v_{Man} - v_{Woman} \approx v_{King} - v_w \quad w = ?$$

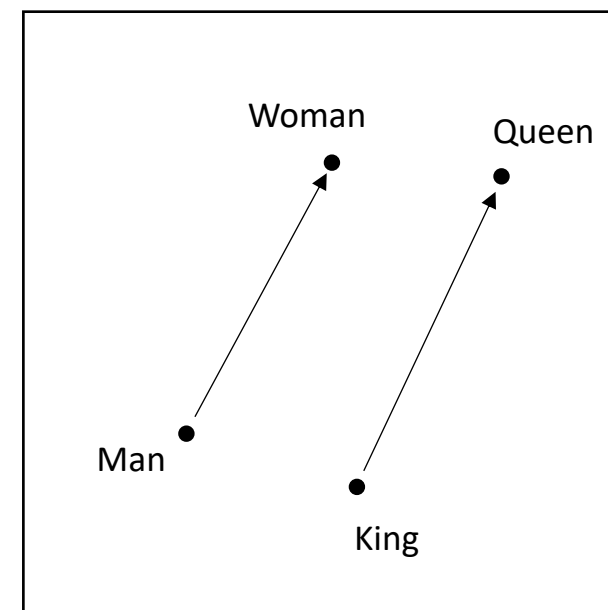
Find the word w so that:

$$\arg \max_w \text{sim}(v_w, v_{King} - v_{Man} + v_{Woman})$$

Here, $\text{sim}()$ is a similarity function, for example, cosine similarity

$$\text{sim}(u, v) = \frac{u^T v}{\|u\| \|v\|}$$

Finding the most similar vector v_w will hopefully pick up the word $w = \text{Queen}$



Word Analogy Task (as an interesting application)

Capital-common-countries:

Athens Greece Baghdad **Iraq**
Athens Greece Bangkok **Thailand**
Athens Greece Beijing **China**
Athens Greece Berlin **Germany**
...

Family:

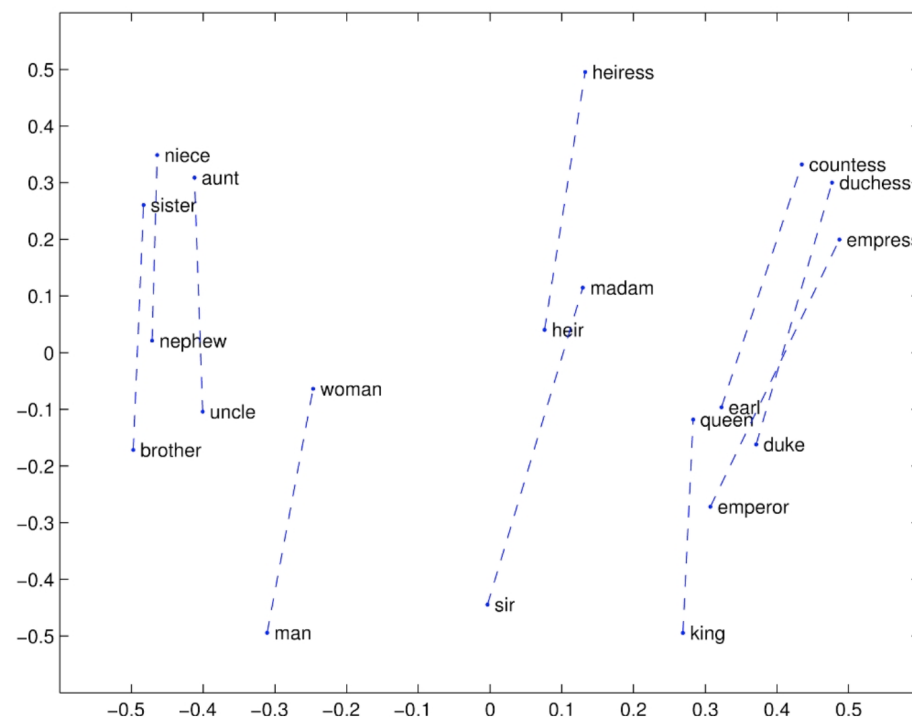
boy girl brother **sister**
boy girl brothers **sisters**
boy girl dad **mom**
boy girl father **mother**
...

Comparative:

bad worse big bigger
bad worse bright brighter
bad worse cheap cheaper
bad worse cold colder
...

City-in-state:

Chicago Illinois Houston **Texas**
Chicago Illinois Philadelphia **Pennsylvania**
Chicago Illinois Phoenix **Arizona**
Chicago Illinois Dallas **Texas**
...



70 - 80 % accuracy reported in Mikolov et al., 2013

Figure from: Pennington et al. (2014). Glove: Global vectors for word representation.

How to use embeddings

- Load pretrained word2vec/glove embeddings to initialize parameters

```
import torch
import torch.nn as nn
import numpy as np
from gensim.models import KeyedVectors

# Load pretrained embeddings (Word2Vec format)
word_vectors = KeyedVectors.load_word2vec_format("path/to/word2vec.bin", binary=True)

# Define vocabulary (example: words mapped to indices)
vocab = {"hello": 0, "world": 1, "goodbye": 2} # Example vocab
vocab_size = len(vocab)
embedding_dim = word_vectors.vector_size # Must match pretrained embedding size

# Initialize an embedding matrix
embedding_matrix = np.zeros((vocab_size, embedding_dim))

# Fill the embedding matrix with pretrained word vectors
for word, idx in vocab.items():
    if word in word_vectors:
        embedding_matrix[idx] = word_vectors[word]
    else:
        embedding_matrix[idx] = np.random.normal(scale=0.6, size=(embedding_dim,)) # F

# Convert to torch tensor
embedding_tensor = torch.tensor(embedding_matrix, dtype=torch.float)

# Initialize nn.Embedding with pretrained weights
embedding_layer = nn.Embedding.from_pretrained(embedding_tensor, freeze=False) # Set 1
```

Binary or text format

Fun Application: Emoji2vec

Eisner, B., Rocktäschel, T., Augenstein, I., Bošnjak, M., & Riedel, S. (2016). emoji2vec: Learning emoji representations from their description. *arXiv preprint arXiv:1609.08359*.



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