Tutorial for chapter 7&8

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Keywords for chapter 7&8

Collision

Work

Power

Instantaneous power

向心的

Centripetal force

small

Infinitesimal 极其微小的

Net work

Stretch

Relaxed length

Compress

Perturbation 扰动

Release

Kinetic energy

Potential energy

Conservative force

Non-conservative force

Dependent

Independent

Elastic potential energy

Mechanical energy

Gravitational Potential

Internal energy

Thermal energy

Isolated system

Closed system

Conservation of energy

Turning point

Stable equilibrium

Unstable equilibrium

Kinetic Energy & Work

Kinetic energy: $K = \frac{1}{2}mv^2$

Work:

$$W = \int_{\vec{r}_1}^{\vec{r}_2} dW = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r},$$

where $dW = \vec{F} \cdot d\vec{r}$ is the work done by the force through an infinitesimal displacement $d\vec{r}$.

Work – Kinetic Energy Theorem:

$$\Delta K = K_f - K_i = W_{\text{net}}$$

动能定理

Work for two Special type force:

$$W = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r} = \vec{F} \cdot \left(\int_{\vec{r}_1}^{\vec{r}_2} d\vec{r} \right)$$
$$= \vec{F} \cdot \vec{d} = Fd\cos\phi$$

Only valid for the force with both magnitude & direction unchanging.

Work Done by a Spring Force

$$W_s = \int_{x_i}^{x_f} \vec{F} \cdot d\vec{x} = \int_{x_i}^{x_f} -kx\,dx$$
$$= \frac{1}{2}k{x_i}^2 - \frac{1}{2}k{x_f}^2$$

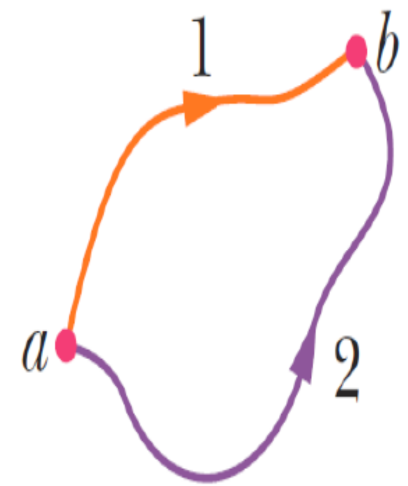
Positions are w.r.t. the relax position of the spring.

Chapter 8 **Potential Energy & Conservation of Energy**

Conservative Force:

The net work done by a conservative force on an object between two points **does NOT Depend** on the **path** taken.两点之间**任意路径**做功相等

Type：Gravitational force, spring force, …



Nonconservative Force:

The net work done by a non-conservative force between two points is **path-dependent.**（两点之间某些路径做功相等也可能是非保守力）

Type：Kinetic frictional force, drag force …

Chapter 8 **Potential Energy & Conservation of Energy**

Potential Energy of a System

Work（W）done by a conservative force causes the energy transfer between the kinetic energy of an object inside a system and the corresponding potential energy（U）of the system.

gravitational potential energy associated with a **particle–Earth system**

$$\Delta U = -W_G = -\int_{y_i}^{y_f} -mgdy = mgy\Big|_{y_i}^{y_f}$$

$$\Rightarrow U_f - U_i = mgy_f - mgy_i$$

$$U = mgy \qquad (U_i = 0 \ \ at \ \ y_i = 0)$$

Elastic potential energy associated with a **block–spring system**

$$\Delta U = -W_S = -\int_{x_i}^{x_f} -kxdx = \frac{1}{2}kx^2\Big|_{x_i}^{x_f}$$

$$\Rightarrow U_f - U_i = \frac{1}{2}kx_f{}^2 - \frac{1}{2}kx_i{}^2$$

$$U = \frac{1}{2}kx^2 \qquad (U_i = 0 \ \ at \ \ x_i = 0)$$

relaxed length

Chapter 8 **Potential Energy & Conservation of Energy**

Conservation of Energy with External Forces

General energy: $E = E_{mec} + E_{th} + E_{other}$

$$E_{transfer} = \Delta E_{mec} + \Delta E_{th} + \Delta E_{other}$$

Considering that the only type of energy transfer to or from a system is due to the net work W done by external forces on the system,

$$\boxed{W = \Delta E_{mec} + \Delta E_{th}}$$

For an isolated system: $0 = \Delta E_{mec} + \Delta E_{th}$

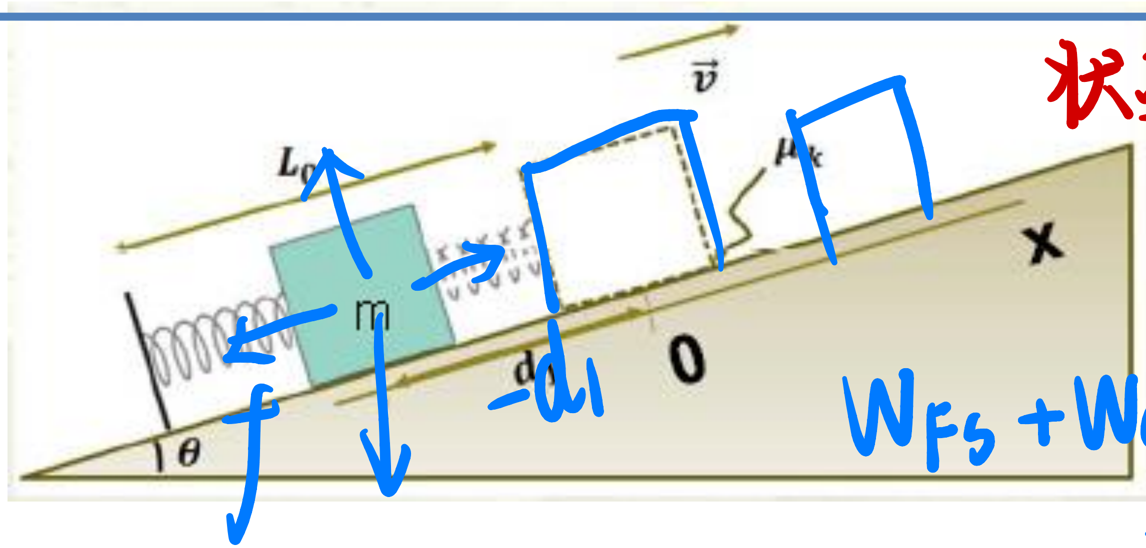
$$\boxed{\Delta E_{th} = f_k d}$$

Effect of the kinetic friction is to **unidirectional transfer the kinetic energy of a** moving object to the thermal energy of the system of the two contact objects

For an object , no internal force, all the forces are external	$\Delta K = K_f - K_i = W_{net}$
For a system only involving conservative internal force	$\Delta E_{mec} = E_{mecf} - E_{meci} = W_{external}$
For a system involving non-conservative internal force	$\Delta E = E_f - E_i = W_{external}$

Attention: The system what you choose is precondition, Different system, different formula

Tutorial Problem 1



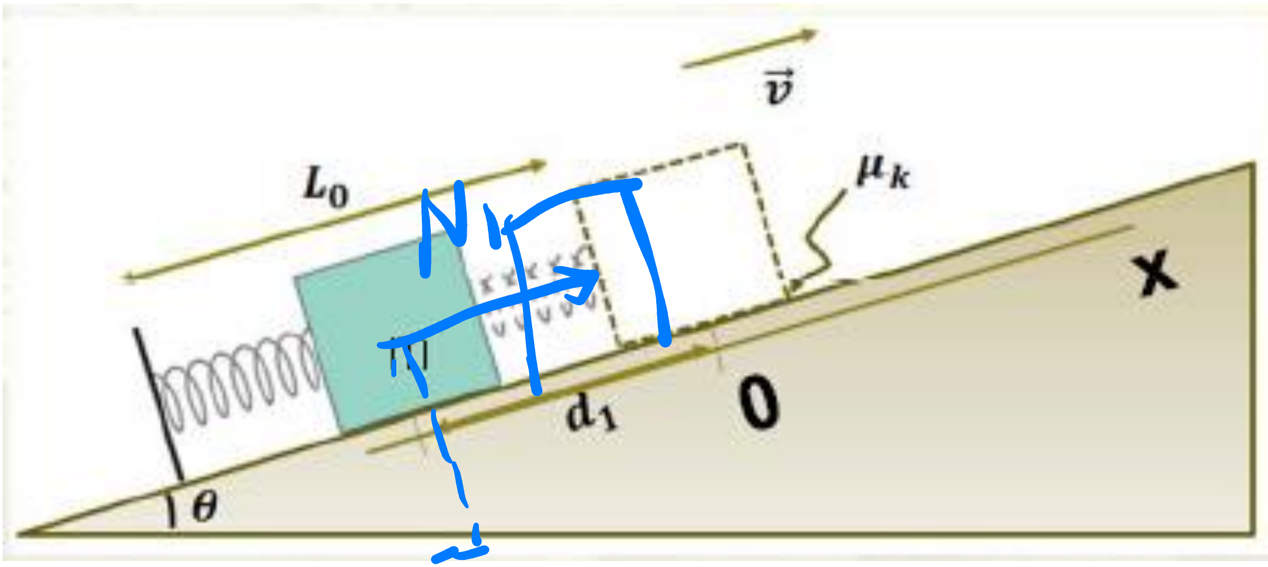
状态（不求与时间关系）

能量

$W_{F_s} + W_G + W_{f_k} + \overset{\to 0}{W_N} = 0$

连接

A massless spring of spring constant $k = 16\ N/m$ and relaxed length L_0 =0.30 m with a block of mass $m = 0.20\ kg$ attached to the free end is placed on a ramp with an inclined angle $\theta = 15°$ *as* shown. The coefficients of the static and kinetic friction between the block and the inclined plane are $\mu_s = 0.20$ and $\mu_k = 0.10$, respectively. Initially the block is pushed so that the spring has a compressed *length* $d_1 = 0.12\ m$. We then release the block and the block begins to move up along the ramp. After passing the relaxed position of the spring, the block continues moving up, stretching the spring.



a) What is the maximum distance upward beyond the relaxed position that the block can reach ?

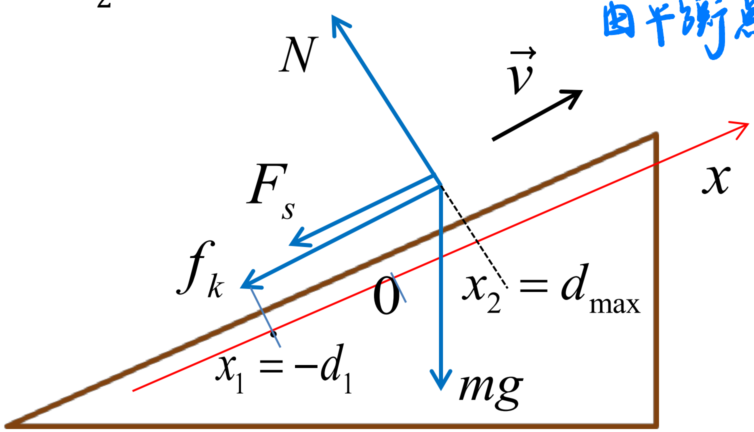
b) Can the block stay at rest there?

(a) On ramp, choose relaxed position of spring as origin, up along ramp as $+x$ axis, at maximum height d_{max}, $v_f = 0$, then $\Delta K = 0$

At $x_2 = dmax$

单向简谐

因平衡点做对称



$N - mg\cos\theta = 0 \quad f_k = \mu_k \cdot N = \mu_k mg\cos\theta$

① By work-K.E. theorem: $\Delta K = W_{net}$

$$W_{F_s} = \tfrac{1}{2}kd_1^2 - \tfrac{1}{2}kd_{\max}^2$$

$$W_G = -mg(d_1 + d_{\max})\cdot\sin\theta$$

$$W_{f_k} = -f_k(d_1 + d_{\max}) = -\mu_k mg\cos\theta(d_1 + d_{\max})$$

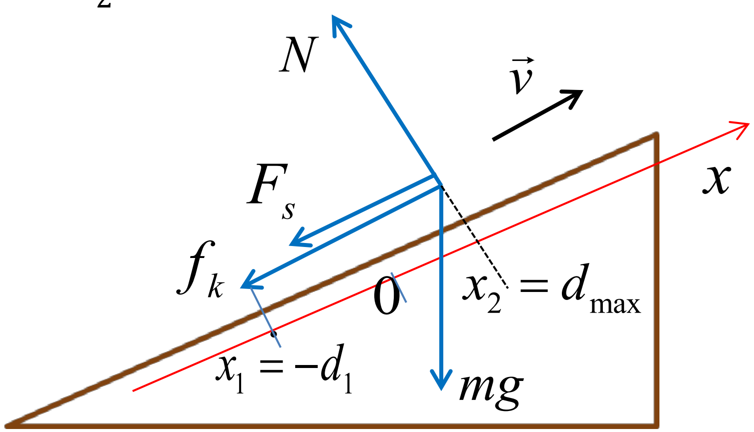
$\Delta K = W_{net} = W_G + W_{F_s} + W_{f_k}$, Normal force does no work

$$\Rightarrow 0 = [\tfrac{1}{2}k(d_1 - d_{\max}) - mg\sin\theta - \mu_k mg\cos\theta](d_1 + d_{\max})$$

$$\Rightarrow d_{\max} = d_1 - \frac{1}{k}2mg(\sin\theta + \mu_k\cos\theta) = 0.033m$$

(a) On ramp, choose relaxed position of spring as origin, up along ramp as $+x$ axis, at maximum height d_{max} , $v_f = 0$, then $\Delta K = 0$

At $x_2 = dmax$



系统内E守恒

② By conservation of energy:

System: Earth + $\underbrace{\overbrace{\text{spring+ block}}^{U_S}}_{U_G}$ + $\underbrace{\overbrace{\text{ramp}}^{斜面}}_{\Delta E_{th}}$

Internal forces cause energy transferred.

$$\Delta U_G = mg(d_1 + d_{\max}) \cdot \sin\theta$$

距离平衡位置 d_{max}

$$\Delta U_S = \tfrac{1}{2}kd_{\max}^2 - \tfrac{1}{2}kd_1^2$$

$$\Delta E_{th} = f_k\,(d_1 + d_{\max}) = \mu_k mg\cos\theta(d_1 + d_{\max})$$

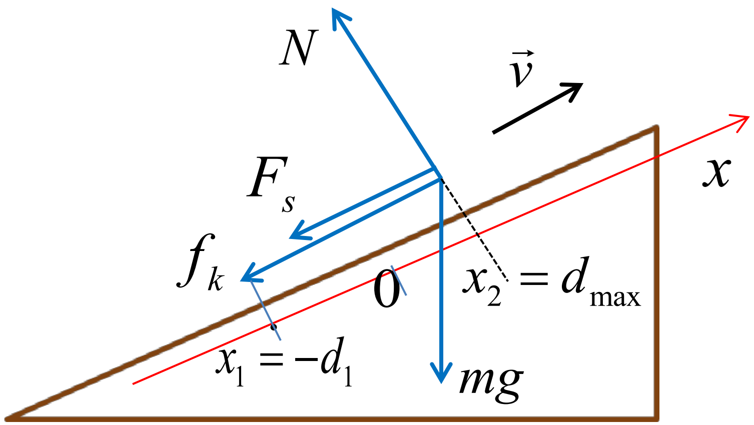
It is an isolated system and have non-conserved internal force (No external forces on it)

$$0 = \Delta E = \Delta E_{mec} + \Delta E_{th} = \Delta K + \Delta U + \Delta E_{th} = 0 + \Delta U_G + \Delta U_S + \Delta E_{th}$$

$$\Rightarrow 0 = \Delta U_G + \Delta U_S + \Delta E_{th}$$

$$\Rightarrow d_{\max} = 0.033m$$

(a) On ramp, choose relax position of spring as origin, up along ramp as $+x$ axis, at maximum height d_{max} , $v_f = 0$, then $\Delta K = 0$



③ By conservation of energy:

System: Earth + spring+ block

External force(kinetic friction) cause energy transferred.

$$W_{ext} = -f_k\,(d_1 + d_{\max}) = -\mu_k mg\cos\theta(d_1 + d_{\max})$$

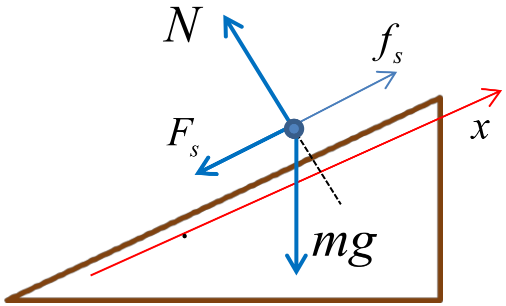
$$\Delta U_G = mg(d_1 + d_{\max})\cdot\sin\theta$$

$$\Delta U_S = \tfrac{1}{2}kd_{\max}^2 - \tfrac{1}{2}kd_1^2$$

$$W_{ext} = \Delta E_{mec} = \Delta K + \Delta U_G + \Delta U_S$$

$$\Rightarrow d_{\max} = 0.033m$$

(b)To be able to stay at rest for ever, requiring the needed static frinction f_s meets $f_s \leq f_{s,max}$



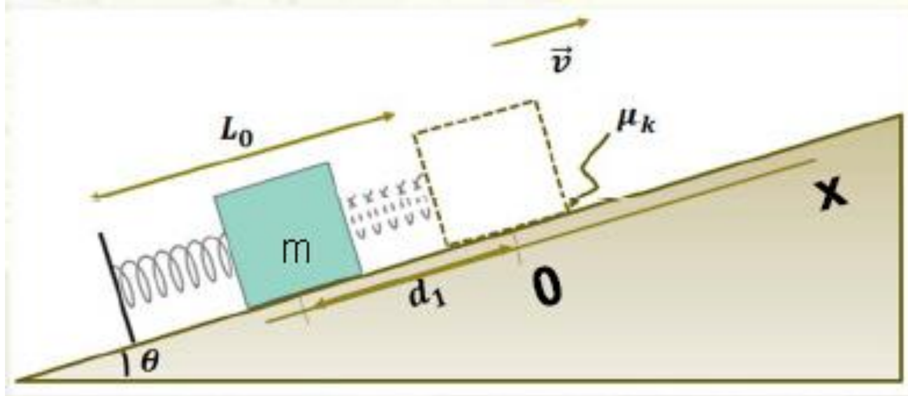
Assume the block stays at rest at the maximum height:

$$f_{s,\max} = \mu_s N = \mu_s mg\cos\theta = 0.379N$$

$$f_s = F_s + mg\sin\theta = kd_{\max} + mg\sin\theta = 1.04N$$

$$\Rightarrow f_s > f_{s,\max}$$

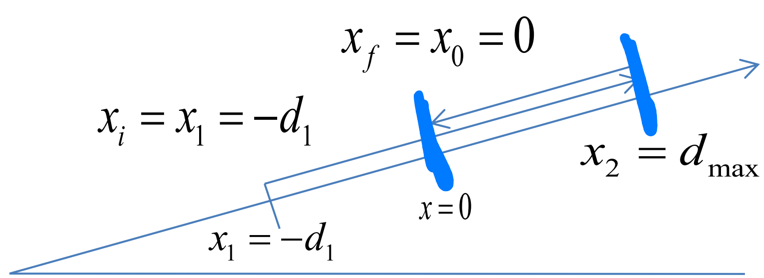
So the maximum static friction is not enough to hold the block at rest and the block will move back.



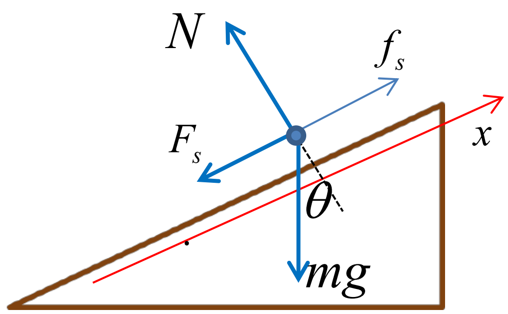
c) If the block can move back from the maximum height and pass the relaxed position again, how much work are done on the block by friction, gravitational force and the spring force **since from the initial position**?

$$the\ route\ of\ the\ block:\ x_1 = -d_1 \rightarrow x_0 = 0 \rightarrow x_2 = d_{max} \rightarrow x_0 = 0$$

$$x_i = x_1 = -d_1; \quad xf = x_0 = 0$$



(c)



$$W_G = -mgd_1 \sin\theta = -0.0609J$$

$$W_{F_s} = \tfrac{1}{2}k \cdot d_1^2 - 0 = 0.1152J$$

$$W_{f_k} = -f_k(d_1 + 2d_{\max}) = -\mu_k mg\cos\theta(d_1 + 2d_{\max}) = -0.0352J$$

- Discussion:
- W_g and W_{Fs} do not depend on the path.

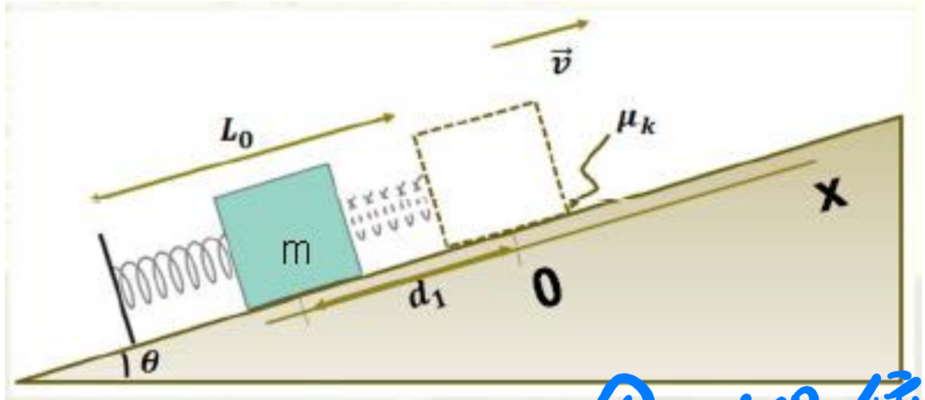
conservative force

- But W_{fk} does depend on the path!

Non-conservative force

- $W_{net} = W_g + W_{Fs} + W_{fk} = 0.0191J > 0.$

Block can continue moving down after passing origin.



① v=0 位置
② 终止条件

最大f静

$f_{s,m} \geq f_s$

d) Where would the block finally stop?

Check the second momentary stop position (below relaxed position) to see if it requires $f_s \leq f_{s,max}$, if okay, then stop there forever. Otherwise, check the third momentary stop position, and ……, until the requirement for $f_s \leq f_{s,max}$ meets.

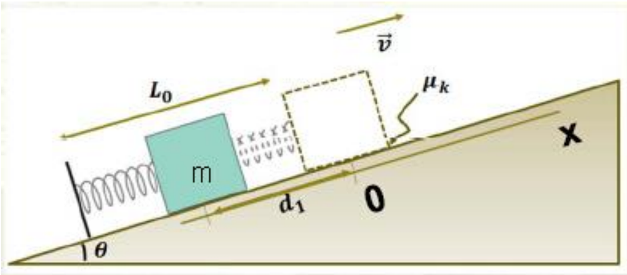
使之平衡所需f

$$x_3 = -0.0728m$$

$$f_{s,\max} = \mu_s N = \mu_s mg\cos\theta = 0.379N$$

$$f_s = k|x_3| - mg\sin\theta = -0.658N$$

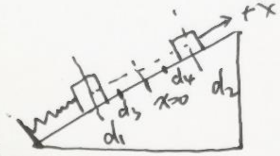
Can't stop there, so we should continue checking the third momentary stop position---



The coefficients of the static and kinetic friction between the block and the inclined plane are $\mu_s = 0.20$ and $\mu_k = 0.10$, respectively. Initially the block is pushed so that the spring has a compressed *length* $d_1 = 0.12\ m$. We then release the block and the block begins to move up along the ramp. After passing the relaxed position of the spring, the block continues moving up, stretching the spring.

Where would the block finally stop?

The momentary stopping position can be get by applying conservation of energy if we choose the block, spring, Earth and ramp as our system. So we have:



$$\frac{1}{2}kd_1^2 = \frac{1}{2}kd_2^2 + mg(d_1+d_2)\sin\theta + \mu_k mg\cos\theta(d_1+d_2) \quad (1)$$

$$\frac{1}{2}kd_2^2 = \frac{1}{2}kd_3^2 - mg(d_2+d_3)\sin\theta + \mu_k mg(d_2+d_3)\cos\theta \quad (2)$$

$$\frac{1}{2}kd_3^2 = \frac{1}{2}kd_4^2 + mg(d_3+d_4)\sin\theta + \mu_k mg(d_3+d_4)\cos\theta \quad (3)$$

$$\cdots\cdots$$

$$\left.\begin{aligned} (1) &\Rightarrow d_1 - d_2 = \frac{2mg}{k}(\sin\theta + \mu_k\cos\theta) \\ (2) &\Rightarrow d_2 - d_3 = \frac{2mg}{k}(-\sin\theta + \mu_k\cos\theta) \\ (3) &\Rightarrow d_3 - d_4 = \frac{2mg}{k}(\sin\theta + \mu_k\cos\theta) \end{aligned}\right\} \Rightarrow \begin{aligned} d_1 - d_2 &= \frac{2mg}{k}(\sin\theta + \mu_k\cos\theta) \\ d_1 - d_3 &= \frac{2mg}{k}(2\mu_k\cos\theta) \\ d_1 - d_4 &= \frac{2mg}{k}(\sin\theta + 3\mu_k\cos\theta) \\ &\cdots\cdots \end{aligned}$$

$$\Rightarrow d_2 = d_1 - \frac{2mg}{k}(\sin\theta + \mu_k\cos\theta) =$$

$$d_3 = d_1 - \frac{2mg}{k}(2\mu_k\cos\theta) =$$

$$d_4 = d_1 - \frac{2mg}{k}(\sin\theta + 3\mu_k\cos\theta) =$$

$$\cdots\cdots$$

(2) The final stop position can be get by juding the needed friction and the maximum static friction, if the magnitude of needed friction is smaller then $f_{s,max}$ then the block will stop otherwise, it will move:

for the $d_2, d_4 \cdots$, the needed friction: $f_n = kd_i + mg\sin\theta \quad i = 2, 4 \cdots$

for the $d_1, d_3 \cdots\cdots$ the needed friction: $f_n = kd_i - mg\sin\theta \quad i = 1, 3 \cdots$

the $f_{s,max} = \mu_s mg\cos\theta$.

Tutorial Problem 2

$h = \cos\theta \cdot R$

$\begin{cases} F_n \text{ 只由G分力提供} \\ \Delta U_G + \Delta E_k = 0 \end{cases}$

$2mg(1-\cos\theta) = \frac{mv^2}{R}$

$mg\cos\theta = \frac{mv^2}{R}$

A boy initially seated on the top of a hemispherical ice mound of radius R =13.8 m. He begins to slide down the ice, with a negligible initial speed. Approximate the ice as being frictionless. At what height does the boy lose contact with the ice?

$2 = 3\cos\theta$

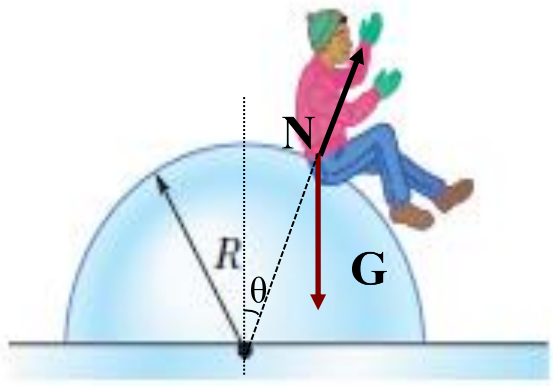
$\cos\theta = \frac{2}{3}$

$h = \frac{2}{3}R$

1, Free-Body Diagram and Motion Analysis:

Circular motion before the boy lose contact with the ice

$$F_C = mg\cos\theta - N = m\frac{v^2}{R}$$



2, Force and Energy Analysis:

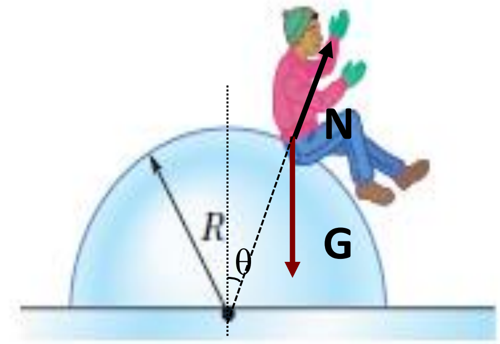
System: Boy + Ice + Earth

External force: None

Internal force: Normal force (no work); Gravitational force is a conservative force

$$W_{external} = \Delta E_{mec} = \Delta U_G + \Delta K = 0$$

$$W_{external} = \Delta E_{mec} = \Delta U_G + \Delta K = 0 \quad \Rightarrow U_{Gf} + K_f = U_{Gi} + K_i$$



$$\left.\begin{array}{ll} U_{Gi} = 0 & U_{Gf} = -mgR(1-\cos\theta) \\ K_i = 0 & K_f = \dfrac{1}{2}mv^2 \end{array}\right\} \Rightarrow mgR(1-\cos\theta) = \frac{1}{2}mv^2$$

$$\Rightarrow \begin{cases} mg\cos\theta - N = m\dfrac{v^2}{R} \ldots\ldots(1) \\ mgR(1-\cos\theta) = \dfrac{1}{2}mv^2 \ldots.(2) \end{cases}$$

At the moment of the boy lose contact with the ice: $N = 0$

$$mg\cos\theta = 2mg(1-\cos\theta) \Rightarrow \cos\theta = \frac{2}{3}$$

$$\boldsymbol{So,\ h = R\cos\theta = \frac{2}{3}R = 9.2m}$$