Tutorial 12 for Calculus I

Sect. 8.3-8.4

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Review of Sect. 8.3

1. Products of Powers of Sines and Cosines: P472

Case 1: If m is odd.

$\int \sin^m x\ dx = \int \sin^{2k+1} x\ dx = \int (\sin^2 x)^k \sin x\ dx$

$= \int -(1-\cos^2 x)^k\ d(\cos x)$

Case 2: If m is even and n is odd.

$\int \sin^m x\ \ \cos^n x\ dx = \int \sin^m x\ \ \cos^{2k+1} x\ dx$

$= \int \sin^m x\ (\cos^2 x)^k \cos x\ dx = \int \sin^m x\ (1-\sin^2 x)^k\ d(\sin x)$

Case 3: If both m and n are even.

$\int \sin^m x\ \ \cos^n x\ dx = \int (\sin^2 x)^k\ (\cos^2 x)^{k_1}\ dx$

$= \int (\frac{1-\cos 2x}{2})^k\ (\frac{1+\cos 2x}{2})^{k_1}\ dx$

Evaluate the integrals:

12. $\int \cos^3 2x \sin^5 2x \; dx$

17. $\int_0^\pi 8 \sin^4 x \; dx$

$\frac{1-\cos 2x}{2}$

67. $\int x \sin^2 x \; dx$

$\frac{1}{4}\int 2x\left(-\frac{1}{2}\right)\cos 2x \, dx$

$-\frac{1}{8}\int u \, d(\sin u)$

$= -\frac{1}{8} u \sin u + \frac{1}{8}\int \sin u \, du$

68. $\int x \cos^3 x \; dx$

$\int \frac{1}{2}x^2 \cdot 3\cos^2 x(-d\sin x)$

Review and Homework of Sect. 8.3

2. Eliminating Square Roots: P472

Use $1 + \cos 2x = 2\cos^2 x; 1 - \cos 2x = 2\sin^2 x, 1 + \sin x = (\sin \frac{x}{2} + \cos \frac{x}{2})^2$

Evaluate the integrals:

24. $\int_0^{\pi} \sqrt{1 - \cos 2x}\ dt$

27. $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\sin^2 x}{\sqrt{1-\cos x}}\ dx$

28. $\int_0^{\frac{\pi}{6}} \sqrt{1 + \sin x}\ dx$

3. Integrals of Powers of $\tan x$ and $\sec x$: P474

Use $\tan^2 x = \sec^2 x - 1, \cot^2 x = \csc^2 x - 1, d(\tan x) = \sec^2 x dx$.

$\frac{\cos^2 x}{\sin^2 x} = \frac{1}{\sin^2 x} - 1$

Evaluate the integrals:

47. $\int \tan^5 x \ dx$

4. Products of Sines and Cosines: P475

Use $\sin mx \ \sin nx = \frac{1}{2}[\cos(m-n)x - \cos(m+n)x]$

$\sin mx \ \cos nx = \frac{1}{2}[\sin(m-n)x + \sin(m+n)x]$

$\cos mx \ \cos nx = \frac{1}{2}[\cos(m-n)x + \cos(m+n)x]$

Evaluate the integrals:

55. $\int \cos 3x \ \cos 4x \ dx$

Review of Sect. 8.4

Trigonometric Substitutions: P474

If the integrals involving $\sqrt{a^2+x^2}, \sqrt{a^2-x^2}, \sqrt{x^2-a^2}$, the most common substitutions are

$x = a\tan\theta$ requires $\theta = \tan^{-1}(\frac{x}{a})$ with $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$x = a\sin\theta$ requires $\theta = \sin^{-1}(\frac{x}{a})$ with $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$

$x = a\sec\theta$ requires $\theta = \sec^{-1}(\frac{x}{a})$ with $0 \leq \theta < \frac{\pi}{2}$ if $\frac{x}{a} \geq 1$;
$\frac{\pi}{2} < \theta \leq \pi$ if $\frac{x}{a} \leq -1$

For $1+\tan^2\theta = \sec^2\theta, 1-\sin^2\theta = \cos^2\theta, \sec^2\theta - 1 = \tan^2\theta$

Evaluate the integrals.

13. $\int \frac{dx}{\cancel{x^2\sqrt{x^2-1}}}$, $x > 1$

$\sec x$ $\tan x$

$\int \frac{\cos x(1-\sin^2 x)}{\sin x}\,dx$

$\int \cot x\,dx - \int \frac{1}{2}\sin 2x\,dx$

17. $\int \frac{x^3\,dx}{\sqrt{x^2+4}}$

$u = x^2+4$

$du = 2x\,dx$

$\frac{(u-4)\frac{du}{2}}{\sqrt{u}}$

21. $\int \sqrt{\frac{x+1}{1-x}}\,dx$

$\int \frac{x+1}{\sqrt{1-x^2}}\,dx$

$-\sqrt{1-x^2} + \sin^{-1} x + C$

44. $\int \frac{\sqrt{1-(\ln x)^2}}{x \ln x}\,dx$

$\int \frac{\sqrt{1-u^2}}{u}\,du$

$\theta = \sin^{-1} u$

$u = \sin\theta$

$\frac{\cos\theta}{\sin\theta}\cos\theta\,d\theta$

$\frac{1-\sin^2\theta}{\sin\theta}\,d\theta$

$\int (\sec\theta - \sin\theta)\,d\theta$

45. $\int \sqrt{\frac{4-x}{x}}\,dx$

$u = 4-x$ $x = 4-u$

$v = u-2$

$-(u-2)^2+4$

$v = 2k$ $dv = 2dk$

$\int \sqrt{\frac{u}{4-u}}\,(-du) = -\int \frac{u}{\sqrt{-u^2+4u}}\,du = -\int \frac{v+2}{\sqrt{4-v^2}}\,dv = -\int \frac{2k+2}{2\sqrt{1-k^2}}\,2dk$