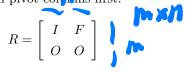
## Problem Set 4 —— Linear Algebra (Spring 2024)

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1. Suppose R is m by n of rank r, with pivot columns first:



- (a) What are the shapes of those four blocks?
- (b) Find a right-inverse B with RB = I if r = m. The zero blocks are gone.
- (c) Find a left-inverse C with CR = I if r = n. The F and O column is gone.
- (d) What is the reduced row echelon form of  $R^T$  (with shapes)?
- (e) What is the reduced row echelon form of  $R^T R$  (with shapes)?
- 2. Suppose you know that the  $3 \times 4$  matrix A has the vector  $\mathbf{s} = (2, 3, 1, 0)$  as the only special solution to  $A\mathbf{x} = \mathbf{0}$ .
  - (a) What is the rank of A and the complete solution to  $A\mathbf{x} = \mathbf{0}$ .
  - (b) What is the exact row reduced echelon form R of A?
  - (c) How do you know that  $A\mathbf{x} = \mathbf{b}$  can be solved for all  $\mathbf{b}$ ?
- 3. Suppose  $A\mathbf{x} = \mathbf{b}$  and  $C\mathbf{x} = \mathbf{b}$  have the same (complete) solutions for every  $\mathbf{b}$ . Is is true that A equals C?
- 4. Which invertible matrices allow A = LU (elimination without row exchanges)?
- 5. Suppose  $Q^T = Q^{-1}$  (transpose equals inverse, so  $Q^TQ = I$ ).
  - (a) Show that the columns  $\mathbf{q}_1, \mathbf{q}_2, \dots, \mathbf{q}_n$  are unit vectors:  $||\mathbf{q}_i||^2 = 1$ .
  - (b) Find a  $2 \times 2$  example with first entry  $q_{11} = \cos \theta$ .