Key words

Diffraction

Illuminate

Overlap

Coincide

Resolvability

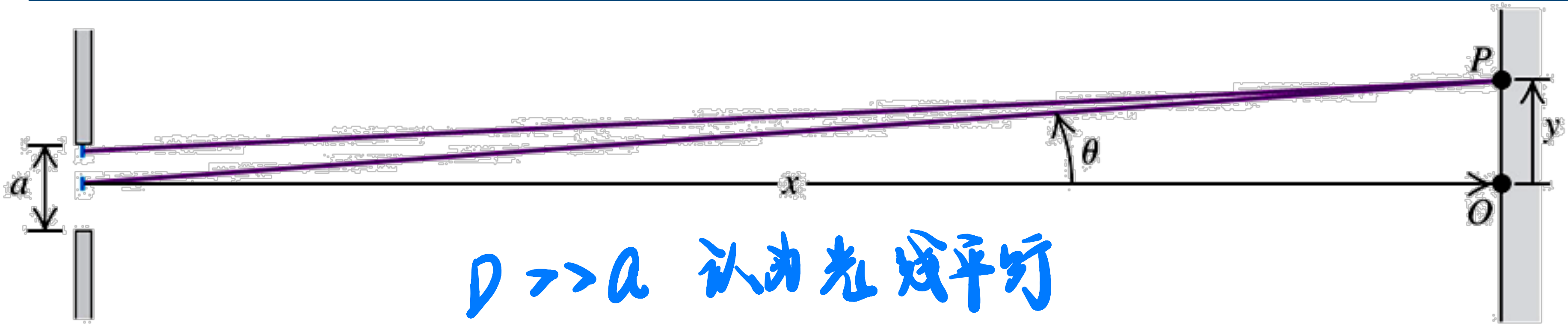
Circular Aperture

Angular separation

Rayleith's Criterion

Envelope

Diffraction (衍射)



$D >> a$ 认为光线平行

A dark fringe occurs whenever :

$$\boxed{\sin\theta = \frac{m\lambda}{a} \qquad (m = \pm 1, \pm 2, \pm 3, \dots)}$$

$\sin\theta < 0.2$

Since $\sin\theta \approx \theta$ when θ is small (θ in radians)

才能小角近似

$$\boxed{\theta = \frac{m\lambda}{a} \qquad (m = \pm 1, \pm 2, \pm 3, \dots) \quad \text{(for small angles } \theta)}$$

$a < \lambda$ 取不到 first min => 全是亮

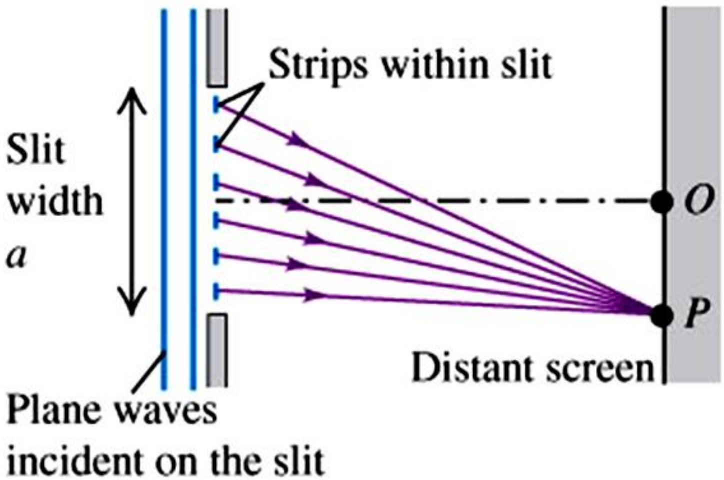
What if $\lambda = a$?

Bright fringe cover the whole screen

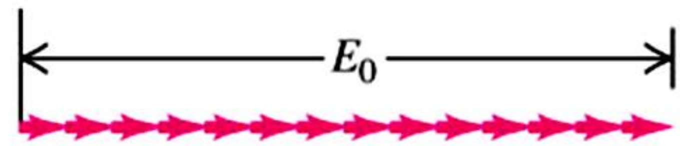
What if $a >> \lambda$?

θ goes to zero, all the fringes are overlapped, so only a point presented on the screen. That what we have seen in daily life.

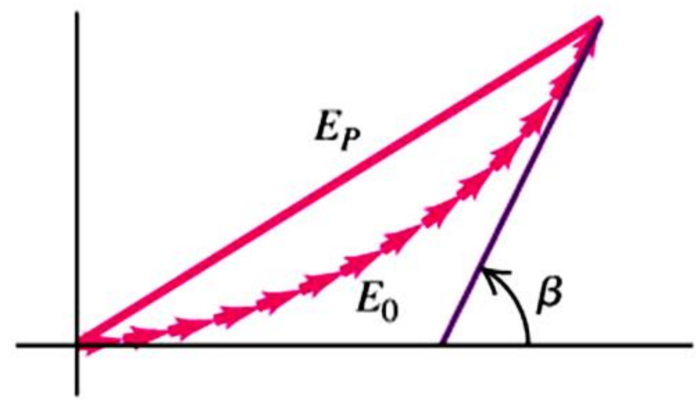
Intensity of the single-slit diffraction pattern



At the center of the diffraction pattern (point O), the phasors are in phase.

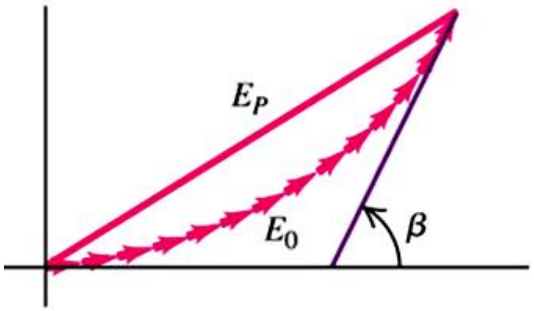


Phasor diagram at a point slightly off the center of the pattern; β = total phase difference between the first and last phasors.



We have to consider the E-field of each point source

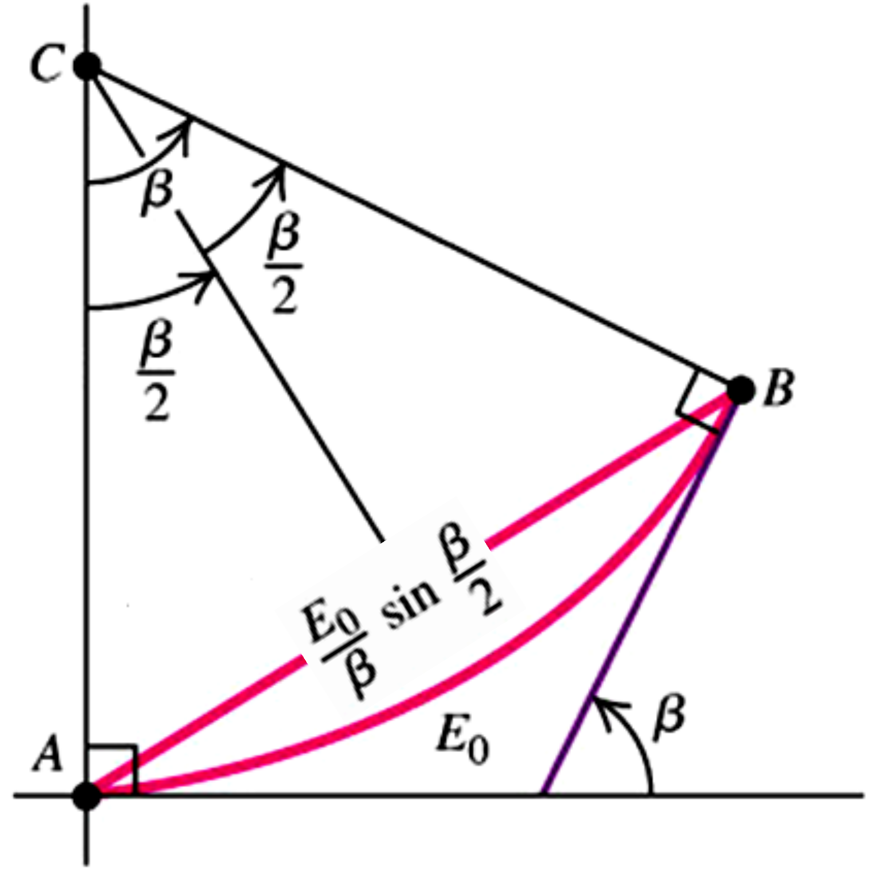
Intensity of the single-slit diffraction pattern



$$\left.\begin{aligned} E_P &= 2R\sin(\frac{\beta}{2}) \\ E_0 &= R\beta = 2R\frac{\beta}{2} \end{aligned}\right\} \Rightarrow \frac{E_P}{E_0} = \frac{\sin(\frac{\beta}{2})}{\frac{\beta}{2}}$$

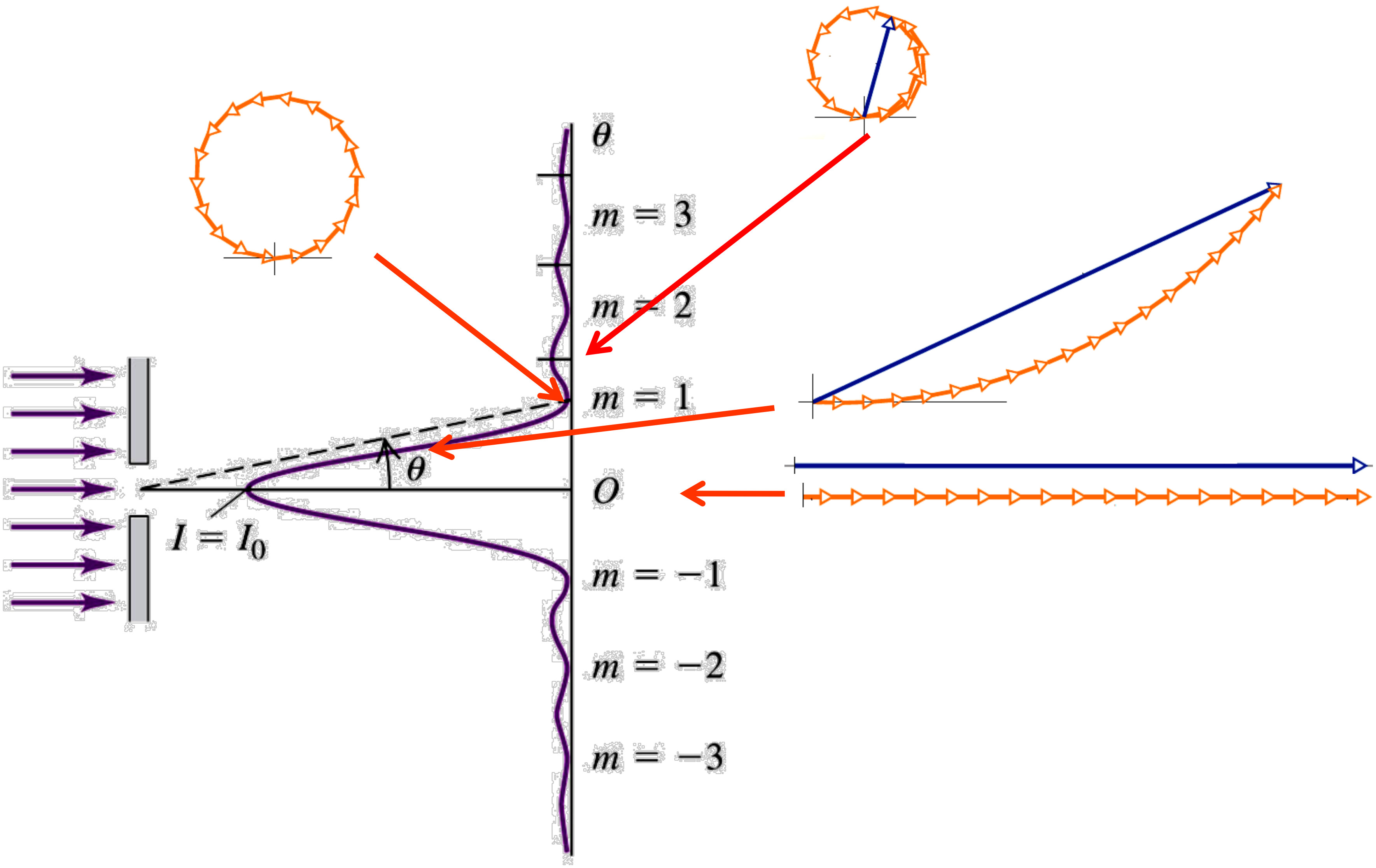
$$\frac{I}{I_0} = (\frac{E_P}{E_0})^2 = (\frac{\sin(\beta/2)}{\beta/2})^2 \Rightarrow I = I_0(\frac{\sin(\beta/2)}{\beta/2})^2$$

$$\alpha = \frac{\beta}{2} = \frac{\pi a \sin\theta}{\lambda}$$



***β* : phase difference in radians between the oscillations at points P resulting from the first and last point source at the slit**

Intensity of the single-slit diffraction pattern



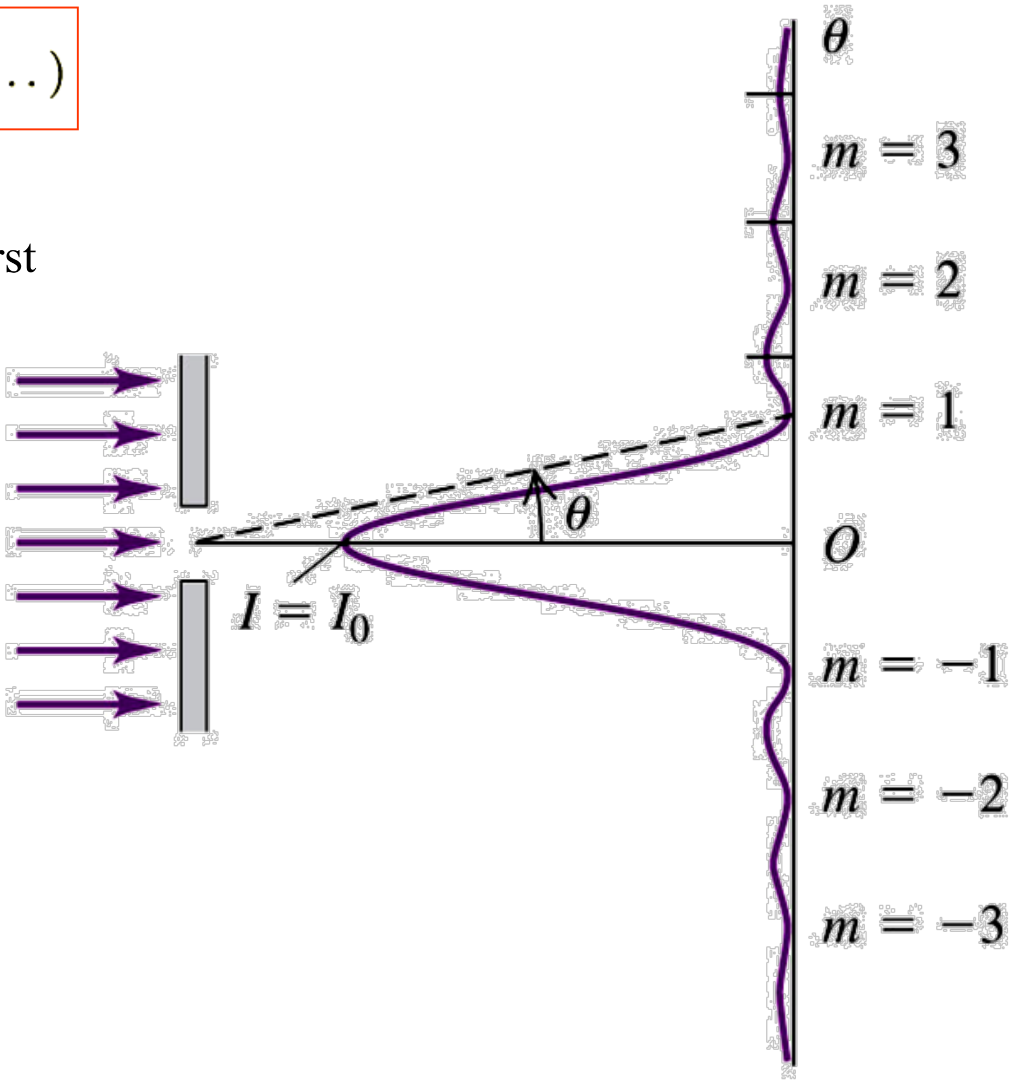
Intensity of the single-slit diffraction pattern

Angular position $\boldsymbol{\theta}$ of the dark fringes :

$$\boxed{\sin\theta = \frac{m\lambda}{a} \qquad (m = \pm 1, \pm 2, \pm 3, \ldots)}$$

Intensity at any point on the screen in terms of phase difference β for the first and last point source at the slit :

$$\boxed{I = I_0 \left[\frac{\sin(\beta/2)}{\beta/2}\right]^2}$$



To relate the two, we have

$$\beta = \frac{2\pi}{\lambda} a \sin\theta$$

Question 1

In the single-slit diffraction experiment of Fig. 36-4, let the wavelength of the light be 500nm, the slit width be $6.00\mu m$, and the viewing screen be at distance D=4.00m. Let a y axis extend upward along the viewing screen, with its origin at the center of the diffraction pattern. Also let I_P represent the intensity of the diffracted light at point P at y=15.0cm.

d a

$\tan\theta = \frac{y}{D}$ → 可近似

$\beta = a\sin\theta \frac{2\pi}{\lambda}$

求 second min 就不能小角近似

$= I_0$

$\frac{I_P}{I_m} = \left(\frac{\sin\frac{\beta}{2}}{\frac{\beta}{2}}\right)^2$

(a) What is the ratio of I_P to the intensity I_m at the center of the pattern?

(b) Determine where point P is in the diffraction pattern (by giving the maximum and minimum) between which it lies, or the two minima between which it lies.

$a\sin\theta = m\lambda$

单缝一般不找极大
除了中央极大

Question 1

(a) Given $y/D = 15/400$, then $\theta = \tan^{-1}(y/D) = 2.15°$. Use of Eq. 36-6 (with a = 6000 nm and λ = 500 nm) leads to

$$\alpha = \frac{\pi a \sin\theta}{\lambda} = \frac{\pi(6000\,\text{nm})\sin 2.15°}{500\,\text{nm}} = 1.414\,\text{rad}.$$

Thus,

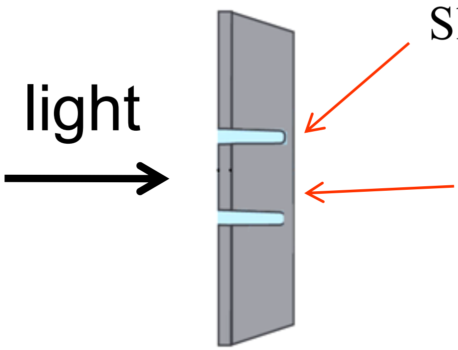
$$\frac{I_p}{I_m} = \left(\frac{\sin\alpha}{\alpha}\right)^2 = 0.487\,.$$

(b) Consider Eq. 36-3 with "continuously variable" m (of course, m should be an integer for diffraction minima, but for the moment we will solve for it as if it could be any real number):

$$m = \frac{a\sin\theta}{\lambda} = \frac{(6000\text{ nm})\sin 2.15°}{500\text{ nm}} \approx 0.45$$

which suggests that the angle takes us to a point between the central maximum ($\theta_{centr} = 0$) and the first minimum (which corresponds to m = 1 in Eq. 36-3).

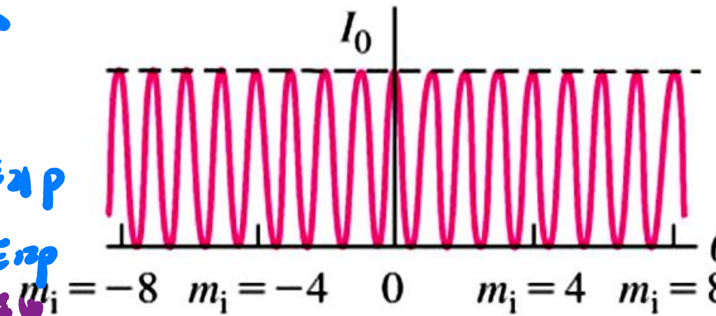
A realistic case of two-slit interference pattern



认为两个缝处 θ 仍一样

$E_p = E_{1p} + E_{2p} \cdots + E_{np} + E'_{1p} + E'_{2p} \cdots + E'_{np}$

$\phi = d\sin\theta$ 要 仅由 d 引起



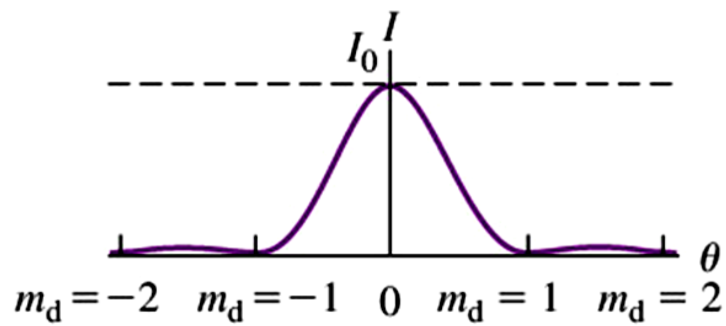
Assume very narrow slit, we have interference pattern :

$d \sin\theta = m\lambda$ (m = **0,** ± 1, ± 2, ….) Bright fringes

$\sin\theta = m\lambda / 4a$

In real, the slit width a will have a diffraction pattern :

$$\sin\theta = \frac{m\lambda}{a} \qquad (m = \pm 1, \pm 2, \pm 3, \ldots) \quad \text{Dark fringes}$$



For intensity, we combine the two :

中心强度

$$I = I_0 \cos^2\frac{\phi}{2}\left[\frac{\sin(\beta/2)}{\beta/2}\right]^2 \quad \text{(two slits of finite width)}$$

(两个缝干涉)

$$\phi = \frac{2\pi d}{\lambda}\sin\theta$$

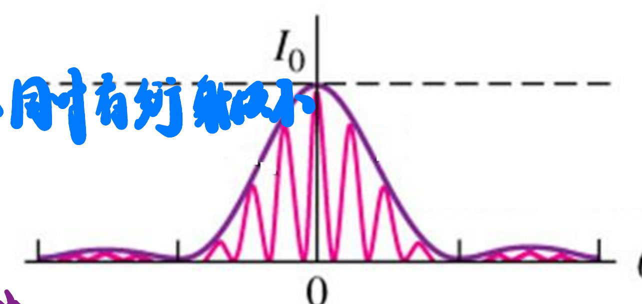
$$\beta = \frac{2\pi a}{\lambda}\sin\theta$$

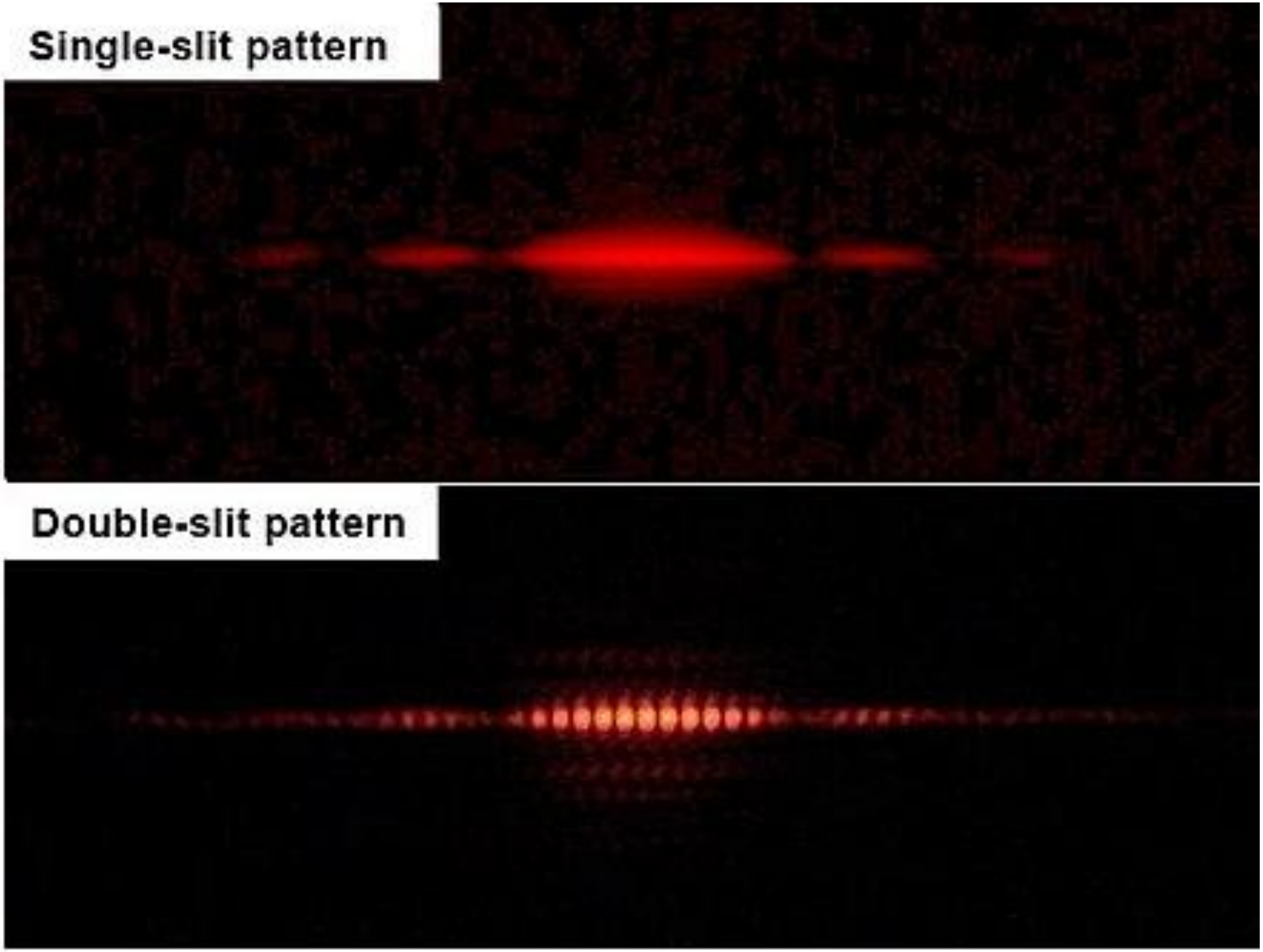
missing order：干涉极大同时有衍射极小

$a\sin\theta = m\lambda$

$d\sin\theta = n\lambda$

$\Rightarrow \frac{d}{a} = \frac{n}{m}$ 为整数比




Single-slit pattern
Double-slit pattern

Question 2

考虑：无穷远不算，衍射取整

λ

Light of wavelength 600 nm is incident normally on a double slits. Two adjacent maxima occur at angles given by $\sin\theta = 0.2$ and $\sin\theta = 0.3$. The fourth-order maxima are missing.

d

$d\sin\theta = m\lambda \Rightarrow d = \frac{1\cdot\lambda}{0.3-0.2} = 6\,\mu m$

(a) What is the separation between adjacent slits?

$d\sin\theta = 4\lambda$ 取 $n=1$ $\quad a$

(b) What is the smallest slit width this slits can have?

$\Rightarrow a\sin\theta = m\lambda \quad a = 1.5\,\mu m$

For that slit width, what are the (c) largest, (d) second largest, and (e) third largest values of the order number m of the maxima produced by the slits ?

$\sin\theta \le 1$

最大找干涉处m取整

$m = \frac{d\sin\theta}{\lambda} = \left[\frac{d}{\lambda}\right] = 10$

但在无穷远处无意义看不到

$m_{max} = 9$

$m_2 = 8\ 7$

8看不到

$m_3 = 6$

考虑正负

(f) Let the viewing screen be at distance D = 50 cm, point P lies at distance y = 10 cm (15) from the center of the pattern, What is the ratio of the intensity I_P at point P to the intensity I_{cen} at the center of the pattern?

Question 2 (Solution)

(a) Let m be the order number for the line with $\sin\theta = 0.2$ and $m + 1$ be the order number for the line with $\sin\theta = 0.3$.

Then, $0.2d = m\lambda$

and $0.3d = (m+1)\lambda$.

$$d = \frac{\lambda}{0.1} = 6\mu m.$$

(b) If a is the smallest slit width for which this order is missing, the angle must be given by $asin\theta = \lambda$.

It is also given by $dsin\theta = 4\lambda$, so

$$a = {}^{d}/_{4} = \frac{6.0\times10^{-6}m}{4} = 1.5\times10^{-6} \text{ m}$$

Question 2 (Solution)

(c) First, we set θ =90° and find the largest value of m for which

$$m\lambda < dsin\theta$$

$$m < \frac{dsin90^0}{\lambda} = 10$$

The highest order seen is the m = 9 order. The fourth and eighth orders are missing, so the observable orders are m = 0, 1, 2, 3, 5, 6, 7, and 9. Thus, the largest value of the order number is m = 9.

(d) Using the result obtained in (c), the second largest value of the order number is m = 7.

(e) Similarly, the third largest value of the order number is m = 6.

Question 2 (Solution)

(f) Let the viewing screen be at distance D = 50 cm, point P lies at distance y = 15 cm from the center of the pattern, What is the ratio of the intensity I_P at point P to the intensity I_{cen} at the center of the pattern?

$$I = I_0 \cos^2\frac{\phi}{2}\left[\frac{\sin(\beta/2)}{\beta/2}\right]^2$$

不可近似

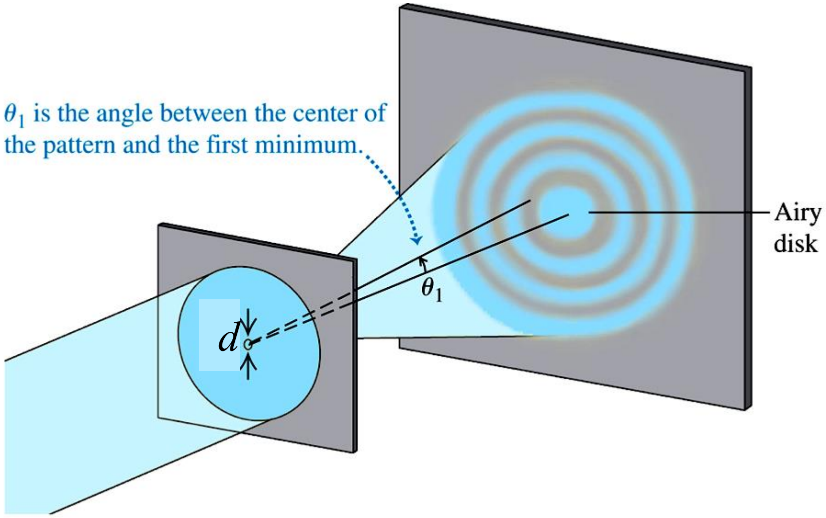
Here, $\tan\theta = \frac{y}{D} = 0.3 \Rightarrow \sin\theta = 0.287$

$$\frac{\phi}{2} = \frac{\pi d \sin\theta}{\lambda} = \pi((0.287)(10) = 2.87\pi$$

$$\frac{\beta}{2} = \frac{\pi a \sin\theta}{\lambda} = \pi((0.242)(\frac{10}{4}) = 0.718\pi$$

$$\Rightarrow \frac{I_P}{I_{cen}} = \cos^2\frac{\phi}{2}[\frac{\sin(\beta/2)}{\beta/2}]^2 = \cos^2(2.87\pi)[\frac{\sin(0.718\pi)}{0.718\pi}]^2 = 0.099 = 0.99\%$$

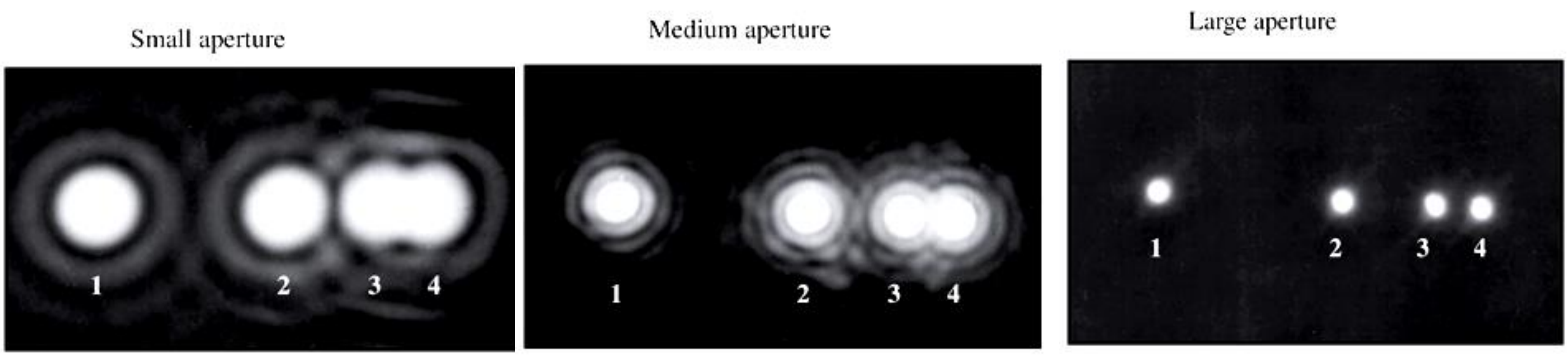
Diffraction by a circular aperture and resolvability



$$\sin\theta = \frac{\lambda}{a} \quad \text{(first minimum—single slit).}$$

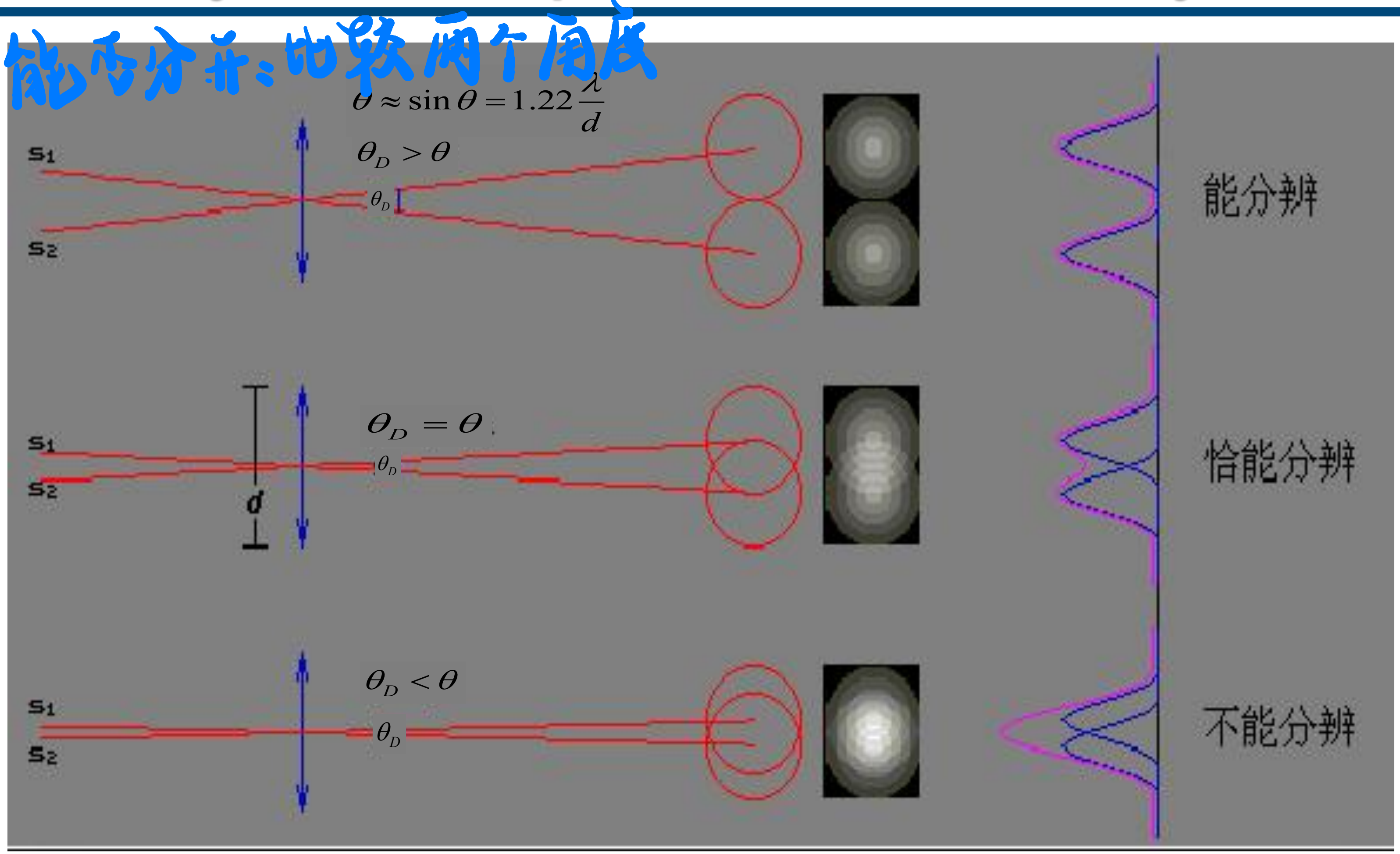
For circular aperture

$$\boxed{\sin\theta = 1.22\frac{\lambda}{d} \quad \text{(first minimum—circular aperture)}}$$



Rayleigh's criterion for resolvability : central maximum of the diffraction pattern of one source is centered on the first minimum of the diffraction patter of the other

Diffraction by a circular aperture and resolvability



Rayleigh's criterion for resolvability : **central maximum of the diffraction pattern of one source is centered on the first minimum of the diffraction patter of the other**

Question 3

Two yellow flowers are separated by 60 cm along a line perpendicular to your line of sight to the flowers. How far are you from then flowers when they are **at the limit of resolution according to the Rayleigh criterion**?

Assume the light from the flowers has a single wavelength of 550 nm and that your pupil has a diameter of 5.5 mm.

$$\theta_0 = \frac{60\,cm}{L} = 1.22\,\frac{550\,nm}{5.5\,mm}$$

Question 3 (Solution)

Following the method of Sample Problem — "Pointillistic paintings use the diffraction of your eye,"

we have

$$\theta = \frac{1.22\lambda}{d} = \frac{D}{L}.$$

Thus we get $L = 4.9 \times 10^3$ m.