

Key words

Interference

Huygens' principle

Propagation

Wave-front/ Wavelet

Wavelength

Index of Refraction

Phase difference

Optical path difference

Double-Slit interference

Monochromatic

Bright/ Dark Fringe

Central maximum

Second side maxima

Coherence/ Incoherence

Intensity

Thin film interference

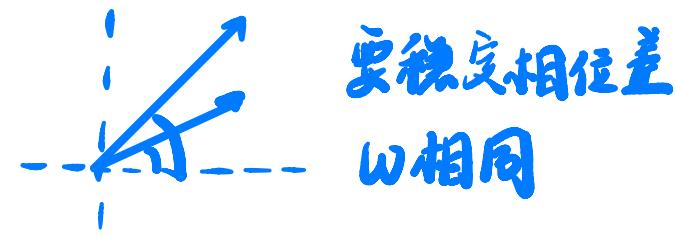
Reflection phase shifts

Michelson's Interferometer

From chapter 33, we know that light is EM wave. If there are two EM waves and they have same ω , same λ in vacuum. When they come to a common point P, the net oscillation of that point is just the superposition of each E oscillation due to each travelling wave

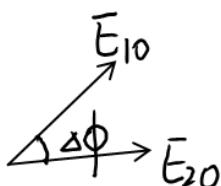
$$\left. \begin{aligned} \vec{E}_1 &= \vec{E}_{10} \cos(k_1 L_1 - \omega t + \phi_1) \\ \vec{E}_2 &= \vec{E}_{20} \cos(k_2 L_2 - \omega t + \phi_2) \end{aligned} \right\} \Rightarrow \vec{E}_P = \vec{E}_1 + \vec{E}_2$$

正弦型于角相图描述



We can apply the **phasor diagram** to deal with the net E field at point P. Let's assume that $\phi_1 = \phi_2$, then the phase difference between the two E at point P is:

$$\Delta\phi = k_1 L_1 - k_2 L_2 = \frac{2\pi L_1}{\lambda_1} - \frac{2\pi L_2}{\lambda_2} = \frac{2\pi}{\lambda} (n_1 L_1 - n_2 L_2) \quad (\lambda_n = \frac{\lambda}{n}) \quad nL \text{ is called optical path}$$

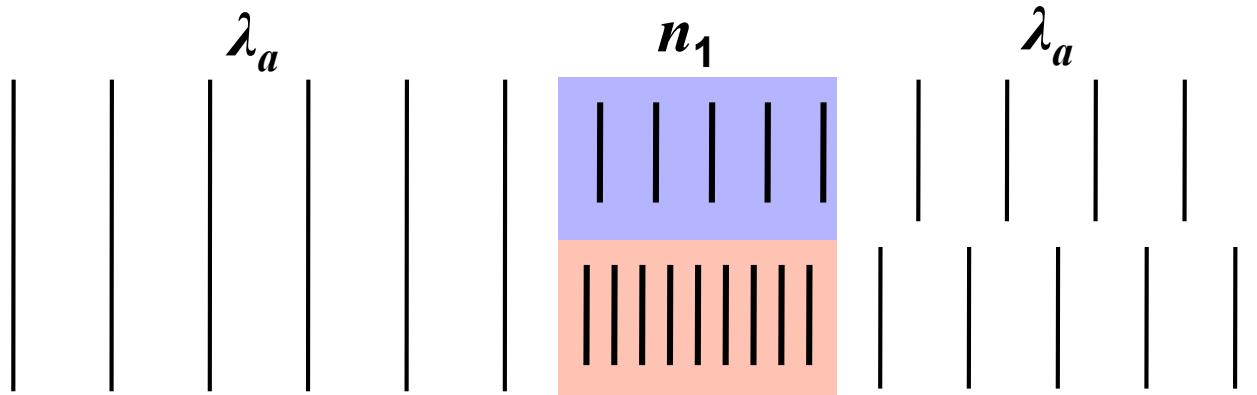


真空传播
In phase: $\Delta\phi = m \times 2\pi \Rightarrow m\lambda$

Out of phase: $n_1 L_1 - n_2 L_2 = (m + \frac{1}{2})\lambda$

$$\Delta\phi = \frac{2\pi}{\lambda} (n_1 L_1 - n_2 L_2) = \begin{cases} m \times 2\pi \Rightarrow n_1 L_1 - n_2 L_2 = m\lambda, \text{ constructive interference} \\ (m + \frac{1}{2}) \times 2\pi \Rightarrow n_1 L_1 - n_2 L_2 = (m + \frac{1}{2})\lambda, \text{ destructive interference} \end{cases}$$

If $n_1 = n_2 = 1$, that means two rays are both in the air, then: $n_1 L_1 - n_2 L_2 = \Delta L$



$$\frac{\text{phase difference}}{2\pi} = \frac{\text{path length difference}}{\lambda}$$

$$\lambda_n = \frac{\lambda}{n}$$

wavelength in vacuum/air
refractive index

$$\text{No. of wavelength } N_1 \text{ in medium 1 : } \frac{L}{\lambda_{n1}} = \frac{Ln_1}{\lambda_a}$$

$$\text{No. of wavelength } N_2 \text{ in medium 2 : } \frac{L}{\lambda_{n2}} = \frac{Ln_2}{\lambda_a}$$

$$\text{Phase difference between the waves } \phi : \frac{\phi}{2\pi} = \frac{Ln_2}{\lambda_a} - \frac{Ln_1}{\lambda_a} = \frac{L(n_2 - n_1)}{\lambda_a}$$

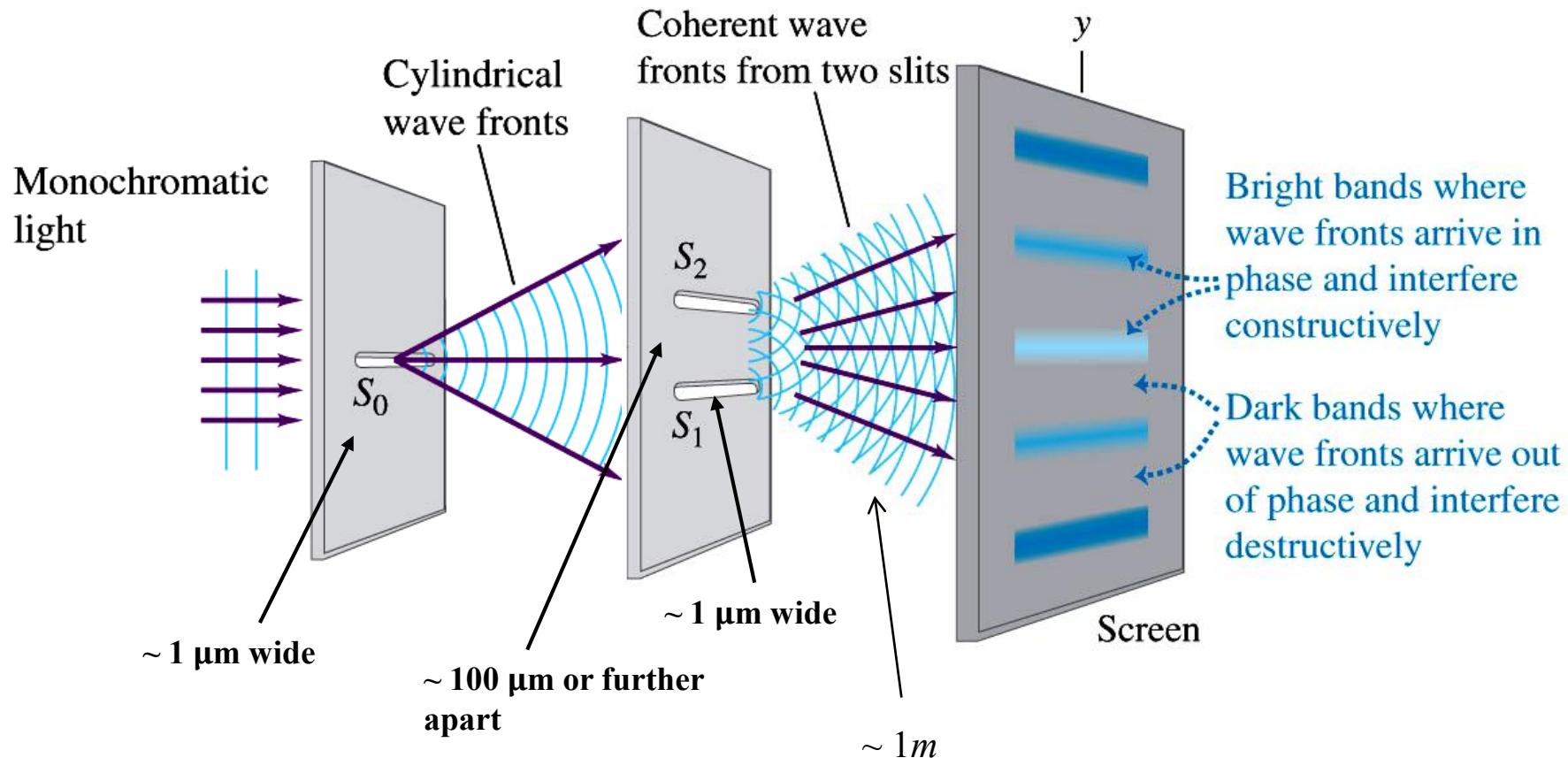
say, if it is 3.6
we take the
decimal fraction,
i.e. 0.6, $\phi=1.2\pi$

Optical path Different

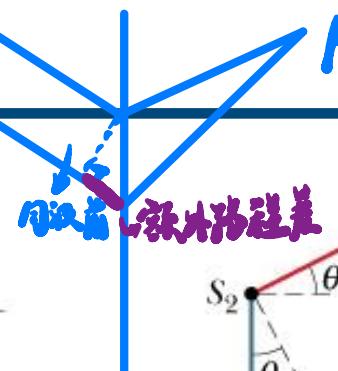
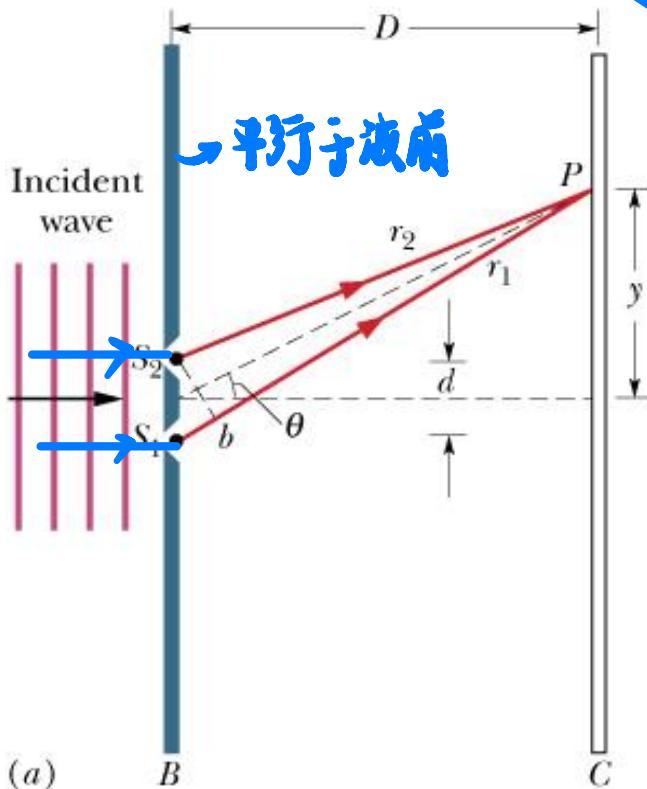
Optical path

Review of Ch35

Interference of light waves passing through two slits



Review of Ch35



Path Length Difference: $\Delta L = d \sin \theta$

Maxima-bright fringes: $m=0$ central maxime
 $m=1$ first maxima
 $d \sin \theta = m\lambda$ for $m = 0, 1, 2, \dots$

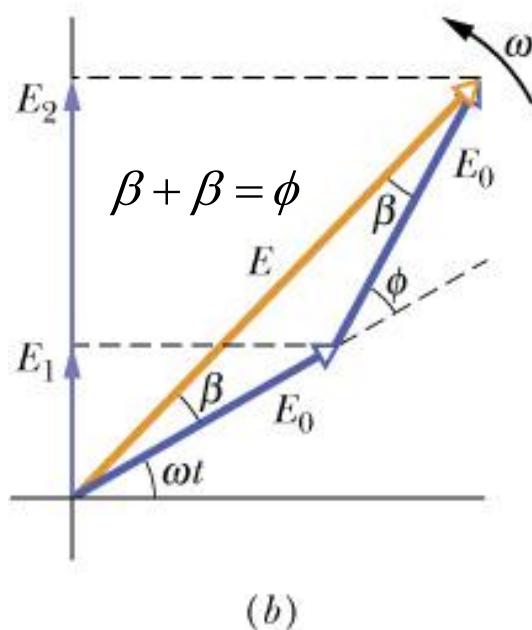
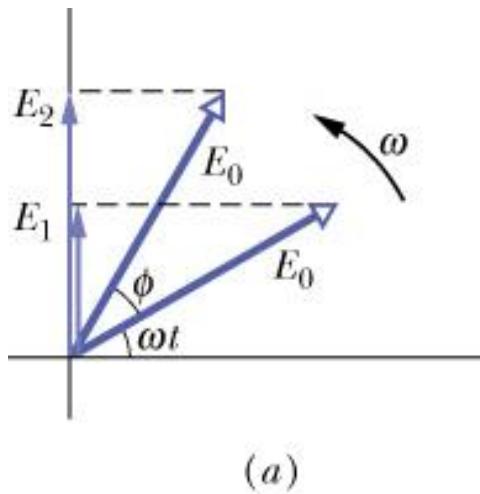
Minima-dark fringes: $m=0$ first dark
 $d \sin \theta = (m + \frac{1}{2})\lambda$ for $m = 0, 1, 2, \dots$

Second order bright fringe
Second side maximum

Second order dark fringe
Second minimum

$$m = 2 \text{ bright fringe at: } \theta = \sin^{-1}(2\lambda/d)$$

$$m = 1 \text{ dark fringe at: } \theta = \sin^{-1}\left(\frac{1.5\lambda}{d}\right)$$



$$E(t) = E_0 \sin \omega t + E_0 \sin(\omega t + \phi)$$

$$E = 2(E_0 \cos \beta) = 2E_0 \cos \frac{1}{2}\phi$$

$$E^2 = 4E_0^2 \cos^2 \frac{1}{2}\phi$$

$$\frac{I}{I_0} = \frac{E^2}{E_0^2} = 4 \cos^2 \frac{1}{2}\phi \rightarrow I = 4I_0 \cos^2 \frac{1}{2}\phi$$

見屏上一點

$$\frac{\text{(phase difference)}}{2\pi} = \frac{\text{(path length difference)}}{\lambda}$$

$$\left(\frac{\text{(phase difference)}}{\lambda} \right) = \frac{2\pi}{\lambda} \left(\frac{\text{(path length difference)}}{\lambda} \right)$$

$$\phi = \frac{2\pi}{\lambda} (d \sin \theta)$$

$$m \lambda = d \sin \theta$$

$$\frac{\phi}{2\pi} \lambda = d \sin \theta$$

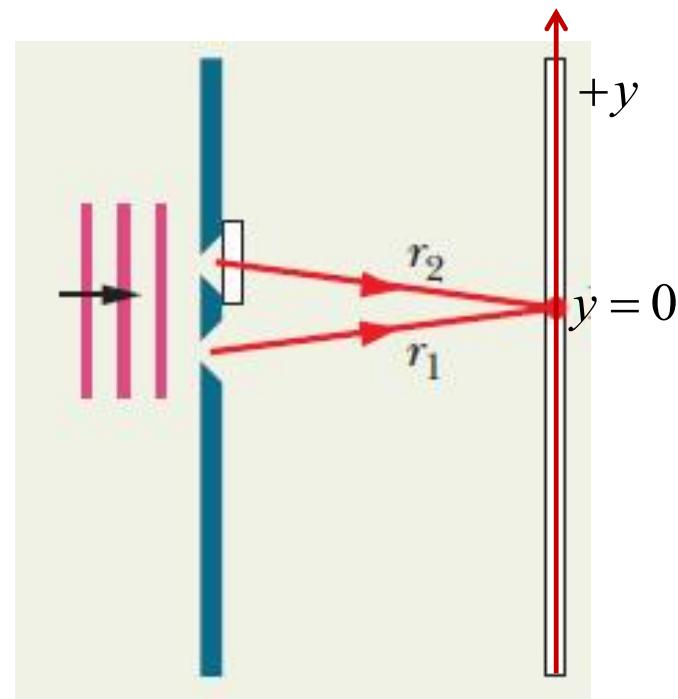
$$\boxed{\phi = \frac{2\pi d}{\lambda} \sin \theta}$$

56 In the double-slit experiment of Fig. 35-10, the viewing screen is at distance $D = 4.00$ m, point P lies at distance $y = 20.5$ cm from the center of the pattern, the slit separation d is $4.50 \mu\text{m}$, and the wavelength λ is 650 nm . (a) Determine where point P is in the interference pattern by giving the maximum or minimum on which it lies, or the maximum and minimum between which it lies. (b) What is the ratio of the intensity I_P at point P to the intensity I_{cen} at the center of the pattern?

Problem 1 of Ch35

A thin flake (薄片) of mica ($n = 1.58$) is used to cover one slit of a double-slit interference arrangement. The central point on the viewing screen is now occupied by what had been the seventh bright side fringe ($m = 7$). If $\lambda = 550 \text{ nm}$

- a) What is the thickness of the mica?
- b) If the slit separation $d=0.12\text{mm}$, and the slit-screen separation $D=55\text{cm}$. Find the position of $m=0$. **可以用y /θ 描述**
- c) Find the intensity at the position of y (Suppose intensity on the screen from each slit is I_0) **假设无反射**



$$\phi = 4\pi$$

a) At the center point, $m=7$, so the phase difference at the center point is:

$$\phi = 2\pi \left(\frac{L}{\lambda_n} - \frac{L}{\lambda} \right) = \frac{2\pi L}{\lambda} (n-1) = m(2\pi) = 7(2\pi) \Rightarrow L = \frac{m\lambda}{n-1} = \frac{7 \times 550 \times 10^{-9}}{1.58-1} = 6.64 \times 10^{-6} m$$

$$Or \quad OPD = (n-1)L = 7\lambda \Rightarrow L = \frac{7\lambda}{n-1} = \frac{7 \times 550 \times 10^{-9}}{1.58-1} = 6.64 \times 10^{-6} m$$

b) At the point of $m=0$, the phase difference is **0**

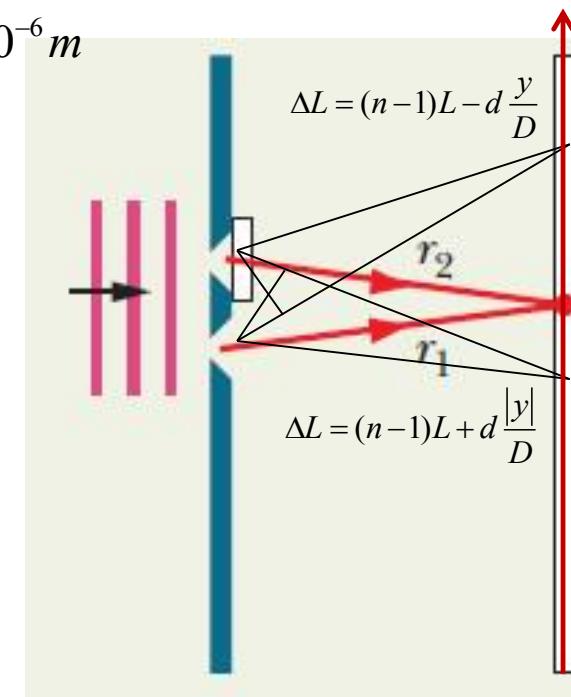
$$\phi = \frac{2\pi L}{\lambda} (n-1) - \frac{2\pi}{\lambda} d \sin \theta = 7(2\pi) - \frac{2\pi}{\lambda} d \sin \theta = 0$$

$$\Rightarrow 7\lambda = d \sin \theta \approx d \tan \theta = d \frac{y_0}{D} \Rightarrow y_0 = \frac{m\lambda D}{d} = 1.76 cm$$

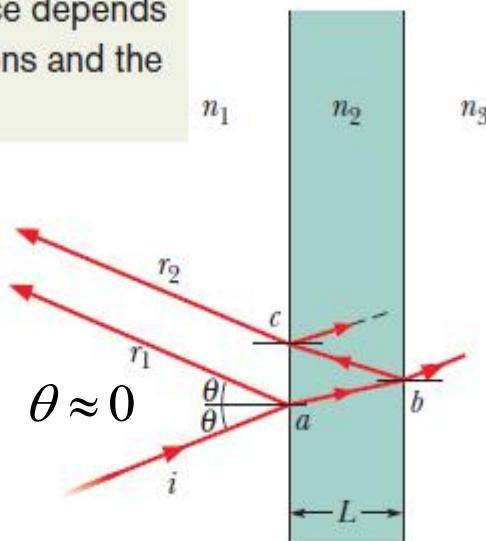
k · DPD

$$c) \phi = \frac{2\pi L}{\lambda} (n-1) - \frac{2\pi}{\lambda} d \sin \theta = 7(2\pi) - \frac{2\pi}{\lambda} d \frac{y}{D}$$

$$I = 4I_0 \cos^2(\phi/2)$$



The interference depends on the reflections and the path lengths.



For constructive interference :

$$2L = (m + \frac{1}{2}) \frac{\lambda}{n_2} \quad \text{for } m = 0, 1, 2, \dots$$

For destructive interference :

$$2L = m \frac{\lambda}{n_2} \quad \text{for } m = 0, 1, 2, \dots$$

Condition: $n_1 > n_2 < n_3$ or $n_1 < n_2 > n_3$

Reflection phase shift

$n_1 > n_2$ No phase change

$n_1 = n_2$ No phase change

$n_1 < n_2$ Phase change, 0.5 wavelength

Consider reflection and the path lengths, the Optical path difference between the incident light and reflection light is: $2L n_2 - \frac{\lambda}{2}$

仅反射时有半波损失

$\frac{1}{2}$: 1个 phase shift

0 : 2个 phase shift

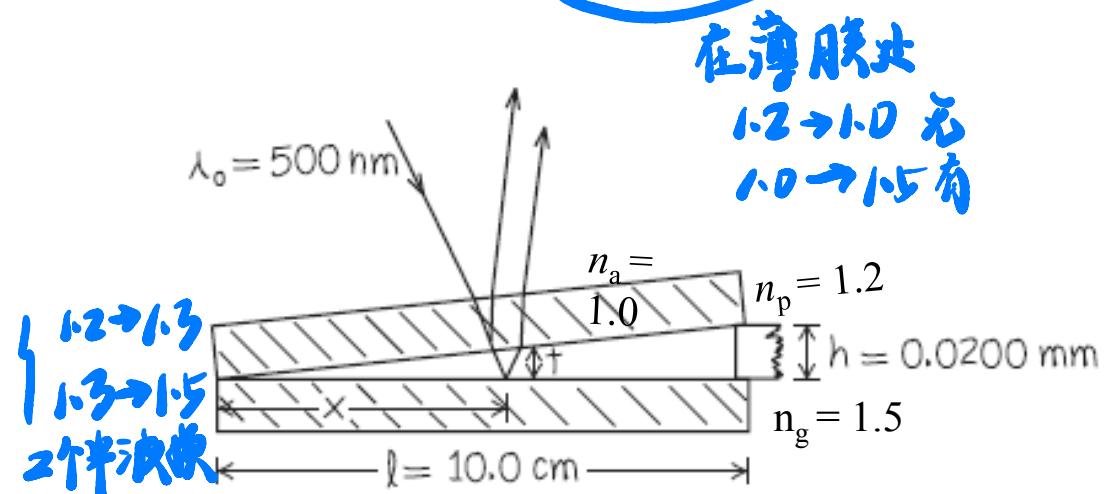
if $L \ll \lambda$

we can ignore the thickness of the film always
destructive interference as the phase difference is always 1/2

Problem 2

Suppose a plastic plate ($n_p = 1.2$) and a glass plate ($n_g = 1.5$) in figure are two microscope slides 10.0 cm long. At one end they are in contact; at the other end they are separated by a piece of paper 0.0200 mm thick. Assume monochromatic light with a wavelength in air of $\lambda_0 = 500 \text{ nm}$ and the light rays are almost perpendicular to the film. Seen by reflection,

- Is the fringe at the line of contact bright or dark?
- How many bright fringes along the glass length?
- What happens if the set up is immersed in the water ($n_w = 1.3$)



Problem 2

We'll consider only interference between the light reflected from the upper and lower surfaces of the **air wedge** between the microscope slides. [The top slide has a relatively great thickness, about 1 mm, so we can ignore interference between the light reflected from its upper and lower surfaces]. The wave reflected from the lower surface undergoes a π rad phase change while the wave reflected from the upper surface does not.

- a) At the line of contact, the thickness of the wedge is 0, so the phase difference between the two rays is π , thus there is a dark fringe at the line of contact.



- b) At a place where the thickness of the wedge is d , the condition for a maximum intensity is:

$$2d = (m + 1/2)\lambda_0 \Rightarrow m = 2d / \lambda_0 - 1/2$$

$$d = 0.02\text{mm} \Rightarrow m_{\max} = [79.5] = 79$$

Since the first bright fringe corresponds to $m = 0$, $m = 79$ corresponds to the 80 bright fringes.

Problem 2

c) Now we need consider the interference pattern formed by waves reflected from the upper and lower surfaces of the **water wedge**. The wave reflected from the lower surface undergoes a π rad phase change and the wave reflected from the upper surface also undergoes another π rad phase change.

At the line of contact, the thickness of the wedge is still 0, but the two rays now are in phase, thus there is a bright fringe at the line of contact.

At a place where the thickness of the wedge is d , the condition for a maximum intensity now should be:

$$2d = m\lambda_w \Rightarrow m = 2n_w d / \lambda_0$$

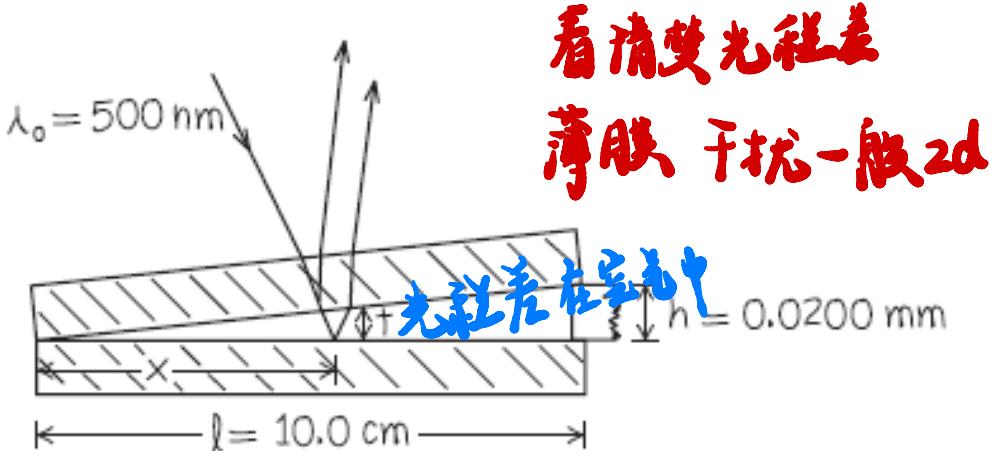
$$d = 0.02\text{mm} \Rightarrow m_{\max} = 104$$

Since the first bright fringe corresponds to $m = 0$, $m = 104$ corresponds to the 105 bright fringes.

Problem 2

Suppose a plastic plate ($n_p = 1.2$) and a glass plate ($n_g = 1.5$) in figure are two microscope slides 10.0 cm long. At one end they are in contact; at the other end they are separated by a piece of paper 0.0200 mm thick. Assume monochromatic light with a wavelength in air of $\lambda_0 = 500 \text{ nm}$ and the light rays are almost perpendicular to the film. Seen by transmission,

- d) Is the fringe at the line of contact bright or dark?
- e) How many bright fringes along the glass length?



Problem 2

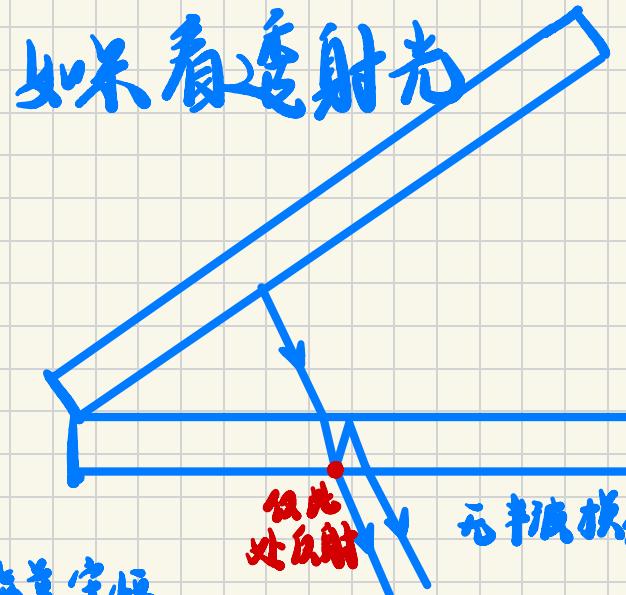
We'll consider only interference between the light reflected from the upper and lower surfaces of the **air wedge** between the microscope slides. [The top slide has a relatively great thickness, about 1 mm, so we can ignore interference between the light reflected from its upper and lower surfaces]. The wave reflected from the lower surface undergoes a π rad phase change while the wave reflected from the upper surface does not.

- a) At the line of contact, the thickness of the wedge is 0, so the phase difference between the two rays is π , thus there is a dark fringe at the line of contact.
- b) At a place where the thickness of the wedge is d , the condition for a maximum intensity is:

$$2d = (m + 1/2)\lambda_0 \Rightarrow m = 2d / \lambda_0 - 1/2$$

$$d = 0.02\text{ mm} \Rightarrow m_{\max} = [79.5] = 79$$

Since the first bright fringe corresponds to $m = 0$, $m = 79$ corresponds to the 80 bright fringes.

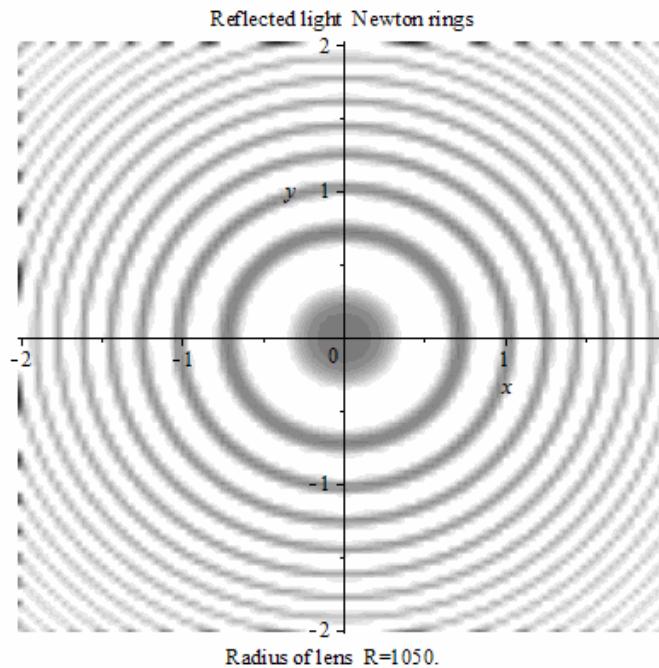
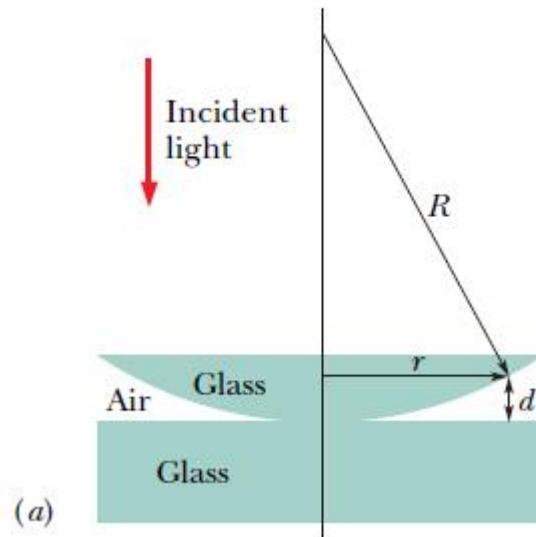


能量守恒



Problem 3 of Ch35

Figure *a* shows a lens with radius of curvature R lying on a flat glass plate and illuminated from above by light with wavelength λ . Figure *b* (a photograph taken from above the lens) shows that circular interference fringes (called *Newton's rings*) appear, associated with the variable thickness d of the air film between the lens and the plate. Find the radii r of the interference maxima. Assuming $r/R \ll 1$



Problem 3 of Ch35

Consider the interference pattern formed by waves reflected from the upper and lower surfaces of the air wedge. The wave reflected from the lower surface undergoes a π rad phase change while the wave reflected from the upper surface does not. At a place where the thickness of the wedge is d , the condition for a maximum in intensity is

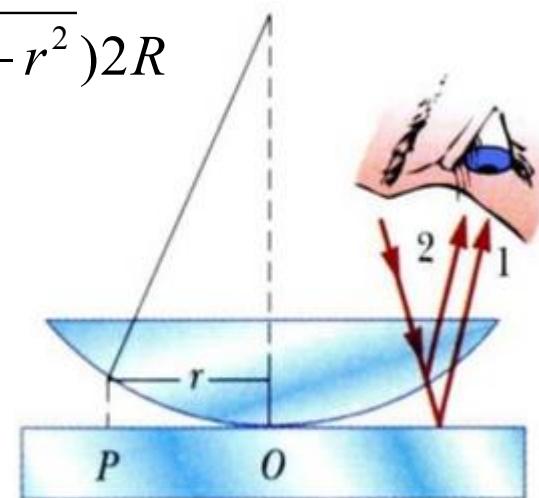
$$2d = (m + 1/2)\lambda \quad \text{for } m = 0, 1, 2, \dots$$

$$d = R - \sqrt{R^2 - r^2}$$

$$r^2 = (R - \sqrt{R^2 - r^2})(R + \sqrt{R^2 - r^2}) \cong (R - \sqrt{R^2 - r^2})2R$$

$$\Rightarrow R - \sqrt{R^2 - r^2} = d = \frac{r^2}{2R}$$

$$\Rightarrow \frac{r^2}{2R} = (2m + 1)\lambda/4 \Rightarrow r = \sqrt{\frac{(2m + 1)R\lambda}{2}}$$



Problem 3 of Ch35

If the radius of curvature R of the lens is 5.0 m and the lens diameter is 18mm.

- (a) How many bright rings are produced? Assume that $\lambda = 589\text{nm}$
- (b) How many bright rings would be produced if the arrangement were immersed in water ($n=1.33$)?

Problem 3 of Ch35

a) $m = \frac{r^2}{R\lambda} - \frac{1}{2} = \frac{(9 \times 10^{-3} \text{ m})^2}{(5.0 \text{ m})(589 \times 10^{-9} \text{ m})} - \frac{1}{2} = 27$

Since the first bright fringe corresponds to $m = 0$, $m = 27$ corresponds to the 28 bright fringe.

b) We now replace λ by $\lambda_n = \lambda/n_w$

$$m_n = \frac{r^2}{R\lambda_n} - \frac{1}{2} = \frac{n_w r^2}{R\lambda} - \frac{1}{2} = \frac{(1.33)(9 \times 10^{-3} \text{ m})^2}{(5.0 \text{ m})(589 \times 10^{-9} \text{ m})} - \frac{1}{2} = 36.$$

This corresponds to the 37 bright fringe.

Problem 3 of Ch35

c) If the radii of the m th and $(m+20)$ th bright rings are measured and found to be 0.162 and 0.368 cm, respectively, in light of wavelength $\lambda = 546$ nm. Calculate the radius of curvature of the lower surface of the lens

when m is changed to $m + 20$, r becomes r' , so

$$\frac{r^2}{R} = (m + 1/2)\lambda \Rightarrow \begin{cases} m = \frac{r^2}{R\lambda} - \frac{1}{2} \\ m + 20 = \frac{r'^2}{R\lambda} - \frac{1}{2} \end{cases}$$

$$\Rightarrow R = \frac{r'^2 - r^2}{20\lambda} = 100\text{cm}$$