# Tutorial for chapter 15

# Key words

Oscillation
Oscillator
Simple Harmonic motion (SHM)
Period
Periodic
Frequency
Angular frequency
Sinusoidal
Amplitude
Phase
Phase angle
Effective
Stretch
Compress

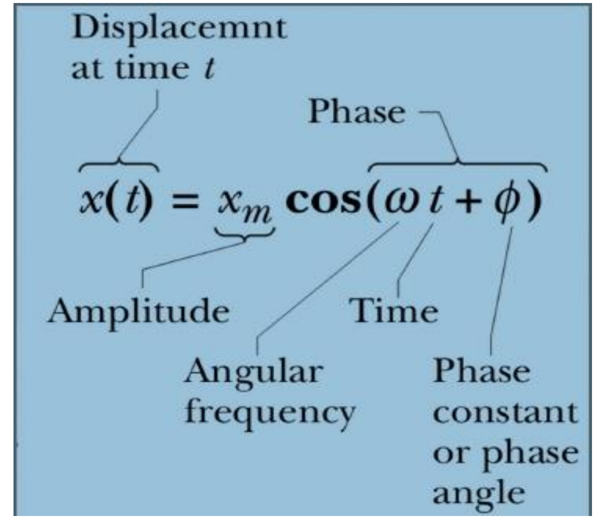
Suspension system
Restoring force
Damp
Pendulum
Torsion pendulum
Twist
Pivot
Suspension
Resonance
Exponential
Decay
Approximation
Projection

# Chapter 15 Oscillation—linear SHM

Periodic motion: $x(t) = x(t+T)$

$$x_m \cos(\omega t + \phi) = x_m \cos(\omega(t+T) + \phi)$$

$$\omega(t+T) = \omega t + 2\pi \Rightarrow \boxed{\omega = \frac{2\pi}{T} = 2\pi f}$$




$$v(t) = \frac{dx(t)}{dt} = \frac{d}{dt}[x_m \cos(\omega t + \phi)]$$

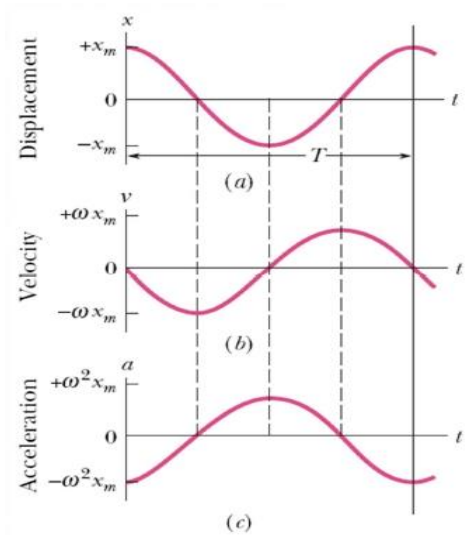
$$\Rightarrow \boxed{v(t) = -\omega x_{\mathrm{m}} \sin(\omega t + \phi)}$$

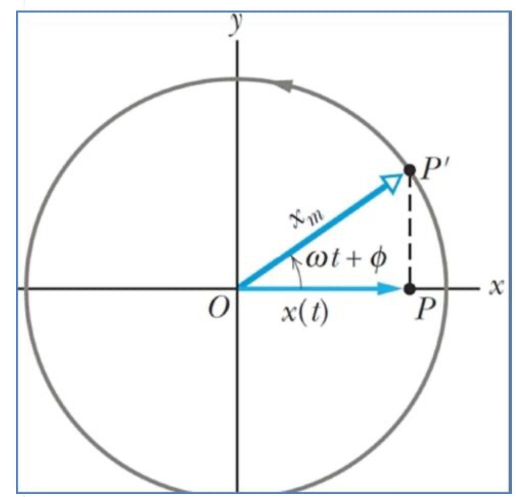
$$= \omega x_{\mathrm{m}} \cos(\omega t + \phi + \frac{\pi}{2})$$

$$a(t) = \frac{\mathrm{d}v(t)}{\mathrm{d}t} = \frac{d^2 x(t)}{dt^2}$$

$$\Rightarrow \boxed{a(t) = -\omega^2 x_{\mathrm{m}} \cos(\omega t + \phi)}$$

$$= -\omega^2 x(t)$$

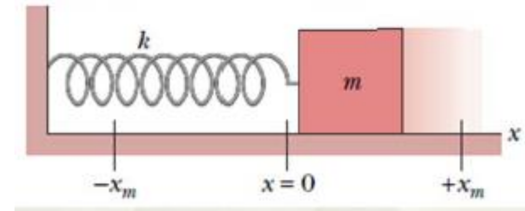


# Chapter 15 Oscillation

$$F = ma \Rightarrow -kx = m\frac{d^2x}{dt^2} \Rightarrow \frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

$$\omega = \sqrt{\frac{k}{m}}$$




$$\Rightarrow \frac{d^2x}{dt^2} + \omega^2 x = 0 \Rightarrow x(t) = x_{\mathrm{m}}\cos(\omega t + \phi)$$

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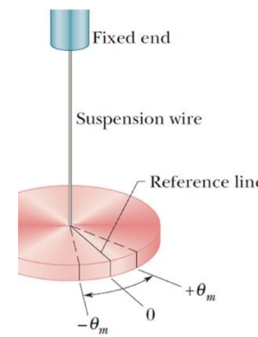
$$\tau = I\alpha \Rightarrow -\kappa\theta = I\frac{d^2\theta}{dt^2} \Rightarrow \frac{d^2\theta}{dt^2} + \frac{\kappa}{I}\theta = 0$$

$$\omega = \sqrt{\frac{\kappa}{I}}$$




$$\Rightarrow \frac{d^2\theta}{dt^2} + \omega^2\theta = 0 \Rightarrow \theta(t) = \theta_{\mathrm{m}}\cos(\omega t + \phi)$$

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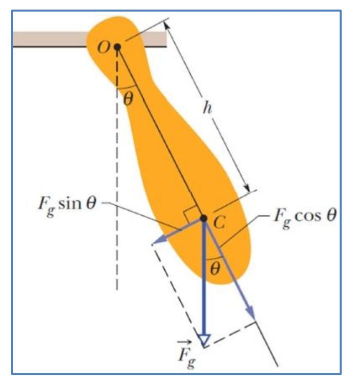
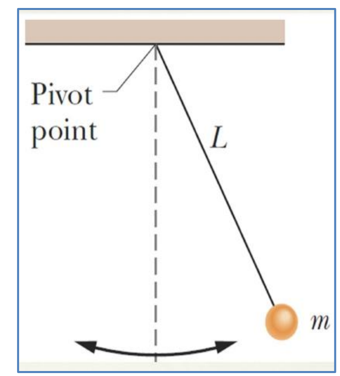
$$-Mgh\theta = -\kappa\theta = I\frac{d^2\theta}{dt^2} \Rightarrow \frac{d^2\theta}{dt^2} + \frac{\kappa}{I}\theta = 0$$

$$\Rightarrow \frac{d^2\theta}{dt^2} + \omega^2\theta = 0 \Rightarrow \theta(t) = \theta_{\mathrm{m}}\cos(\omega t + \phi)$$



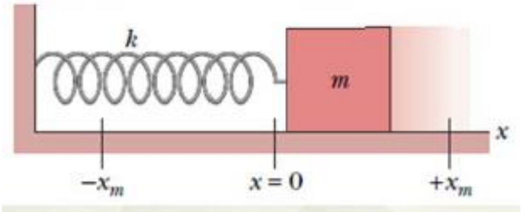

$$\omega = \sqrt{\frac{\kappa}{I}} = \sqrt{\frac{\mathrm{Mgh}}{I}}$$




$$\omega = \sqrt{\frac{Mgl}{ml^2}} = \sqrt{\frac{g}{l}}$$

# Chapter 15 Oscillation—Conservation of energy

$$\frac{1}{2}mv^2+\frac{1}{2}kx^2=\frac{1}{2}kx_{\max}^2=\frac{1}{2}mv_{\max}^2=\frac{1}{2}m\omega^2x_{\max}^2$$

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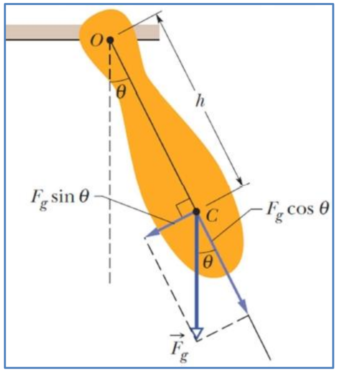
$$\frac{1}{2}I\Omega^2+Mgh(1-\cos\theta)=\frac{1}{2}I\Omega^2+\frac{1}{2}Mgh\theta^2$$

$$=\frac{1}{2}Mgh\theta_{\max}^2=\frac{1}{2}I\Omega_{\max}^2=\frac{1}{2}I\omega^2\theta_{\max}^2$$

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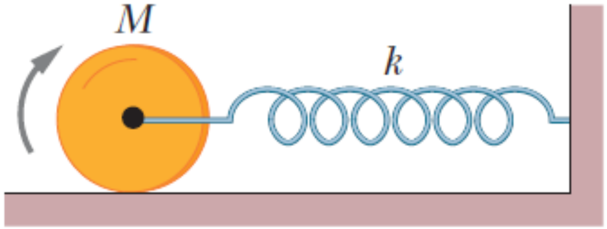
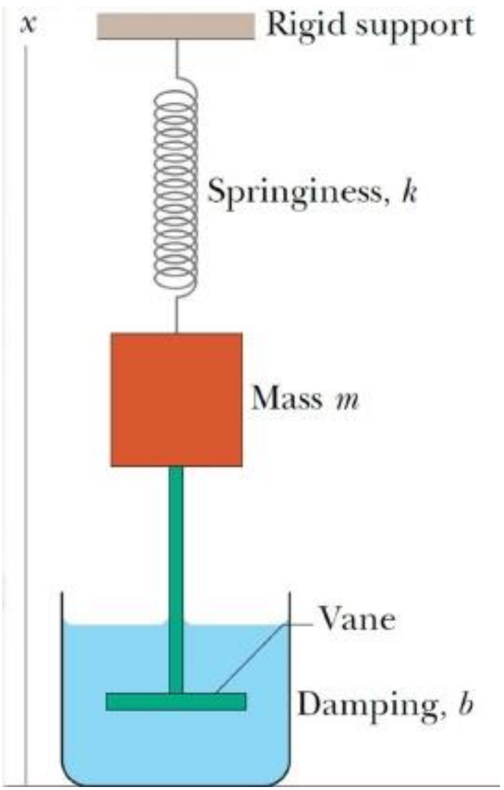
$$\frac{1}{2}mv_{\text{com}}{}^2+\frac{1}{2}I\Omega^2+\frac{1}{2}kx^2=\frac{1}{2}kx_{\max}^2$$

$$=\frac{1}{2}mv_{\max}^2+\frac{1}{2}I\Omega_{\max}^2=\frac{1}{2}(m+\frac{I}{R^2})v_{\max}^2$$

$$=\frac{1}{2}(m+\frac{I}{R^2})\omega^2x_{\max}^2$$



# Chapter 15 Oscillation-Damped SHM and Resonance




$$F_{net} = -bv - kx = ma$$

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$$

$$\boxed{x(t) = x_m e^{-bt/2m}\cos(\omega' t + \phi)},$$

$$\boxed{\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}} \quad \text{and} \quad \boxed{E(t) \approx \tfrac{1}{2}kx_m^2 e^{-bt/m}}$$

**Forced Oscillation and Resonance**

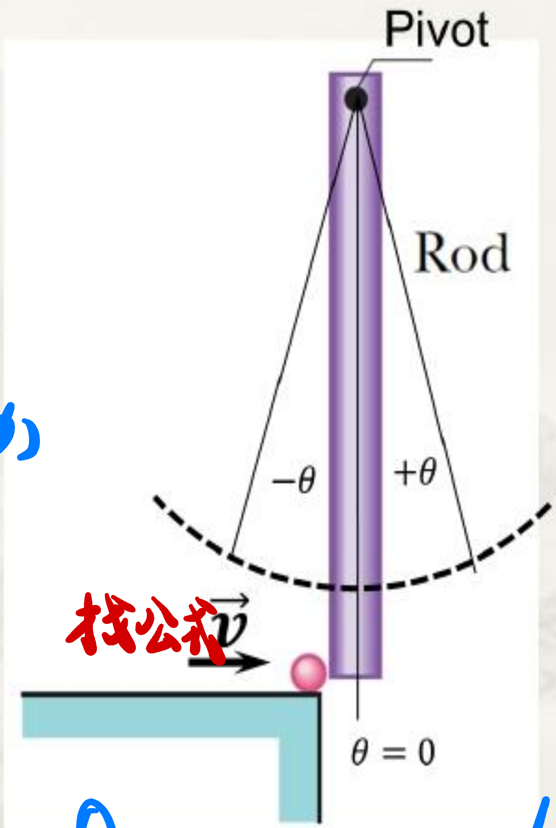
$$\boxed{\omega_d = \omega_0 \qquad \text{(resonance)}}$$

# Chapter 15 Tutorial Problem 1

A uniform rod of length L=1.0m and mass M=1.0kg is rest vertically with one end fixed at a pivot point as shown in the figure. The rod is able to swing about the pivot in a vertical plane. A putty particle of mass m=0.5kg moves horizontally at a speed of v=0.5m/s towards the other end of the rod, and collides with the rod at time t=0. After collision, the particle sticks to the end of the rod and swings together with the rod in SHM. The angular displacement of this SHM follows $\theta(t)= \theta_m cos(\omega t + \varphi)$

极小角

$x(t) = x_m \cos(\omega t + \phi)$




找公式

Find: (a) the angular speed $\Omega$ of the rod-particle system immediately after the collision.

角动量

求导 $\omega\theta_m$

(b) The phase angle $\varphi$ $-\frac{\pi}{2}$, angular frequency $\omega$, and the amplitude $\theta_m$ *of the* SHM oscillation.

$\Omega(t) = -\omega\theta_m \sin(\omega t + \phi)$

在 $t = 0$ 是 $\Omega > 0$

$\phi = -\frac{\pi}{2}$

(a) Conservation of angular momentum :

$$L_i = mLv = L_f = I\Omega_m$$

$$I = \underbrace{I_{rod} + I_{particle}} = \left[\frac{1}{12}ML^2 + M\left(\frac{L}{2}\right)^2\right] + mL^2 = (\frac{M}{3} + m)L^2$$

$$\Rightarrow \Omega_m = mLv / I = \frac{mLv}{(\frac{M}{3} + m)L^2} = \frac{mv}{(\frac{M}{3} + m)L} = 0.3 rad / s$$

(b) Find phase angle, angular frequency and amplitude:

(i) $$\theta(t)=\theta_m\cos(\omega t+\varphi)$$

$$\left.\begin{array}{l} t=0 \\ \theta=0 \end{array}\right\} \Rightarrow \theta(0)=\theta_m\cos(\varphi)=0 \Rightarrow \cos\varphi=0 \Rightarrow \varphi=\pm\frac{\pi}{2}$$

But as t increase, θ is positive. so, $\varphi=-\frac{\pi}{2}$

找特殊点、代入

(ii) $$\omega=\sqrt{\frac{(m+M)gh}{I}},$$ * h is the distance of COM of rod-particle to pivot.

重心质心

$$(m+M)h=M(\frac{L}{2})+mL \Rightarrow h=(\frac{m+M/2}{m+M})L$$

$$\omega=\sqrt{\frac{(m+M/2)g}{(m+M/3)L}}=3.43rad/s$$

(iii) At t=0

$$\Omega = \frac{d\theta}{dt} = -\omega\theta_m \sin(-\frac{\pi}{2})$$

$$\Rightarrow \Omega_m = \omega\theta_m \rightarrow \theta_m = \frac{\Omega_m}{\omega} = 0.087 rad$$

You can also use the conservation of energy:

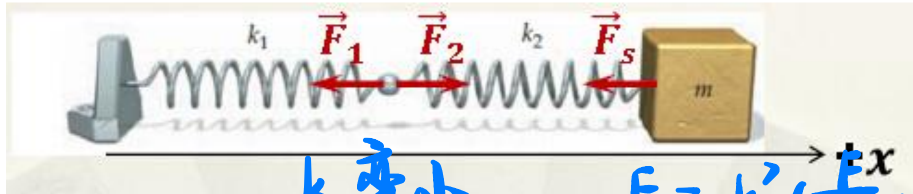
$$\frac{1}{2} I\Omega^2 = mgh_x + \frac{Mgh_x}{2}$$

$$h_x = (1-\cos\theta_m)L = 2\sin^2(\theta_m/2)L \approx \frac{\theta_m{}^2}{2} L$$

$h_x$ is height increasing of particle m.

**Thus, we can get the $\theta_m$=0.087rad=$5^0$**

# Effective Spring Constant

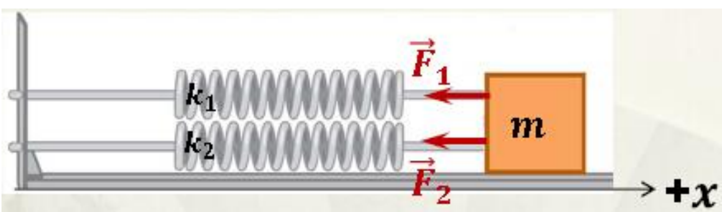


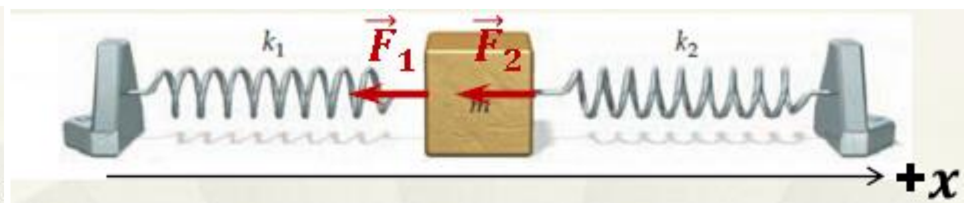

k 变小

$F = k'\left(\frac{F}{k_1} + \frac{F}{k_2}\right)$

$k' = \frac{k_1 k_2}{k_1 + k_2}$

$$\boldsymbol{F}_{\mathbf{net}} = -F_s = -k_2 x_2 = -\frac{k_1 k_2}{k_1 + k_2} x = -\boldsymbol{k}_{\boldsymbol{eff}} * \boldsymbol{x} \Rightarrow \frac{\mathbf{1}}{\boldsymbol{k}_{\boldsymbol{eff}}} = \frac{\mathbf{1}}{\boldsymbol{k}_{\mathbf{1}}} + \frac{\mathbf{1}}{\boldsymbol{k}_{\mathbf{2}}}$$




$$\boldsymbol{F}_{\mathbf{net}} = -F_1 - F_2 = -k_1 x_1 - k_2 x_2 = -(k_1 + k_2)x = -\boldsymbol{k}_{\boldsymbol{eff}} * \boldsymbol{x}$$

$$\Rightarrow \boldsymbol{k}_{\boldsymbol{eff}} = \boldsymbol{k}_{\mathbf{1}} + \boldsymbol{k}_{\mathbf{2}}$$

# Tutorial Problem 2

Figure shows block1 sliding to right over a frictionless elevated surface at a speed of $v$ = 8.0m/s. At t = 0, the block undergoes an elastic collision with stationary block2, which is attached to two parallel springs of spring constants $k_1$ = 400N/m, and $k_2$ = 600N/m with same relax-length. Assume that the springs do not affect the collision. After collision, block2 oscillates in SHM and block1 slides back.

k

(i) At some moment t, the position (measured from the relaxation position of springs), velocity, and acceleration of block2 are x = 0.1m, $v = -\sqrt{30}$m/s, and a = $-100$m/s$^2$, respectively. Find

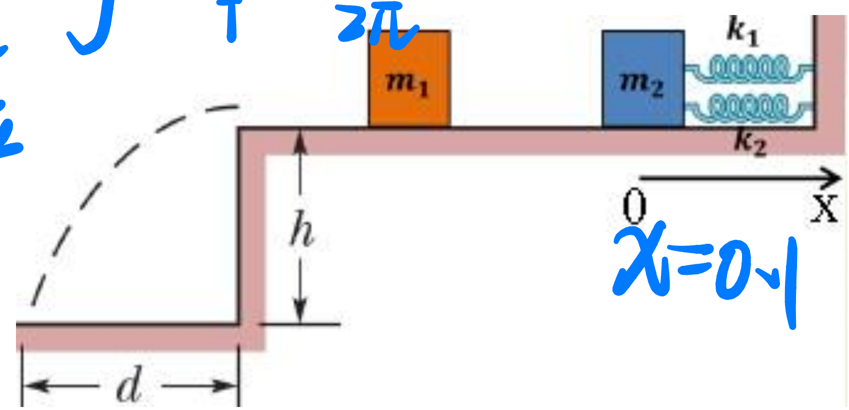
(a) the frequency of oscillation $\omega=\sqrt{\frac{k}{m_2}}$ $f=\frac{1}{T}=\frac{\omega}{2\pi}$

(b) ~~the mass $m_2$ of block2~~

(c) the amplitude $x_m$ of the oscillation 振幅

(d) the phase angle $\varphi$ of the oscillation.

从何时 t=0




$x=0.1$

For spring-block2: effective spring constant k

$$F_{net} = -k_1x - k_2x = -(k_1 + k_2)x = -kx$$

$so,\ k = k_1$+$k_2$=1000N/m

(a) x(t)=$x_m cos$(ω$t$+$\varphi$)..........(1)

v(t)=−ω$x_m sin$(ω$t$+$\varphi$).......(2)

a(t)=−$\omega^2 x_m cos$(ω$t$+$\varphi$)....(3)

$\frac{a}{x}$=$-\omega^2$ ⟹ ω=2π$f$=$\sqrt{-ax}$= 31.6 rad/s

$f$=$\frac{\omega}{2\pi}$=5.03Hz

(b) $\omega = \sqrt{\frac{k}{m_2}}$ $\therefore\ m_2 = \frac{k}{\omega^2} = 1.0kg$

(c) $(1)^2 + (2)^2$

$$\Rightarrow x_m = \sqrt{(\frac{v}{\omega})^2 + x^2} = \sqrt{(\frac{-\sqrt{30}}{\sqrt{1000}})^2 + 0.1^2} = 0.2m$$

(d) At t=0, block2 is at the equilibrium position, x=0

$$x(t) = x_m \cos(\omega t + \varphi)$$

$$t = 0, x = 0 \Rightarrow x(0) = x_m \cos(\omega 0 + \varphi) = x_m \cos\varphi = 0$$

$$\Rightarrow \varphi = \pm\frac{\pi}{2}$$

$\varphi = \frac{\pi}{2}$ $or$ $-\frac{\pi}{2}$ But as t increase, x is positive. so, $\varphi = -\frac{\pi}{2}$

(ii) After the collision, the block1 slides off the opposite end of the elevated surface, landing a distance d from the base of that surface after falling height h=4.9m.

Find: (e): the speed $v_1$ of block1 after the collision

(f): the mass $m_1$ of block1

(g): the value of $d$

(2) Conservation of momentum

逐字读

$$m_1v=-m_1v_1+m_2v_2 \quad ......(i)$$

Conservation of energy (kinetic energy)

完全弹性碰撞

$$\frac{1}{2}m_1v^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \ ......(ii)$$

From (i) and (ii), we have: $v_1 = v - v_2$

$$v_2(t=0)=-\omega x_m sin(\omega 0-\pi/2)= \omega\, x_m =6.32 m/s$$

(e) $v_1 = v - v_2 = 8 - 6.32 = 1.68$ m/s (f): from (i), we have $m_1 = \frac{v_2}{v+v_1} m_2 = \frac{6.32}{8+1.68} \times 1 = 0.65\text{kg}$

(g): block1 as projectile:

y direction: $h = \frac{1}{2} g t^2, \quad so, \quad t = \sqrt{\frac{2h}{g}}$

x direction: $d = v_1 t = v_1 \sqrt{\frac{2h}{g}} = 1.68\text{m}$