Key words of chapter 29

Law of Biot and Savart 毕奥萨法尔定律

Ampere’s Law 安培定理

Long Straight Wire 长直导线

Circular Arc 圆弧

Solenoids 直螺线管

Toroids 环形螺线管

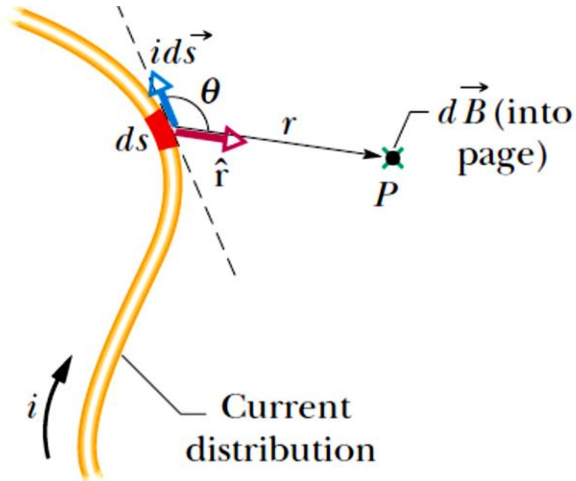
Crooked wire 弯曲导线

Amperian loop 安培回路

Biot-Savart law

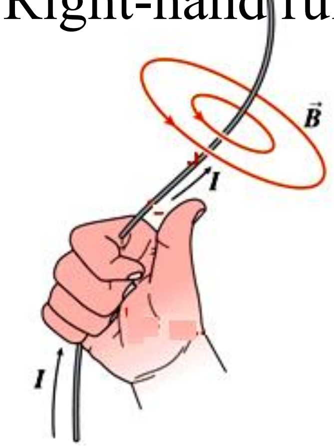
$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i\,d\vec{s}\times\hat{r}}{r^2}$$

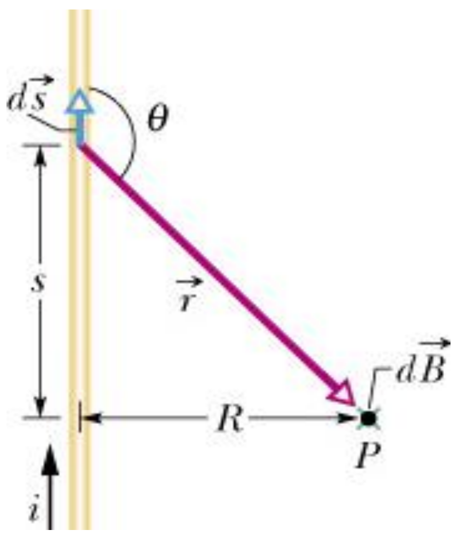
$$dB = \frac{\mu_0}{4\pi}\frac{ids\sin\theta}{r^2}$$



✓ 右手定则判导线 B

Right-hand rule

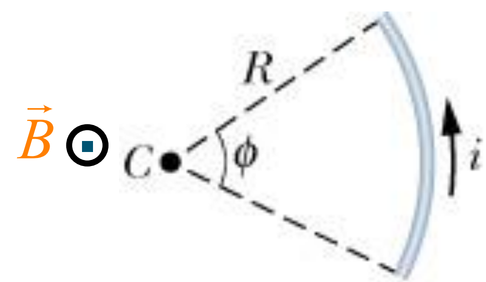




$$dB = \frac{\mu_0}{4\pi}\frac{ids\sin\theta}{r^2}$$

$$= \frac{\mu_0 i}{4\pi}\int_{-\infty}^{\infty}\frac{Rds}{\left(s^2+R^2\right)^{3/2}}$$

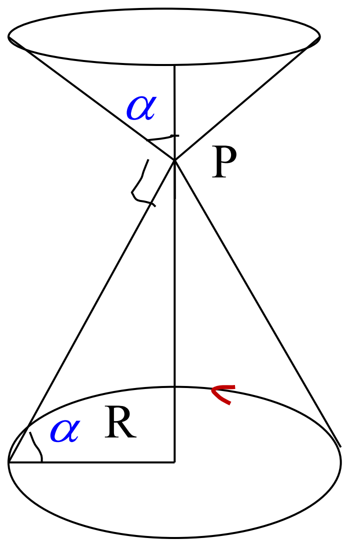
$$= \frac{\mu_0 i}{2\pi R}$$



$$dB = \frac{\mu_0}{4\pi}\frac{ids\sin 90°}{r^2} = \frac{\mu_0}{4\pi}\frac{iR}{R^2}d\phi$$

$$\Rightarrow B = \frac{\mu_0}{4\pi}\frac{i}{R}\int_0^{\phi}d\phi = \frac{\mu_0}{4\pi}\frac{i\phi}{R}$$

Biot-Savart law--circular Loop

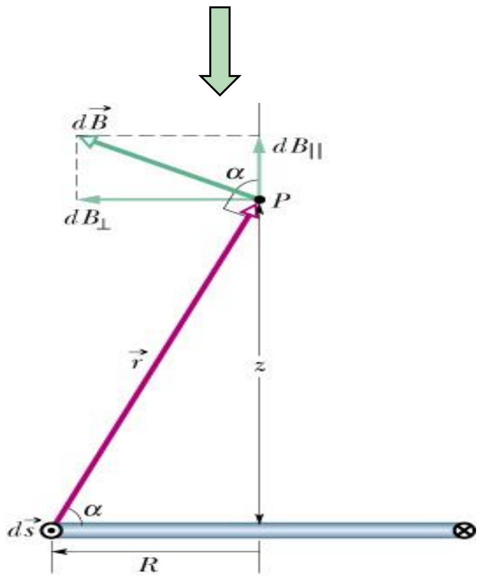


$$r=\sqrt{R^2+z^2} \qquad dB_{\parallel}=dB\cos\alpha=\frac{\mu_0 i}{4\pi}\frac{ds\cos\alpha}{r^2} \qquad \cos\alpha=\frac{R}{r}$$

$$\rightarrow dB_{\parallel}=\frac{\mu_0 iR}{4\pi}\frac{ds}{r^3}=\frac{\mu_0 iR}{4\pi}\frac{ds}{\left(R^2+z^2\right)^{3/2}} \qquad B=\int dB_{\parallel}$$

$$B=\frac{\mu_0 iR}{4\pi\left(R^2+z^2\right)^{3/2}}\int ds=\frac{\mu_0 iR}{4\pi\left(R^2+z^2\right)^{3/2}}(2\pi R)=\frac{\mu_0 iR^2}{2\left(R^2+z^2\right)^{3/2}}$$

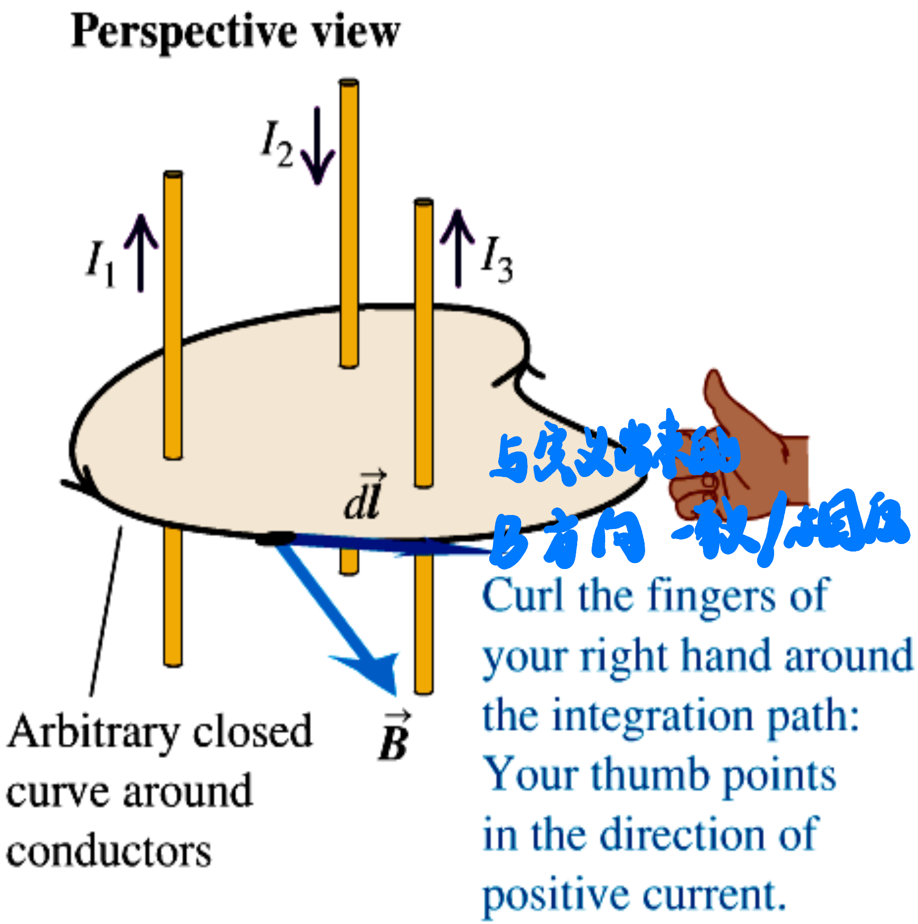
(像电偶极子电场)



$$\boxed{z>>R\Rightarrow B=\frac{\mu_0 iR^2}{2z^3}=\frac{\mu_0 i\pi R^2}{2\pi z^3}=\frac{\mu_0\mu}{2\pi z^3}}$$

$$\Rightarrow \boxed{\vec{B}=\frac{\mu_0\vec{\mu}}{2\pi z^3}}$$

Ampere's law



闭合路径　　　　闭合面积

$$\boxed{\oint \vec{B}\cdot d\vec{l} = \mu_0 i_{enc}} = \mu_0 \int_A \vec{J}\cdot d\vec{A}$$

(闭合电流环路/无限长直导线成立)

$$\boxed{\oint \vec{B}\cdot d\vec{l} = \mu_0 (I_1 - I_2 + I_3)}$$

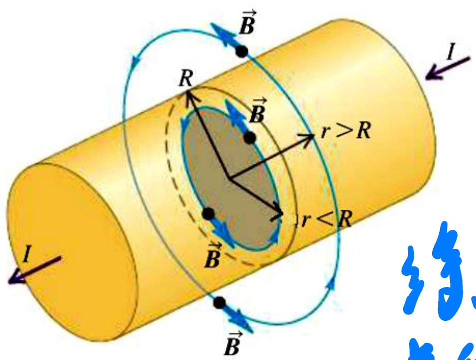
1) Currents are steady
2) No magnetic materials are present
3) No time-varying E-fields are present

Ampere's law

柱对称性 — 绕轴转不变

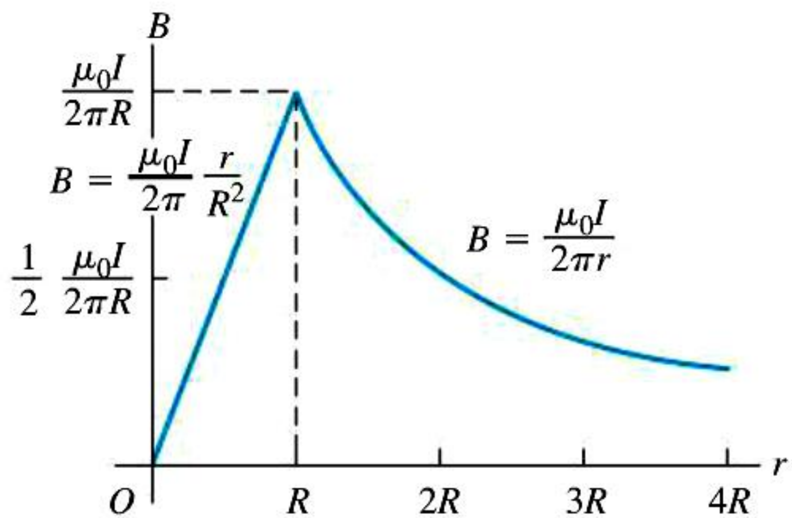
方向只与r有关

A long Current Cylinder



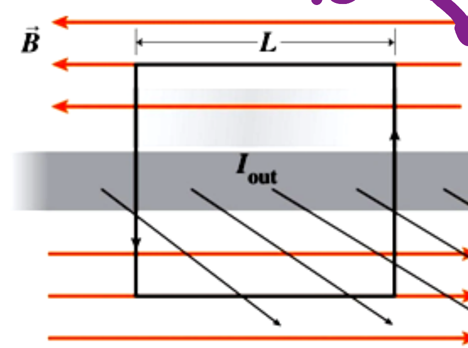
须知道磁场长什么样

$$B(2\pi r) = \mu_0 I_{enc}$$



A Large Current Sheet with uniform current density J_S

先知道B场形状



竖直方向上抵消

$\oint \vec{B}\cdot d\vec{l} = 2BL$
$= \mu_0 JL$
$B = \frac{1}{2}\mu_0 J$

B is // to sheet & $\perp I_{out}$

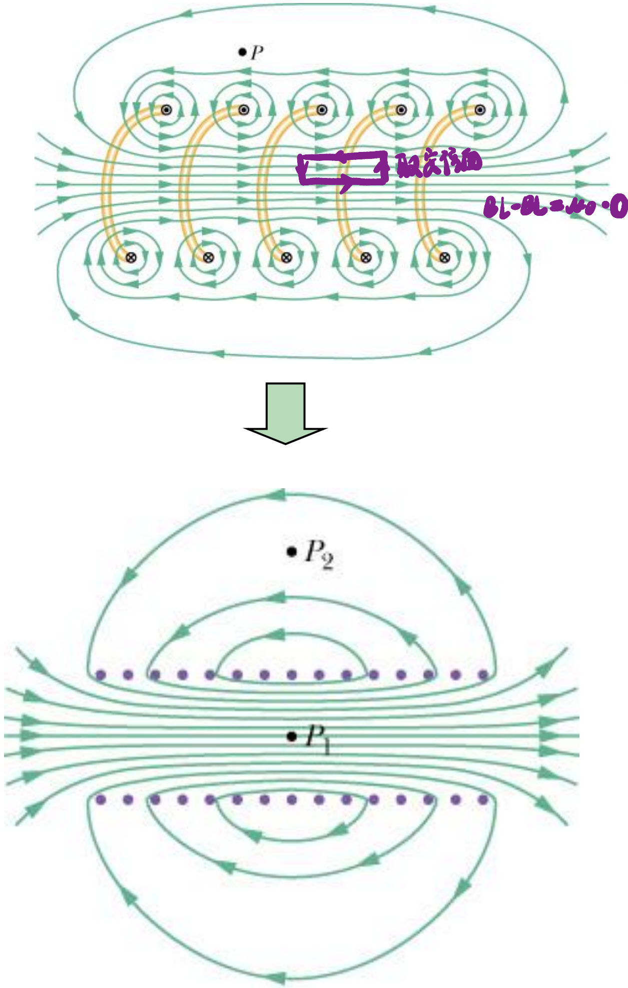
$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$$

$$B\,L + 0 + B\,L + 0$$

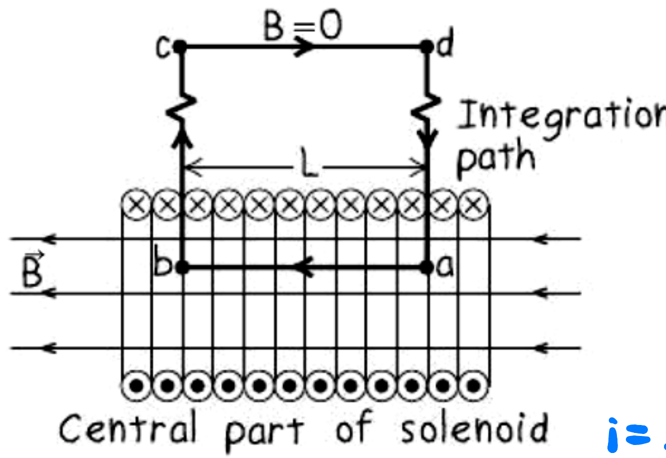
$$= \mu_0 I_{enc} = \mu_0 J_S\, L$$

$$\boxed{\Rightarrow B = \frac{1}{2}\mu_0 J_S}$$

Coil length is much longer than the coil diameter



B-field is uniform inside the solenoid and zero outside



$i=\frac{dq}{dt}$

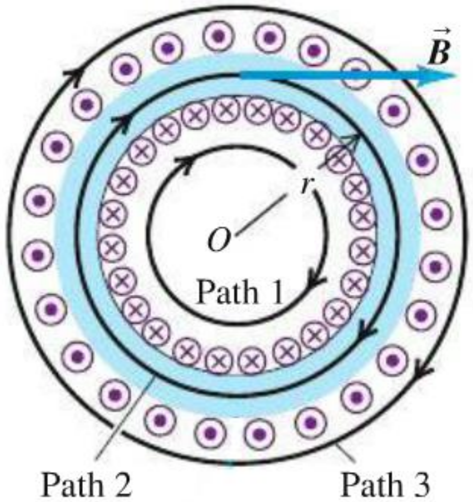
给到类似螺线：可以带电圆柱体旋转

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$$

单位长度匝数

$$BL = \mu_0 n L I$$

Cylindrical symmetry:



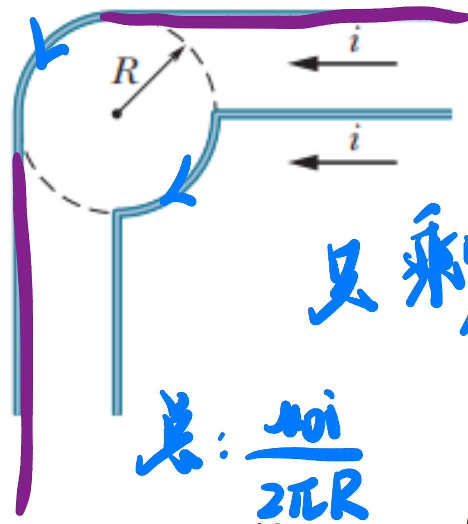
Path 1 : no current enclosed, $\boldsymbol{B = 0}$

Path 3 : net current enclosed is zero, $\boldsymbol{B = 0}$

Path 2 : $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$

$$2\pi r B = \mu_0 N I$$

$$\boxed{B = \frac{\mu_0 N I}{2\pi r} \quad \text{(toroidal solenoid)}}$$



其他抵消了

只剩2个直导线的贡献

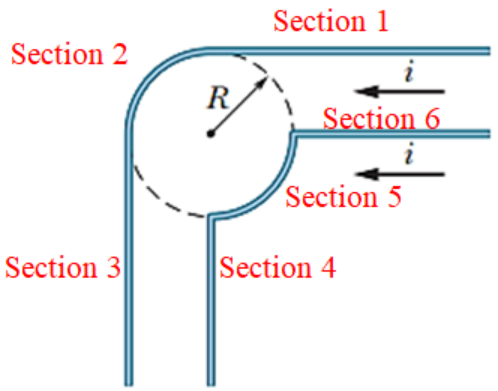
半无限长

总: $\frac{\mu_0 i}{2\pi R}$

1个半无限长贡献 $\frac{\mu_0 i}{2\pi R} \times \frac{1}{2}$

In Figure, two infinitely long wires carry equal currents i. Each follows a 90° arc on the circumference of the same circle of radius R. Find the magnetic field at the center of the circle.

Solution 1 of Ch29



We refer to the center of the circle as C. we see that:

the current in the straight segments that are collinear with C (the section 4 and section 6) do not contribute to the field there.

The right-hand rule indicates that the currents in the two arcs (section 2 and section 5) contribute to the field at C:

$$B_4 + B_6 = \frac{\mu_0 i(\pi/2)}{4\pi R} - \frac{\mu_0 i(\pi/2)}{4\pi R} = 0$$

Thus, the nonzero contributions come from those straight segments that are not collinear with C (section 1 and section 3). Both contribute fields pointing out of the page according to the right hand rule. For a "semi-infinite" current carrying wire, from the symmetry, we know the magnetic field is exactly half of B field due to the infinite long one. We can apply Ampere's law to get the B field created by infinite long current carrying wire, that is:

$$B \cdot 2\pi r = \mu_0 i \Rightarrow B = \frac{\mu_0 i}{2\pi r} \Rightarrow B_{semi} = \frac{1}{2}\frac{\mu_0 i}{2\pi r} = \frac{\mu_0 i}{4\pi r}$$

So we get the net field is: $B_{\text{net}} = B_1 + B_2 + B_3 + B_4 + B_5 + B_6 = 2 \times \frac{\mu_0 i}{4\pi R} = \frac{\mu_0 i}{2\pi R}$

direction: out of page

Problem 2 of Ch29

The current density J inside a long, solid, cylindrical wire of radius a = 3.1mm is in the direction of the central axis, and its magnitude varies linearly with radial distance r from the axis according to $J = J_0 r/a$, where $J_0 = 310\ \mathrm{A/m^2}$. Find the magnitude of the magnetic field at following position:

(a) $r = 0$

(b) $r = a/2$

(c) $r = a$

(d) $r = 4a$

$$\oint Bl = \mu_0 \int_0^r J(r)\, 2\pi r' dr'$$

$$B 2\pi r = \mu_0 \int_0^r \frac{J_0 r'}{a} 2\pi r' dr'$$

Problem 2 of Ch29

Cylindrical symmetry, Applying Ampere's law inside the cylinder:

$$\oint \vec{B}_r \cdot d\vec{l} = \mu_0 I_{enc}$$

$$B_r \cdot 2\pi r = \mu_0 i_r = \mu_0 \int_A J dS = \mu_0 \int_0^r \frac{J_0 r'}{a} \underbrace{2\pi r' dr'}_{dA} \Rightarrow B_r = \frac{\mu_0 J_0 r^2}{3a}$$

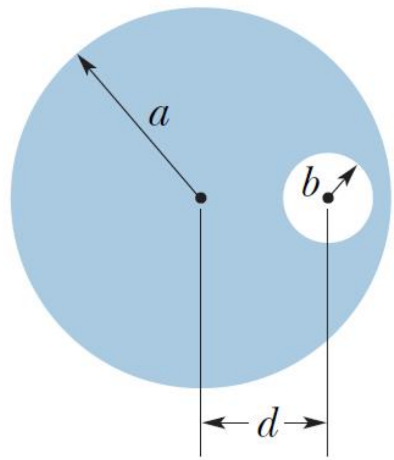
$$r = 0, \qquad B = 0$$

$$r = a/2, \qquad B(r)\Big|_{r=a/2} = \frac{\mu_0 J_0 (a/2)^2}{3a} = 1.0\times 10^{-7}\ \mathrm{T}$$

$$r = a, \qquad B(r)\Big|_{r=a} = \frac{\mu_0 J_0 (a)^2}{3a} = 4.0\times 10^{-7}\ \mathrm{T}$$

$$r = 4a, \qquad B = \frac{\mu_0 I_{enc}}{2\pi r} = \frac{\mu_0 (2\pi J_0 a^2)}{3(2\pi r)} = \frac{\mu_0 (J_0 a^3)}{3(4a)} = 1.0\times 10^{-7}\ T$$

Problem 3 of Ch29



补上对称性

A cross section of a long cylindrical conductor of radius a = 4.00cm containing a long cylindrical hole of radius b = 1.50 cm. The central axes of the cylinder and hole are parallel and are distance d = 2.00 cm apart; current i = 5.25 A is uniformly distributed over the tinted area.

(a) Find the magnitude of the magnetic field at the center of the hole

(b) Find the magnetic field at any point in the hole.

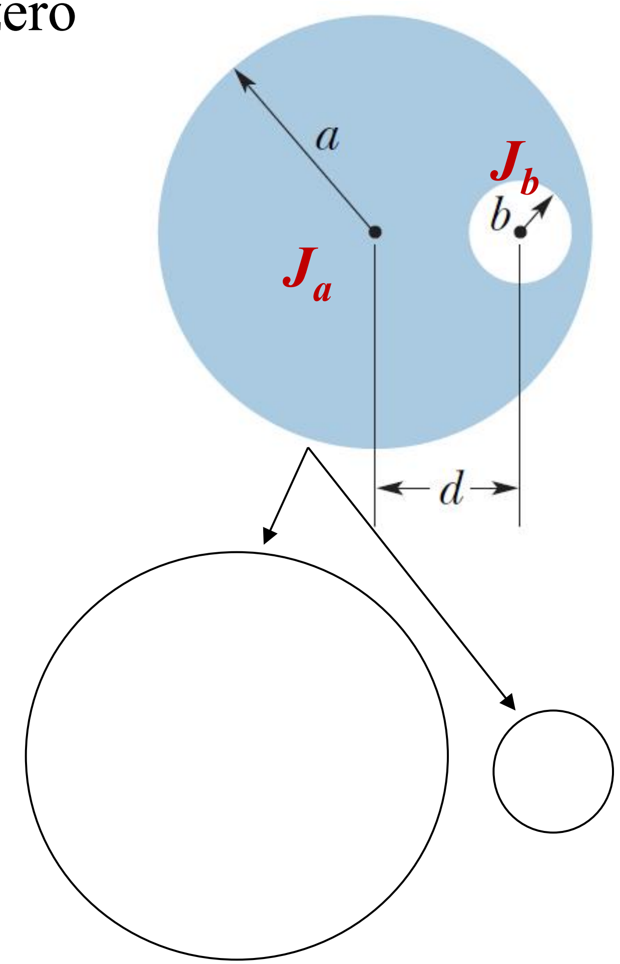
Problem 3 of Ch29

The total current in the region of the hole is zero

$$J_a = \frac{i}{S} = \frac{i}{\pi(a^2 - b^2)} = -J_b = J$$

$$I_a = J_a S_{r_a} = \frac{i r_a^{\,2}}{(a^2 - b^2)}, \quad r_a < a$$

$$I_b = J_b S_{r_b} = -\frac{i r_b^{\,2}}{(a^2 - b^2)}, \quad r_b < b$$

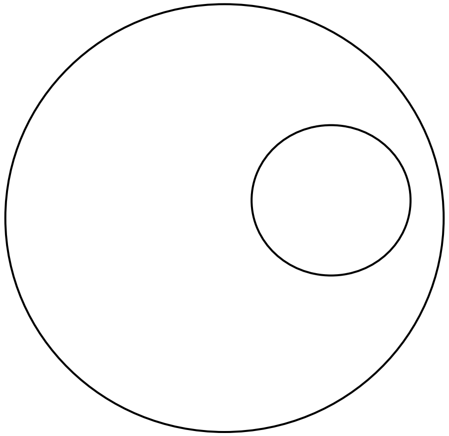


Problem 3 of Ch29

The magnetic field at a point within the hole is the sum of the B fields due to this two current distributions:

$$\vec{B} = \vec{B}_1 + \vec{B}_2 \qquad B_1 = \frac{\mu_0 i r_1}{2\pi(a^2 - b^2)}$$

$$B_2 = \frac{\mu_0 i r_2}{2\pi(a^2 - b^2)}$$



a) the magnitude of the B field at the center of the hole

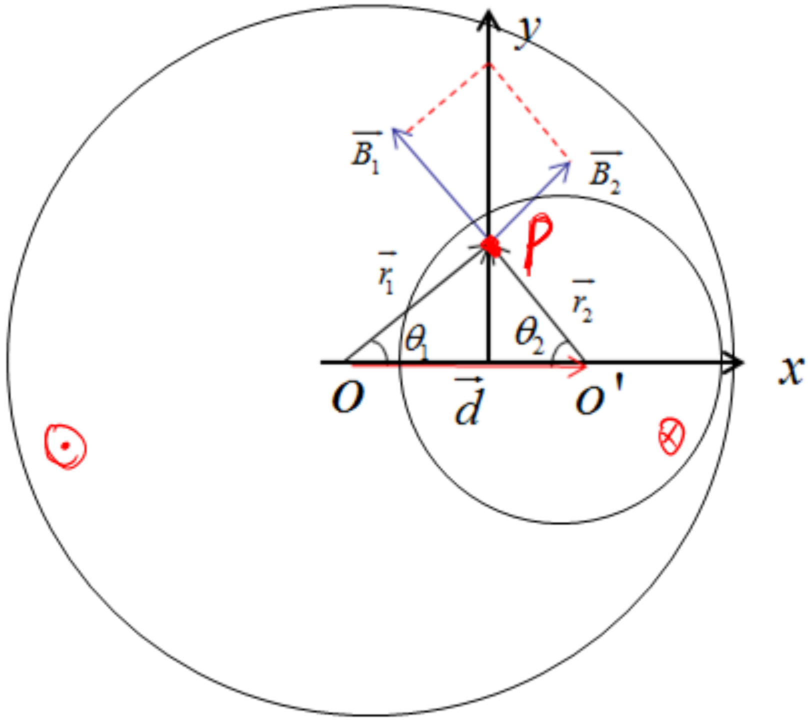
$$r_b = 0 \Rightarrow \vec{B}_2 = 0$$

$$\Rightarrow B = \frac{\mu_0 i d}{2\pi(a^2 - b^2)} = \frac{(4\pi\times 10^{-7}\,\mathrm{T\cdot m/A})\big(5.25\,\mathrm{A}\big)(0.0200\,\mathrm{m})}{2\pi[(0.0400\,\mathrm{m})^2 - (0.0150\,\mathrm{m})^2]} = 1.53\times 10^{-5}\,\mathrm{T}$$

Problem 3 of Ch29

c) Any point in the hole

$$\vec{B} = \vec{B}_1 + \vec{B}_2$$

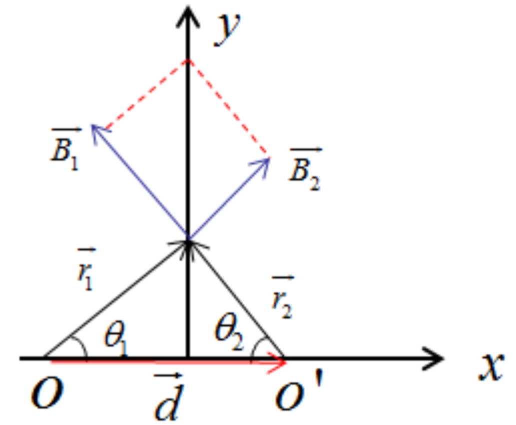


Problem 3 of Ch29

c) Any point in the hole

$$B_1 = B_{r_a} = \frac{\mu_0 i r_a}{2\pi(a^2-b^2)} = \frac{\mu_0 i r_1}{2\pi(a^2-b^2)} = \mu_0 J r_1 / 2$$

$$B_2 = B_{r_b} = \frac{\mu_0 i r_b}{2\pi(a^2-b^2)} = \frac{\mu_0 i r_2}{2\pi(a^2-b^2)} = \mu_0 J r_2 / 2$$



$$B_x = B_2 \sin\theta_2 - B_1 \sin\theta_1$$
$$= \frac{1}{2}\mu_0 J(r_2 \sin\theta_2 - r_1 \sin\theta_1) = 0$$

$$B_y = B_1 \cos\theta_1 - B_2 \cos\theta_2$$
$$= \frac{1}{2}\mu_0 J(r_1 \cos\theta_1 - r_2 \cos\theta_2) = \frac{1}{2}\mu_0 J d$$

$$\vec{B}_1 = \frac{\mu_0}{2}\vec{J}\times\vec{r}_1$$

$$\vec{B}_2 = \frac{\mu_0}{2}(-\vec{J})\times\vec{r}_2$$

$$\vec{B}_1 + \vec{B}_2 = \frac{\mu_0}{2}\vec{J}\times(\vec{r}_1 - \vec{r}_2) = \frac{\mu_0}{2}(\vec{J}\times\vec{d})$$

Conclusion: for the hollow cylinder, the B-field within the hollow is a constant