Tutorial 04 for Calculus I

Sect. 4.1-4.3

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Review of Sect. 3.8-4.3

1. Section 4.1: Definition of absolute extreme values, critical point, local extreme values.
2. Section 4.2: The Rolle's Theorem, and The Mean Value Theorem.
3. Section 4.3: The definition of monotonic functions, and the first Derivative Test

Theorem (The extreme value theorem)

If f is continuous on a closed interval $[a, b]$, *then f attains both an absolute maximum value M and an absolute minimum value m in* $[a, b]$. *P200*

Remark: Maximum and minimum values are called extreme values. P199

Review of Sect. 4.1

内部才 critical

1. $\boxed{\text{An interior point}}$ of the domain of a function f where f' is zero or undefined is a critical point of f. P203

2. How to find the absolute extrema;

If f is a continuous function on a finite closed Interval:

(1) Evaluate f at all critical points (interior points where $f' = 0$ or f' is undefined) and endpoints,

(2) Take the largest and smallest of these values.

3. Local extreme values: A function f has a local maximum value (local minimum value) or relative extrema at a point c within its D if $f(x) \leq f(c)$ $(f(x) \geq f(c))$ for all $x \in D$ lying in some open interval containing c. P201

Review of Sect. 4.2

Theorem (Rolle's Theorem)

Suppose that $y = f(x)$ is continuous at every point of the closed interval $[a, b]$ and differentiable at every point of its interior (a, b). If $f(a) = f(b)$, then there is at least one number c in (a,b) at which $f'(c) = 0$. P207

Remark: Rolle's Theorem my be combined with the Intermediate Value Theorem to show when there is only one real solution of an equation $f(x) = 0$.

Theorem (The Mean Value Theorem)

Suppose $y = f(x)$ is continuous at every point of the closed interval $[a, b]$ and differentiable on the interval's interior (a, b). Then there is at least one point c in (a,b) at which

$$\frac{f(b) - f(a)}{b - a} = f'(c).$$

Remark: Do you know how to prove it ?

专题一: 单调性,极值与最值.

延伸: (1) $f(x), g(x)$ are positive and differentiable functions, and $f'(x)g(x) - f(x)g'(x) < 0$, then if $a < x < b$, we know (　　)

(A) $f(x)g(b) > f(b)g(x)$　　(B) $f(x)g(a) > f(a)g(x)$

(C) $f(x)g(x) > f(b)g(b)$　　(D) $f(x)g(x) > f(a)g(a)$

(2) $f(x)$ is differentiable, and $f(x)f'(x) > 0$, then (　　)

(A) $f(1) > f(-1)$　　(B) $f(1) < f(-1)$

(C) $|f(1)| > |f(-1)|$　　(D) $|f(1)| < |f(-1)|$

(3) Let $c > 0$. How many real roots are there for the equation $x^3 - 6x^2 + 9x + c = 0$ (　　)

A. 0　　B. 1　　C. 2　　D. 3

Homework of Section 4.1, 4.3

专题一: 单调性, 极值与最值.

延伸: (4) The equation $|x|^{\frac{1}{4}} + |x|^{\frac{1}{2}} - \cos x = 0$ in $(-\infty, \infty)$ ( )

A. has no zero root.

B. has exactly one zero root.

C. has exactly two zero roots.

D. has infinite zero root.

振荡 $f'(x) = \cos\frac{1}{x}$

(5) If $f'(c) > 0$, then $f(x)$ is strictly increasing in some neighborhood of c.

(6) $f(x)$ has local extreme at $x = x_0$, then $f'(x_0) = 0$. ✗

Homework of Section 4.1, 4.3

专题一: 单调性, 极值与最值.

$\frac{f(x) - f(0)}{x^2} = 1$

延伸:

(7) $f(x)$ is continuous at the neighbourhood of $x = 0$, and

邻域 $f(x) > 0$

$f(0) = 0$, $\lim\limits_{x \to 0} \frac{f(x)}{1 - \cos x} = 2$, then $f(x)$ (　　) at $x = 0$.

$f(x) > 0$, $1 - \cos x > 0$

A. is not differentiable　　　$f(0)$　　　B. is differentiable and $f'(0) \neq 0$

C. has local maximum　　　极小　　　D. has local minimum

(8) $f'(x)$ is continuous on $[a, b]$, and $f'(a) > 0$, $f'(b) < 0$, then the wrong statement is (　　).

A. at least exist a point $x_0 \in (a, b)$, such that $f(x_0) > f(a)$.

B. at least exist a point $x_0 \in (a, b)$, such that $f(x_0) > f(b)$.

C. at least exist a point $x_0 \in (a, b)$, such that $f'(x_0) = 0$.

D. at least exist a point $x_0 \in (a, b)$, such that $f(x_0) = 0$.

专题一: 单调性, 极值与最值.

延伸: (9) Let $f(x) = x\sin x + \cos x$, then the right statement is ( ).

A. $f(0)$ is local maximum, $f(\frac{\pi}{2})$ is local minimum.

B. $f(0)$ is local minimum, $f(\frac{\pi}{2})$ is local maximum.

C. $f(0)$ is local maximum, $f(\frac{\pi}{2})$ is local maximum.

D. $f(0)$ is local minimum, $f(\frac{\pi}{2})$ is local minimum.

(10) Suppose the tangent line at the inflection point (1, 1) for the function $\underline{y = ax^3 + bx^2 + cx}$. is horizontal. Find a, b and c.

(11) The function $y = f(x)$ is defined by the equation $y^3 + xy^2 + x^2y + 6 = 0$, find the extreme values of $f(x)$.

专题二：中值定理.

例: (1) Show that the function $f(t)=\sec t-\frac{1}{t^3}+5,(0,\frac{\pi}{2})$ has exactly one zero in the given interval $(0,\frac{\pi}{2})$.(书本25)

(2) Suppose that $f'(x)\le 1$ for $1\le x\le 4$, show that $f(4)-f(1)\le 3$. (书本61)

(3) Suppose that $0<f'(x)<\frac{1}{2}$ for all x-values, show that $f(-1)<f(1)<2+f(-1)$. (书本62)

(4) Show that $|\cos x-1|\le|x|$ for all values.(书本63)

(5) Show that for any numbers a and b, the sine inequality $|\sin b-\sin a|\le|b-a|$ is true.(书本64)

专题二: 中值定理.

不遗漏!

延伸: (1) If $f(x)$ is differentiable on (a,b), and $a<x_1<x_2<b$, then there exists at least a point c such that (   )

A. $f(b)-f(a)=f'(c)(b-a) \quad (a<c<b)$.

B. $f(b)-f(x_1)=f'(c)(b-x_1) \quad (x_1<c<b)$.

C. $f(x_2)-f(x_1)=f'(c)(x_2-x_1) \quad (x_1<c<x_2)$.

D. $f(x_2)-f(a)=f'(c)(x_2-a) \quad (a<c<x_2)$.

(2) $f(x)$ is continuous on $[0,3]$, and differentiable on $(0,3)$, $f(0)+f(1)+f(3)=3, f(3)=1$. Show that there must exist some $c\in(0,3)$ such that $f'(c)=0$;