Tutorial 03 for Calculus I

Sect. 3.4-3.9

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Review Sect. Sect. 3.4-3.9

- Section 3.4: The Derivative as a Rate of Change. (基本不考)
- Section 3.5: Derivatives of trigonometric functions: $\sin x$, $\cos x$, $\tan x$, $\cot x$, $\sec x$ and $\csc x$.
- Section 3.6: The Chain Rule.
- Section 3.7: Implicit differentiation, Derivatives of higher order.
- Section 3.8: Find a rate of change from other known rates of change.
- Section 3.9: Linearization, Differentials, Estimating with differentials.

Review of Sect. 3.4

1. Speed is the absolute value of velocity: speed$=|v(t)|=|\frac{ds}{dt}|$. P142

2. Jerk is the derivative of acceleration with respect to time:
$j(t)=\frac{da}{dt}=\frac{d^3s}{dt^3}$. P143

3. Derivatives in Economics: the cost of production $c(x)$ is a function of x, then $\frac{dc}{dx}=\lim\limits_{h\to 0}\dfrac{c(x_0+h)-c(x_0)}{h}$=the marginal cost of production. P146
Sometimes the marginal cost of production is loosely defined to be the extra cost of producing one additional unit: $\frac{\Delta c}{\Delta x}=\frac{c(x+1)-c(x)}{1}$
which is approximated by the value of $\frac{dc}{dx}$ at x.

Derivatives of trigonometric functions:

$\frac{d}{dx}(\sin x) = \cos x;\quad \frac{d}{dx}(\cos x) = -\sin x;$

$\frac{d}{dx}(\tan x) = \sec^2 x;\quad \frac{d}{dx}(\cot x) = -\csc^2 x;$

$\frac{d}{dx}(\sec x) = \sec x \tan x;\quad \frac{d}{dx}(\csc x) = -\csc x \cot x.$ P151-154

Review of Sect. 3.6

Theorem (The Chain Rule)

If $f(u)$ is differentiable at the point $u = g(x)$ and $g(x)$ is differentiable at x, then the composite function $(f \circ g)(x) = f\big(g(x)\big)$ is differentiable at x, and

$$(f \circ g)'(x) = f'\big(g(x)\big) \cdot g'(x).$$

Remark: Let $f(x) = x^2$ and $g(x) = |x|$. Then the composites $(f \circ g)(x) = (g \circ f)(x) = x^2$ are both differentiable at $x = 0$ even though g itself is not differentiable at $x = 0$. Does this contradicts the chain rule ?

Review of Sect. 3.7

- Steps to compute implicit differentiation: Equations of the form $F(x, y) = 0$.
  1. Differentiate both side of the equation with respect to x, treating y as a differentiable function of x.
  2. Collect the terms with dy/dx on one side and solve for dy/dx.
- Derivatives of higher order.
- Tangent and normal lines.

Review of Sect. 3.9

1.Linearization: If f is differentiable at x=a, then the approximating function $L(x) = f(a) + f^{'}(a)(x - a)$ is the lineralization of f at a. The approximation $f(x) \approx L(x)$ of f by L is the standard linear approximation of f at a. P180

2. Differentials: Let $y = f(x)$ be a differentiable function. The differential dx is an independent variable. The differential dy is $dy = f^{'}(x)dx$. P182

3.Estimating with differentials: The approximation $\triangle y \approx dy$ can be used to estimate $f(a + dx)$ when $f(a)$ is know and dx is small. P184

专题一: 导数相关的判断题.

Determine if the following statement is correct or not, and state your reasons.

(1) Let $f = g + h$, if f has derivative at $x = x_0$, then g, h have derivatives at $x = x_0$.

(2) Let $f = g + h$, if g has derivative at $x = x_0$ and h is not differentiable at $x = x_0$, then f is not differentiable at $x = x_0$.

(3) Let $f = g \cdot h$, if f has derivative at $x = x_0$, then g, h have derivatives at $x = x_0$.

(4) Let $f = g \cdot h$, if g has derivative at $x = x_0$ and h is not differentiable at $x = x_0$, then f is not differentiable at $x = x_0$

专题二：极限与导数的定义.

例: $\lim\limits_{\theta\to\pi/4}\frac{\tan\theta-1}{\theta-\frac{\pi}{4}}$. (书本50)

延伸: (1) $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\cos(\pi\tan x)+1}{x^2-\pi^2/16}$.

(2) $\lim\limits_{x\to a}\dfrac{\sin x-\sin a}{x-a}$.

$\dfrac{\text{上差}}{\text{下差}}$ 的式子视作导数

专题三：链式法则.

例：Find $\frac{dy}{dx}$.

(1) $y = \sec(\sqrt{x})\tan(\frac{1}{x})$.(书本38)

(2) $y = \sqrt{3x + \sqrt{2 + \sqrt{1 - x}}}$. (书本58)

延伸：(1) Find $\frac{d^2y}{dx^2}$ when $y = f(f(x))$.

(2) Find $\frac{dy}{dx}$ when $y = \sin(\sin(\sin x))$.

专题四: 高阶导.

延伸: (1) Let $f(x) = \begin{cases} x^2 \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ Then the largest positive integer n, for which $f^{(n)}(0)$ exists, is

(A) 1 (B) 2 (C) 3 (D) 4

(2) Compute the 100th derivative of the following function: $y = x^2 \sin x$.

(3) Let $f(x) = \frac{1}{1+x^2}$, then find $f^{(3)}(0)$.

(4) Let $f(x) = \frac{1}{2x+3}$, then find $f^{(n)}(0)$.

Homework of Section 3.7

专题四: 隐函数求导.

例1: $y\sin(\frac{1}{y}) = 1 - xy$, find $\frac{dy}{dx}$. (书本13)

例2: If $xy + y^2 = 1$, find the value of $\frac{d^2y}{dx^2}$ at the point $(0, -1)$. (书本21)

专题五: 变化率.

注意项

例1: A draining conical reservoir: Water is flowing at the rate of 50 $\mathrm{m}^3/\mathrm{min}$ from a shallow concrete conical reservoir (vertex down) of base radius 45 m and height 6 m.

a. How fast (centimeters per minute) is the water level falling when the water is 5 m deep?

b. How fast is the radius of the water's surface changing then ? Answer in centimeters per minute. (书本28)

Homework of Section 3.8

专题五: 变化率.

例2: Baseball players: A baseball diamond is a square 27 m on a side. A player runs from first base to second at a rate of 5 m/s.

(1)At what rate is the player's distance from third base changing when the player is 9 m from first base?

(2)At what rates are angles θ_1 and θ_2 (see the figure) changing at that time?

(3)The player slides into second base at the rate of 4.5 m/s. At what rates are angles θ_1 and θ_2 changing as the player touches base?. (书本43)

专题六: 线性化.

$(1+x)^n \quad x \to 0$

$1+nx$

例1: Estimate $\sqrt[3]{1.009}$. (书本15)

例2: Estimate the volume of the material in a cylindrical shell with length 30 cm, radius 6 cm, and shell thickness 0.5 cm. (书本43)

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