Tutorial 08 for Calculus I

Sect. 6.1-6.5

Sun Lulu

sunll@sustech.edu.cn

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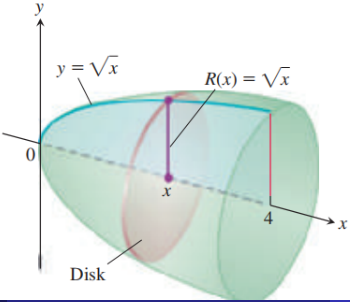
1. Solids of Revolution: The Disk Method

(1)Volume by Disk for Rotation About the x-axis:

$V = \int_a^b A(x)dx = \int_a^b \pi[R(x)]^2 dx$. P330

(2)Volume by Disk for Rotation About the y-axis:

$V = \int_c^d A(y)dy = \int_c^d \pi[R(y)]^2 dy$. P332



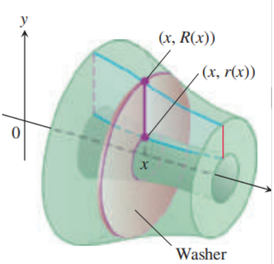
2. The Washer Method

(1)Volume by Washers for Rotation About the x-axis:

$V = \int_a^b A(x)dx = \int_a^b \pi([R(x)]^2 - [r(x)]^2)dx$. P333

(2)Volume by Washers for Rotation About the y-axis:

$V = \int_c^d A(y)dy = \int_c^d \pi([R(y)]^2 - [r(y)]^2)dy$.



Review of Sect. 6.2

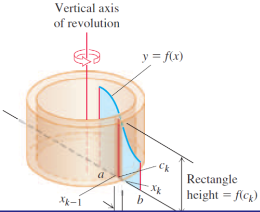
Solids of Revolution: The shell Method

The volume of the solid generated by revolving the region between the x-axis and the graph of a continuous function $y = f(x) \geq 0$, $L \leq a \leq x \leq b$, about a vertical line $x = L$ is

$$V = \int_a^b 2\pi \begin{pmatrix} shell \\ radius \end{pmatrix} \begin{pmatrix} shell \\ height \end{pmatrix} dx = 2\pi \int_a^b (x - L) f(x)\, dx$$

,

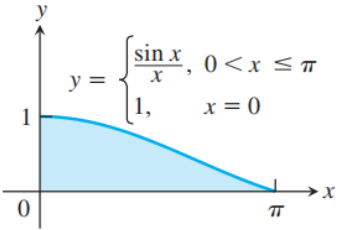
x is the thickness variable.



13. Let $f(x) = \begin{cases} \frac{\sin x}{x}, & 0<x\leq \pi \\ 1, & x= 0 \end{cases}$

a. Show that $xf(x) = \sin x, 0 \leq x \leq \pi$.

b. Find the volume of the solid generated by revolving the shaded region about the y-axis in the accompanying figure.



30. Compute the volume of the solid generated by revolving the triangular region bounded by the lines $2y = x + 4$, $y = x$, and $x = 0$ about

a. the x-axis using the washer method.

b. the y-axis using the shell method.

c. the line $x = 4$ using the shell method.

d. the line $y = 8$ using the washer method.

Homework of Section 6.2

42. A Bundt cake, well known for having a ringed shape, is formed by revolving around the y-axis the region bounded by the graph of $y = \sin(x^2 - 1)$ and the x-axis over the interval $1 \le x \le \sqrt{1 + \pi}$. Find the volume of the cake.

Arc Length:

1.If f' is continuous on $[a,b]$, then the length (arc length) of the curve $y=f(x)$ from the point $A=(a,f(a))$ to the point $B=(b,f(b))$ is the value of the integral $L=\int_a^b\sqrt{1+[f'(x)]^2}dx=\int_a^b\sqrt{1+(\frac{dy}{dx})^2}dx$. P346

Notice:$L=\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}L_k=\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\sqrt{(\Delta x_k)^2+(\Delta y_k)^2}$

$$=\lim_{n\to\infty}\sum_{k=1}^{n}\sqrt{(\Delta x_k)^2+(f'(c_k)\Delta x_k)^2}=\lim_{n\to\infty}\sum_{k=1}^{n}\sqrt{1+[f'(c_k)]^2}\Delta x_k.$$

If g' is continuous on $[c,d]$, then the length (arc length) of the curve $x=g(y)$ from the point $A=(g(c),c)$ to the point $B=(g(d),d)$ is the value of the integral $L=\int_c^d\sqrt{1+[g'(y)]^2}dy=\int_c^d\sqrt{1+(\frac{dx}{dy})^2}dy$. P348

Areas of Surfaces of Revolution:

1. If the function $f(x) \geq 0$ is continuously differentiable on [a,b], the area of the surface generated by revolving the graph of $y = f(x)$ about the x-axis is $S = \int_a^b 2\pi y\sqrt{1 + (\frac{dy}{dx})^2}dx = \int_a^b 2\pi f(x)\sqrt{1 + [f'(x)]^2}dx$. P352

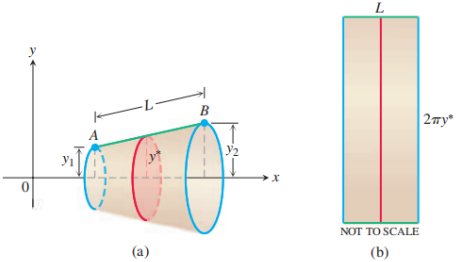


FIGURE 6.29 (a) The frustum of a cone generated by rotating the slanted line segment AB of length L about the x-axis has area $2\pi y^* L$. (b) The area of the rectangle for $y^* = \frac{y_1 + y_2}{2}$, the average height of AB above the x-axis.

2. If the function $x = g(y) \geq 0$ is continuously differentiable on [c,d], the area of the surface generated by revolving the graph of $x = g(y)$ about the y-axis is $S = \int_c^d 2\pi x \sqrt{1 + (\frac{dx}{dy})^2} dy = \int_c^d 2\pi g(u) \sqrt{1 + [g'(y)]^2} dy$. P353

11. Find the length of the curve $x = \int_0^y \sqrt{\sec^4 t - 1} dt, -\frac{\pi}{4} \leq y \leq \frac{\pi}{4}$.

5 .Find the area of the surface generated by revolving the curve $x^{\frac{1}{2}} + y^{\frac{1}{2}} = 3$ from (4,1) to (1,4), about the x-axis.

In Exercises 1–8:

a. Set up an integral for the area of the surface generated by revolving the given curve about the indicated axis.

T **b.** Graph the curve to see what it looks like. If you can, graph the surface too.

T **c.** Use your utility's integral evaluator to find the surface's area numerically.

Review of Sect. 6.5

Work: The work done by a variable force $F(x)$ in the direction of motion along the x-axis from $x = a$ to $x = b$ is

$$W = \lim_{n\to\infty} \sum_{k=1}^{n} F(c_k)\Delta x_k = \int_a^b F(x)dx.$$ P357

Homework of Section 6.5

16. Pumping a half-full tank

    Suppose that, instead of being full, the tank in Example 5 is only half full. How much work does it to take to pump the remaining oil to a level 1 m above the top of the tank?

EXAMPLE 5 The conical tank in Figure 6.39 is filled to within 2 m of the top with olive oil weighing $0.9\ \mathrm{g/cm^3}$ or $8820\ \mathrm{N/m^3}$. How much work does it take to pump the oil to the rim of the tank?

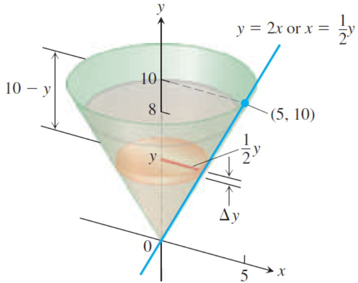


FIGURE 6.39 The olive oil and tank in Example 5.

21. Emptying a water reservoir We model pumping from spherical containers the way we do from other containers, with the axis of integration along the vertical axis of the sphere. Use the figure here to find how much work it takes to empty a full hemispherical water reservoir of radius 5 m by pumping the water to a height of 4 m above the top of the reservoir. Water weighs 9800 $\mathrm{N/m^3}$.