Tutorial 11 for Calculus I

Sect. 8.1-8.2

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Using Basic Integration Formulas

Evaluate the integrals.

9. $\int \frac{dz}{e^z+e^{-z}}$ $\frac{e^z dz}{e^{2z}+1}$ $u=e^z$ $du=e^z dz$

27. $\int \frac{2dx}{x\sqrt{1-4\ln^2 x}}$ $2\ln x = u$ $2\frac{1}{x}dx = du$ $\frac{du}{\sqrt{1-u^2}}$ $\sin^{-1} u$

33. $\int_{-1}^{0} \sqrt{\frac{1+y}{1-y}}\, dy$ $\frac{1+y}{\sqrt{1-y^2}}dy$

$du = \sec^2 x\, dx$

38. $\int \frac{d\theta}{\cos\theta - 1}$ $\frac{d\frac{\theta}{2}}{\cos^2\frac{\theta}{2}}$ $\frac{du}{\cos^2 u}$ $\sec^2 u\, du$

$\int \frac{(\tan^2 x+1)\, dx}{(2\tan^2 x+1)}$

40. $\int \frac{\sqrt{x}}{1+x^3}\, dx.$ $u = x^{\frac{3}{2}}$ $du = \frac{3}{2}x^{\frac{1}{2}}dx$ $\frac{2}{3}du = \sqrt{x}dx$ $\frac{\frac{2}{3}du}{1+u^2}$ $\frac{1}{2}+\frac{\frac{1}{2}}{2u^2+1}$

48. Use the substitution $u = \tan x$ to evaluate the integral $\int \frac{dx}{1+\sin^2 x}$. $\int \frac{du}{2u^2+1}$

$\tan^{-1} u = x$

$dx = \frac{1}{u^2+1}du$

Review of Sect. 8.2

Integration by Parts:

1. $\int u dv = uv - \int v du$ P464

2. $\int_a^b f(x)g'(x)dx = f(x)g(x)]_a^b - \int_a^b f'(x)g(x)dx$ P467

Evaluate the integrals.

18. $\int (r^2 + r + 1)\, e^r \, dr$ $\quad d(e^r)$

48. $\int_0^{\frac{\pi}{2}} x^3 \cos 2x \, dx$ $\quad \frac{1}{16}\int_0^{\pi} u^3 \, d(\sin u)$

$= \frac{1}{16} u^3 \sin u \Big]_0^{\pi} + \frac{1}{16}\int_0^{\pi} 3u^2 \, d(\cos u)$

$\int_1^e d(\frac{1}{4}x^4) \ln x$

6. $\int_1^e x^3 \ln x \, dx$ $= \frac{1}{4}x^4 \ln x \Big]_1^e - \int_1^e \frac{1}{4}x^4 \frac{1}{x} dx$

33. $\int x (\ln x)^2 \, dx$ $\quad \int d(\frac{1}{2}x^2)(\ln x)^2$

49. $\int_{\frac{2}{\sqrt{3}}}^{2} t \sec^{-1} t \, dt$ $\quad = (\ \ ) - \int x \ln x \cdot dx$

$= \int d(\frac{1}{2}t^2) \sec^{-1} t$

$= (\ \ ) - \int \frac{1}{2}t^2 \frac{1}{\sqrt{1-t^2}} dt$ $\quad \theta = \sin^{-1} t$ $\quad t = \sin\theta$

$\int \frac{1}{2}\sin^2\theta \, d\theta$

$F'(x) = f(x)$

$= xF(x) - \int_a^b d(\frac{1}{2}x^2) F(x)$

$\int_a^b x \, d(F(x)) - aF(x)\Big]_a^b$

23. $\int e^{2x} \cos 3x \, dx$

$\ln x = u$

29. $\int \sin(\ln x) \, dx$ $\quad e^u = x$ $\quad e^u du = dx$

41. $\int \sin 3x \cos 2x \, dx$

$\frac{1}{2}(\sin 5x + \sin x)$ $\quad \int_x^b f(t)dt$

69. Show that $\int_a^b (\int_x^b f(t) \, dt) \, dx = \int_a^b (x - a) f(x) \, dx$.

11. Compute the derivatives of the following functions:

(4) $y = \arccos(e^{-t})$ (5) $y = x^{\arctan x} \quad (x > 0)$

12. Find $\frac{dy}{dx}$ if $y = \int_{x^2+1}^{2x^2+3} t \tan\sqrt{x+t}\ dt$.

15. Prove the following identities:

(1) $2\arctan\sqrt{x} - \arcsin\frac{x-1}{x+1} = \frac{\pi}{2} \qquad (x \geq 0)$.

(2) $\int_0^x \int_0^u f(t)\ dt\ du = \int_0^x f(u)(x-u)\ du$, f is continuous.

18. Assume f(x) is continuous and $\int_0^x tf(2x-t)\ dt = \frac{1}{2}\arctan(x^2)$ with $f(1) = 1$. Compute $\int_1^2 f(x)\ dx$. 积分求导公式

19. Given the following function $f(x) = \begin{cases} \frac{\ln(1+ax^3)}{x-\arcsin x}, & \text{x<0} \\ 0, & \text{x=0} \\ \frac{e^{ax}+x^2-ax-1}{x\sin\frac{x}{4}}, & \text{x>0} \end{cases}$

(1) If $f(x)$ is continuous at $x=0$, find the value of a.

(2) If $f(x)$ has a removable discontinuity at $x=0$, find a.

4. Compute the following limits:

(2) $\lim\limits_{x\to 0} \dfrac{(1+x)^{\frac{1}{x}} - e}{x}$

(3) $\lim\limits_{x\to\infty} (\pi - 2\arctan x)\ln x$

(5) $\lim\limits_{x\to 0} \dfrac{e^x - e^{-x} - 2x}{x\sin(x^2)}$

(6) $\lim\limits_{x\to 0} \dfrac{x\cot x - 1}{x^2\ln(1+x)}$

(7) $\lim\limits_{x\to 0} \cot x\left(\frac{1}{\sin x} - \frac{1}{x}\right)$

(11) $\lim\limits_{x\to 0} \dfrac{x\arcsin^2 x}{\sin x - x}$

(12) $\lim\limits_{x\to 0}\left(\frac{1}{\arcsin^2 x} - \frac{1}{x^2}\right)$

(16) $\lim\limits_{x\to 0^+} \dfrac{\arctan^3\sqrt{x}}{\ln(1+\sqrt{x})\sin x}$

(19) $\lim\limits_{x\to +\infty} \dfrac{\int_1^x \left(t^2(e^{\frac{1}{t}} - 1) - t\right)\, dt}{x^2\ln(1+\frac{1}{x})}$

(20) $\lim\limits_{x\to 0}\left(\frac{1}{e^x - 1} - \frac{1}{\ln(1+x)}\right)$