Chapter 12 Equilibrium & Elasticity

A body in equilibrium:

1. The linear momentum $\vec{P}$ of its center of mass is constant.

$$\vec{F}_{net} = \frac{d\vec{P}}{dt} \Rightarrow \vec{F}_{net} = 0 \text{ (condition for translational equilibrium)}$$

2. The angular momentum $\vec{L}$ about **any** point (including its center of mass) is constant.

$$\vec{\tau}_{net} = \frac{d\vec{L}}{dt} \Rightarrow \vec{\tau}_{net} = 0 \text{ (condition for rotational equilibrium)}$$

Static Equilibrium

P= 0 and L= 0 (about *any point)*

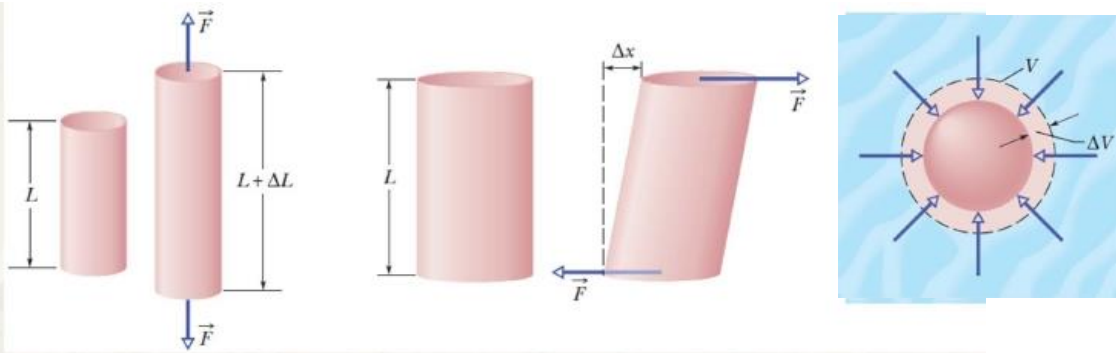
Dynamic Equilibrium:

P=constant, L=0
L=constant, P=0
P=constant , L=constant,

Balance of forces	Balance of torques
$F_{net,x} = 0$	$\tau_{net,x} = 0$
$F_{net,y} = 0$	$\tau_{net,y} = 0$
$F_{net,z} = 0$	$\tau_{net,z} = 0$

→ ***For any Fixed point***

Chapter 12 Equilibrium & Elasticity



- The stress is the deforming force per unit area.
- The strain is the unit deformation.
- The modulus of elasticity is :

Stress = modulus x strain

Tension /Compression

$$\frac{F}{A} = E\frac{\Delta L}{L}$$

远小于L

不完全是刚体

$F = \left(\frac{EA}{L}\right)\Delta L$

$\Rightarrow F = k\Delta L$

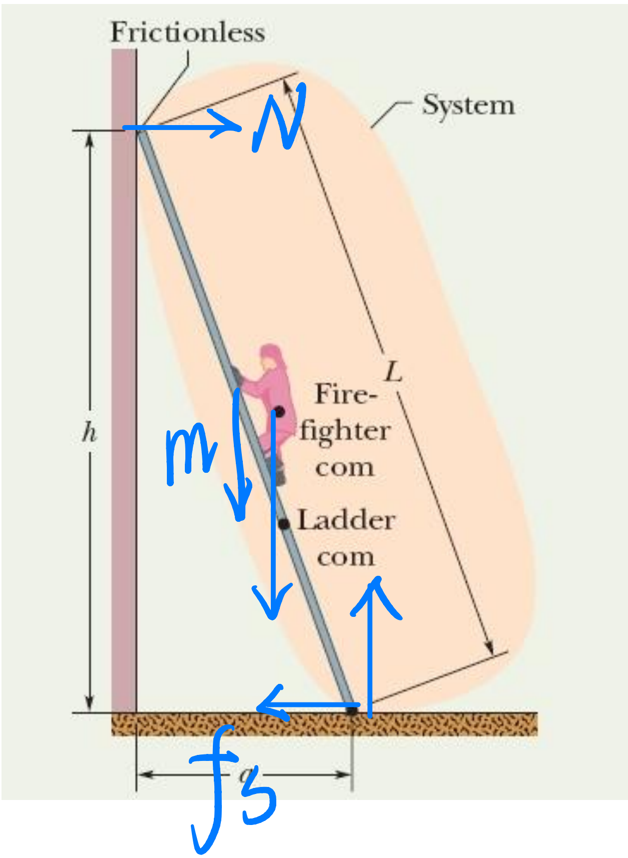
Shearing

$$\frac{F}{A} = G\frac{\Delta x}{L}$$

Hydraulic Stress

$$\frac{F}{A} = \Delta p = B\frac{\Delta V}{V}$$

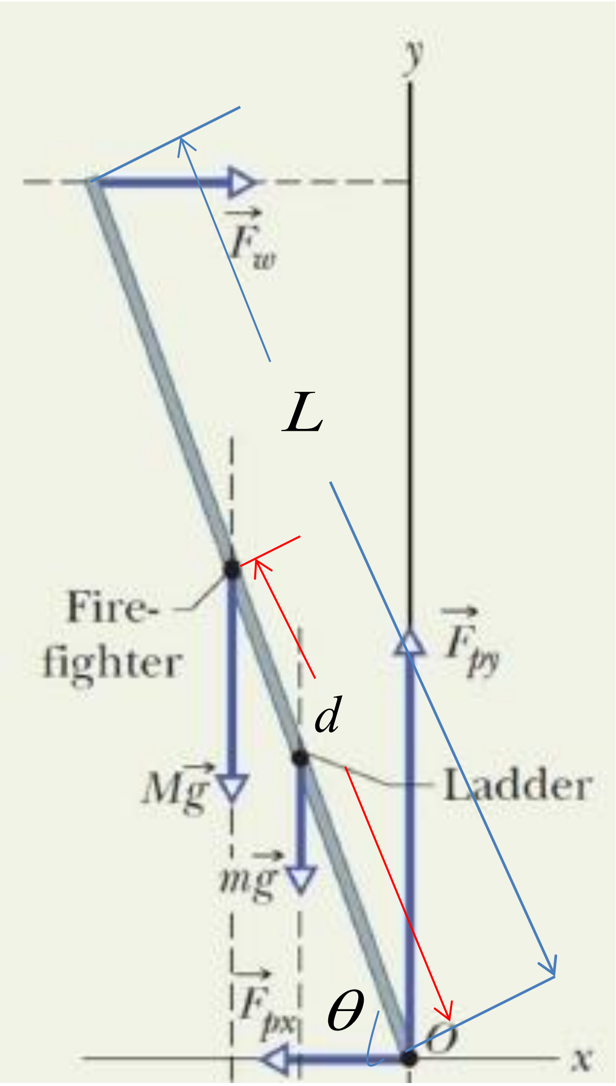
Chapter 12 Tutorial Problem 1



A ladder **of length L = 12 m** and mass **m = 45 kg** leans against a slick (frictionless) wall. Its upper end is at height **h = 9.3 m** above the pavement on which the lower end is supported. It's center of mass is ***L/3*** *from the* lower end. Let the coefficient of static friction μ_s between the ladder and the pavement **be 0.53**. A firefighter of **mass M =72kg** climbs the ladder.

How **far (in percent) up** the ladder must the firefighter go to put the ladder on the verge of sliding?

Chapter 12 Tutorial Problem 1



Let ***d*** be the distance the firefighter up the ladder, Then we need find the value of d/L

Choose a rotation axis through **o point** in the picture.

Since the system is **in static equilibrium**, the net force acting on the system is zero:

Force Balance:

$$F_{net,x} = 0, \Rightarrow F_w = F_{P_x} \leq \mu_s F_{P_y} \text{....(1)}$$

$$F_{net,y} = 0, \Rightarrow F_{P_y} = (M+m)g \text{....(2)}$$

Chapter 12 Tutorial Problem 1

In addition, **the net torque about *O*** (contact point between the ladder and the pavement) must also be zero.

Torque Balance:

$$\tau_{net,o} = 0 \Rightarrow -F_w L \sin\theta + Mgd\cos\theta + \frac{L}{3} mg\cos\theta = 0\ldots(3)$$

Since the ladder is on the verge of sliding, Therefore:

$$F_{P_x} = \mu_s F_{P_y} = \mu_s (M+m)g\ldots(4)$$

$$\tan\theta = \frac{h}{\sqrt{L^2 - h^2}}$$

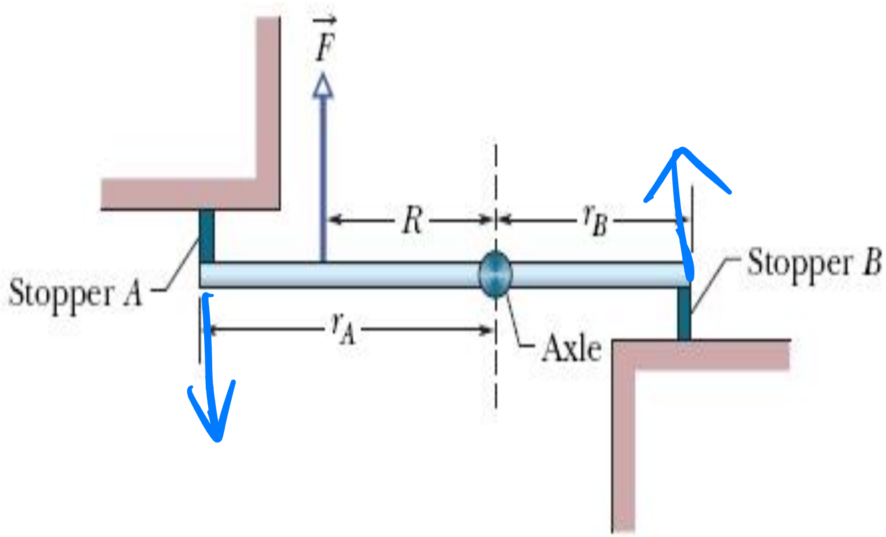
According to (1),(2),(3) and (4),

$$\Rightarrow -\mu_s (M+m)gL\sin\theta + Mgd\cos\theta + \frac{L}{3} mg\cos\theta = 0$$

$$\Rightarrow \frac{d}{L} = \frac{\mu_s (M+m)\tan\theta}{M} - \frac{m}{3M} = 0.845 \approx 85\%$$

Chapter 12 Tutorial Problem 2

As shown in the figure, an overhead view of **a rigid rod** that turns about a vertical axle until the identical rubber stoppers *A and B* are forced against rigid walls at distances ***r_A* =7.0 cm** *and* ***r_B* =4.0 cm** from the axle. Initially the stoppers touch the walls without being compressed. Then force of **magnitude F=220 N** is applied perpendicular to the rod at a **distance *R* 5.0 cm** *from the axle.*



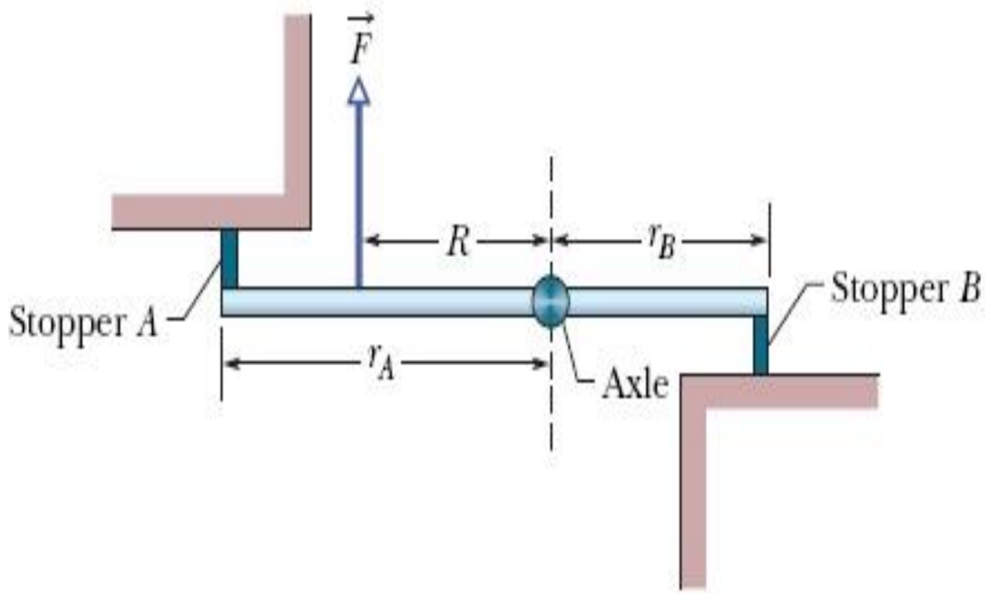
What is the magnitude of the force compressing (a) stopper A and (b) stopper B ?

Chapter 12 Tutorial Problem 2

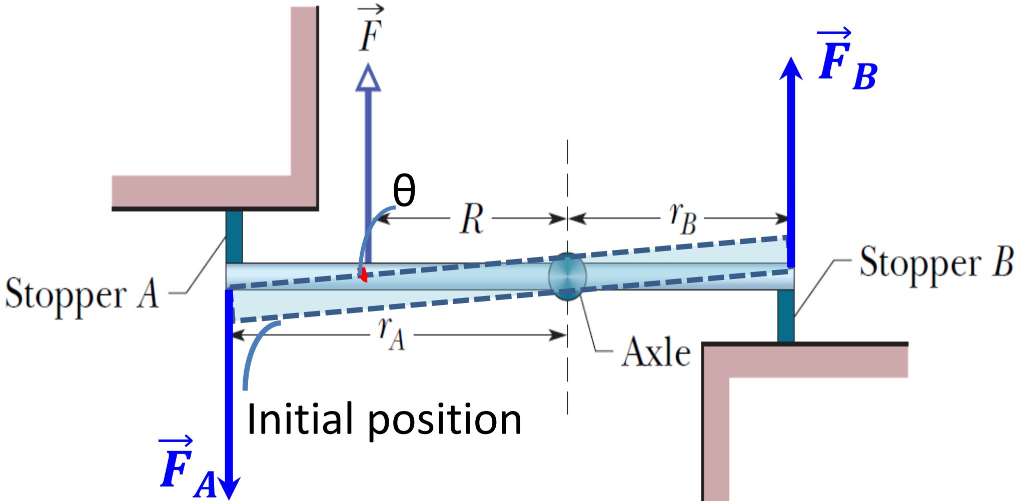
1) The Stopper A and B will be compressed or pulled after applying the force?

2) The Force F_A and F_B **in terms of** ΔL_A **and** ΔL_B

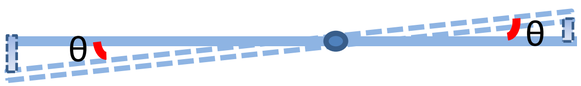
3) The relation between ΔL_A **and** ΔL_B **?**



Chapter 12 Tutorial Problem 2



Both stoppers are compressed after applying $\vec{F}$ with the compressed length ΔL_A and ΔL_B. **The rod is a rigid rod and it has no deformation,** when the rod rotates a (presumably small) angle θ (in radians), then



$$\frac{\Delta L_A}{\Delta L_B} = \frac{r_A}{r_B}$$

$$\Delta L_A \ll L, \Delta L_A = r_A \theta$$

$$\Delta L_B \ll L, \Delta L_B = r_B \theta$$

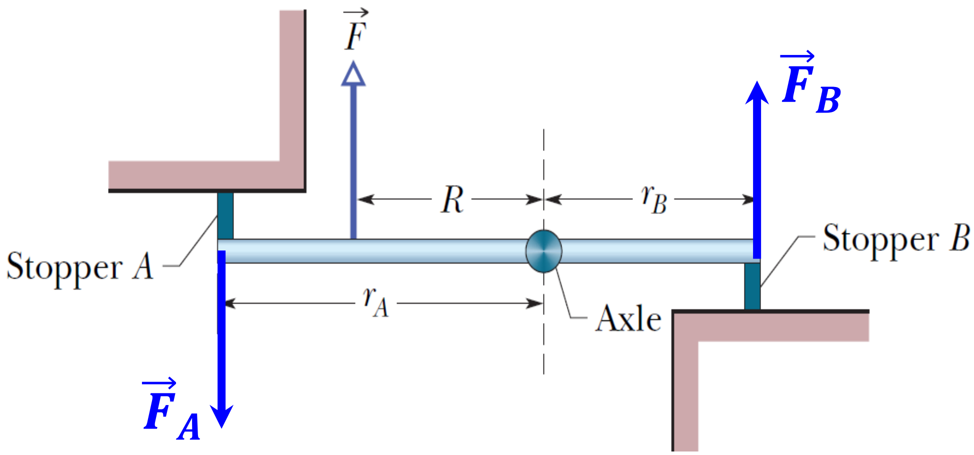
$$\frac{F_A}{A} = E\frac{\Delta L_A}{L}$$

$$\frac{F_B}{A} = E\frac{\Delta L_B}{L}$$

$$\frac{F_A}{F_B} = \frac{r_A}{r_B} \quad \ldots (1)$$

Tension and Compression

Chapter 12 Tutorial Problem 2



Equilibrium of torques about the axle requires:

$F_A r_A + F_B r_B - FR = 0$ ... (2) **Balance of Torque**

$$\Rightarrow F_A = \frac{Rr_A}{r_A^2 + r_B^2} F = \frac{(5.0\text{ cm})(7.0\text{ cm})}{(7.0\text{ cm})^2 + (4.0\text{ cm})^2}(220\text{ N}) = 118\text{ N} \approx 1.2\times10^2\text{ N}.$$

(1) + (2)

$$\Rightarrow F_B = \frac{Rr_B}{r_A^2 + r_B^2} F = \frac{(5.0\text{ cm})(4.0\text{ cm})}{(7.0\text{ cm})^2 + (4.0\text{ cm})^2}(220\text{ N}) = 68\text{ N}.$$

Chapter 12 Tutorial Problem 2

Equilibrium of forces about the axle requires:

$$F + F_B - F_A + F_{\text{axle}} = 0$$ **Balance of Force**

$\Rightarrow$ $F_{\text{axle}} = -170$ N (magnitude 170 N, direction opposite to applied $\vec{F}$)