



南方科技大学
SOUTHERN UNIVERSITY OF SCIENCE AND TECHNOLOGY

考试科目: 高等数学(上) A

开课单位: 数学系

考试时长: 120 分钟

命题教师: 高等数学出题组

题号	1	2	3	4	5	6	7	8	9
分值	15 分	15 分	10 分	10 分	10 分	12 分	12 分	10 分	6 分

本试卷共 9 道大题, 满分 100 分. (考试结束后请将试卷、答题本、草稿纸一起交给监考老师)

注意: 本试卷里的中文为直译 (即完全按英文字面意思直接翻译), 所有数学词汇的定义请参照教材 (Thomas' Calculus, 13th Edition) 中的定义. 如果其中有些数学词汇的定义不同于中文书籍 (比方说同济大学的高等数学教材) 里的定义, 以教材 (Thomas' Calculus, 13th Edition) 中的定义为准.

1. (15pts) **Multiple Choice Questions:** (only one correct answer for each of the following questions.)

- (1) The number of real roots for $x^3 - 12x + 19 = 0$ is
(A) 0. (B) 1. (C) 2. (D) 3.
- (2) Let $f(x), g(x)$ be differentiable functions that are always greater than zero. If $f'(x)g(x) - f(x)g'(x) < 0, \forall x \in [a, b]$, then _____ for $a < x < b$.
(A) $f(x)g(b) > f(b)g(x)$ (B) $f(x)g(a) > f(a)g(x)$
(C) $f(x)g(x) > f(b)g(b)$ (D) $f(x)g(x) > f(a)g(a)$
- (3) If a function $f(x)$ is continuous at $x = 0$ and $\lim_{x \rightarrow 0} \frac{f(x)}{x} = 2$ then
(A) $f(0) = 1$ and $f'(0) = 2$. (B) $f(0) = 0$ and $f'(0) = 0$.
(C) $f(0) = 0$ and $f'(0) = 2$. (D) None of (A), (B) and (C) is correct.
- (4) If $f(x)$ is differentiable, and $\alpha = f(x + \Delta x) - f(x) - f'(x)\Delta x$, then
(A) $\lim_{\Delta x \rightarrow 0} \frac{\alpha}{\Delta x} = 0$. (B) $\lim_{\Delta x \rightarrow 0} \frac{\alpha}{\Delta x} = 1$.
(C) $\lim_{\Delta x \rightarrow 0} \frac{\alpha}{(\Delta x)^2} = 1$. (D) $\lim_{\Delta x \rightarrow 0} \frac{\alpha}{\Delta x} = 0$.
- (5) If $f(x) = |x|g(x)$ is differentiable at $x = 0$, then we must have
(A) $\lim_{x \rightarrow 0^+} g(x) = \lim_{x \rightarrow 0^-} g(x)$. (B) $\lim_{x \rightarrow 0} g'(x) = g'(0)$.
(C) $\lim_{x \rightarrow 0^+} g(x) = -\lim_{x \rightarrow 0^-} g(x)$. (D) $\lim_{x \rightarrow 0} g'(x) = g(0)$.

2. (15 pts) Fill in the blanks.

- (1) If $(-1, 0)$ is an inflection point on the curve $y = x^3 + ax^2 + bx + 1$, then $b =$ _____.

- (2) Let $f(x) = x(x+1)(x+2)\cdots(x+n)$, then $f'(0) = \underline{\hspace{2cm}}$.
- (3) If $f(x) = \sqrt{x\sqrt{\sin x}}$, then $f'(x) = \underline{\hspace{2cm}}$.
- (4) $\lim_{n \rightarrow \infty} \left(\frac{1^5}{n^6} + \frac{2^5}{n^6} + \cdots + \frac{(n-1)^5}{n^6} \right) = \underline{\hspace{2cm}}$.
- (5) The asymptotes of the graph of function $f(x) = x + x \sin \frac{1}{x}$ are $\underline{\hspace{2cm}}$.
3. (10 pts) You are planning to close off a corner of the first quadrant with a line segment 20 units long running from $P(a,0)$ to $Q(0,b)$. what is the largest area that $\triangle OPQ$ (O is the origin) can have, and what are its dimensions ?
4. (10 pts) Let $y^3 + y = 2 \cos x$, find $\left. \frac{dy}{dx} \right|_{x=0}$ and $\left. \frac{d^2y}{dx^2} \right|_{x=0}$.
5. (10 pts) The region is bounded by the x -axis, the curve $f(x) = \begin{cases} \frac{\tan^2 x}{x}, & 0 < x \leq \frac{\pi}{4} \\ 0, & x = 0 \end{cases}$, and the line $x = \frac{\pi}{4}$. Find the volume of the solid generated by revolving the region about the y -axis.
6. (12 pts) Compute the following integrals:
- (1) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{\cos x - \cos^3 x} dx$.
- (2) $\int_{\frac{3}{2}}^4 \frac{x+1}{\sqrt{2x+1}} dx$.
7. (12 pts) Find the limits (**Do not use the L'Hopital's rule**):
- (1) $\lim_{x \rightarrow 1} \frac{(1 - \sqrt{x})(1 - \sqrt[3]{x})}{(1 - x^2)^2}$.
- (2) $\lim_{x \rightarrow 0} \frac{\tan x - \sin x}{\sin(x^3)}$.
8. (10 pts) Find the equation for the tangent line for the curve $y = 1 + x + \int_0^x \cos((x-t)^2) dt$ at the point $(0,1)$.
9. (6 pts) Find the absolute minimum value for $f(x) = |\sin x + \cos x + \tan x + \cot x + \sec x + \csc x|$.

一、 (15分) 单项选择题:

- (1) 方程 $x^3 - 12x + 19 = 0$ 的实根的个数为
(A) 0. (B) 1. (C) 2 (D) 3.
- (2) 函数 $f(x), g(x)$ 为恒正可微函数, 且满足 $f'(x)g(x) - f(x)g'(x) < 0, \forall x \in [a, b]$. 则当 $a < x < b$ 时, 必有
(A) $f(x)g(b) > f(b)g(x)$ (B) $f(x)g(a) > f(a)g(x)$
(C) $f(x)g(x) > f(b)g(b)$ (D) $f(x)g(x) > f(a)g(a)$
- (3) 函数 $f(x)$ 在 $x = 0$ 处连续, 且满足 $\lim_{x \rightarrow 0} \frac{f(x)}{x} = 2$. 则
(A) $f(0) = 1$, 且 $f'(0) = 2$. (B) $f(0) = 0$, 且 $f'(0) = 0$.
(C) $f(0) = 0$, 且 $f'(0) = 2$. (D) 前面 3 个选项都不对.
- (4) 设函数 $f(x)$ 可导, $\alpha = f(x + \Delta x) - f(x) - f'(x)\Delta x$, 则
(A) $\lim_{\Delta x \rightarrow 0} \frac{\alpha}{\Delta x} = 0$. (B) $\lim_{\Delta x \rightarrow 0} \frac{\alpha}{\Delta x} = 1$.
(C) $\lim_{\Delta x \rightarrow 0} \frac{\alpha}{(\Delta x)^2} = 1$. (D) $\lim_{\Delta x \rightarrow 0} \frac{\Delta x}{\alpha} = 0$.
- (5) 若函数 $f(x) = |x|g(x)$ 在 $x = 0$ 处可导, 则必有
(A) $\lim_{x \rightarrow 0^+} g(x) = \lim_{x \rightarrow 0^-} g(x)$. (B) $\lim_{x \rightarrow 0} g'(x) = g'(0)$.
(C) $\lim_{x \rightarrow 0^+} g(x) = -\lim_{x \rightarrow 0^-} g(x)$. (D) $\lim_{x \rightarrow 0} g'(x) = g(0)$.

二、 (15分) 填空题:

- (1) 若曲线 $y = x^3 + ax^2 + bx + 1$ 有拐点 $(-1, 0)$, 则 $b =$ _____.
- (2) 设 $f(x) = x(x+1)(x+2) \cdots (x+n)$, 则 $f'(0) =$ _____.
- (3) 若 $f(x) = \sqrt{x\sqrt{\sin x}}$, 则 $f'(x) =$ _____.
- (4) $\lim_{n \rightarrow \infty} \left(\frac{1^5}{n^6} + \frac{2^5}{n^6} + \cdots + \frac{(n-1)^5}{n^6} \right) =$ _____.
- (5) 曲线 $f(x) = x + x \sin \frac{1}{x}$ 的 (所有) 渐近线为 _____.

三、 (10分) 两个点 $P(a, 0)$ 和 $Q(0, b)$ 与原点 $O(0, 0)$ 组成一个三角形. 若线段 PQ 的长度为 20, 则 $\triangle OPQ$ 的最大面积为多少? 此时 a 和 b 的值分别是多少?

四、 (10分) 已知曲线方程为 $y^3 + y = 2 \cos x$, 求 $\left. \frac{dy}{dx} \right|_{x=0}$ 和 $\left. \frac{d^2y}{dx^2} \right|_{x=0}$.

五、 (10分) 已知区域 R 由 x 轴, 直线 $x = \frac{\pi}{4}$ 和曲线 $f(x) = \begin{cases} \frac{\tan^2 x}{x}, & 0 < x \leq \frac{\pi}{4} \\ 0, & x = 0 \end{cases}$ 所围成. 把区域 R 绕 y 轴旋转, 求此旋转体的体积.

六、 (12分) 计算下列积分:

- (1) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{\cos x - \cos^3 x} dx.$
- (2) $\int_{\frac{3}{2}}^4 \frac{x+1}{\sqrt{2x+1}} dx.$

七、 (12分) 求极限(不准使用洛必达法则):

$$(1) \lim_{x \rightarrow 1} \frac{(1 - \sqrt{x})(1 - \sqrt[3]{x})}{(1 - x^2)^2}.$$

$$(2) \lim_{x \rightarrow 0} \frac{\tan x - \sin x}{\sin(x^3)}.$$

八、 (10分) 求曲线 $y = 1 + x + \int_0^x \cos((x-t)^2) dt$ 在点 $(0, 1)$ 处的切线方程.

九、 (6分) 求函数 $f(x) = |\sin x + \cos x + \tan x + \cot x + \sec x + \csc x|$ 的全局极小值 (即最小值)