22-23 大物期末: HWX

1. 设 v_1, v_2. 由动能定理:

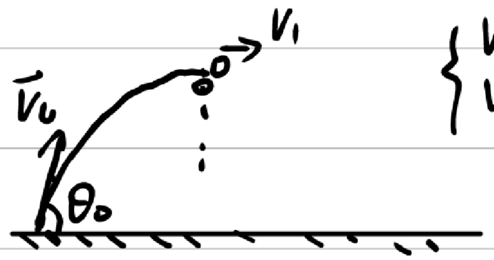
$$-mg(h+L\sin\theta)-\mu_k mg\cos\theta L = 0-\frac{1}{2}mv_1^2$$

$$\Rightarrow v_1=\sqrt{2g(h+L\sin\theta)+2\mu_k gL\cos\theta} = 7.19\ m/s$$

$$mg(h+L\sin\theta)-\mu_k mg\cos\theta\cdot L = \frac{1}{2}mv_2^2-0$$

$$\Rightarrow v_2=\sqrt{2g(h+L\sin\theta)-2\mu_k gL\cos\theta} = 6.44\ m/s$$

2



$$\begin{cases} v_y = v_0\sin\theta_0 - gt \\ v_x = v_0\cos\theta_0 \end{cases}$$

$$\begin{cases} x = v_0\cos\theta_0 t \\ y = v_0\sin\theta_0 t - \frac{1}{2}gt^2 \end{cases}$$

当 $v_y=0 \Rightarrow t=\frac{v_0\sin\theta_0}{g}$ $\quad x_1=\frac{v_0^2\sin\theta_0\cos\theta_0}{g}$ $\quad y=\frac{v_0^2\sin^2\theta_0}{2g}$

设第二块碎片速度 v_1. 动量守恒:

$$m v_x = \frac{m}{2}\cdot 0+\frac{m}{2}v_1$$

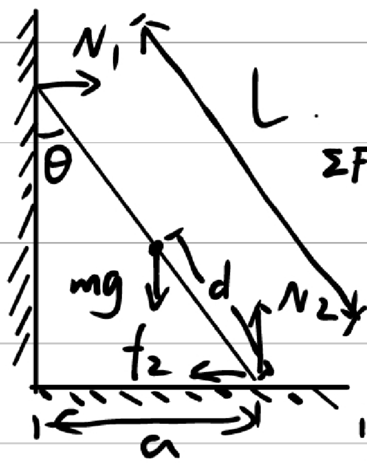
$$\Rightarrow v_1 = 2v_x = 2v_0\cos\theta_0$$

$$\begin{cases} y = \frac{v_0^2\sin^2\theta_0}{2g}-\frac{1}{2}gt^2 \\ x = v_1 t \end{cases}$$

当 $y=0 \Rightarrow t=\frac{v_0\sin\theta_0}{g}$ $\quad x_2=\frac{2v_0^2\sin\theta_0\cos\theta_0}{g}$

故 $s = x_1+x_2 = \frac{3v_0^2\sin\theta_0\cos\theta_0}{g} = 119\ m.$

3.



受力分析如图:

$$\Sigma F=0 \Rightarrow \begin{cases} N_2 = mg & ① \\ N_1 = f_2 & ② \end{cases}$$

$\Sigma M = 0$

$$\Rightarrow mgd\cdot\sin\theta = N_1\cdot L\cos\theta \quad ③$$

临界: $f_2=\mu_s N_2$ ④

①②③④ $\Rightarrow \mu_s = \frac{f_2}{N_2}=\frac{d}{L}\tan\theta = \frac{d}{L}\frac{a}{\sqrt{L^2-a^2}} = 0.217$

4\. (a) 设 ω. 角动量守恒:

$$mv_0\frac{L}{2}=\left[2M\cdot\left(\frac{L}{2}\right)^2+m\left(\frac{L}{2}\right)^2\right]\omega$$

$$\Rightarrow \omega=\frac{2mv_0}{(2M+m)L}=0.16\,rad/s$$

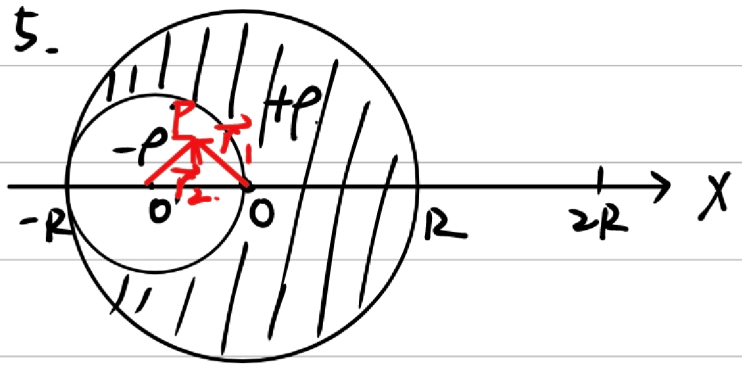
(b) $E_{ki}=\frac{1}{2}mv_0^2=0.4J$

$$E_{kf}=\frac{1}{2}2M\left(\frac{\omega L}{2}\right)^2+\frac{1}{2}m\left(\frac{\omega L}{2}\right)^2$$

$$=\frac{m^2v_0^2}{2(2M+m)}\approx 4.9\times10^{-3}J$$

$$\xi=\frac{E_{kf}}{E_{ki}}=\frac{1}{81}\approx 0.012$$

5\.



看作 $+\rho$ 半径 R 大球, $-\rho$ 半径 $\frac{R}{2}$ 小球.

$$\rho=\frac{M}{\frac{4}{3}\pi R^3}\qquad M_+=M,\ M_-=-\rho\frac{4}{3}\pi\left(\frac{R}{2}\right)^3=-\frac{M}{8}$$

(1) 在 $x=2R$ 处.

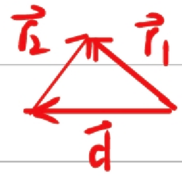
$$F_1=\frac{GM_+m}{(2R)^2}+\frac{GM_-m}{(2R+\frac{R}{2})^2}=\frac{23}{100}\frac{GMm}{R^2}\approx 1.75\times10^{-8}N$$

(2) 在空腔内部

$$\vec{F}(r)=\frac{-G\frac{M}{\frac{4}{3}\pi R^3}\frac{4}{3}\pi r^3\cdot m}{r^2}\hat{r}=-\frac{GMm}{R^3}\vec{r}$$

$$\vec{F}_2=\vec{F}_++\vec{F}_-=-\frac{GMm}{R^3}\vec{r}_1-\frac{G\cdot(-\frac{1}{8}M)\cdot m}{(\frac{R}{2})^3}\vec{r}_2$$

$$=-\frac{GMm}{R^3}(\vec{r}_1-\vec{r}_2)$$



如图, $\vec{r}_1-\vec{r}_2=\vec{d}$.

故 $\vec{F}_2=-\frac{GMm}{R^3}\vec{d}$.

$$|\vec{F}_2|=\frac{GMm}{2R^2}=3.81\times10^{-8}N$$

6. 设物块2质量 m_2.

$$T=2\pi\sqrt{\frac{m_2}{k}} \Rightarrow m_2=k\left(\frac{T}{2\pi}\right)^2=0.75\,kg$$

设物块1反弹速度 v_1. 物块2. v_2.

动量守恒:

$$m_1v_0=m_2v_2-m_1v_1.$$

能量守恒:

$$\frac{1}{2}m_1v_0^2=\frac{1}{2}m_1v_1^2+\frac{1}{2}m_2v_2^2.$$

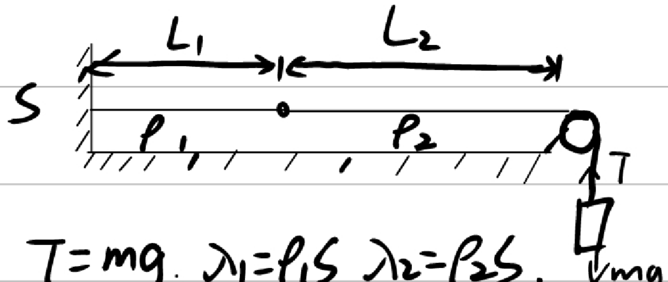
联立解得 $v_1=\frac{m_2-m_1}{m_2+m_1}v_0$

$$=1.80\,m/s$$

之后做平抛.

$$\begin{cases} h=\frac{1}{2}gt^2 \\ d=v_1t \end{cases} \Rightarrow d=v_1\sqrt{\frac{2h}{g}}=1.8\,m.$$

7.



$T=mg.$ $\lambda_1=\rho_1S$ $\lambda_2=\rho_2S.$

$$v_1=\sqrt{T/\lambda_1}=\sqrt{\frac{mg}{\rho_1S}}$$

$$v_2=\sqrt{T/\lambda_2}=\sqrt{\frac{mg}{\rho_2S}}$$

$$\lambda_1=v_1/f \quad \lambda_2=v_2/f.$$

$$L_1=n_1\lambda_1 \quad L_2=n_2\lambda_2.$$

$$\Rightarrow \frac{n_1}{n_2}=\frac{L_1}{L_2}\cdot\frac{\lambda_2}{\lambda_1}=\frac{L_1}{L_2}\cdot\sqrt{\frac{\rho_1}{\rho_2}}=\frac{2}{5}.$$

$$故\begin{cases} n_1=2k & k=1,2,3\cdots \\ n_2=5k & k=1,2,3\cdots \end{cases}$$

$$f=\sqrt{\frac{mg}{\rho_1S}}\,\frac{2k}{L_1} \quad k=1,2,3\cdots$$

故第二最低频率 ($k=2$) 为

$$f=\frac{4}{L_1}\sqrt{\frac{mg}{\rho_1S}}=1157.7\,Hz.$$

8.

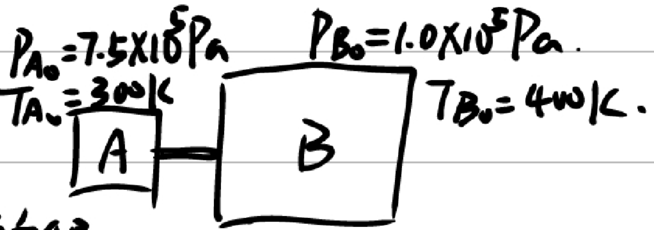
(a) 由多普勒效应：

$$f_{u.s}=\frac{V+V_{us}}{V-V_F}f_0=1023\ Hz$$

(b) 由多普勒效应：

$$f_F=\frac{V+V_F}{V-V_{us}}f_{u.s}=1047\ Hz$$

9. $V_B=3V_A$



(a). 初始. 理想气体状态方程：

$$\begin{cases}P_{A_0}V_A=\nu_A RT_A\\ P_{B_0}V_B=\nu_B RT_B\end{cases}\Rightarrow\begin{cases}\nu_A=\dfrac{P_{A_0}V_A}{RT_A}\\ \nu_B=\dfrac{P_{B_0}V_B}{RT_B}=\dfrac{3P_{B_0}V_A}{RT_B}\end{cases}$$

平衡压强 P. 有：

$$\begin{cases}PV_A=\nu_A' RT_A\\ PV_B=\nu_B' RT_B\end{cases}\qquad \nu_A'+\nu_B'=\nu_A+\nu_B$$

$$\Rightarrow P=\frac{P_{A_0}T_B+3P_{B_0}T_A}{T_B+3T_A}=3.0\times10^5\ Pa$$

(b) $E_{int.i}=\frac{3}{2}\nu_A RT_A+\frac{3}{2}\nu_B RT_B=\frac{3}{2}P_{A_0}V_A+\frac{3}{2}P_{B_0}V_B=\frac{3}{2}V_A(P_{A_0}+3P_{B_0})$.

$=1.575\times10^6\ J$

$E_{int\,f}=\frac{3}{2}\nu_A' RT_A+\frac{3}{2}\nu_B' RT_B=6PV_A=1.8\times10^6\ J$.

$W=0$.

由热一律 $Q=W+\Delta E=2.25\times10^5\ J$.

10. $V_c=8V_b$

$P_b=10.0\ atm$

$V_b=1.0\times10^{-3}\ m^3$

$a\to b$. 等容过程：$W=0$.

$$\frac{P_b}{T_b}=\frac{P_a}{T_a}\qquad P_bV_b=\nu RT_b$$

$Q_1 = \nu C_V (T_b - T_a)$

$b \to c$ 绝热 $Q = 0$.

$P_b V_b^{\gamma} = P_a V_c^{\gamma}$ $\gamma = \frac{5}{3}$

$c \to a$. 等压过程.

$\frac{V_c}{T_c} = \frac{V_b}{T_a}$.

$Q_2 = \nu C_p (T_a - T_c)$

联立得 $Q_1 = \nu \frac{3}{2} R \left(1 - \frac{P_a}{P_b}\right) \frac{P_b V_b}{\nu R}$.

$= \frac{3}{2} P_b V_b \left[1 - \left(\frac{V_b}{V_c}\right)^{\gamma}\right] = 1472\,J > 0$.

$Q_2 = \nu \cdot \frac{5}{2} R \left(1 - \frac{V_c}{V_b}\right) \frac{P_a}{P_b} \cdot \frac{P_b V_b}{\nu R}$.

$= \frac{5}{2} P_b V_b \left(1 - \frac{V_c}{V_b}\right) \left(\frac{V_b}{V_c}\right)^{\gamma} = -554\,J < 0$.

综上 (a) $Q_{吸} = 1472\,J$ (b) $Q_{放} = 554\,J$.