Chapter 17 KEY WORDS

Bulk modulus (B) 体积模量

Wavefronts 波阵面

Propagation 传播

Medium 媒介

Sound wave 声波

Compression 压缩

Expansion 膨胀

Pressure amplitude 声压振幅

Displacement amplitude 位移振幅

Path length difference 路程差

Interference 干涉

Fully constructive interference 完全相长干涉

Fully destructive interference 完全相消干涉

Beats 拍

Intensity 强度

Isotropic 各向同性的

Sound level 声级

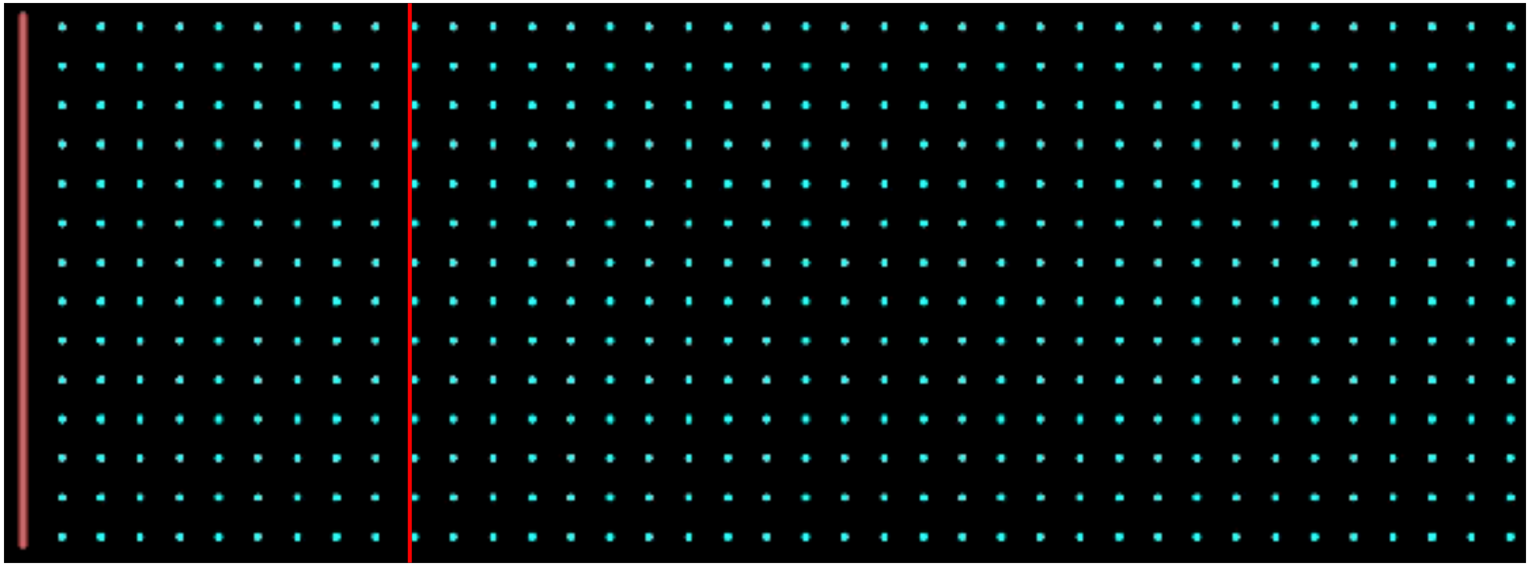
Decibels (dB) 分贝

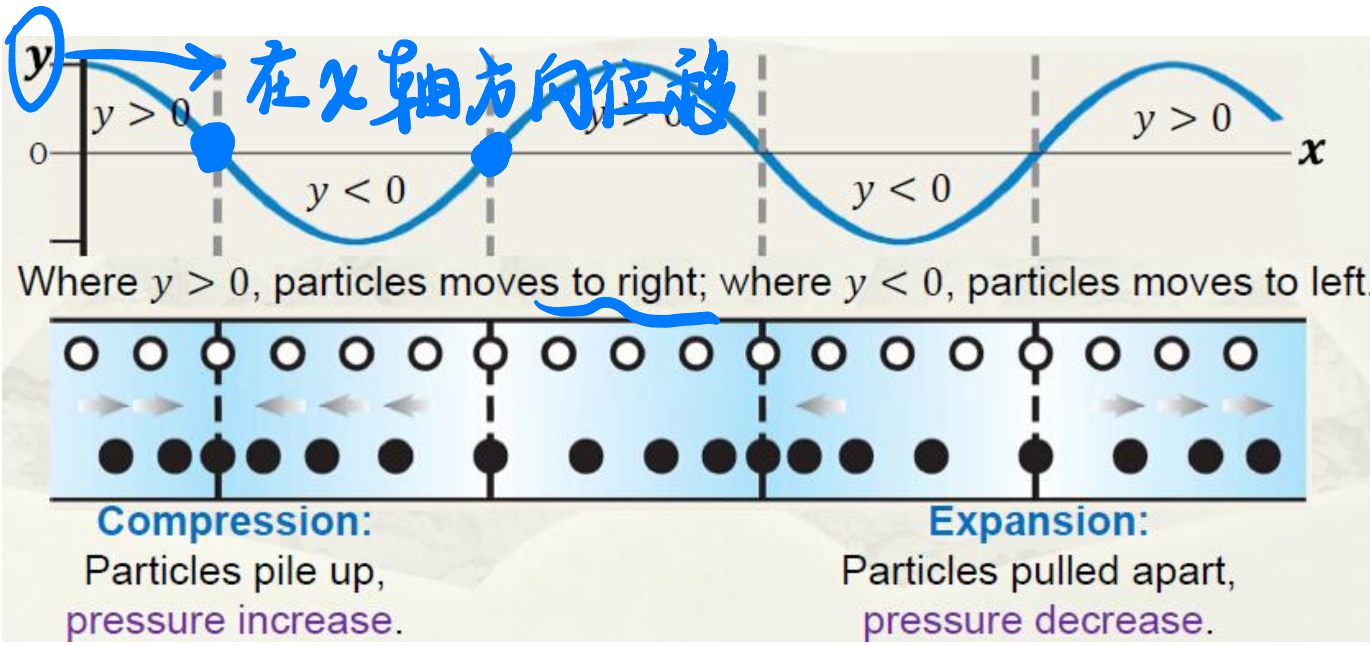
Doppler effect 多普勒效应

Detector 探测器

Source 源

Pipe 管





$$s = s_m \cos(kx-\omega t) \Rightarrow \frac{\partial s}{\partial x} = -ks_m \sin(kx-\omega t)$$

不同位置压强不同

$$\Delta p = -B\frac{\partial s}{\partial x} = Bks_m \sin(kx-\omega t) = \Delta p_m \sin(kx-\omega t)$$

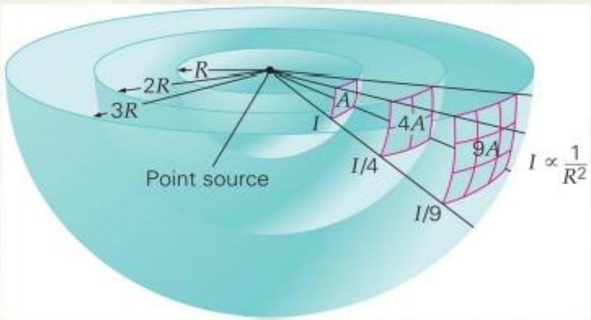
不同时间振动速度不同

$$u = \frac{\partial s}{\partial t} = \omega s_m \sin(kx-\omega t) \qquad a = \frac{\partial u}{\partial t} = -\omega^2 s_m \cos(kx-\omega t)$$

Chapter 17 Wave II--Wave speed and intensity

Wave speed: $$\boxed{v = \sqrt{\frac{B}{\rho}}}$$

- B: bulk modulus of fluid.
- ρ: equilibrium density of the fluid.



$$I = \frac{P_{avg}}{A}$$ SI units: $\mathbf{W/m^2}$

- For an isotropic point source with source power P_s:

$$I(r) = \frac{P_s}{4\pi r^2}$$

Sound Level

Sound intensity in decibel scale:

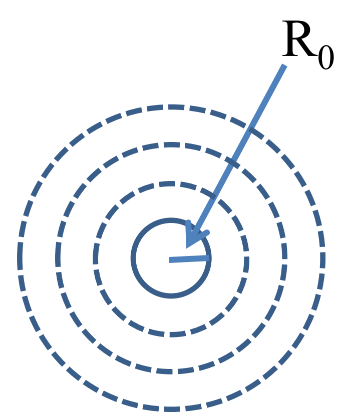
$$\beta = (10\ \mathrm{dB})\log\frac{I}{I_0}$$

with $I_0 = 10^{-12}\ \mathrm{W/m^2}$

For sinusoidal sound wave: $$\boxed{I = \frac{P_{avg}}{A} = \frac{1}{2}\rho v \omega^2 {s_m}^2}$$

A spherical speaker of radius $R_0 = 2cm$ is vibrating at frequency $f = 1.0\ kHz$ with an amplitude of $S_0 = 0.02mm$. The density of air is $\rho = 1.3kg/m^3$ and the speed of sound in air is $v = 340m/s$. Assume that the air molecules around the speaker are vibrating with the same frequency and amplitude as the speaker. The speaker radiates sound uniformly into sphere. Find:

(a) The power P_s of the speaker
(b) Sound intensity and
(c) pressure amplitude Δp_m at $r = 50m$ from the center of the speaker



Solution

$$I=\frac{1}{2}\rho v\omega^2 S_0^2=\frac{P_s}{A}=\frac{P_s}{4\pi R_0^2}\Rightarrow P_s=2\pi\rho v\omega^2 S_0^2 R_0^2=1.75\times 10^{-2}\,\mathrm{W}$$

$$I(r)=\frac{P_s}{4\pi r^2}=5.58\times 10^{-7}\,\mathrm{W/m^2}$$

每个波前的P等 $P=IA$

$$\left.\begin{aligned} I(r)&=\frac{1}{2}\rho v\omega^2 S_m^2\\ \Delta p_m&=\rho v\omega S_m\end{aligned}\right\}\Rightarrow I=\frac{1}{2}\frac{(\Delta p_m)^2}{\rho v}\Rightarrow \boxed{\Delta p_m}=\sqrt{2\rho v I(r)}=0.022\,\mathrm{Pa}$$

Things to master:

- **Sound intensity from isotropic and symmetrical-shaped source**
- **Intensity and pressure amplitude of sinusoidal sound wave**

Chapter 17 Wave II---

Same Frequency, In phase:

$k=\frac{2\pi}{\lambda}$

Oscillation equation of point P

$$\left.\begin{aligned} s_1 &= s_m\cos(kx_1-\omega t) \\ s_2 &= s_m\cos(kx_2-\omega t)\end{aligned}\right\} \Rightarrow s(P,t)=s_1+s_2=2s_m\cos\left(\frac{k\Delta L}{2}\right)\cos(\omega t-\tilde{\phi})$$

Fully **constructive** interference:

$$\Delta L=n\lambda \quad (n=1,2,3....)$$

Fully **destructive** interference:

$$\Delta L=(2n-1)\frac{\lambda}{2}=k\frac{\lambda}{2} \quad (k=1,3,5....)$$

Beat Frequency:

Intensity fluctuates with beat frequency:

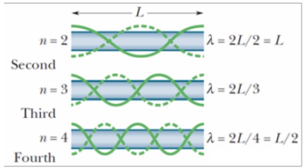
$$\left.\begin{aligned} s_1 &= s_m\cos(\omega_1 t) \\ s_2 &= s_m\cos(\omega_2 t)\end{aligned}\right\} \Rightarrow \omega_{beat}=\Delta\omega=\left|\omega_1-\omega_2\right|; f_{beat}=\Delta f=\left|f_1-f_2\right|$$

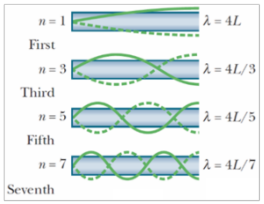
Standing wave:

Open end: Anti-node

Closed end: Node

$$\lambda=\frac{2L}{n} \quad (n=1,2,...) \qquad \lambda=\frac{4L}{n} \quad (n=1,3,...) \qquad \lambda=\frac{2L}{n} \quad (n=1,2,...)$$

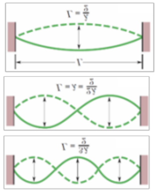




1st

2nd

3rd



Chapter 17 Tutorial Problem 2

Two speakers separated by distance d_1**=2.00m** are **in phase**. Assume the amplitudes of the sound waves from the speakers are approximately the same at the listener's ear at distance d_2**=3.75m** directly in front of one speaker. Consider the full audible range for normal hearing, 20 Hz to 20 kHz. Speakers and listener are all under water. Water's density and bulk modulus are ρ**=1.0×10³kg/m³** and B**=2.0×10⁹ Pa**, respectively.

$f = \frac{v}{\lambda}$ $\frac{\lambda}{2} = \Delta x$

a) What is the lowest frequency $f_{\min,1}$ that gives minimum signal (destructive interference) at the listener's ear?

b) By what number must $f_{\min,1}$ be multiplied to get the second lowest frequency $f_{\min,2}$ that gives minimum signal.

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c) What is the lowest frequency $f_{\max,1}$ that gives maximum signal (constructive interference) at the listener's ear?

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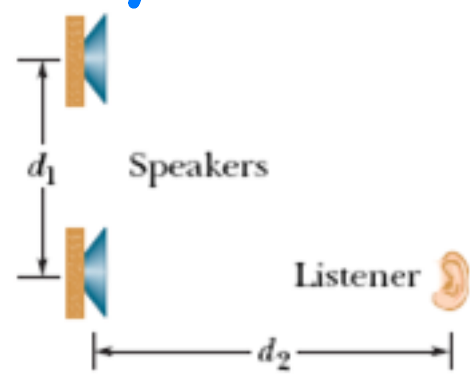


Solution

Wave 1: $s_1(x_1,t) = s_m \cos(kx_1 - \omega t + \phi_1)$
Wave 2: $s_2(x_2,t) = s_m \cos(kx_2 - \omega t + \phi_2)$

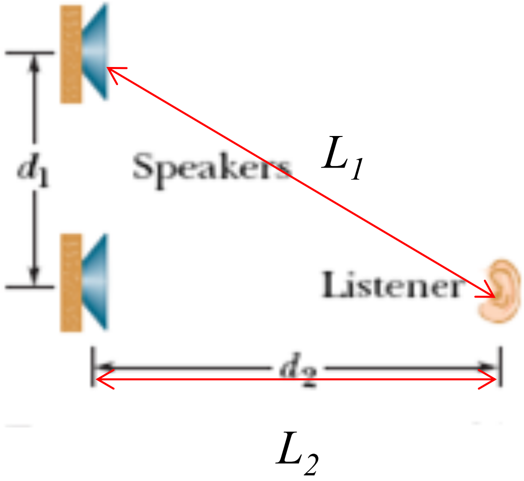
$$s(P,t) = s_1 + s_2 = \left[2\cos\left(\frac{k\Delta L + \Delta\phi}{2}\right) s_m\right] \cos(\omega t - \tilde{\phi})$$

$$s(P,t) = \left[2\cos\left(\frac{k\Delta L + \Delta\phi}{2}\right) s_m\right] \cos(\omega t - \tilde{\phi}) = A_m \cos(\omega t - \tilde{\phi}')$$

Oscillation amplitude at point P: $A_m(P) = \left|2\cos\left(\frac{k\Delta L + \Delta\phi}{2}\right) s_m\right|$

Path length difference of two sound wave from the two speakers to the listener is:

$$\Delta L = L_1 - L_2 = \sqrt{d_1^2 + d_2^2} - d_2$$



For a minimum signal (destructive interference) at the listener,

$$\Delta L = (2m-1)\frac{\lambda}{2}, for \ \ m = 1,2,3\ldots$$

Sound wave speed

$$v = \sqrt{\frac{B}{\rho}}$$

$$\Rightarrow f_{\min,m} = \frac{v}{\lambda} = \frac{(2m-1)}{2(\sqrt{d_1^2 + d_2^2} - d_2)}\sqrt{\frac{B}{\rho}} = (2m-1)(1414\text{Hz}), for\ m = 1,2,3\ldots$$

Solution

Similarly, for a maximum signal (constructive interference) at the listener,

$$\Delta L = m\lambda, \; for \;\; m = 0,1,2\ldots$$

$$v = \sqrt{\frac{B}{\rho}}$$

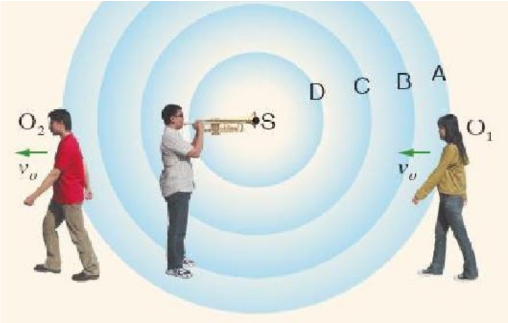
$$\Rightarrow f_{\max,m} = \frac{v}{\lambda} = \frac{2m}{2(\sqrt{d_1^2 + d_2^2} - d_2)}\sqrt{\frac{B}{\rho}} = 2m(1414\text{Hz}), \; for \;\; m = 1,2\ldots(\text{since } \Delta L > 0, \;\; m \neq 0)$$

a) When m=1, the lowest frequency that gives minimum signal is f=1414Hz

b) When m=2, the second lowest frequency that gives minimum signal is
$f = 3 \times 1414$ Hz, thus the factor is 3.

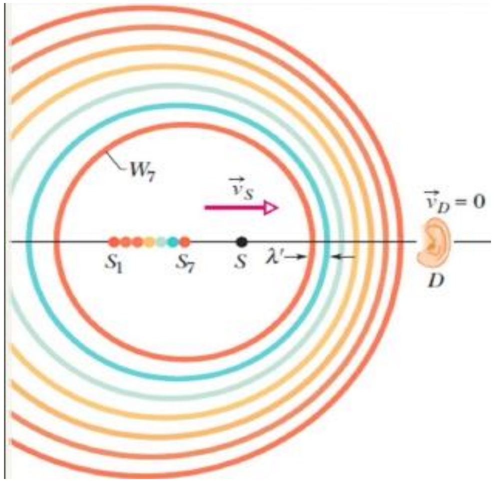
c) When m=1, the lowest frequency that gives maximum signal is f=2828Hz

Relative to Medium: Moving Detector, Stationary Source



Frequency felt by detector: $f' = \frac{v'}{\lambda'} = f\frac{v \pm v_D}{v}$

Relative to Medium: Moving Source, Stationary Detector



Frequency felt by detector: $f' = \frac{v'}{\lambda'} = f\frac{v}{v \pm v_S}$

$$\boxed{f' = \frac{N}{t} = \frac{v'}{\lambda'} = f\frac{v \pm v_D}{v \pm v_S}}$$

Toward → f Increase
Away → f Decrease

Chapter 17 Tutorial Problem 3

A B

wind

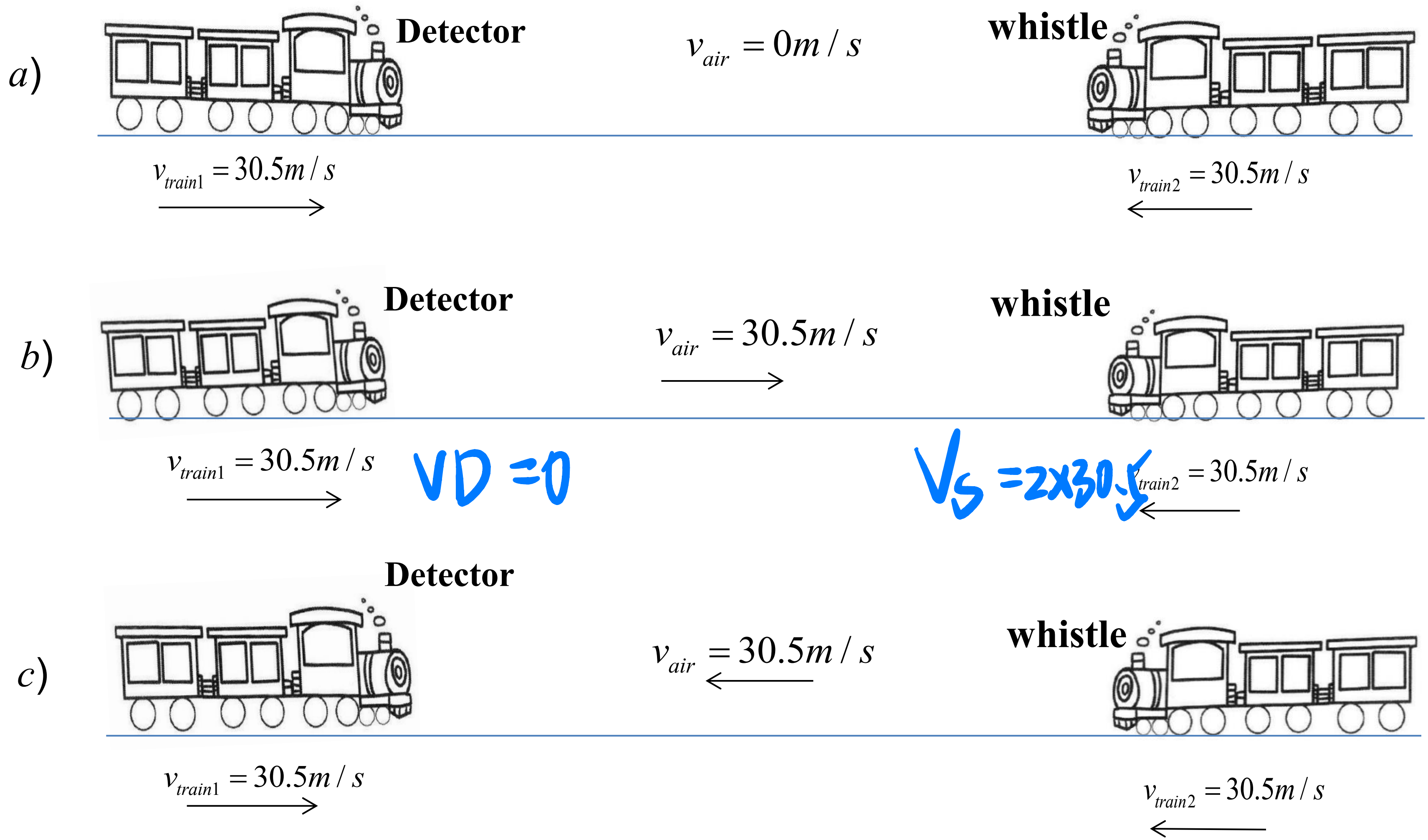
Two trains are traveling toward each other at 30.5 m/s relative to the ground. One train is blowing a whistle at 500 Hz.

a) What frequency is heard on the other train in still air?

b) What frequency is heard on the other train if the wind is blowing at 30.5 m/s toward the whistle and away from the listener?

c) What frequency is heard if the wind direction is reversed?

Solution



$$f' = f\frac{v \pm v_D}{v \pm v_s}$$

v: the speed of the sound wave

v_D: the speed of the detector relative to the medium

v_s: the speed of the source relative to the medium

Toward → f Increase; Away → f Decrease

Solution

$f' = f\dfrac{v \pm v_D}{v \pm v_s}$ with $f = 500$ Hz and $v = 343$ m/s. We choose signs to produce $f' > f$.

a) The frequency heard in still air is

$$f' = (500\,Hz)\left(\frac{343\,m/s + 30.5m/s}{343\,m/s - 30.5m/s}\right) = 598\,Hz$$

参考系

b) In a frame of reference where the air seems still, the velocity of the detector is 30.5 – 30.5 = 0, and that of the source is 2(30.5). Therefore

$$f' = (500\,Hz)\left(\frac{343\,m/s + 0}{343\,m/s - 2\times 30.5m/s}\right) = 608\,Hz$$

c) We again pick a frame of reference where the air seems still. Now, the velocity of the source is 30.5 – 30.5 = 0, and that of the detector is 2(30.5). Consequently,

$$f' = (500\,Hz)\left(\frac{343\,m/s + 2\times 30.5m/s}{343\,m/s - 0}\right) = 589\,Hz$$

Things to master:

- Determine speeds relative to medium
- Determine speeds of source, detector and their signs