Calculus I　复习专题一　第二和七章　极限与连续性

1. (2019年期末) Determine whether the following statements are true or false? No justification is necessary.

(1) If $|f(x)|$ is continuous at $x = a$, then so is $(f(x))^2$.

(2) Suppose that $f(a) = g(a) = 0$, that f and g are differentiable on an open interval I containing a, and that $g'(x) \neq 0$ on I if $x \neq a$. If $\lim_{x\to a} \frac{f'(x)}{g'(x)}$ does not exist, then neither does $\lim_{x\to a} \frac{f(x)}{g(x)}$.

2.(2022年期末) Let $f(x) = \frac{\tan x}{|x|(x-\frac{\pi}{2})^4}$. Which of the following statements must be correct?

(A) f is continuous at $x = 0$ and f has a jump discontinuity at $x = \frac{\pi}{2}$.

(B) f has a jump discontinuity at $x = 0$ and f is continuous at at $x = \frac{\pi}{2}$.

(C) f has an infinite discontinuity at $x = 0$ and f has an oscillating discontinuity at $x = \frac{\pi}{2}$.

(D) f has a jump discontinuity at $x = 0$ and f has an infinite discontinuity at $x = \frac{\pi}{2}$.

3. (2021年期末) If $f(x) = \frac{2+e^{\frac{1}{x}}}{1+e^{\frac{1}{x}}}$, if $x \neq 0$; $f(x) = 0$ if $x = 0$. Then at $x = 0$, it is a

(A) jump discontinuity.　(B) removable discontinuity.

(C) infinite discontinuity.　(D) continuous point

4. (2019年期末) If $f(x) = \frac{\ln|x|}{|x-1|}\sin x$, then the function $f(x)$ has

(A) 1 removable discontinuity and 1 jump discontinuity.

(B) 2 removable discontinuities.

(C) 1 removable discontinuity and 1 infinite discontinuity.

(D) 2 jump discontinuities.

5. (2019年期末) Suppose $\lim_{x\to 0^+} f(x) = a$, $\lim_{x\to 0^-} f(x) = b$, then $\lim_{x\to 0^-}(f(x - \sin x) + 2f(x^2 + x)) =$

(A) $a + 2b$.　(B) $b + 2a$.　(C) 3a.　(D) 3b .

6. (2022年期末) The number of asymptotes of $y = e^{\frac{1}{x^2}} \arctan \frac{x^2+x+1}{(x-1)(x+2)}$ is

(A) 1　(B) 2　(C) 3　(D) 4

7. (2022年期末) Suppose that $a < 0 < b$, and $f(x)$ is continuous on (a, b). Let $F(x) = \frac{\int_0^x tf(t)\,dt}{x}$, if $x \neq 0$; $F(x) = 0$ if $x = 0$. Which of the following statements must be correct?

(A) F is differentiable on (a, b) and F' is not continuous at $x = 0$.

(B) F is differentiable on (a, b) and F' is continuous at $x = 0$.

(C) F is not differentiable on (a, b) and F is continuous at $x = 0$.

(D) None of the above statements is correct.

8. (2021年期末) If $\lim_{x\to\infty}(\frac{x+a}{x-a})^x = 8$, then $a =$ (　　).

9. (2022年期末) Evaluate the following limits: $\lim_{x\to 0^+} \frac{\ln \tan 7x}{\ln \tan 2x}$.

10. (2021年期末) Evaluate the following limits.

(1) $\lim_{x\to 0} \frac{(1+x)^{\frac{1}{x}} - e}{x}$

(2) $\lim_{x\to 0} \frac{3\sin x + x^2 \cos \frac{1}{x}}{(1+\cos x)\ln(1+x)}$

11. (2020年期末) Evaluate the following limits: $\lim_{x\to 0} \left(\frac{\ln(1+x)}{x}\right)^{\frac{1}{e^x - 1}}$.

12. (2019年期末) Evaluate the following limits.

(1) $\lim_{x\to 0} \frac{\tan^{-1} x - x}{x \tan^2 x}$.

(2) $\lim_{x\to \infty} \frac{(x+100)^{100x}}{x^{100x}}$