Calculus I　复习专题三　　第五和八章　　积分

知识点一: 积分的定义.

$\frac{1-\cos(2+2x)}{2}$

$\int_0^1 x\frac{1}{2}-\frac{x}{2}\cos(2+2x)\,dx$　分开做两部

1.(2022年期末) Evaluate the following limits: $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{k}{n^2}\sin^2(1+\frac{k}{n})$.

$u=2+2x$　$du=2dx$

$=\int_0^1\frac{1}{2}x-\int_0^1\frac{x}{2}\cos(2+2x)\,dx$

2.(2021年期末) $\lim_{n\to\infty}\frac{1}{n}(\sqrt{1+\cos\frac{\pi}{n}}+\sqrt{1+\cos\frac{2\pi}{n}}+\cdots+\sqrt{1+\cos\frac{n\pi}{n}})=(\quad)$.

$=\frac{1}{\pi}\int_0$

$=\int_2^4\frac{u-2}{4}\cos u\,\frac{du}{2}$

3.(2020年期末) Evaluate the following limits.

$\lim_{n\to+\infty}(\frac{n}{2n^2+3n+1^2}+\frac{n}{2n^2+6n+2^2}+\cdots+\frac{n}{2n^2+3nk+k^2}+\cdots+\frac{n}{2n^2+3n^2+n^2})$.

$(2n+1)(n+1)$　$(2n+2)(n+2)$　$(n+k)(n+k)$

$=\frac{1}{8}\int_2^4 u\cos u\,du-\int_2^4\frac{1}{4}\cos u\,du$

$=\frac{1}{8}(u\sin u+\cos u)\Big|_2^4-\frac{1}{4}\sin u\Big|_2^4$

知识点二: 积分的性质(填空选择题).

4.(2022年期末) If $\int_0^1\frac{e^x}{x+1}\,dx=a$, then $\int_0^1\frac{e^x}{(x+1)^2}\,dx=(\quad)$.

5.(2021年期末) If $\int_0^x f(t)\,dt=\frac{x^4}{2}$, then $\int_0^4\frac{1}{\sqrt{x}}f(\sqrt{x})\,dx=$

$u=\sqrt{x}$　$du=\frac{1}{2\sqrt{x}}dx$

(A) 8.　　(B) 16.　　(C) 128.　　(D) 256

$2\int_0^2 f(u)\,du$

6.(2021年期末) Let $f(x)$ be a continuous function on $[-a,a], a>0$, then $\int_{-a}^{a}f(x)\,dx=$

(A) $\int_0^a(f(x)+f(-x))\,dx$　　(B) $\int_0^a(f(x)-f(-x))\,dx$　　(C) 0　　(D) $2\int_0^a f(x)\,dx$

7.(2020年期末) If $f(x)$ is continuous on $(-\infty,\infty)$, which of the following statements is wrong ?

(A) $\int_0^1 f(x)\,dx=\int_0^1 f(t)\,dt$.　　(B) $\int_0^1 f(x)\,dx=\int_0^1 f(\sin x)\,d(\sin x)$.

(C) $d(\int_0^x f(t)\,dt)=f(x)\,dx$.　　(D) $d(\int_0^{x^2} f(t)\,dt)=f(x^2)\,d(x^2)$.

8.(2019年期末) Let $f(x)$ be a continuous function, and a is a nonzero constant. Which of the following function is an odd function ?

(A) $\int_a^x(\int_0^u tf(t^2)\,dt)\,du$.　　(B) $\int_0^x(\int_a^u f(t^3)\,dt)\,du$.

(C) $\int_0^x(\int_a^u tf(t^2)\,dt)\,du$.　　(D) $\int_a^x(\int_0^u (f(t))^2\,dt)\,du$.

9.(2021年期末) The average value for $f(x)=\cos^4 x$ on $[0,\pi]$ is (　).

10.(2020年期末) The average value for $f(x)=\sin^3 x$ on $[0,\pi]$ is (　).

11.(2021年期末) Using Simpson's Rule with $n=4$ to estimate $\int_2^4\frac{1}{x-1}\,dx$, the approximation is (　).

4 2 4

1　2　4

$\frac{\Delta x}{3}(\quad)$

知识点三: 积分的计算.

12.(2022年期末) Evaluate the integrals.

(1) $\int x\tan^2 x\,dx$.　$(\sec^2 x-1)$　$\int x\,d(\tan x)$　$\ln|\sec x|$

(2) $\int_1^{\sqrt{3}}\frac{1}{x(1+x^2)^2}\,dx$.　$u=x^2$　$du=2x\,dx$　$\int_1^3\frac{\frac{1}{2}du}{u(1+u)^2}$

(3) $\int\frac{\ln(1-x^2)}{x^2\sqrt{1-x^2}}\,dx$.

13.(2021年期末) Evaluate the integrals.

(1) $\int_{\frac{1}{e}}^{e}\frac{\ln^2 x}{x}\,dx$　$d(\ln x)$　$u=\ln x$　$x=\frac{1}{e}$　$u=-1$　$u=1$

(2) $\int_1^{\sqrt{2}}\frac{1}{x^3\sqrt{x^2-1}}\,dx$

(3) $\int\frac{1}{(1+x+x^2)^2}\,dx$

14.(2020年期末) Evaluate the integrals.

(1) $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \csc^3 x\, dx$.

(2) $\int \sqrt{\frac{x}{x-2}}\, dx$, where $x > 2$.

(3) $\int_1^e \ln^3 x\, dx$

15.(2019年期末) Evaluate the integrals.

(1) $\int \frac{dx}{\sqrt{1+e^x}}$.

$u = \sqrt{1+e^x}$

(2) $\int \frac{3x+6}{(x-1)^2(x^2+x+1)}\, dx$.

$du = \frac{e^x}{2\sqrt{1+e^x}}\, dx$

(3) $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \tan^2 x \sec x\, dx$

$= \int \frac{2du}{u^2-1}$

16.(2020年期末) Let $f(n) = \sum_{m=1}^{n} \int_0^m \cos \frac{2\pi n[x+1]}{m}\, dx$, here $[x+1]$ is the largest integer which is less than or equal to $x+1$. Evaluate $f(2021)$.

知识点四: 反常积分的计算和判断.

17. Evaluate the integrals.

(1) $\int_0^{+\infty} \frac{\tan^{-1} e^x}{e^{2x}}\, dx$. (2022年期末)

(2) $\int_1^{\infty} \frac{1}{x^6(x^5+4)}\, dx$. (2021年期末)

(3) $\int_1^{+\infty} \frac{(x+2)\ln(x^2+1)}{x^3}\, dx$. (2020年期末)

(4) $\int_{\frac{1}{2}}^{\frac{3}{2}} \frac{1}{\sqrt{|x-x^2|}}\, dx$. (2020年期末)

$= \frac{\pi}{2} - 1$

$= \frac{1}{2}\ln 2$

$= \ln 2$

$= 3\ln 2 + \frac{\pi}{2}$

18.(2022年期末) Which of the following improper integrals is divergent ?

(A) $\int_{-1}^{1} \ln|x|\, dx$ (B) $\int_0^1 \frac{e^x}{\sqrt{1-x}}\, dx$ (C) $\int_0^{+\infty} \frac{1}{x^2}\, dx$ (D) $\int_e^{+\infty} \frac{1}{x\ln^2 x}\, dx$

19.(2021年期末) Find all values for p such that the improper integral $\int_0^{\infty} \frac{e^{-x}}{x^p}\, dx$ converges.

20.(2020年期末) If the improper integral $\int_0^{+\infty} \frac{\tan^{-1}(x^2)}{x^k}\, dx$ converges, then the constant k must satisfy

∞ $k>1$ $k-2<1$

(A) $k < 1$. (B) $k > 3$. (C) $1 < k < 2$. (D) $1 < k < 3$.