

Tutorial for Chapter 20

Reversible process	Heat engine
Irreversible process	Refrigerator
Reservoir	Carnot engine
Closed system	Carnot refrigerator
Entropy	Thermal efficiency
Multiplicity	
Isothermal process	
Isochoric process	
Isobaric process	
Adiabatic process	
Free expansion	

Chapter 20 Entropy

Entropy S :

Quantity to describe disorder in a system.

S is a state function:

Entropy of a closed system only depends on its state, just like $(p, V, T(E_{\text{int}}))$.

- **Path-independence:** Change of entropy of a *closed* system between two states does **NOT** depend on the path taken.

△ S 与过程无关，和初末状态有关
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- To find for an irreversible process between the initial (i) and the final (f) states, replace that process with any reversible process that connects the same two states.

$$\Delta S_{i \rightarrow f} = \int_i^f \frac{dQ_{\text{reversible}}}{T}$$

$$dE = Q - pdV$$
$$dQ = nC_V dT + pdV$$

Chapter 20 Calculate ΔS --Thermodynamic Processes

$$dQ = dE_{\text{int}} + dW = nC_V dT + pdV$$

$$\Rightarrow dS = \int_i^f \frac{dQ}{T} = \int_i^f \frac{nC_V dT + pdV}{T} \Rightarrow \Delta S = nC_V \ln \frac{T_f}{T_i} + nR \ln \frac{V_f}{V_i}$$

任意路径成立
但为准静态路径算
再 $PV=nRT$

Isothermal process:

$$T_i = T_f \Rightarrow \Delta S = nR \ln \frac{V_f}{V_i} = \frac{Q}{T}$$

Adiabatic process:

$$dQ \equiv 0 \Rightarrow \Delta S = 0$$

Isochoric process:

$$V_i = V_f \Rightarrow \Delta S = nC_V \ln \frac{T_f}{T_i} = nC_V \ln \frac{p_f}{p_i}$$

Isobaric process:

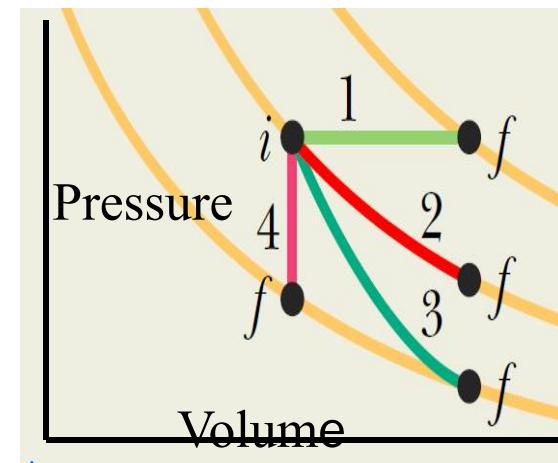
$$p_i = p_f \Rightarrow \Delta S = nR \ln \frac{V_f}{V_i} + nC_V \ln \frac{T_f}{T_i} = nC_p \ln \frac{T_f}{T_i}$$

Free expansion – irreversible process:

$$\Delta S = nR \ln \frac{V_f}{V_i} = nR \ln \frac{p_i}{p_f}$$

对应等温线始端2点

$Q=0$, but ΔS is not 0. Same to isothermal process



Chapter 20 Calculate ΔS --Heat absorption

Phase change at transition temperature T :

$$Q = Lm$$



$$\Delta S = \frac{Lm}{T}$$

对于 $Q = mc\Delta T$
不可逆过程
热接触不能控制

Same phase, change in temperature $\Delta T = T_f - T_i$:

但是 可逆
 $dQ = cmdT$

每次与一个极近热源接触

时刻准静态

$$\Delta S = \int \frac{dQ}{T} = cm \ln \left(\frac{T_f}{T_i} \right)$$

Phase change and temperature change:

$$\Delta S = \sum \Delta S_{\text{same-phase}} + \sum \Delta S_{\text{phase-change}}$$

Chapter 20 Entropy—2nd Law of Thermodynamics

$$\Delta S \geq 0$$

isolated 独立系统

- If a process **occurs in a ~~closed~~ system**, the entropy of the system increases for irreversible processes and remains constant for reversible processes. It never decreases.
- In the real world, due to friction, turbulence, and other dissipative factors, all processes are irreversible with entropy increase.
- * Reversible processes with $\Delta S = 0$ are idealizations.

Chapter 20 Tutorial Problem 1

设有n mol理想单原子气体与外界绝热，初始时系统在外界压强 p_0 下处于平衡态 (p_0, V_0) . 现将外界压强突然变成 $2p_0$ ，并维持恒定，系统达到新的平衡态 (p, V, T) , 请确定 V, T 以及该过程中系统的熵变。

$$\text{For the ideal gas: } p_0 V_0 = nRT_0 \quad (1)$$

$$2p_0 V = nRT \quad (2)$$

Constant external pressure and no heat exchange: 待考题了

$$nC_V \Delta T = \Delta E_{\text{int}} = Q - W_{\text{by}} = -W_{\text{by}} = W_{\text{on}} = 2p_0(V - V_0) \Rightarrow \frac{3}{2}nR(T - T_0) = 2p_0V - 2p_0V_0 \quad (3)$$

$$\text{Combine (1)&(2)&(3), we have: } V = \frac{7}{10}V_0, \quad T = \frac{7}{5}T_0 = \frac{7}{5} \frac{P_0 V_0}{nR}$$

Entropy is a state function and the change in entropy does not depend on the specific process, so for this irreversible process, the entropy change is:

$$\Delta S = nC_V \ln \frac{T_f}{T_i} + nR \ln \frac{V_f}{V_i} = n \frac{3}{2} R \ln \frac{7}{5} + nR \ln \frac{7}{10} = 0.148nR$$

Chapter 20 Tutorial Problem 2

An insulated Thermos contains **150 g** of water at **80.0°C**. You put in a **12.0 g** ice cube at **0°C** to form a system of **ice + original water**.

$$L_F = 333 \text{ kJ/kg}, \quad C_w = 4187 \text{ J/kg} \cdot K$$

$$Q = c_{\text{m,original}}(T_f - T_i) + L_m + c_{\text{m,ice}}(T_f - 0) = 0$$

- (a) What is the equilibrium T_f temperature of the system? **68.2°C**
- (b) What are the entropy changes of the water that was originally the ice cube as it melts and as it warms to the equilibrium temperature?
- (c) What is the entropy change of the original water as it cools to the equilibrium temperature?
- (d) What is the net entropy change of the ice + original water system as it reaches the equilibrium temperature?

Chapter 20 Tutorial Problem 2

Solution (a):

First we need judge the final equilibrium state for **ice + original water system**.

If the temperature of the water just decreases from 80°C to zero (no phase change), it will rejects heat

$$Q_w = m_w c \Delta T = 0.15 \text{ kg} \times 4187 \text{ J/kg.K} \times 80 \text{ K} = 50244 \text{ K}$$

If the ice all melts, it will absorb heat:

$$Q_I = m_I L_F = 0.012 \text{ kg} \times 333 \text{ KJ/kg} = 3.996 \text{ KJ} = 3996 \text{ J}$$

So all the ice will melt. For the ice and water system $\Sigma Q = 0$

$$m_I L_F + cm_I (T_F - 0^{\circ}\text{C}) + cm_w (T_F - 80^{\circ}\text{C}) = 0 \Rightarrow T_F = 68.2^{\circ}\text{C} = 341.35 \text{ K}$$

Chapter 20 Tutorial Problem 2

Solution (b):

The process of ice at 0°C turning to water at 0°C involves an entropy change:

$$\Delta S_1 = \frac{Q}{T} = \frac{L_F m_I}{273.15} = 14.6 \text{ J/K}$$

用开尔文温度

The process of $m = 0.012 \text{ kg}$ of water warming from 0°C to 68.2 °C involves an entropy change :

$$\Delta S_2 = \int_{273.15}^{341.35} \frac{cm_I dT}{T} = cm_I \ln\left(\frac{341.35}{273.15}\right) = 11.2 \text{ J/K}$$

Solution (c):

The cooling of the original water involves an entropy change:

$$\Delta S_3 = \int \frac{dQ}{T} = \int_{353.15}^{341.35} \frac{cm_w dT}{T} = cm_w \ln\left(\frac{341.35}{353.15}\right) = -21.4 \text{ J/K}$$

Chapter 20 Tutorial Problem 2

Solution (d):

The net entropy change in this experiment is found by summing the previous results:

$$\Delta S_{sum} = \Delta S_1 + \Delta S_2 + \Delta S_3 = 4.40 \text{ J/K} > 0$$

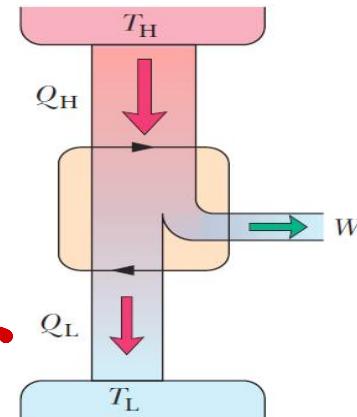
Chapter 20 Entropy--Heat Engines

- Efficiency of engine:

对外做功
净功就是面积

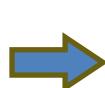
$$\varepsilon = \frac{\text{energy we get}}{\text{energy we pay for}} = \frac{|W|}{|Q_H|}$$

w=Q_{cyc}
从可吸的热 对于循环
Q=w



Ideal Carnot Engine:

$$\Delta S = \frac{Q_H}{T_H} + 0 + \frac{-|Q_L|}{T_L} + 0$$

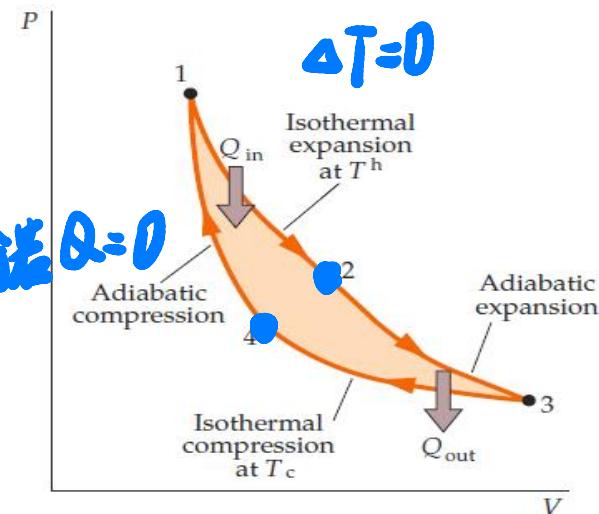


$$\frac{|Q_H|}{T_H} = \frac{|Q_L|}{T_L}$$

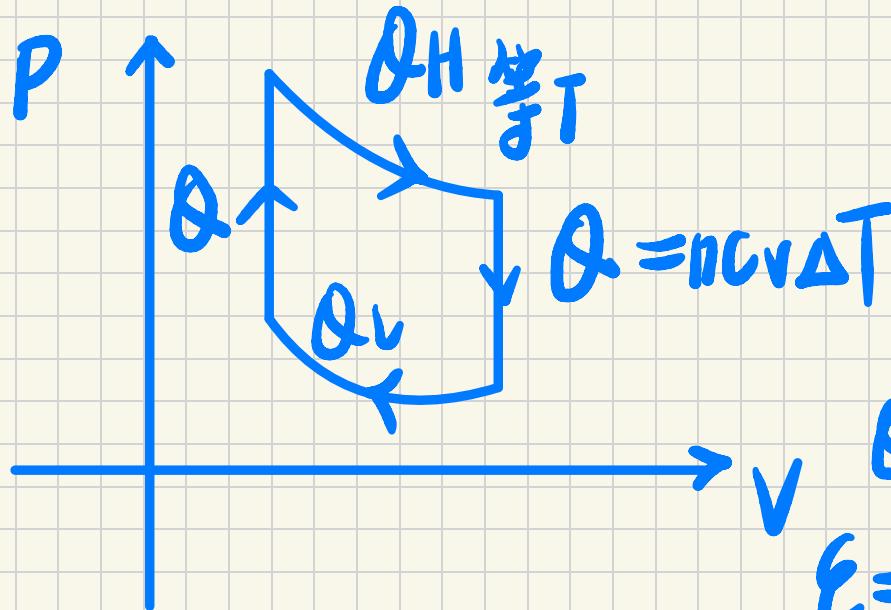
*只适用卡诺 $\theta=0$



$$\varepsilon = \frac{W}{Q_H} = \frac{Q_H - |Q_L|}{Q_H} = 1 - \frac{|Q_L|}{Q_H} = 1 - \frac{T_L}{T_H}$$



斯特林热机:



$$\epsilon = \frac{w}{q} = \frac{Q_H - |Q_L| + q}{Q_H + q}$$

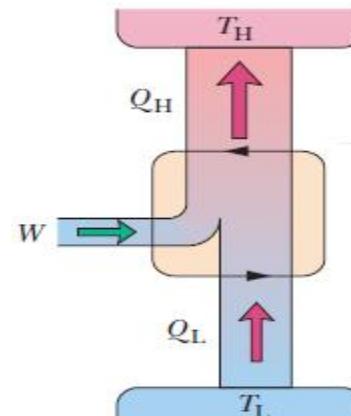
$w = Q_{net}$

pay → 吸热总和

Chapter 20 Entropy--Refrigerators

- Coefficient of performance of refrigerator:

$$K = \frac{\text{what we want}}{\text{what we pay for}} = \frac{|Q_L|}{|W|}$$



Ideal Carnot Refrigerator:

*卡诺制冷机

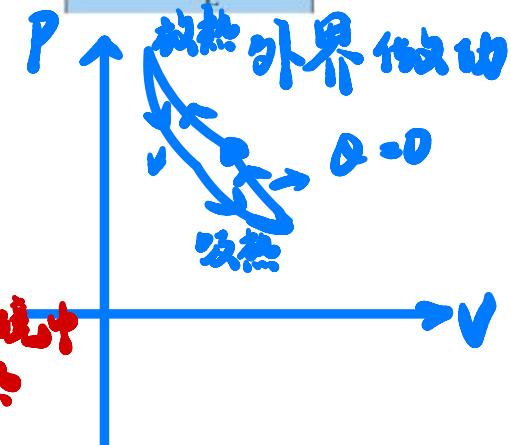
$$\Delta S = \frac{-|Q_H|}{T_H} + 0 + \frac{|Q_L|}{T_L} + 0$$

$$\frac{|Q_H|}{T_H} = \frac{|Q_L|}{T_L}$$

$$K = \frac{|Q_L|}{|Q_H| - |Q_L|}$$

$$K_C = \frac{T_L}{T_H - T_L}$$

热机吸收低温环境中的热



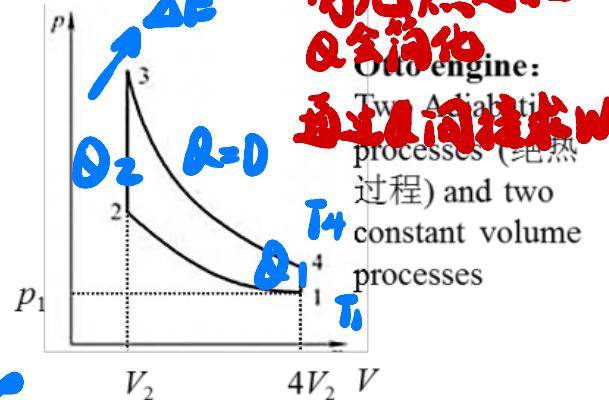
Chapter 20 Tutorial Problem 3

Figure shows a pV -diagram for an idealized Otto cycle taken by 1.0 mol of an ideal diatomic gas with rotation but no vibration $\underline{\underline{V_{14} = 4V_{23} = 4V_2}}$, $T_1 = 300 \text{ K}$ and $T_4 = 400 \text{ K}$.

- Find the temperatures T_3 and T_2 .
- Find the change in K_{tran} (the kinetic energy of the translational motion of the molecules) as it goes from state 2 to 3.
- Find the change in entropy as it goes from state 2 to 3.
- Find efficiency of this idealized heat engine ($n=W/Q$).

$$\eta = \frac{Q}{Q_{\text{pay}}} = \frac{Q_1 + Q_2}{Q_2}$$

Q_2 放出的热
 Q_{pay}



Chapter 20 Tutorial Problem 3

For the diatomic gas: $C_V = \frac{5R}{2}$ $C_p = \frac{7R}{2}$, $\gamma = 1.4$

a) Find the temperatures T_3 and T_2 . We apply the adiabatic process equation and ideal gas law to get the temperatures.

$$\left. \begin{array}{l} T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1} \\ T_3 V_3^{\gamma-1} = T_4 V_4^{\gamma-1} \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} T_2 = T_1 \left(\frac{V_1}{V_2} \right)^{\gamma-1} = (300K)(4)^{0.4} = 522.3K \\ T_3 = T_4 \left(\frac{V_4}{V_3} \right)^{\gamma-1} = (400K)(4)^{0.4} = 696.4K \end{array} \right.$$

b) Find the change in K_{tran} (the kinetic energy of the translational motion of the molecules) as it goes from state 2 to 3.

$$K_t = n \frac{3}{2} R (T_3 - T_2) = \frac{3R}{2} (696.4 - 522.3) = 2170.2J$$

Chapter 20 Tutorial Problem 3

c) Find the change in entropy as it goes from state 2 to 3. we start from the definition of the change in entropy:

$$\Delta S = \int_2^3 \frac{dQ}{T} = \int_2^3 \frac{nC_V dT}{T} = nC_V \ln \frac{T_3}{T_2} = nC_V \ln \frac{T_4}{T_1} = \frac{5R}{2} \ln \frac{4}{3} = 5.98 \text{ J/K}$$

d) Find efficiency of this idealized heat engine ($\eta=W/Q$).

$$\eta = \frac{W}{Q} = \frac{Q_{23} + Q_{41}}{Q_{23}} = 1 + \frac{Q_{41}}{Q_{23}} = 1 - \frac{Q_{14}}{Q_{23}} = 1 - \frac{nC_V(T_4 - T_1)}{nC_V(T_3 - T_2)} = 1 - \frac{1}{4^{0.4}} = 0.43$$