线性代数复习2

2.6~3.3

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O N E

1

linear transformations

Linear transformations

additivity（相加性）

T(v1+v2)=T（v1）+T（v2)

homogeneity(齐次性)

T(cv)=cT(v)

origin

T(0)=0

prove T is a linear transformation?

T is a linear transformation?

2)=T（v1）+T（v2）

cT(v)

考试流程（每学期必考）

1.找一组/证明是一组 **基**

2.证明是一组**线性变换**

3.**矩阵表示**

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6. (8 points) Let $\mathbb{R}^{2\times 2}$ be the vector space consisting of all 2×2 real matrices. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and

$$E = \left\{ E_{11} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, E_{12} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, E_{21} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, E_{22} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}.$$

(a) Show that E is a basis for $\mathbb{R}^{2\times 2}$.

- linear independence
- E spans $\mathbb{R}^{2\times 2}$.

(b) Show that $T : \mathbb{R}^{2\times 2} \to \mathbb{R}^{2\times 2}, X \mapsto XA$ is a linear transformation.

- $T(X+Y) = T(X) + T(Y)$,
- $T(\lambda X) = \lambda T(X)$.

(c) Find the matrix representation of T with respect to the ordered basis $E_{11}, E_{12}, E_{21}, E_{22}$.

设 $\mathbb{R}^{2\times 2}$ 为所有 2×2 实矩阵构成的向量空间. 设 $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$,

$$M = \begin{bmatrix} a & c & 0 & 0 \\ b & d & 0 & 0 \\ 0 & 0 & a & c \\ 0 & 0 & b & d \end{bmatrix}.$$

$$E = \left\{ E_{11} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, E_{12} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, E_{21} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, E_{22} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}.$$

(a) 证明: E 为 $\mathbb{R}^{2\times 2}$ 的一组基.

(b) 证明: $T : \mathbb{R}^{2\times 2} \to \mathbb{R}^{2\times 2}, X \mapsto XA$ 为线性变换.

(c) 求 T 在有序基 $E_{11}, E_{12}, E_{21}, E_{22}$ 下的矩阵表示.

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5. (15 points) Let $M_{2\times 2}(\mathbb{R})$ be the vector space of 2×2 real matrices. Let

$$A=\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix},\ B=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix},\ C=\begin{bmatrix}0 & 0\\ 1 & 0\end{bmatrix}.$$

Consider the map

$$T: M_{2\times 2}(\mathbb{R})\to\mathbb{R}^3,\ T(X)=\begin{bmatrix}tr(A^TX)\\ tr(B^TX)\\ tr(C^TX)\end{bmatrix},$$

for any 2×2 real matrix X, where $tr(D)$ denotes the trace of a matrix D.

The trace of an $n\times n$ matrix D is defined to be the sum of all the diagonal entries of D, in other words,

$$tr(D)=d_{11}+d_{22}+\cdots+d_{nn}.$$

(a) Show that T is a linear transformation.

(b) Find the matrix representation of T with respect to the ordered basis

$$v_1=\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix},\ v_2=\begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix},\ v_3=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix},\ v_4=\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}$$

for $M_{2\times 2}(\mathbb{R})$ and the standard basis

$$e_1=\begin{bmatrix}1\\ 0\\ 0\end{bmatrix},\ e_2=\begin{bmatrix}0\\ 1\\ 0\end{bmatrix},\ e_3=\begin{bmatrix}0\\ 0\\ 1\end{bmatrix}$$

for $\mathbb{R}^3$.

(c) Can we find a matrix X such that $T(X)=\begin{bmatrix}1\\ -2\\ 1\end{bmatrix}$? If yes, please find one such matrix. Otherwise, give an explanation.

T W O

2

orthogonality

orthogonality

The inner product xTy is zero if and only if x and y are orthogonal vectors.

If xTy > 0, their angle is less than 90°. If xTy < 0, their angle is greater than 90 °

Two subspaces V and W of the same space Rn are orthogonal if every

vector v in V is orthogonal to every vector w in W:

Orthogonality

inner product: uTv

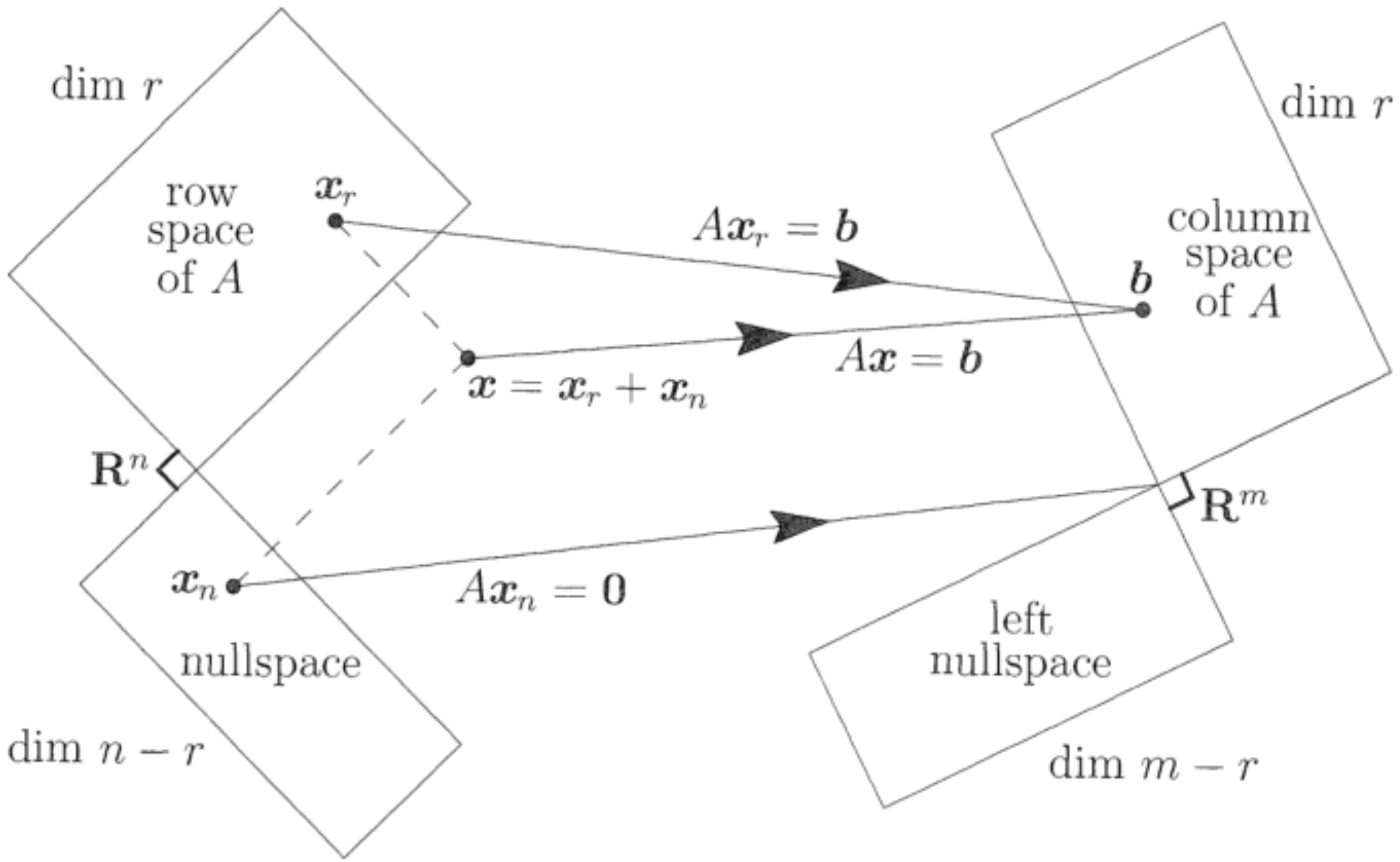
length ||x||2: x12+x22+...+xn2=xTx

orthogonal vectors : uTv=vTu=0

orthonormal basis : 正交基

orthogonal spaces : v in V & w in W, vTw=0

C(A)&N(AT) C(AT)&N(A)



Every matrix transforms its row space onto its column space.

(1) Let $u, v \in \mathbb{R}^n$ with $||u|| = 2$, $||v|| = 4$ and $u^T v = 6$. Then $||3u - v|| =$_

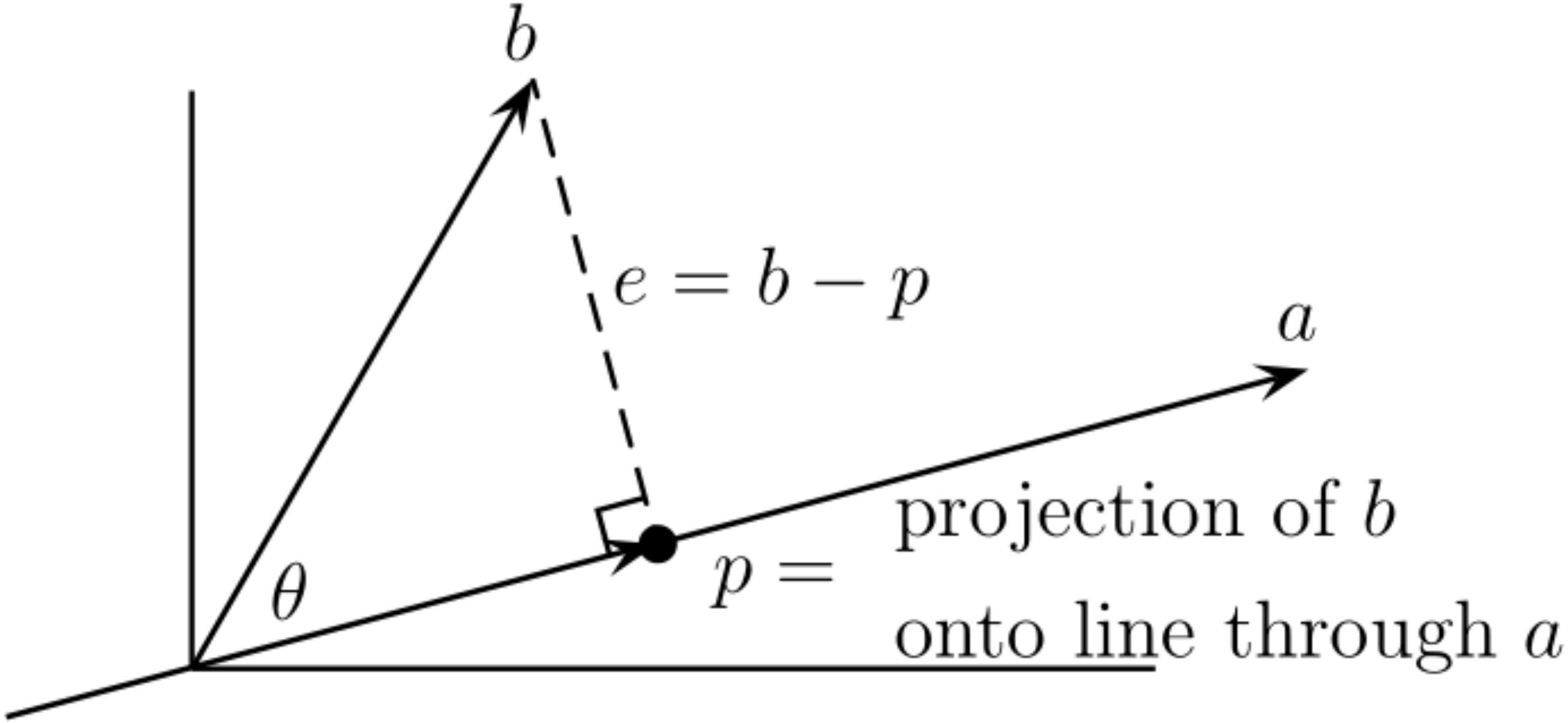
设 $u, v \in \mathbb{R}^n$ 且 $||u|| = 2$, $||v|| = 4$ 以及 $u^T v = 6$. 则 $||3u - v|| =$______

T H E R E

3

Projection

Projection



Projection

$projection\ m$

$schwarz\ inequality$

F O U R

4

Least squares

THANKS