# Tutorial for chapter 5

# Key words of chapter 5

Net force

Superposition 重合

Interact

Equilibrium

Stationary

Inertial reference frame

Relative motion

Free body diagram

Gravitational force

Normal force

Negligible

Tension

Frictional force (friction)

Frictionless

Ignore

Massless

Tangent

Ramp 斜坡

Inclined

External force

Internal force

Orthogonal axis

O 直角的

# Review-Newton's three laws

- **Newton's three mechanics laws**

  only valid in inertial reference frame (like ground), tension, normal force, gravitational force, frictional force…

  - Newton's **First Law** ( Law of Inertia)

  - Newton's **Second Law** $\vec{F}_{net} = m\vec{a}$

$$F_{net,x} = ma_x \qquad F_{net,y} = ma_y \qquad F_{net,z} = ma_z$$

  - Newton's **Third Law**

When two bodies interact, the forces on the bodies from each other are always equal in magnitude and opposite in direction.

# Review—Applying Newton's second law

**Applying Newton's second law to relate the net force and the motion of moving objects:**

1, Identify the target body (or bodies)

2, Draw the F.B.D for the targets ( tension, normal force, gravitational force, frictional force---)

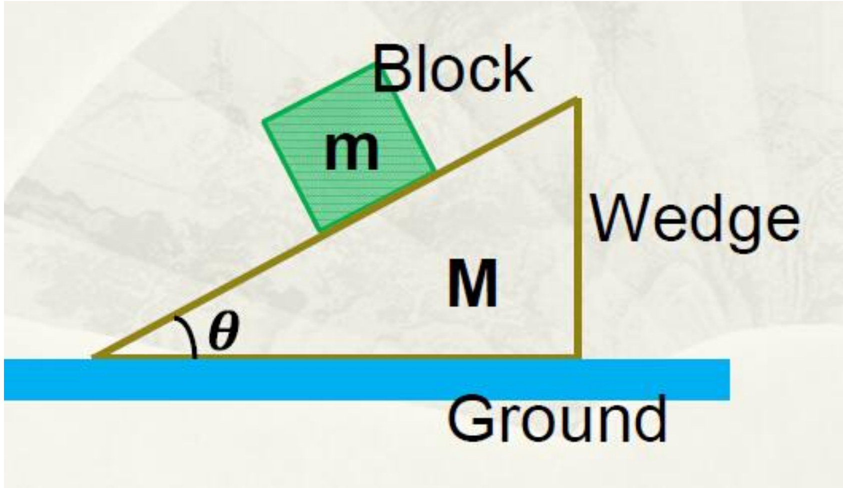
3, Set up a proper coordinates system (be convenient for solving problem)

4, Resolving forces along axes , write component equations of Newton's second law and solve the quantities what you want

5, When component equations of Newton's second law are not enough to figure out the unknown quantities, you have to find other equations (for example, the relationship between accelerations) by analyzing the motion of each object.

# Tutorial Problem 1

As shown in the figure, a block of **mass *m*** is placed on the inclined plane of a wedge. The wedge is then placed on the ground. The mass of wedge is **M** and the angle between its inclined surface and ground is **θ**.




Given the inclined surface of the wedge and the ground surface are **both frictionless**, find **accelerations** of the wedge and the block **with respect to the ground.**

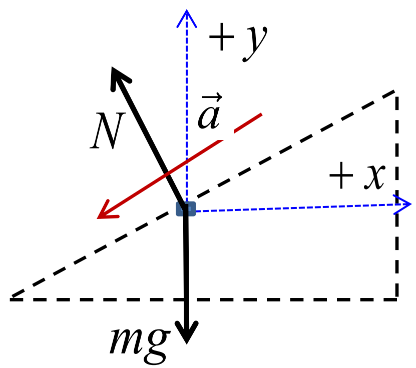
# Solution 1

Known: $m$, $M$, inclined angle $\theta$

Find: Acceleration $\vec{a}_{B,G}$ with respect to ( w.r.t) ground

Acceleration $\vec{b}$ with respect to ground

**F.B.D of Block**

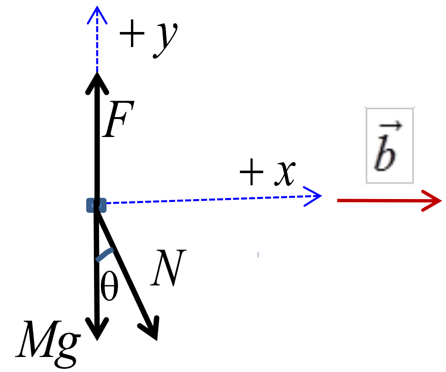



No motion w.r.t wedge $\perp$ inclined plane

**w.r.t wedge** is // inclined plane

$\vec{a}$ point down along inclined plane

**F.B.D of wedge**




Perpendicular to the ground: no motion

So $\vec{b}$ **is parallel to the ground**

**Assuming** $\vec{b}$ **point +*x* direction**

**Set up a coordinates system in which right and vertical up are positive direction**

# Solution 1

Applying Newton's 2nd law to the wedge:

$$N_x = N\sin\theta = Mb............................(1)$$

Applying Newton's 2nd law to the block:

$$\vec{N} + \vec{G} = m\vec{a}$$

**WRONG!!**

$\vec{a}$ is w.r.t wedge, which moves with acceleration, so wedge is **not** an **inertial reference** frame. **Newton's Law can only be applied in inertial reference frame, like ground**

# Solution 1

According to relative motion:

$$\vec{a}_{B,G} = \vec{a}_{B,W} + \vec{a}_{W,G} \Rightarrow \vec{a}_{B,G} = \vec{a} + \vec{b} \qquad \vec{a}_{B,G} \text{ is acceleration of block w.r.t ground}$$

Applying Newton's 2nd law for the block:

$$\vec{N} + \vec{G} = m\vec{a}_{B,G}$$

$x$ component: $N_x + G_x = ma_{B,Gx} = m(a_x + b_x)$

$$-N\sin\theta + 0 = m(-a\cos\theta + b) \text{..........................} (2)$$

y component: $N_y + G_y = ma_{B,Gy} = m(a_y + b_y)$

$$N\cos\theta - mg = m(-a\sin\theta + 0) \text{..........................} (3)$$

$$N_x = N\sin\theta = Mb \text{....................................} (1)$$

# Solution 1

Solving eqs (1), (2'), (3')

$$\begin{cases} a = \dfrac{m+M}{M+m\sin^2\theta} g\sin\theta \\ b = \dfrac{m\cos\theta}{M+m\sin^2\theta} g\sin\theta \\ N = \dfrac{M}{M+m\sin^2\theta} mg\cos\theta \end{cases}$$

Discussion:

$M \to \infty$, Wedge almost stuck on the ground, block slides down like along a fixed inclined plane, we see:

$a \to g\sin\theta$

$b \to 0$

$N \to mg\cos\theta$

$$\alpha_x = a_x + b_x = -a\cos\theta + b = -\frac{M\cos\theta}{M+m\sin^2\theta} g\sin\theta$$

$$\alpha_y = a_y + b_y = -a\sin\theta + 0 = -\frac{m+M}{M+m\sin^2\theta} g\sin^2\theta$$

$$\tan\phi = \frac{\alpha_y}{\alpha_x} = \frac{m+M}{M}\tan\theta > \tan\theta$$

$$\alpha = \sqrt{\alpha_x^2 + \alpha_y^2} = \sqrt{M^2\cos^2\theta + (m+M)^2\sin^2\theta}\,\frac{g\sin\theta}{M+m\sin^2\theta}$$

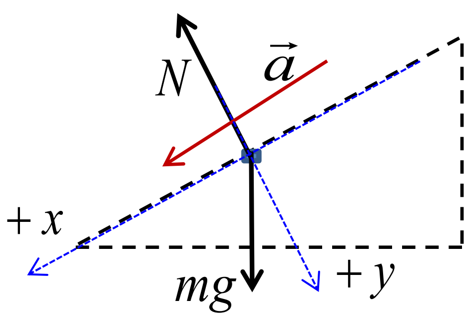
$\phi > \theta$, $\vec{\alpha}$ is not parallel to the inclined plane unless $M \to \infty$

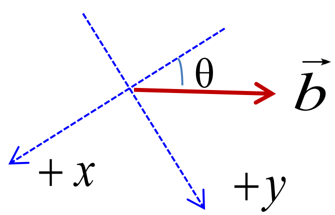
# Solution 1

Alternatively, we can use the coordinate system with axis ⊥ *and* ∥ inclined plane for the block , i.e $x$-axis down along ramp. In this coordinate system:

$\vec{a}$ is in positive $x$ direction.

$\vec{b}$ has two components: $b_x = -b\cos\theta,\quad b_y = b\sin\theta$




Applying Newton's 2nd law:

$$\vec{N} + \vec{G}_g = m\vec{a}$$

$x$ component: $N_x + G_x = ma_{BG,x} = m(a_x + b_x)$

$$0 + mg\sin\theta = m(a - b\cos\theta)..............(2')$$

y component: $N_y + G_y = ma_{BG,y} = m(a_y + b_y)$

$$-N + mg\cos\theta = m(0 + b\sin\theta)...........(3')$$

$$N_x = N\sin\theta = Mb......................(1)$$

# Solution 1

Solving eqs (1), (2'), (3'):

$$a = \frac{m+M}{M+m\sin^2\theta} g\sin\theta$$

$$b = \frac{m\cos\theta}{M+m\sin^2\theta} g\sin\theta$$

$$N = \frac{M}{M+m\sin^2\theta} mg\cos\theta$$

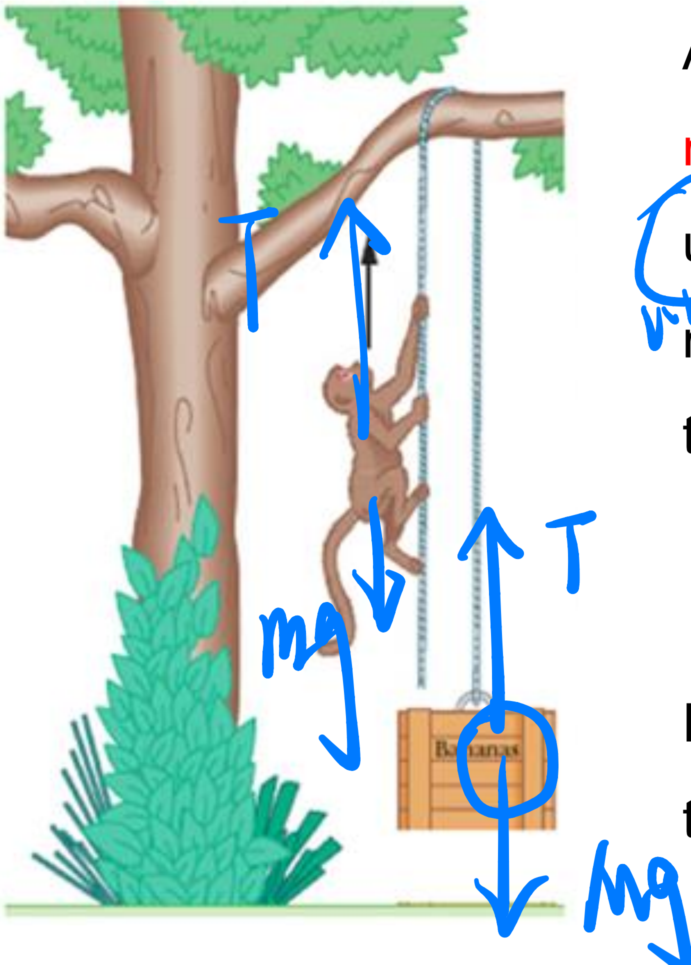
$$a_{BG,x} = a_x + b_x = a - b\cos\theta = g\sin\theta$$

$$a_{BG,y} = a_y + b_y = a + b\sin\theta = g\sin\theta \frac{m\cos\theta\sin\theta}{M+m\sin^2\theta}$$

$a_{BG} = \sqrt{\alpha_x^2 + \alpha_y^2}$ will give the same answer as in the previous coodinate system

**Different coordinates system result the same answer**

# Tutorial Problem 2




As shown in the figure, a monkey of mass m is climbing on a massless rope with an up acceleration $b$ relative to the rope. The rope runs over a frictionless tree limb with the other end tied with a crate of mass M.

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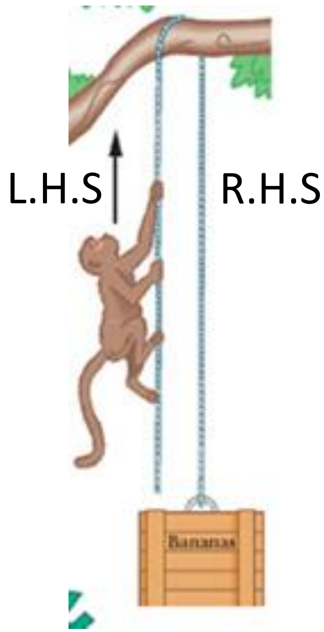
Find the accelerations of the monkey and the crate with respect to the ground.

# Solution 2



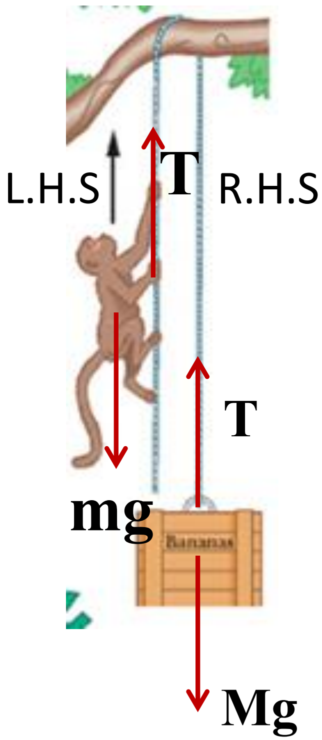

**Known**: Mass of Monkey **m**,
Mass of crate **M**,
Monkey's acceleration with respect to rope $\vec{b}$

**Find:** Acceleration of create respect to ground $\vec{a}_{CG}$
Acceleration of monkey respect to ground $\vec{a}_{MG}$

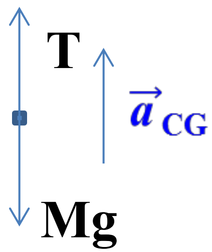
We take the upward to be the positive direction for the monkey and crate. Assuming acceleration of the crate points upward, so does acceleration of the rope with respect to the ground at R.H.S of the tree limb, then the acceleration of the rope with respect to the ground at L.H.S of the tree limb is **?**

$$\boxed{\vec{a}_{RG} = -\vec{a}_{CG}}$$

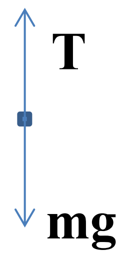
# Solution 2




F.B.D of crate:




F.B.D of monkey:




Applying Newton's 2$^{nd}$ law:

$$T - Mg = Ma_{CG} \text{.................}(1)$$

$$T - mg = ma_{MG} \text{.................}(2)$$

According to relative motion:

$$\vec{a}_{MG} = \vec{a}_{MR} + \vec{a}_{RG} = \vec{b} - \vec{a}_{CG}$$

$$\Rightarrow a_{MG} = b - a_{CG} \text{.................}(3)$$

# Solution 2

$$(1)+(2)+(3) \quad \Rightarrow a_{CG} = \frac{m-M}{m+M}g + \frac{m}{m+M}b$$

$$a_{MG} = b - a_{CG} = \frac{M-m}{m+M}g + \frac{M}{m+M}b$$

Discussion:

1) Given b=g/2, M=2m, we have:

$a_{CG} = -g/6$ w.r.t ground, crate's acceleration of g/6, downward

$a_{MG} = 2g/3$ w.r.t ground, monkey's acceleration of $2g/3$, upward

2) Given b=g/2, M=m, we have

$a_{CG} = g/2$ w.r.t ground, crate's acceleration of $g/2$, upward

$a_{MG} = g/2$ w.r.t ground, monkey's acceleration of $g/2$, upward

## Summary:

**1) Newton's Law can only be applied in inertia reference frame**

we must use the acceleration with respect to the inertia reference frame, like ground.

**2) Choose proper coordinate:**

Different coordinates system must result the same answer, but a proper coordinates system might be much more convenient in solving problems.

# Tutorial for chapter 6

# keywords

- Static  friction ,
- Kinetic friction,
- Coefficient of friction,
- Fluid,
- Drag force,
- Terminal speed,
- Cross-sectional area,
- Spring force,
- Hooke’s law,
- Microscopic,
- Weld,
- Contact,
- Verge,
- sky diver,
- flat track,
- banked track.

# Review

- Friction

  static frictional force and its maximum magnitude, magnitude of kinetic frictional force in relation with normal force,..

- Drag Force

  Terminal speed

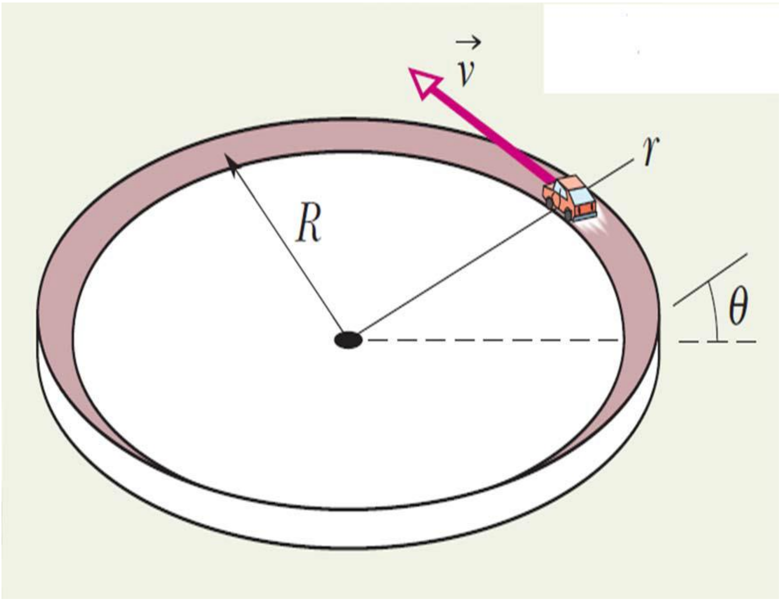
- Spring Force

  Hooke's law

- Centripetal Force in Uniform Circular Motion

  The radial component of the net force causing centripetal acceleration

# Problem 3




A race car runs around a banked circular track with constant speed $v$. The bank inclines with an angle respect to the level. The mass of car is $m$ and the coefficient of the static friction between car's tires and the bank surface is $\mu_s$, The radius of the circular track is $R$.

**Determine the maximum and the minimum speed for the car to remain in the track.**

《工作记录》

http://wanyi0523.photo.pconline.com.cn 婉铱视角VICTORY

2010. 06. 20 中国人民解放军总后勤部定远汽车试验场

# Solution1:

**Direction of the static friction is undetermined. It could either up or down along the banked track.**

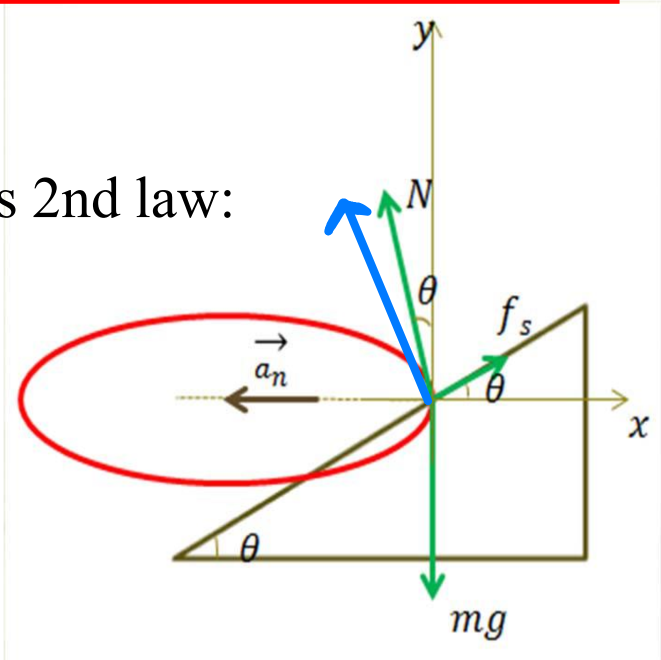
**1) $f_s$ up along the banked track**

We can get two equations applying Newton's 2nd law:

$$y:\quad N\cos\theta + f_s\sin\theta - mg = 0\cdots(1)$$

$$x: -N\sin\theta + f_s\cos\theta = -m\frac{v^2}{R}\cdots(2)$$

符号




**Centripetal force is due to a combination of the horizontal components of the normal force and the static friction.**

# Solution1:

$$(1)+(2)\Rightarrow \mathrm{m}\frac{v^2}{R}=mg\tan\theta-f_s\frac{1}{\cos\theta}$$ as $f_s$ increase, $v$ decrease.

When $f_s = f_{s,max} = \mu_s N$,we have minimum $v$

$$Eq.(1)\Rightarrow \frac{f_{s,\max}}{\mu_s}\cos\theta-mg+f_{s,\max}\sin\theta=0$$

$$f_{s,\max}=mg\Big/\left(\frac{\cos\theta}{\mu_s}+\sin\theta\right)$$

$$\Rightarrow m\frac{v_{\min}^2}{R} = mg\tan\theta - f_{s,\max}\frac{1}{\cos\theta}$$

$$= mg(\tan\theta - \frac{1}{\cos\theta}\frac{1}{\frac{\cos\theta}{\mu_s}+\sin\theta})$$

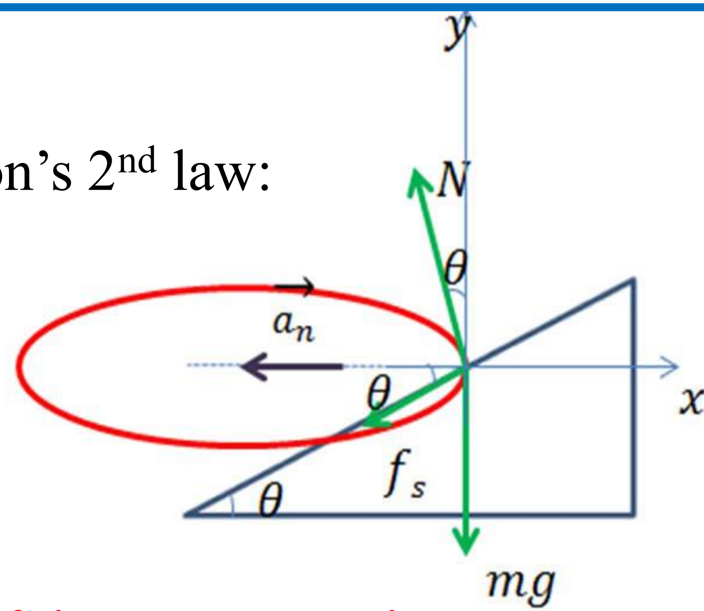
$$= mg\frac{\tan\theta - \mu_s}{1+\mu_s\tan\theta}$$

$$\therefore v_{\min} = \sqrt{gR\frac{\tan\theta - \mu_s}{1+\mu_s\tan\theta}}$$

# Solution1:

## 2) $f_s$ down along the banked track

We can get two equations applying Newton's 2nd law:




$$x: \; -N\sin\theta - f_s\cos\theta = -m\frac{v^2}{R} \cdots (2)$$

$$y: \quad N\cos\theta - f_s\sin\theta - mg = 0 \cdots (1)$$

$$(1),(2) \Rightarrow m\frac{v^2}{R} = mg\tan\theta + f_s\frac{1}{\cos\theta}$$ as $f_s$ increase, $v$ increase.

When $f_s = f_{s.\max} = \mu_s N$, we have maximum $v$.

$$\text{Eq.}(1) \Rightarrow \frac{f_{s.\max}}{\mu_s}\cos\theta\text{-}mg\text{-}f_{s.\max}\sin\theta = 0$$

Therefore, $f_{s.\max} = mg/(\dfrac{\cos\theta}{\mu_s} - \sin\theta)$

$$\Rightarrow m\frac{v_{\max}^2}{R} = mg\tan\theta + f_{s,\max}\frac{1}{\cos\theta}$$

$$= mg(\tan\theta + \frac{1}{\cos\theta}\frac{1}{\cos\theta/\mu_s - \sin\theta}) = \mathrm{mg}\frac{\tan\theta + \mu_s}{1 - \mu_s\tan\theta}$$

$$\therefore v_{\max} = \sqrt{gR\frac{\tan\theta + \mu_s}{1 - \mu_s\tan\theta}}$$

# summary

**Exercise: Ch6-56**

$$v_{\min} = \sqrt{gR\frac{\tan\theta - \mu_s}{1+\mu_s\tan\theta}}$$

$$v_{\max} = \sqrt{gR\frac{\tan\theta + \mu_s}{1-\mu_s\tan\theta}}$$

极端
情况讨论

We can also meets the conditions of $\mu_s < tan\theta$ and $\mu_s < cot\theta$

If static friction is negligible as the sample problem 6-2 discussed in class, we get the speed $v_{max} = \sqrt{gRtan\theta}$

Discussion: As the speed of the car increases from the minimum to a maximum, the static friction evolves from a maximum magnitude up along the ramp to a maximum magnitude down along the banked track.