Calculus I　复习专题二　　第三和四章　　导数

知识点一: 用导数的定义求导.

1.(2020年期末) let $f(x) = x^4 \sin\frac{1}{x}$, if $x \neq 0$; $f(x) = 0$, if $x = 0$. Then the largest positive integer n, for which $f^{(n)}(0)$ exists, is
(A) 1.　　(B) 2.　　(C) 3.　　(D) 4.

2.(2020年期末) If $f''(a)$ exists, and $f'(a) \neq 0$, then $\lim\limits_{x\to a}(\frac{1}{f'(a)(x-a)} - \frac{1}{f(x)-f(a)}) =$ (　　).

3.(2019年期末) Let g be a function that is differentiable throughout an open interval containing the origin. Suppose g has the following properties:
i) $g(x+y) = \frac{g(x)+g(y)}{1-g(x)g(y)}$ for all real numbers x, y, and $x+y$ in the domain of g.
ii) $\lim\limits_{h\to 0} g(h) = 0$.
iii) $\lim\limits_{h\to 0} \frac{g(h)}{h} = 1$.
Find $g(x)$.

4.(2019年期末) If the function $f(x) = a \cdot \sin x$ if $x \leq \frac{\pi}{4}$; $f(x) = 1 + b \cdot \tan x$ if $\frac{\pi}{4} < x < \frac{\pi}{2}$ is differentiable at $x = \frac{\pi}{4}$, find the values of a and b.

知识点二: 复合函数和隐函数求导.

5.(2021年期末) If the line $y = x$ is tangent to the curve $y = \log_a x$, find the value of a.

6.(2020年期末) Function $f(x) = x^2$ has a tangent line $y = Kx - 1$ if $K =$ (　　), or (　　).

7.(2019年期末) The point P(a, b) lies on the curve $l : (y-x)^3 = y + x$, and the slope of the tangent line of l at P(a, b) is 3. Find the values of a and b.

8.(2022年期末) If $f(x) = (1+x)(1+2x)\cdots(1+10x)$, then $f'(0) =$ (　　).

9.(2021年期末) If $f(x) = (x^2+1)(x^2+2)(x^2+3)(x^2+4)$, then $f^{(6)}(0) =$ (　　).

10.(2022年期末) If $f(x) = \arctan\frac{1+x}{1-x}$, then $f'(0) =$ (　　).

11.(2020年期末) Assume that $f'(0) = 3, f''(0) = 5, f'(1) = -4$, and $f''(1) = -7$. Let $g(x) = f(\ln x)$. Then $g''(1) =$ (　　).

12.(2019年期末) If $g(x)$ is one-to-one, and $g(1) = 3, g(3) = 1, g'(1) = 4, g'(3) = 28$, then $(g^{-1})'(3) =$
(A) $\frac{1}{4}$.　　(B) $\frac{1}{28}$.　　(C) $\frac{1}{3}$.　　(D) 4

13.(2020年期末) Let $y = (\cos x)^x$ for $0 < x < \frac{\pi}{2}$, then $y'(x) =$ (　　).

14.(2022年期末) Assume that f is differentiable at $x = 1$, and $f(1) = 1, f'(1) = 2$. Find the value of $\lim\limits_{n\to\infty}(f(1+\frac{1}{n}))^n$.

知识点三: 单调性, 极值, 拐点.

15.(2019年期末) Determine whether the following statements are true or false? No justification is necessary.
(1) If $k > 0$, then $\ln^{100} x < x^{0.0001} < 2^{kx}$ for sufficiently large x.
(2) If the graph of a differentiable function $f(x)$ is concave up on an open interval (a, b), then $f(x)$ has a local minimum value at a point $c \in (a, b)$ if and only if $f'(c) = 0$.

16.(2022年期末) If $f(x) = \frac{ax+b}{x^2-1}$ has a local extreme of 1 at $x = 3$, then
(A) $a = 3, b = -1$. (B) $a = 4, b = -4$. (C) $a = 5, b = -7$. (D) $a = 6, b = -10$.

17.(2021年期末) If the function $f(x)$ has the third derivative at $x = x_0$, and $f'(x_0) = f''(x_0) = 0, f^{(3)}(x_0) > 0$, then
(A) $f(x)$ has a local minimum at x_0. (B) $f(x)$ has a local maximum at x_0.
(C) $f(x)$ has no local extremum at x_0. (D) None of (A), (B) and (C) is correct.

18.(2020年期末) If $f(x)$ is twice-differentiable on $(-\infty, \infty)$, and $g(x) = (1-x)f(0)+xf(1)$, then which of the following statements is correct on $(0, 1)$?
(A) $f(x) > g(x)$ if $f'(x) > 0$. (B) $f(x) > g(x)$ if $f'(x) < 0$.
(C) $f(x) > g(x)$ if $f''(x) > 0$. (D) $f(x) > g(x)$ if $f''(x) < 0$.

19.(2021年期末) A isosceles triangle is to be inscribed in a circle of radius R. What is the largest perimeter possible for the isosceles triangle? Please provide the reason.

20.(2020年期末) (1) For $y = \frac{x^2+1}{x+1}$, identify the coordinates of any local and absolute extreme points and inflection points that may exist.
(2) Sketch the graph of the above function. (Please identify all the asymptotes and some specific points, such as local maximum and minimum points, inflection points, and intercepts.)

知识点四: 求方程的根.

21.(2022年期末) The number of the real roots for the equation $x^3 - 4x^2 + x + 1 = 0$ is ( ).

22.(2021年期末) The number of real roots in $(0, 1)$ for $5x - 2 - \int_0^x \frac{dt}{1+t^8} = 0$ is
(A) 0. (B) 1. (C) 2. (D) greater than 2.

23.(2020年期末) The number of the real roots for the equation $x^3 - 3x + 3 = 0$ is
(A) 0. (B) 1. (C) 2. (D) 3

24.(2019年期末) Let $c > 0$. How many real roots are there for the equation $x^3 - 6x^2 + 9x + c = 0$?
(A) 0. (B) 1. (C) 2. (D) 3.

知识点五: 微积分基本定理.

25.(2020年期末) Find $\frac{dy}{dx}$ if $y = \int_{x^2+1}^{2x^2+3} t \tan\sqrt{x+t}\, dt$.

26.(2019年期末) Find $f'(2)$ if $f(x) = e^{g(x)}$ and $g(x) = \int_2^{\frac{x^2}{2}} \frac{t}{1+t^4}\, dt$.

27.(2019年期末) $f(x)$ is differentiable, and $f'(x) > 0$ on $(0, +\infty)$. Let $F(x) = \int_{\frac{1}{x}}^1 xf(u)\, du + \int_1^{\frac{1}{x}} \frac{f(u)}{u^2}\, du$

(1) Identify the open intervals on which $F(x)$ is decreasing and the open intervals on which $F(x)$ is increasing.

(2) Find the open intervals on which the graph of $y = F(x)$ is concave up and the open intervals on which it is concave down.

知识点六: 其他知识点.

28.(2022年期末) Assume $f(x)$ is continuous on $[0, 1]$ and differentiable on $(0, 1)$. If $f(0) = f(1) = 0$, $f(\frac{1}{2}) = 1$, prove that:

(1) there exists $c \in (\frac{1}{2}, 1)$, , such that $f(c) = c$.

(2) For any real number k, there always exists $\xi \in (0, c)$, such that $f'(\xi) - k[f(\xi) - \xi] = 1$.