Tutorial for chapter 11

Some keywords

Rolling

Translation

Wheel

Soccer

Combination

Smooth rolling

Slipping

Symmetric 匀称的

Angular Momentum

Precession rate

Torque

Rotation

Cylinder

Uniform

Conservation

Non-smooth rolling

Without slipping

Dynamics

Gyroscope 陀螺仪

Combined Motion of a rigid body (*Translation + Rotation)*

* 非质点都在平动 + 转动 (定轴)

判断

Kinetic Energy:

$$K = \frac{1}{2} M v_{\text{com}}^2 + \frac{1}{2} I_{\text{com}} \omega^2 = K_{\text{tran}}^{\text{com}} + K_{\text{rot}}^{\text{com}}$$

轴过质心

Torque:

$$\tau_{\text{net,ext}}^z = I_{\text{com}} \alpha_z$$

- Net torque **about COM axis**: $\tau_{\text{net,ext}}^z$
- Rotational inertial **about COM axis**: I_{com}
- Angular acceleration **about COM axis**: α_z

Krot + Ktra

功的效果也是两部分

Work:

$$W_{\text{net,ext}} = \int \vec{F}_{\text{net,ext}} \cdot d\vec{r}_{\text{com}} + \int \tau_{\text{net,ext}}^z d\theta$$

Work-Kinetic Theorem:

$$W_{\text{net,ext}} = \Delta K_{\text{tran}}^{\text{com}} + \Delta K_{\text{rot}}^{\text{com}} = \Delta K$$

General Strategies for Rigid Body in Combined Motion

Dynamics:

$$\begin{cases} \vec{F}_{\text{net,ext}} = M\dfrac{d\vec{v}_{\text{com}}}{dt} = M\vec{a}_{\text{com}} \\ \tau^{z}_{\text{net,ext}} = I_{\text{com}}\alpha_z \end{cases}$$

forces,
$\vec{\boldsymbol{a}}_{\mathbf{com}}$**, and** $\boldsymbol{\alpha}$

Kinematics: (If $\vec{\boldsymbol{a}}_{\mathbf{com}}$ and/or $\boldsymbol{\alpha}$ are constant)

$v_{\text{com}} = v_{\text{com},0} + a_{\text{com}}t$	$\omega = \omega_0 + \alpha t$
$\Delta x_{\text{com}} = v_{\text{com},0}t + \frac{1}{2}a_{\text{com}}t^2$	$\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
$2a_{\text{com}}\Delta x_{\text{com}} = v^2_{\text{com}} - v^2_{\text{com},0}$	$2\alpha\Delta\theta = \omega^2 - \omega_0^2$

$\boldsymbol{\omega}$, $\boldsymbol{v}_{\mathbf{com}}$,
$\Delta\boldsymbol{x}_{\mathbf{com}}$,
$\Delta\boldsymbol{\theta}$

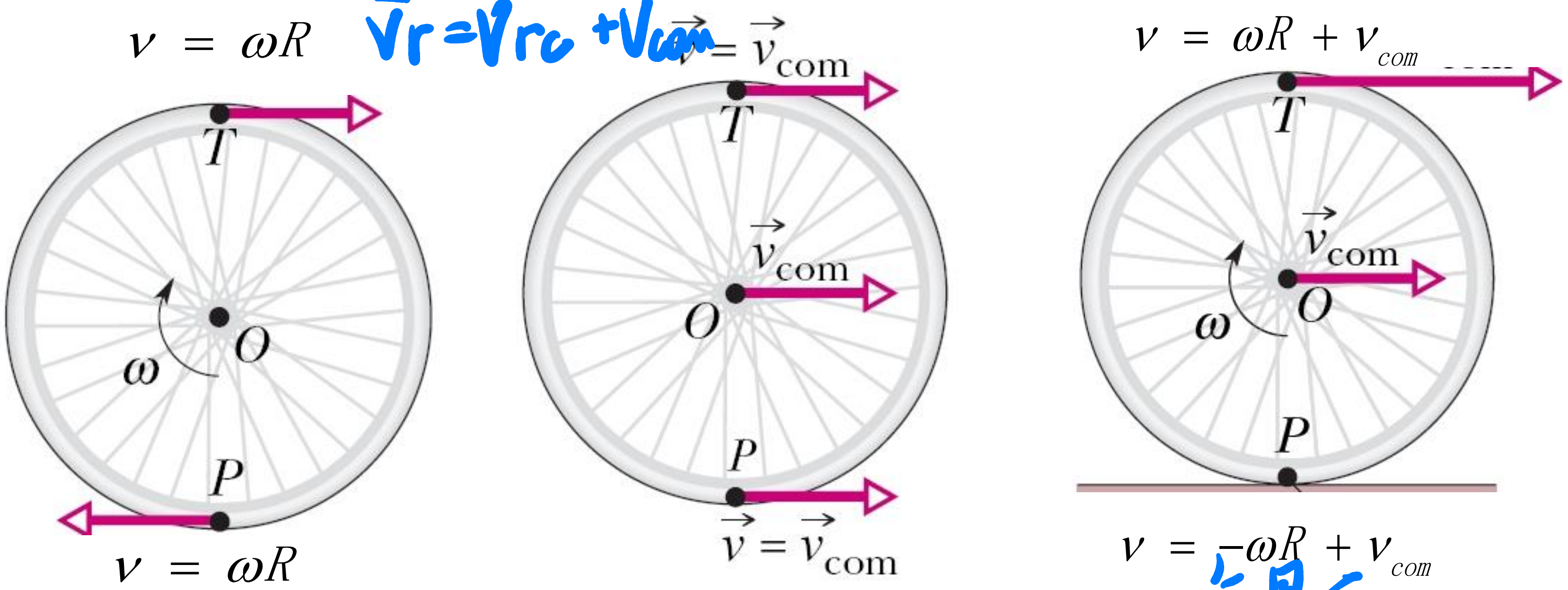
Chapter 11 Rolling

Rolling of a wheel (or other round body)

$$K = \frac{1}{2} I_{com}\, \omega^2 + \frac{1}{2} M v_{com}^2$$

Pure rotation + Pure translation = Rolling

$\vec{v}_r = \vec{v}_{rc} + \vec{v}_{com}$



天然成立 矢量和

No slipping ,static friction, Smooth Rolling → $v = v_{com} - \omega R = 0$ f静

求导: $a_{com} = \alpha R$

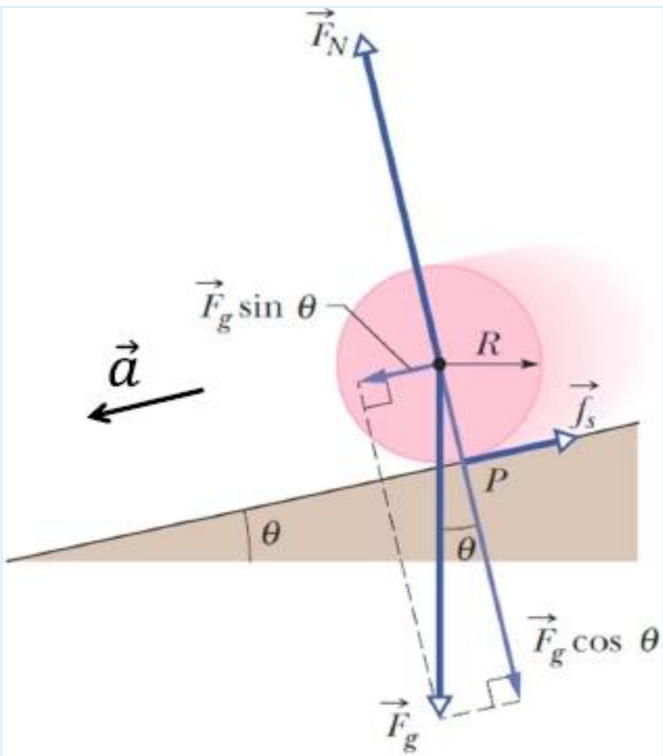
Slipping ,Kinetic friction, Non Smooth Rolling → $v = v_{com} - \omega R \neq 0$ 打滑

可前可后

Smooth Rolling along a Ramp

Pure rotation + **Pure translation** = **Rolling**

$$v = v_{com} - \omega R = 0 \Rightarrow \omega R = v_{com} \qquad s = \theta R \qquad a_{com} = \alpha R$$



$$\tau_{net} = R f_s = I_{com}\alpha \quad \text{(rotation)}$$

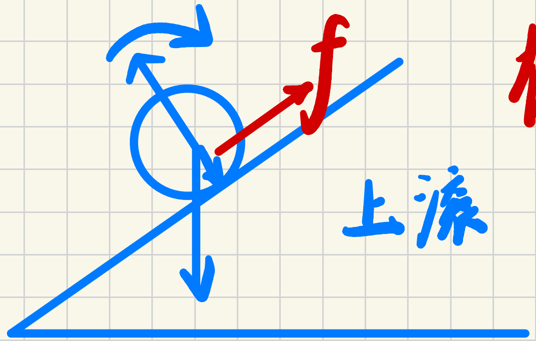
$$mg\sin\theta - f_s = ma \quad \text{(translation)}$$

Smooth rolling: $a = R\alpha$

$$\boxed{a_{com} = \frac{g\sin\theta}{1 + I_{com}/MR^2}} \quad \text{down ramp}$$

$$\boxed{f_s = I_{com}\frac{a_{com}}{R^2} = \frac{\gamma}{1+\gamma} mg\sin\theta}$$

Defining: $\gamma = \dfrac{I}{mR^2}$

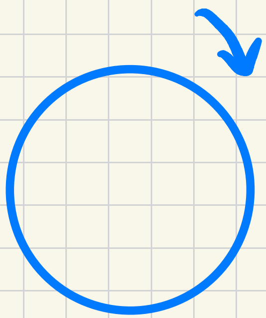


有f才有ω改变

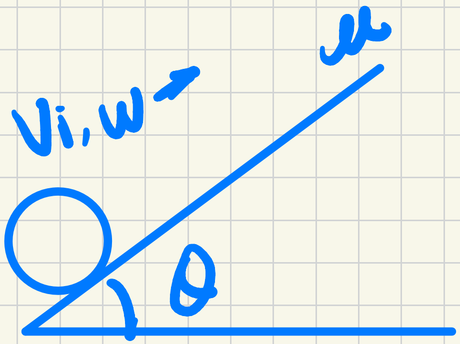
Smooth Rolling

$v\downarrow = \omega\downarrow R$

$\tau\downarrow$



$*f_{static}$ $\begin{cases} \text{与外力相反} \\ \text{与外力矩相反} \end{cases}$



f_s

mg

$\mu \to \infty$ 一直 SR

$\mu \to 0$

无 f

$\to \omega$ 不变

mg

最高点 $V_{com}=0$

$\omega=0$

E 全变 E_p

只有平动动能变 E_p

快不快 → 加速度

Smooth vs. Non-smooth Rolling

- For a smooth rolling motion without external force or torque, there is no friction, $s = R\theta$, $a = \alpha = 0$, and at point P, $v_{com} = v_t = R\omega$.

- For a smooth rolling motion with external force or torque, $s = R\theta$ and at point P, $v_{com} = v_t = R\omega$. There exists a static friction f_s and $a \neq 0, \alpha \neq 0$ with $a = R\alpha$. The f_s does not generate "heat".

- For a non-smooth rolling motion with sliding, there exists a kinetic friction f_k and $a \neq 0, \alpha \neq 0$ with $a = \gamma R\alpha$. The f_k generates "heat", reducing body's kinetic energy.

Chapter 11 Tutorial Problem 1

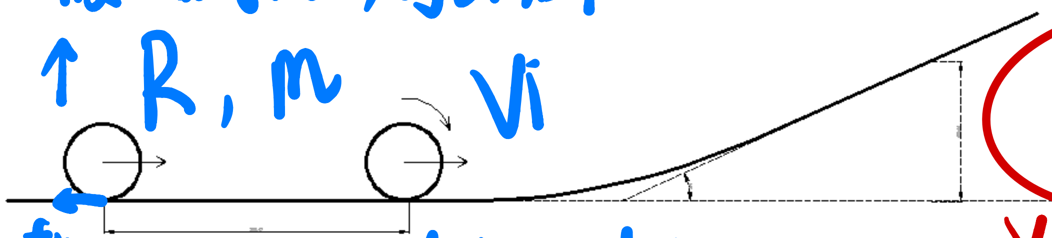
A uniform ball of radius $R = 0.2\ m$, mass m = 2.0kg , is kicked with initial speed of its center of mass $v_i = 5.0\ m/s$ and initial angular speed $\omega_i = 0$, The ball first rolls with sliding (non-smooth rolling) along the horizontal ground surface until a moment when its angular speed is increased enough. Then the ball starts roll smoothly and moves uphill along a ramp with inclined angle $\theta = 20°$. The coefficients of friction between the ball and the ground are $\mu_k = 0.2$ for kinetic friction and $\mu_s = 0.4$ for static friction. Find:

过质心

(1)The angular speed ω of the ball when it begins the smooth rolling.
(2)The increase ΔE_{th} in thermal energy of the ball and the ground due to sliding.
(3)The sliding distance S of the ball's center during the non-smooth rolling.
(4)The maximum height h of the ball on the ramp.
(5)The magnitude of the angular acceleration α of the ball on the ramp.

接触点相对向右，到SR后 $V_P=0$

SR后仍在平地：



无f!

V=ωR 后不做变化

Chapter 11 Tutorial Problem 1

(1) The motion of the ball can be treated as the combination motion of the linear motion of the center of mass and the rotational motion about the center of mass, so apply newton's second law:

$$\left.\begin{aligned} f_k &= ma \\ f_k R &= I\alpha \end{aligned}\right\} \Rightarrow a = \frac{I}{mR}\alpha$$

$$v_i - v_{com} = at$$

$$\omega = \alpha t$$

Smooth Rolling: $\omega R = v_{com}$

Therefore: $$\omega = \frac{v_i}{R(1+{}^{I}\!/\!_{mR^2})} = \frac{5}{0.2\times(1+\frac{2}{5})} rad/s = 17.9\, rad/s$$

只与形状有关

Chapter 11 Tutorial Problem 1

(2) By conservation of energy:

$$\Delta E_{th} = K_i - K_f = \frac{1}{2}mv_i^2 - (\frac{1}{2}mv_{com}^2 + \frac{1}{2}I\omega^2)$$

$$= \frac{1}{2}mv_i^2 - (\frac{1}{2}m\omega^2R^2 + \frac{1}{2}I\omega^2)$$

$\Delta E_{th} = f_k d$ 不可用

$$= \frac{1}{2}mv_i^2 - (\frac{1}{2}m\omega^2R^2 + \frac{1}{2}\frac{2}{5}mR^2\omega^2)$$

$$= \frac{1}{2}mv_i^2 - \frac{1}{2}m\omega^2R^2\frac{7}{5}$$

$W_{f_k} = f_k R\Delta\theta - f_k R S$ 如

$$= \frac{1}{2}mv_i^2 - \frac{1}{2}m(\frac{v_i}{1+I/mR^2})^2\frac{7}{5}$$

$$= 7.1J$$

不可以: $\Delta E_{th} = f_k \cdot d$ X

SR中 f_s 使得 $W_{f_s}=0$ 做功一正一负

$W_{f_k} = -f_k \cdot s + f_k R\Delta\theta$

$v_f^2 - v_i^2 = 2as$

$\omega_f^2 - \omega_i^2 = 2\alpha\Delta\theta$ (加速转为正)

Chapter 11 Tutorial Problem 1

(3) $$v_{com}{}^2 - v_i{}^2 = -2aS_{com}$$

$$S_{com} = \frac{v_i{}^2 - v_{com}{}^2}{2a} = \frac{v_i{}^2 - (\omega \mathrm{R})^2}{2\mu_k g} = 3.1m$$

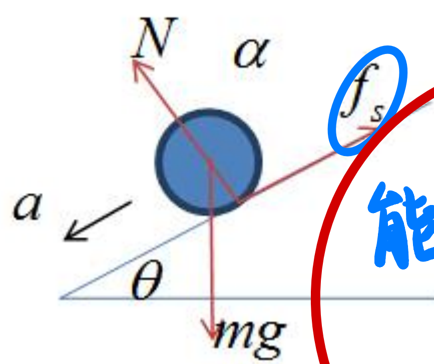
(4) By conservation of mechanic energy:

$$mgh = \frac{1}{2}mv_{com}^2 + \frac{1}{2}I\omega^2$$

$$h = \frac{1}{mg}(\frac{1}{2}mR^2 + \frac{1}{2}I)\omega^2 = \frac{I + mR^2}{2mg}\omega^2 = 0.91m$$

Chapter 11 Tutorial Problem 1

(5)

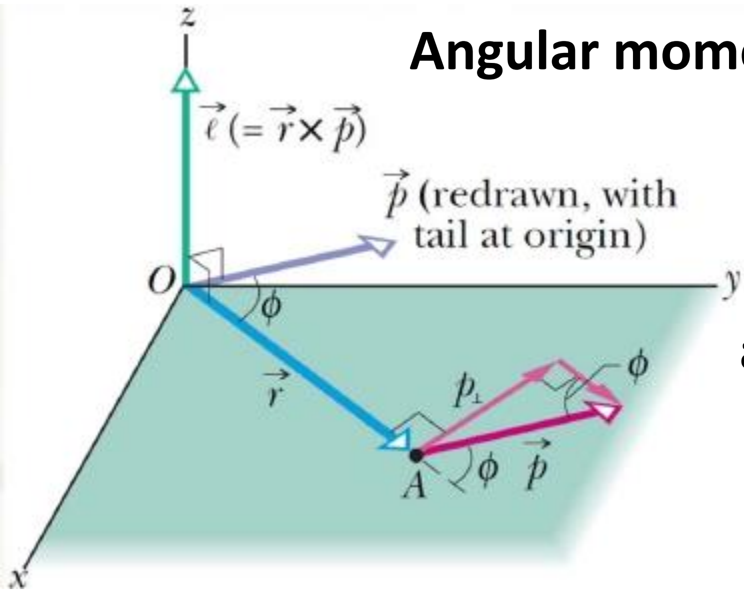


能不能提供足够 f_s

$$\left.\begin{aligned} f_s \cdot R &= I\alpha \\ mg\sin\theta - f_s &= ma \\ a &= \alpha R \end{aligned}\right\} \Rightarrow \alpha = \frac{g\sin\theta}{R(1+I/mR^2)} = \frac{9.8\times\sin 20^0}{0.2\times(1+\frac{2}{5})} rad/s = 12.0 rad/s$$

$$f_s = \frac{I\alpha}{R} = \frac{mg\sin\theta}{(mR^2/I+1)} = \frac{2}{7} mg\sin 20° < \mu_s N = \mu_s mg\cos 20^0$$

Chapter 11 Angular Momentum



Angular momentum is defined with respect to a point (origin)

$$\vec{\ell} = \vec{r} \times \vec{p} = m(\vec{r} \times \vec{v})$$

a system of particles about a point (origin):

$$\vec{L} = \vec{\ell}_1 + \vec{\ell}_2 + \vec{\ell}_3 + \cdots + \vec{\ell}_n = \sum_{i=1}^{n} \vec{\ell}_i$$

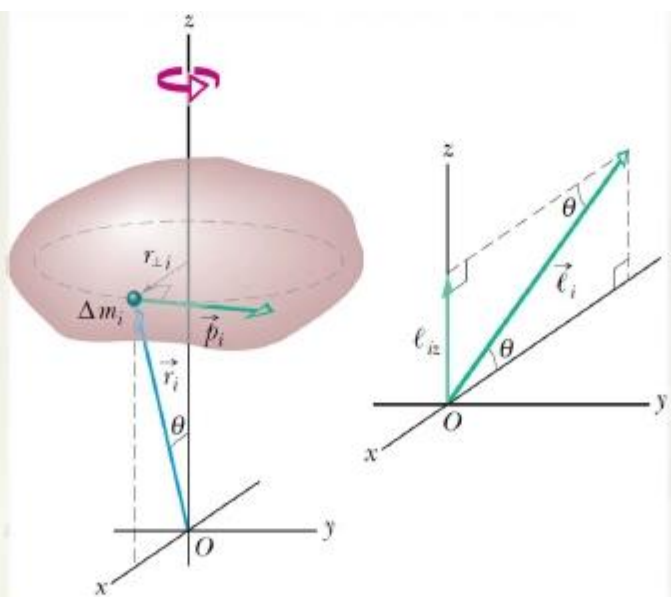
$$l_i = (r_i)(p_i)(\sin 90^\circ) = (r_i)(\Delta m_i v_i)$$

$$l_{iz} = l_i \sin\theta = (r_i \sin\theta)(\Delta m_i v_i) = r_{\perp i} \Delta m_i v_i$$

Because $v = \omega r_\perp$, we get

$$L_z = \sum_{i=1}^{n} l_{iz} = \sum_{i=1}^{n} \Delta m_i v_i r_{\perp i} = \sum_{i=1}^{n} \Delta m_i (\omega r_{\perp i}) r_{\perp i}$$

$$= \omega\left(\sum_{i=1}^{n} \Delta m_i r_{\perp i}^{\;2}\right) = I\omega$$



$$\boxed{L_z = I\omega \qquad \text{(rigid body, fixed axis)}}$$

Chapter 11 Angular Momentum

Newton's Second Law in Angular Form

$$\vec{\tau}_{\text{net}} = \frac{d\vec{\ell}}{dt}$$

Conservation of Angular Momentum

$$\tau_{net} = 0 \Rightarrow \frac{d\vec{L}}{dt} = 0 \Rightarrow L_i = L_f$$

$$I_i\omega_i = I_f\omega_f.$$ **Rigid Body, fixed axis**

More corresponding Variables and Relations for Translational and Rotational Motion

Translational		Rotational	
Force	$\vec{F}$	Torque	$\vec{\tau}\ (= \vec{r} \times \vec{F})$
Linear momentum	$\vec{p}$	Angular momentum	$\vec{\ell}\ (= \vec{r} \times \vec{p})$
Linear momentum[b]	$\vec{P}\ (= \Sigma \vec{p}_i)$	Angular momentum[b]	$\vec{L}\ (= \Sigma \vec{\ell}_i)$
Linear momentum[b]	$\vec{P} = M\vec{v}_{com}$	Angular momentum[c]	$L = I\omega$
Newton's second law[b]	$\vec{F}_{net} = \dfrac{d\vec{P}}{dt}$	Newton's second law[b]	$\vec{\tau}_{net} = \dfrac{d\vec{L}}{dt}$
Conservation law[d]	$\vec{P}$ = a constant	Conservation law[d]	$\vec{L}$ = a constant

[a]See also Table 10-3.
[b]For systems of particles, including rigid bodies.
[c]For a rigid body about a fixed axis, with L being the component along that axis.
[d]For a closed, isolated system.

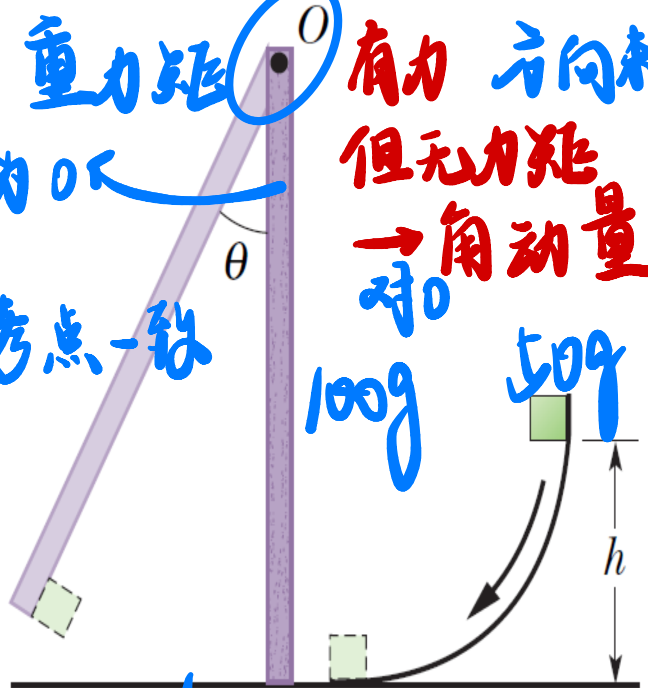
Chapter 11 Tutorial Problem 2

注：重力矩也为0

参考点一致

有力 方向未知 但无力矩 → 角动量守恒

对O



A small 50g block slides down a ~~frictionless surface~~ through height h=20cm and then sticks to a uniform rod of mass 100g and length 40cm. The rod pivots about point O through angle θ before momentarily stopping. Find θ.

$\vec{F}_{net} = \frac{dp}{dt}$

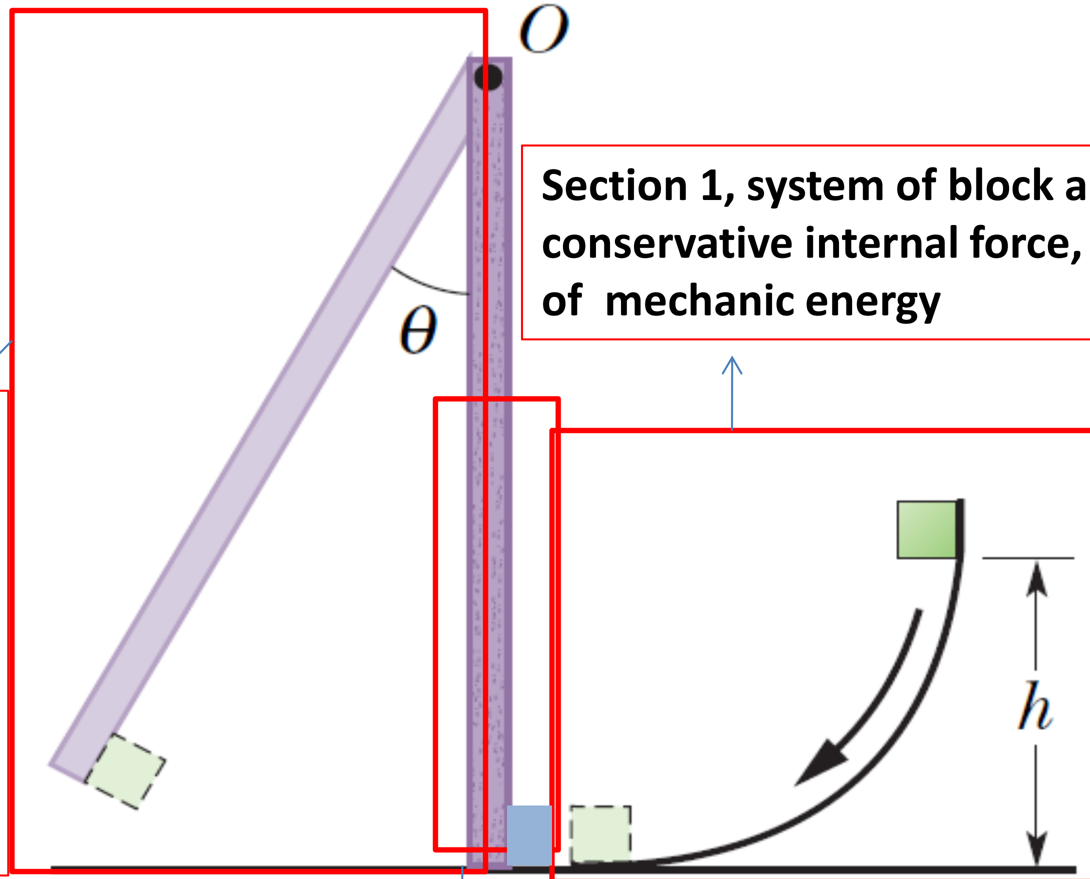
$lmv = I\omega$

$I = I_p + I_{rod}$ → 平行轴定理

$= ml^2 + \frac{1}{12}ml^2 + m(\frac{l}{2})^2$

再θ 机械能守恒

Chapter 11 Tutorial Problem 2



Section 1, system of block and earth, conservative internal force, conservation of mechanic energy

Section 2, Collision: NO external torque about point O, conservation of angular momentum

Section 3, system of block rod ,and earth, conservative internal force, conservation of mechanic energy

Chapter 11 Tutorial Problem 2

Section 1: System: Earth + Block, conservative internal force.
Mechanical energy conservation applied to the block (before impact):

$$\boxed{mgh = \frac{1}{2}mv_f^2 \Rightarrow v_f = \sqrt{2gh}}$$

Section 2: Collision: NO external torque.

Assume the *clockwise* sense as positive, so that the angular velocities (and angles) in this problem are positive. The collision is described by angular momentum, **with respect to the point O,** conservation:

$$\boxed{L_i^z = mv_f d = L_f^z = \left(I_{\text{rod}} + md^2\right)\omega_f}$$

where I_{rod} is found using Table 10-2(e) and the parallel axis theorem:

$$I_{\text{rod}} = \frac{1}{12}Md^2 + M\left(\frac{d}{2}\right)^2 = \frac{1}{3}Md^2.$$

Chapter 11 Tutorial Problem 2

The angular velocity of the system immediately after the collision:

$$\omega_f = \frac{md\sqrt{2gh}}{(Md^2/3) + md^2}$$

which means the system has kinetic energy $K = \left(I_{\text{rod}} + md^2\right)\omega_f^{\,2}/2$

Section 3: System: Block + rod + earth, conservative internal force.

By conservation of mechanic energy: Kinetic energy of the system will turn into potential energy in the final position, where the block has reached height H (relative to the lowest point) and the center of mass of the stick has increased its height by $H/2$. From trigonometric considerations, we note that $H = d(1 - \cos\theta)$, so we have

$$\frac{1}{2}\left(I_{\text{rod}} + md^2\right)\omega^2 = mgH + Mg\frac{H}{2} \Rightarrow \frac{1}{2}\frac{m^2d^2(2gh)}{(Md^2/3)+md^2} = \left(m + \frac{M}{2}\right)gd(1-\cos\theta)$$

Chapter 11 Tutorial Problem 2

from which we obtain

$$\theta = \cos^{-1}\left(1 - \frac{m^2 h/d}{(m+M/2)\,(m+M/3)}\right) = \cos^{-1}\left(1 - \frac{h/d}{(1+M/2m)\,(1+M/3m)}\right)$$

$$= \cos^{-1}\left(1 - \frac{(20\text{ cm}/\ 40\text{ cm})}{(1+1)(1+2/3)}\right) = \cos^{-1}(0.85)$$

$$= 32°.$$

Alternatively, we can use Work-kinetic theorem

$$W_{net} = (mgd + \frac{1}{2}Mgd)(\cos\theta - 1) = \Delta K$$

$$W_{net} = \int \vec{F}\cdot d\vec{s} = \int_{h_{1i}}^{h_{1f}} -mgdh + \int_{h_{2i}}^{h_{2f}} -Mgdh = -mg\Delta h_1 - Mg\Delta h_2$$

$$= -(mgH + Mg\frac{H}{2}) = (mgd + \frac{1}{2}Mgd)(\cos\theta - 1)$$