Tutorial 07 for Calculus I
Sect. 5.5-5.6

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Review Sect. 5.5-6.1

- Section 5.5 Substitution in Indefinite Integrals
- Section 5.6 Substitution in Definite Integrals, Areas Between Curves.

Review of Sect. 5.5-5.6

专题一: 积分的计算:换元法 :

(1) Substitution in Indefinite Integrals:If $u = g(x)$ is a differentiable function whose range is an interval I, and f is continuous on I, then $\int f(g(x))g'(x)dx = \int f(u)du$. P305

(2)Substitution in Definite Integrals: If g' is continuous on the interval [a,b] and f is continuous on the range of $g(x) = u$, then $\int_a^b f(g(x)) \cdot g'(x)dx = \int_{g(a)}^{g(b)} f(u)du$. P310

Remark: The Integrals of $\sin^2 x$ and $\cos^2 x$: $\sin^2 x = \frac{1-\cos 2x}{2}$, $\cos^2 x = \frac{1+\cos 2x}{2}$, $\cos 2x = \cos^2 x - \sin^2 x$. P307

The Integrals of $\tan^2 x$ and $\cot^2 x$: $\tan^2 x = \sec^2 x - 1$, $\cot^2 x = \csc^2 x - 1$.

Homework of Section 5.5

专题一: 积分的计算:换元法 $du = \frac{1}{2\sqrt{x}}\,dx$

例: (1) $\int \frac{1}{\sqrt{x}(1+\sqrt{x})^2}\,dx$ (书本21)

u

(2) $\int \frac{\sin(2x+1)}{\cos^2(2x+1)}\,dx$ (书本31)

(3) $\int \frac{18\tan^2 x \sec^2 x}{(2+\tan^3 x)^2}\,dx$ (书本51)

(4) $\int \frac{\sin\sqrt{x}}{\sqrt{x\cos^3\sqrt{x}}}\,dx$ (书本54)

(5) $\int \sqrt{\sin x}\cos^3 x\ dx$ (书本22)

(6) $\int \frac{1}{x^3}\sqrt{\frac{x^2-1}{x^2}}\ dx$ (书本40)

u

(7) $\int 3x^5\sqrt{x^3+1}\,dx$ (书本48)

u $du = \cos\frac{1}{x}\cdot -\frac{1}{x^2}\,dx$

(8) $\int \frac{1}{x^2}\sin\frac{1}{x}\cos\frac{1}{x}\ dx$

专题一: 定积分的计算:换元法.

例: (1) $\int_0^1 t\sqrt{4+5t}\,dt$ (书本11)

(2) $\int_{\pi}^{\frac{3\pi}{2}} \cot^5(\frac{t}{6})\sec^2(\frac{t}{6})\,dt$ (书本18)

(3) $\int_0^{\sqrt[3]{\pi^2}} \sqrt{t}\cos^2(t^{\frac{3}{2}})\,dt$ (书本23)

Homework of Section 5.6

专题一: 积分的计算:换元法.

例: (1) Suppose that $F(x)$ is an antiderivative of $f(x)=\frac{\sin x}{x}, x>0$. Express $\int_1^3 \frac{\sin 2x}{x}dx$ in terms of F. (书本83)

(2) Show that if f is continuous, then $\int_0^1 f(x)dx=\int_0^1 f(1-x)dx$. (书本84)

(3) If f is a continuous function, find the value of the integral $I=\int_0^a \frac{f(x)dx}{f(x)+f(a-x)}$ by making the substitution $u=a-x$ and adding the resulting integral to I. (书本87)

(4) By using a substitution, prove that all positive numbers x and y, $\int_x^{xy} \frac{1}{t}dt=\int_1^y \frac{1}{t}dt$. (书本88)

换元 改变上下限

专题一: 积分的计算:换元法.

延伸:

降幂

u 三角函数 倍角

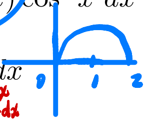
1. Evaluate the definite integral.

   (1) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\cancel{x^3} + \sin^2 x) \cos^2 x \; dx$ (2) $\int_{-\pi}^{\pi} (\cancel{\sin^3 x} + \sqrt{\pi^2 - x^2}) \; dx$

   $(u-1)du$ $(2-x)(x)$

   (3) $\int_0^2 x\sqrt{2x - x^2} \; dx$

   换上下限 $u = \frac{1}{2}\pi$ 另 $u = 1-x$ $du = -dx$



$2\int_0^{2x} f(u)du + 2x \cdot 2f(2x) - 2\cdot 2x f(2x)$

$-2\int_0^{x} f(u)du$

$= 2\int_x^{2x} f(u)du$

2. $f(x)$ is continuous on $[a, b]$, prove that $\int_a^b f(x) \; dx = \int_a^b f(a + b - x) \; dx$.

   $2x - t = u$ | $du = -dt$

3. $f(x)$ is continuous, and $\int_0^x tf(2x - t) \; dt = \frac{1}{2} \tan^{-1} x^2$, $f(1) = 1$, find $\int_1^2 f(x) \; dx$.

   $\int_{2x}^{x} (2x-u) f(u) \, (-du)$

   $\int_x^{2x} (2x-u) f(u) du$

4. Show that $\int_x^1 \frac{dt}{1+t^2} = \int_1^{\frac{1}{x}} \frac{dt}{1+t^2}$.

   $= 2x\int_0^{2x} f(u)du - \int_0^{2x} uf(u)du - \left(2x\int_0^{x} f(u)du - \int_0^{x} uf(u)du\right)$

专题二：曲线所围城的面积与积分.

Areas Between Curves:

1. If f and g are continuous with $f(x) \geq g(x)$ throughout $[a, b]$, then the area of the region between the curves $y = f(x)$ and $y = g(x)$ from a to b is the integral of $(f - g)$ from a to b: $A = \int_a^b [f(x) - g(x)]dx$. P313

2. Integration with Respect to y: If f and g are continuous with $f(y) \geq g(y)$ throughout $[c, d]$, then the area of the region between the curves $x = f(y)$ and $x = g(y)$ from c to d is the integral of $(f - g)$ from c to d: $A = \int_c^d [f(y) - g(y)]dy$. P313

专题二：曲线所围城的面积与积分.

例: Find the area of the region enclosed by $x = y^2 - 1$ and $x = |y|\sqrt{1 - y^2}$. (书本57)