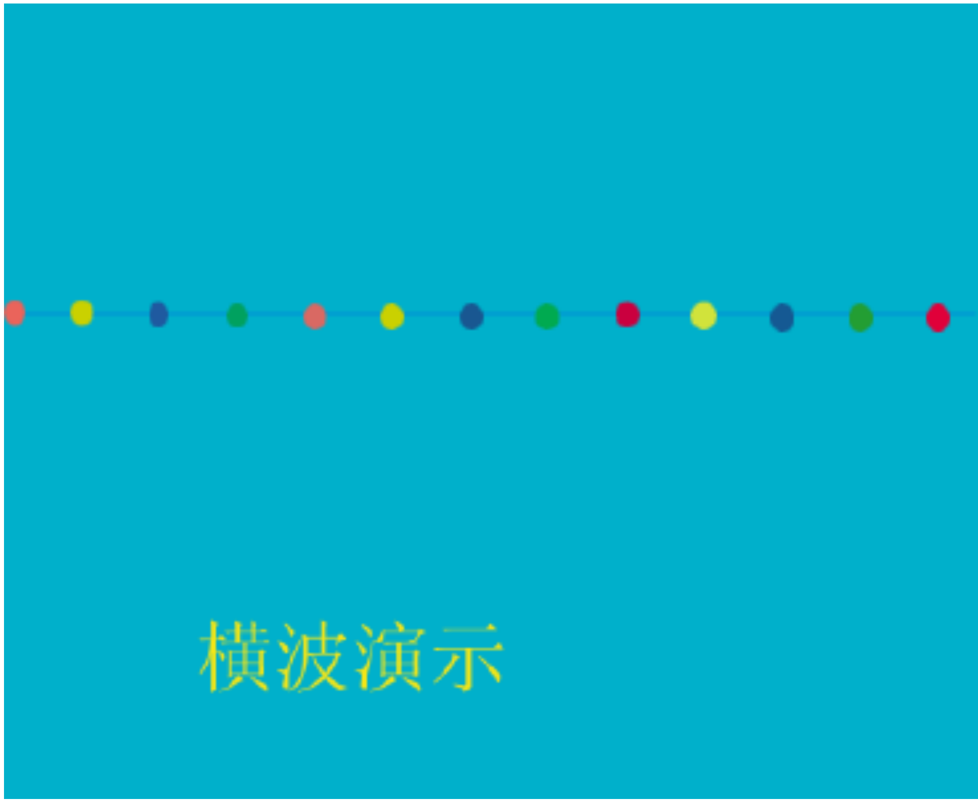
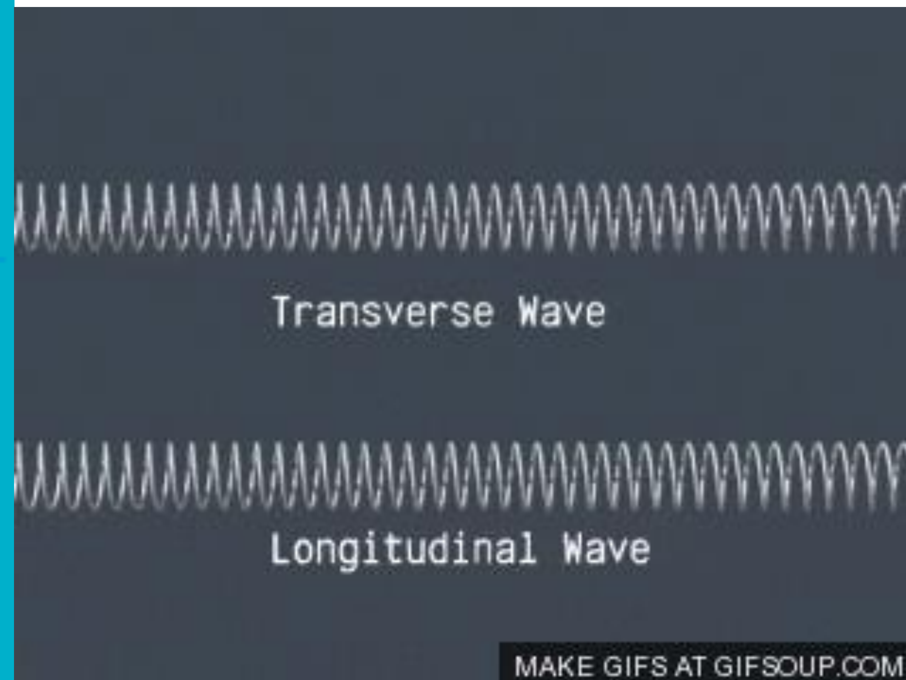
Tutorial for chapter 16

Transverse wave	Node
Longitudinal wave	Antinode
Sinusoidal	Resonance
Wavelength	Oscillation mode
Traveling wave	Fundamental mode
Interference	First harmonic
Standing wave	Second harmonic


横波演示

Transverse Wave
Longitudinal Wave
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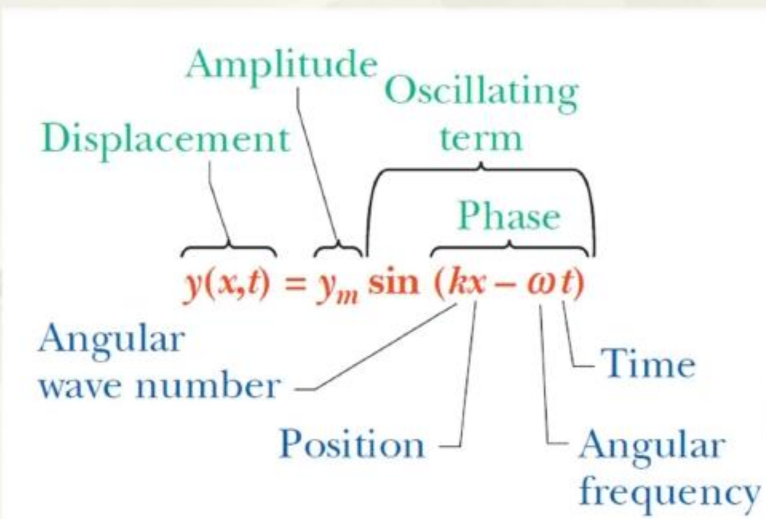
Wave function

$$\frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} = \frac{\partial^2 y}{\partial x^2}$$

$$\boxed{y(x,t) = f(x \pm vt)}$$

- $+$ moving left
- $-$ moving right

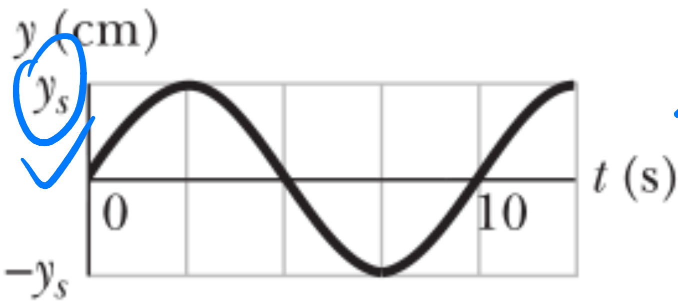
$$y(x,t) = y_m \sin(kx - \omega t)$$



- Period T of sinusoidal wave: time for oscillation in one cycle (wave shape repetition).
- Angular frequency ω: $\omega = \frac{2\pi}{T}$
- Frequency f:
$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$
$$y(x,t) = y(x, t+T) = y(x+\lambda, t)$$
- $$v = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda f$$

$$y(x,t) = y_m \sin(kx - \omega t) = y_m \sin\left\{k\left[x - \left(\frac{\omega}{k}\right)t\right]\right\} = f(x - vt)$$

Problem 1



$x = 0$ m

$\sin \omega t$

λ

A sinusoidal transverse wave of wavelength 18 cm travels along a string in the positive direction of an x axis. The displacement y of the string particle at $x = 0$ is given in figure as a function of time t. The scale of the vertical axis is set by $y_s = 4.0$ cm. The wave equation is to be in the form $y(x, t) = y_m \sin(kx \pm \omega t + \phi)$.

a) At $t = 0$, is a plot of y versus x in the shape of a positive sine function or a negative sine function?

b) What are y_m, k, ω, ϕ, the sign in front of ω, and the speed of the wave?

c) What is the transverse velocity of the particle at $x = 0$ when $t = 5.0$ s?

Problem 1

A general expression for a sinusoidal wave traveling along the $+x$ direction is

$$y(x,t) = y_m \sin(kx - \omega t + \phi) \,.$$

(a) The figure shows that at $x = 0$, $y(0,t) = y_m \sin(-\omega t + \phi)$ is a positive sine function, that is, $y(0,t) = +y_m \sin \omega t$. Therefore, the phase constant must be $\phi = \pi$. At $t = 0$, we then have

$$y(x,0) = y_m \sin(kx + \pi) = -y_m \sin kx$$

which is a **negative sine function**.

From the figure we see that the amplitude is $y_m = 4.0$ cm.

The angular wave number is given by $k = 2\pi/\lambda = \pi/9.0 = 0.35$ rad/cm.

The angular frequency is $\omega = 2\pi/T = \pi/5 = 0.63$ rad/s.

As found in part (a), the phase is $\phi = \pi$

The sign is **minus** since the wave is traveling in the $+x$ direction.

Since the frequency is $f = 1/T = 0.10$ Hz, the speed of the wave is $v = f\lambda = 1.8$ cm/s.

From the results above, the wave may be expressed as

Problem 1

$$y(x,t) = 4.0\sin\left(\frac{\pi x}{9.0} - \frac{\pi t}{5} + \pi\right) = -4.0\sin\left(\frac{\pi x}{9} - \frac{\pi t}{5}\right).$$

Taking the derivative of y with respect to t, we find

$$u(x,t) = \frac{\partial y}{\partial t} = 4.0\left(\frac{\pi}{5}\right)\cos\left(\frac{\pi x}{9.0} - \frac{\pi t}{5}\right)$$

which yields $u(0, 5.0) = -2.5$ cm/s.

Wave along string---Wave speed and power

Wave speed $$\boxed{v = \sqrt{\frac{\tau}{\mu}}}$$

tension

linear density $\mu = \frac{M}{L}$

Power for sinusoidal wave $$P(t) = \mu v \omega^2 y_m^2 \cos^2(kx - \omega t)$$

Average power for sinusoidal wave $$P_{avg} = \frac{1}{2}\mu v \omega^2 y_m^2$$

Interference of Waves

If two sinusoidal waves of the same amplitude and wavelength travel in the ***same*** direction along a stretched string (so the same wave speed and frequency), they interfere to produce a resultant sinusoidal wave traveling in that direction.

Wave 1: $y_1(x,t) = y_m \sin(kx - \omega t)$

Wave 2: $y_2(x,t) = y_m \sin(kx - \omega t + \phi)$

Superposition of displacement: $y'(x,t) = y_1(x,t) + y_2(x,t)$

Resultant wave: $$\boxed{y'(x,t) = [2y_m \cos\tfrac{1}{2}\phi] \sin(kx - \omega t + \tfrac{1}{2}\phi)}$$

Standing Wave

If two sinusoidal waves of the same amplitude and wavelength travel in the *opposite* direction along a stretched string (so the same wave speed and frequency), their interference with each other produces a standing wave.

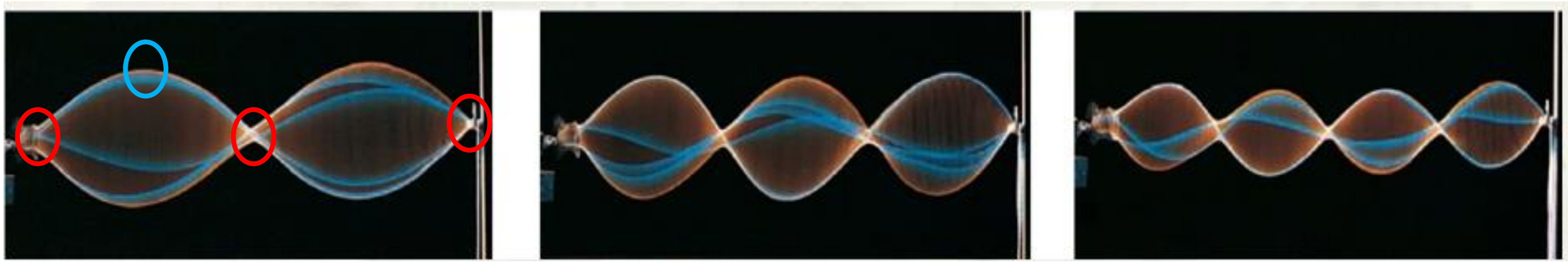
Wave 1: $y_1(x,t) = y_m \sin(kx - \omega t)$

Wave 2: $y_2(x,t) = y_m \sin(kx + \omega t)$

Superposition of displacement: $y'(x,t) = y_1(x,t) + y_2(x,t)$

Resultant wave: $$\boxed{y'(x,t) = [2y_m \sin kx] \cos \omega t}$$

Standing Wave



■ **Nodes**: Some points never move.

$\sin kx = 0$ ➡ $\boxed{x = n\frac{\lambda}{2}}$ $n = 0, 1, 2, \ldots\ldots$ (location of nodes)

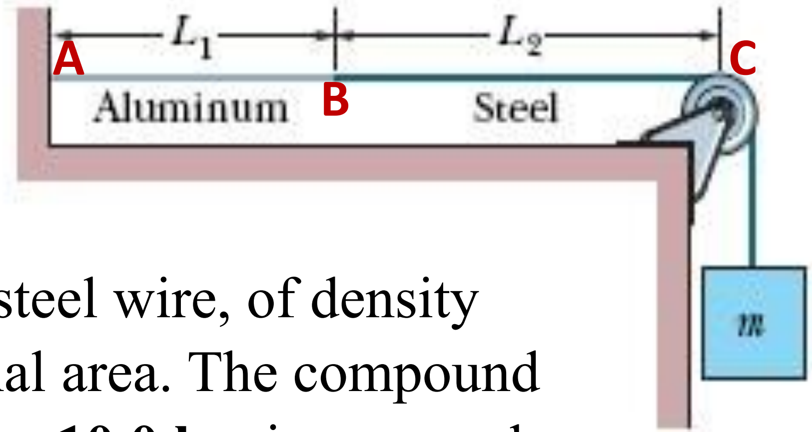
■ **Antinodes**: Some points oscillate with maximum amplitude

$|\sin kx| = 1$ ➡ $\boxed{x = \left(n + \frac{1}{2}\right)\frac{\lambda}{2}}$ (location of antinodes)

"hard" boundary: Displacement is fixed to be zero, Node.

"free" boundary :Displacement can be the maximum. Anti-node

Problem 3

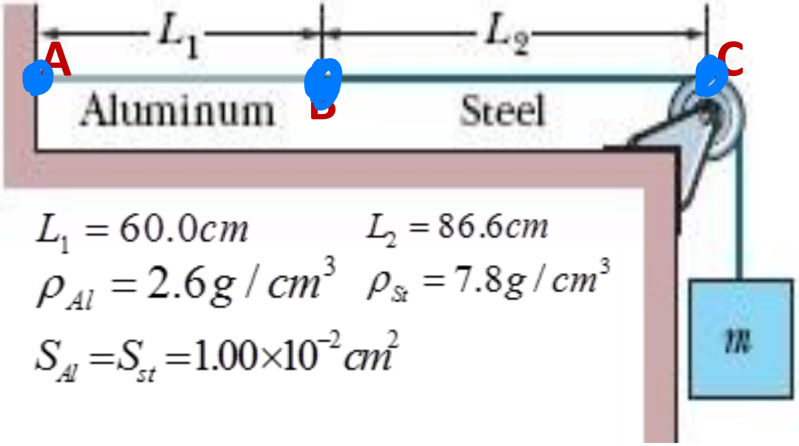


In Fig. 16-42, an aluminum wire, of length, $\boldsymbol{L_1}$**=60.0cm** , cross-sectional area $\boxed{1.00\times10^{-2}\,cm^2}$ and density $\boxed{2.6g/cm^3,}$ is joined to a steel wire, of density $\boxed{7.8g/cm^3}$ and the same cross-sectional area. The compound wire , loaded with a block of mass $\boldsymbol{m}$ **= 10.0 kg**, is arranged so that the distance from the joint to the supporting pulley is

$\boldsymbol{L_2}$**=86.6cm** from the joint to the supporting . Transverse waves are set up on the wire by an external source of variable frequency; **a node is located at the pulley**.

Find the **lowest frequency** that generates **a standing wave** having the **joint as one of the nodes**.

Problem 3



1, Which positions must be the nodes in this problem:

Point A
Point B: the joint
Point C: the pulley

2, Which variable is same for both sections of the wire :

Frequency
velocity
wave length

3, The formula of wavelength for the two sections of the wire

$\boxed{\lambda = 2L/n}$ **Between two fix nodes**

4, The tension on the two sections of the wire and the linear density of wire

$\boxed{\tau = mg}$ $M = \rho V = \rho SL = \mu L \Rightarrow \mu = \rho S$

Problem 3

$$v = \frac{\sqrt{\tau}}{\sqrt{\mu}} = \frac{\sqrt{m\,g}}{\sqrt{\rho\,S}}\ldots\ldots\ldots\ldots\ldots(1)$$

$$f = \frac{v}{\lambda} = \frac{v_1}{\lambda_1} = \frac{v_2}{\lambda_2}\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

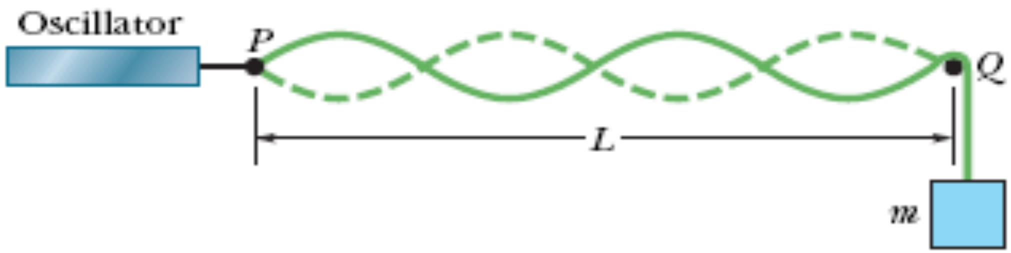
$$\Rightarrow \frac{\sqrt{mg}}{\sqrt{\rho_1 S}}\frac{n_1}{2L_1} = \frac{\sqrt{mg}}{\sqrt{\rho_2 S}}\frac{n_2}{2L_2}$$

$$\Rightarrow \frac{n_2}{n_1} = \frac{L_2\sqrt{\rho_2}}{L_1\sqrt{\rho_1}} = \frac{(0.866\,\mathrm{m})\sqrt{7.80\times10^3\,\mathrm{kg/m}^3}}{(0.600\,\mathrm{m})\sqrt{2.60\times10^3\,\mathrm{kg/m}^3}} = 2.50 = \frac{5}{2}$$

So The smallest integers that have this ratio are n_1 = 2 and n_2 = 5

$$\Rightarrow f = \frac{n_1}{2L_1}\sqrt{\frac{mg}{\rho_1 A}} = \frac{2}{2(0.600\,\mathrm{m})}\sqrt{\frac{(10.0\,\mathrm{kg})(9.80\,\mathrm{m/s}^2)}{(2.60\times10^3\,\mathrm{kg/m}^3)(1.25\times10^{-6}\,\mathrm{m}^2)}} = 289\,\mathrm{Hz}$$

Problem 4



In Fig. 16-41, a string, tied to a **sinusoidal oscillator** at P and running over a support at Q, is stretched by a block of mass m. The separation L between P and Q **is 1.20 m**, and the frequency ***f*** of the oscillator is fixed at **120 Hz**. The amplitude of the motion at **P** is small enough for that point to be considered **a node**. **A node also exists at Q**. A **standing wave** appears when the mass of the hanging block is **286.1 g or 447.0 g**, **but not for any intermediate mass**.

What is the linear density of the string?

Problem 4

The tension on the string: $\boxed{\tau = mg}$

$$f = \frac{v}{\lambda} = \frac{\sqrt{\tau}}{\sqrt{\mu}}\frac{n}{2L} = \frac{\sqrt{mg}}{\sqrt{\mu}}\frac{n}{2L}$$

The wavelength between the two fix point P and Q: $\boxed{\lambda = 2L/n}$

$$\Rightarrow m = \frac{4L^2\mu f^2}{n^2 g} \Rightarrow \frac{m_1}{m_2} = \frac{n_2{}^2}{n_1{}^2}$$

$m_1 = 447.0g$

$m_2 = 286.1g$

Standing wave can not appears for any intermediate mass: $\boxed{n_2 - n_1 = 1}$

$$\Rightarrow \frac{m_1}{m_2} = \frac{n_2{}^2}{n_1{}^2} = \frac{(n_1+1)^2}{n_1{}^2} = \frac{447g}{286.1g} = 1.56 \Rightarrow n_1 = 4$$

$$f = \frac{\sqrt{mg}}{\sqrt{\mu}}\frac{n}{2L} \Rightarrow \mu = \frac{n_1^2 m_1 g}{4f^2L^2} = 0.845 \text{ g/m}$$