



南方科技大学  
SOUTHERN UNIVERSITY OF SCIENCE AND TECHNOLOGY

考试科目: 高等数学(上) A

开课单位: 数学系

考试时长: 120 分钟

命题教师: \_\_\_\_\_

题号	1	2	3	4	5	6	7	8	9
分值	15 分	15 分	10 分	10 分	10 分	10 分	10 分	10 分	10 分

本试卷共 9 道大题, 满分 100 分. (考试结束后请将试卷、答题本、草稿纸一起交给监考老师)

注意: 本试卷里的中文为直译(即完全按英文字面意思直接翻译), 所有数学词汇的定义请参照教材(Thomas' Calculus, 13th Edition)中的定义。如果其中有些数学词汇的定义不同于中文书籍(比方说同济大学的高等数学教材)里的定义, 以教材(Thomas' Calculus, 13th Edition)中的定义为准。

1. (15 pts) Multiple Choice Questions: (only one correct answer for each of the following questions.)

(1) Let  $f(x) = |x| \sin x$ . The greatest value of  $n$ , for which  $f^{(n)}(0)$  exists, is \_\_\_\_\_.

- (A) 0 (B) 1 (C) 2 (D) 3

(2) If  $\lim_{x \rightarrow \infty} \left( \frac{x^2+1}{x+1} - ax - b \right) = \frac{1}{2}$ , then the values of  $a, b$  are \_\_\_\_\_.

- (A)  $a = 1, b = -\frac{3}{2}$  (B)  $a = -1, b = \frac{3}{2}$   
(C)  $a = -1, b = 1$  (D)  $a = 1, b = -1$ .

(3) The average value of function  $g(x) = x^2 + 6$ , for  $0 \leq x \leq 6$  is \_\_\_\_\_.

- (A) 12 (B) 18 (C) 16 (D) 10

(4) Which one of the following functions is not differentiable at  $x = 0$ ?

- (A)  $f(x) = |x| \sin |x|$   
(B)  $f(x) = |x| \sin \sqrt{|x|}$   
(C)  $f(x) = \cos |x|$   
(D)  $f(x) = \cos \sqrt{|x|}$

(5) What is the derivative of  $f(x) = \frac{1-\sin x}{1+\sin x}$  at  $x = \pi/6$ ?

- (A)  $\frac{4\sqrt{3}}{9}$  (B)  $-\frac{\sqrt{3}}{3}$  (C)  $-\frac{4}{3\sqrt{3}}$  (D)  $\frac{1}{3}$

2. (15 pts) Please fill in the blank for the questions below.

(1) The integration  $\int_{-\pi/2}^{\pi/2} \sin^5 x \cos^3 x dx$  equals \_\_\_\_\_.

(2) If  $f$  is continuous and  $\int_0^{x^3-1} f(t) dt = x$ , then  $f(7) =$  \_\_\_\_\_.

(3) If  $f'(x) = \left(x + \frac{1}{x}\right)^2$  and  $f(1) = 1$ , then  $f(x) =$  \_\_\_\_\_.

- (4) A particle is moving on the sphere  $x^2 + y^2 + z^2 = 13^2$ . While  $t = t_0$ ,  $x(t_0) = 3$ ,  $y(t_0) = 4$ ,  $z(t_0) = 12$ ,  $x'(t_0) = 4$ ,  $y'(t_0) = 3$ , then  $z'(t_0) = \underline{\hspace{2cm}}$ .

(5)  $\lim_{s \rightarrow a} \frac{\sqrt{s^2+1} - \sqrt{a^2+1}}{s-a} = \underline{\hspace{2cm}}$ .

3. (10 pts) Find the limits (DO NOT apply l'Hôpital's Rule).

(1)  $\lim_{x \rightarrow \infty} \frac{\sqrt{3+x} - \sqrt{x+1}}{x^2 + x - 2}$ .

(2)  $\lim_{x \rightarrow 0} \frac{\cos x - \sec^2 x}{x \sin x}$ .

4. (10 pts) Evaluate the integral.

(1)  $\int_0^{2\pi} |\sin^2 x - \cos^2 x| dx$ .

(2)  $\int_0^1 (x+2)\sqrt{1-x^2} dx$ .

5. (10 pts) Let  $f(x) = \frac{x^3+x-2}{x-x^2}$ .

- (1) Identify the inflection points and local maxima and minima of the function that may exist.

- (2) Identify the horizontal, vertical, and oblique asymptotes that may exist.

- (3) Graph the function.

6. (10 pts)

- (1) Find  $\frac{dy}{dx}$  if

$$y(x) = \int_1^{1+2x} \sqrt{t^2 - 1} dt, \quad x > 0.$$

- (2) Find the equation of the line that is tangent to the curve  $x^2 - y^2 = 9$  at point  $(5, -4)$ .

7. (10 pts) Find the area of the region bounded by curves  $y = x^2$  and  $y = 2x - x^2$ .

8. (10 pts) Find the volume of the solid generated by revolving the region bounded by  $y = 2x - 1$ ,  $y = \sqrt{x}$  and  $y$ -axis about the line  $x = -1$ .

9. (10 pts) Assume that  $f$  is continuous on  $[0, 1]$ . Show that there exists a number  $c \in (0, 1)$  such that  $f(c) = \int_0^1 f(x) dx$ .

一、(15分) 单项选择题: (每题只有一个正确答案.)

(1) 令  $f(x) = |x| \sin x$ . 使得  $f^{(n)}(0)$  存在的  $n$  的最大值为\_\_\_\_\_.

- (A) 0 (B) 1 (C) 2 (D) 3

(2) 若  $\lim_{x \rightarrow \infty} \left( \frac{x^2+1}{x+1} - ax - b \right) = \frac{1}{2}$ , 则  $a, b$  的值为\_\_\_\_\_.

- (A)  $a = 1, b = -\frac{3}{2}$  (B)  $a = -1, b = \frac{3}{2}$   
(C)  $a = -1, b = 1$  (D)  $a = 1, b = -1$

(3) 函数  $g(x) = x^2 + 6$  在  $0 \leq x \leq 6$  上的平均值为\_\_\_\_\_.

- (A) 12 (B) 18 (C) 16 (D) 10

(4) 下列哪个函数在  $x = 0$  处不可微?

- (A)  $f(x) = |x| \sin |x|$   
(B)  $f(x) = |x| \sin \sqrt{|x|}$   
(C)  $f(x) = \cos |x|$   
(D)  $f(x) = \cos \sqrt{|x|}$

(5) 函数  $f(x) = \frac{1-\sin x}{1+\sin x}$  在  $x = \pi/6$  的导数是\_\_\_\_\_.

- (A)  $\frac{4\sqrt{3}}{9}$  (B)  $-\frac{\sqrt{3}}{3}$  (C)  $-\frac{4}{3\sqrt{3}}$  (D)  $\frac{1}{3}$

二、(15分) 填空题.

(1) 定积分  $\int_{-\pi/2}^{\pi/2} \sin^5 x \cos^3 x dx$  的值为\_\_\_\_\_.

(2) 如果  $f$  连续且  $\int_0^{x^3-1} f(t) dt = x$ , 则  $f(7) =$ \_\_\_\_\_.

(3) 如果  $f'(x) = (x + \frac{1}{x})^2$  且  $f(1) = 1$ , 则  $f(x) =$ \_\_\_\_\_.

(4) 一个质点在球面  $x^2 + y^2 + z^2 = 13^2$  上运动. 在  $t = t_0$  时,  $x(t_0) = 3, y(t_0) = 4, z(t_0) = 12, x'(t_0) = 4, y'(t_0) = 3$ , 则  $z'(t_0) =$ \_\_\_\_\_.

(5)  $\lim_{s \rightarrow a} \frac{\sqrt{s^2+1}-\sqrt{a^2+1}}{s-a} =$ \_\_\_\_\_.

三、(10分) 求下列极限 (不要使用洛必达法则).

(1)  $\lim_{x \rightarrow \infty} \frac{\sqrt{3+x} - \sqrt{x+1}}{x^2 + x - 2}$ .

(2)  $\lim_{x \rightarrow 0} \frac{\cos x - \sec^2 x}{x \sin x}$ .

四、(10分) 计算积分.

(1)  $\int_0^{2\pi} |\sin^2 x - \cos^2 x| dx$ .

(2)  $\int_0^1 (x+2)\sqrt{1-x^2} dx$ .

五、(10分) 考虑函数  $f(x) = \frac{x^3+x-2}{x-x^2}$ .

(1) 求所有 (局部) 极值点和拐点.

(2) 求所有水平渐近线、垂直渐近线和斜渐近线.

(3) 作出函数 $f(x)$ 的简略图.

六、(10分)

(1) 求  $\frac{dy}{dx}$ , 这里

$$y(x) = \int_1^{1+2x} \sqrt{t^2 - 1} dt, \quad x > 0.$$

(2) 求曲线 $x^2 - y^2 = 9$ 在点 $(5, -4)$ 处的切线.

七、(10分) 求曲线 $y = x^2$ 和 $y = 2x - x^2$ 所围成的区域的面积.

八、(10分) 曲线 $y = 2x - 1$ ,  $y = \sqrt{x}$ 和 $y$ 轴围成一个区域. 把这个区域绕直线 $x = -1$ 旋转可得一个旋转体, 求此旋转体的体积.

九、(10分) 设函数 $f$ 在区间 $[0, 1]$ 上连续. 证明: 至少存在一点 $c \in (0, 1)$ 使得 $f(c) = \int_0^1 f(x) dx$ .