Key words

Electric field lines

Electric flux

Gauss's Law

Gaussian surface

静电屏蔽

Electrostatic shielding

Insulating sphere

Conducting sphere

Conductor

interior

Cube

Cylinder

Cylindrical symmetry

球对称

Spherical symmetry

平面对称

Planar symmetry

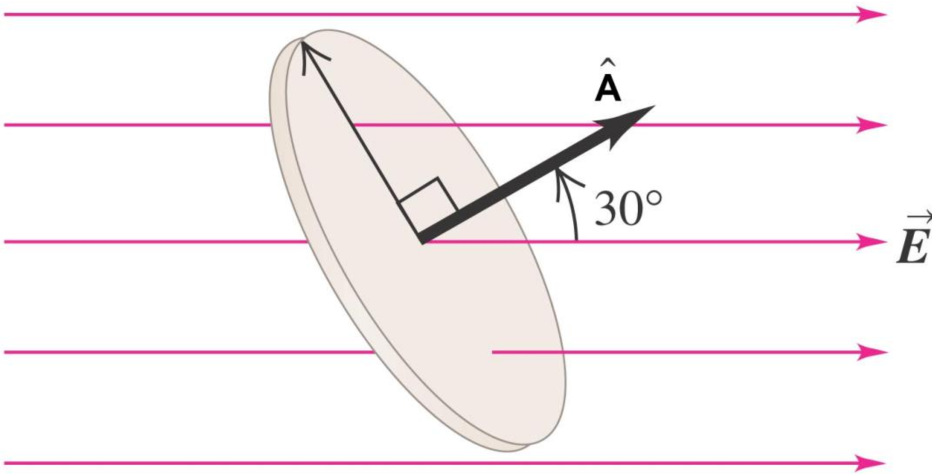
同轴的，同心的

Concentric

exterior

GAUSS'S LAW

electric flux (电通量): $\Phi = EA\cos\theta = \vec{E}\cdot\vec{A}$



$$d\Phi = \vec{E}\cdot d\vec{A} = EdA\cos\theta$$

Gauss' law :

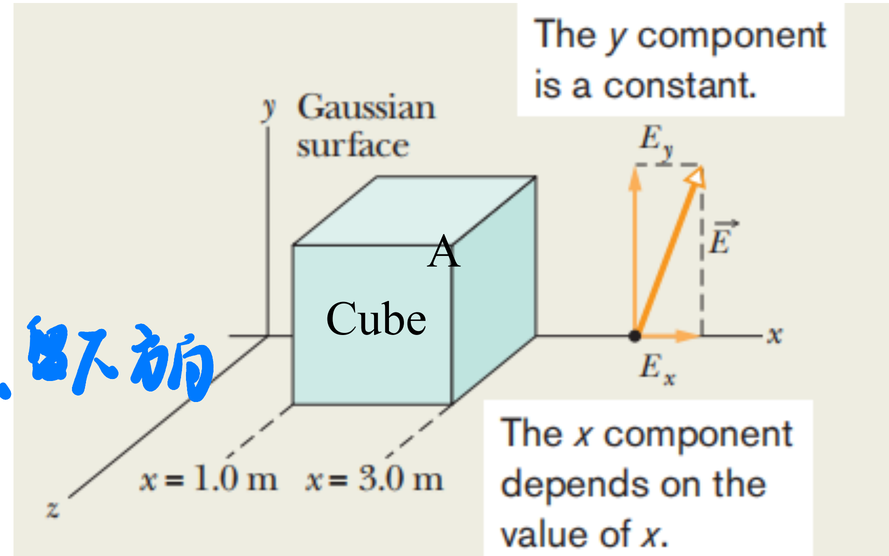
$$\varepsilon_0\Phi = q_{enc} \qquad \text{(Gauss' law)}$$

$$\varepsilon_0\oint \vec{E}\cdot d\vec{A} = q_{enc} \qquad \text{(Gauss' law)}$$

GAUSS'S LAW

$$\vec{E} = ax\hat{i} + b\hat{j}$$

Net Q inside the cube =?



找出面积的方向

$$\int_L \vec{E}\cdot d\vec{A} = \int (a\hat{i} + b\hat{j})\cdot(-\hat{i})dA = -aA$$

$$\int_R \vec{E}\cdot d\vec{A} = \int (3a\hat{i} + b\hat{j})\cdot(\hat{i})dA = 3aA$$

$$\int_T \vec{E}\cdot d\vec{A} = \int (ax\hat{i} + b\hat{j})\cdot(\hat{j})dA = bA$$

$$\int_B \vec{E}\cdot d\vec{A} = \int (ax\hat{i} + b\hat{j})\cdot(-\hat{j})dA = -bA$$

$$\int_f \vec{E}\cdot d\vec{A} = \int (ax\hat{i} + b\hat{j})\cdot(\hat{k})dA = 0$$

$$\int_r \vec{E}\cdot d\vec{A} = \int (ax\hat{i} + b\hat{j})\cdot(-\hat{k})dA = 0$$

$$\oint \vec{E}\cdot d\vec{A} = \int_L \vec{E}\cdot d\vec{A} + \int_R \vec{E}\cdot d\vec{A} + \int_T \vec{E}\cdot d\vec{A}$$

$$+\int_B \vec{E}\cdot d\vec{A} + \int_f \vec{E}\cdot d\vec{A} + \int_r \vec{E}\cdot d\vec{A}$$

$$= -aA + 3aA + bA - bA = 2aA = Q/\varepsilon_0$$

$$Q = 2\varepsilon_0 aA = 8\varepsilon_0 a$$

$$\oint \vec{E}\cdot d\vec{A} = \frac{q}{\varepsilon_0}$$

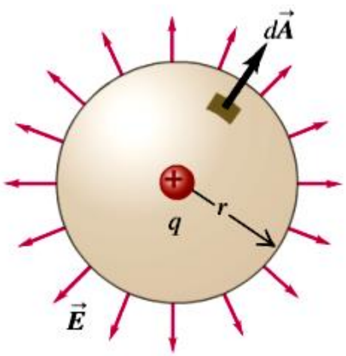
求一类大小

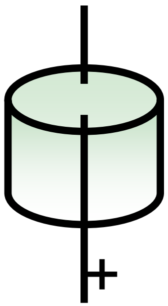
Problem-solving strategy

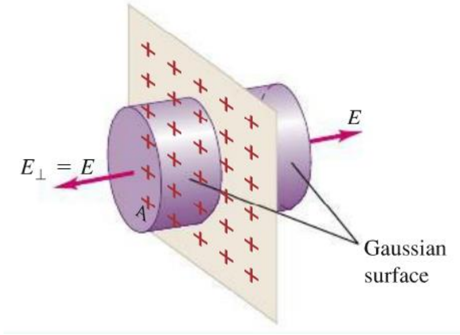
Select the appropriate **closed, imaginary** Gaussian surface.

- For spherical symmetry, use a concentric spherical surface
- For cylindrical symmetry, use a coaxial cylindrical surface with flat ends perpendicular to the axis of symmetry
- For planar symmetry, use a cylindrical surface with its flat ends parallel to the plane

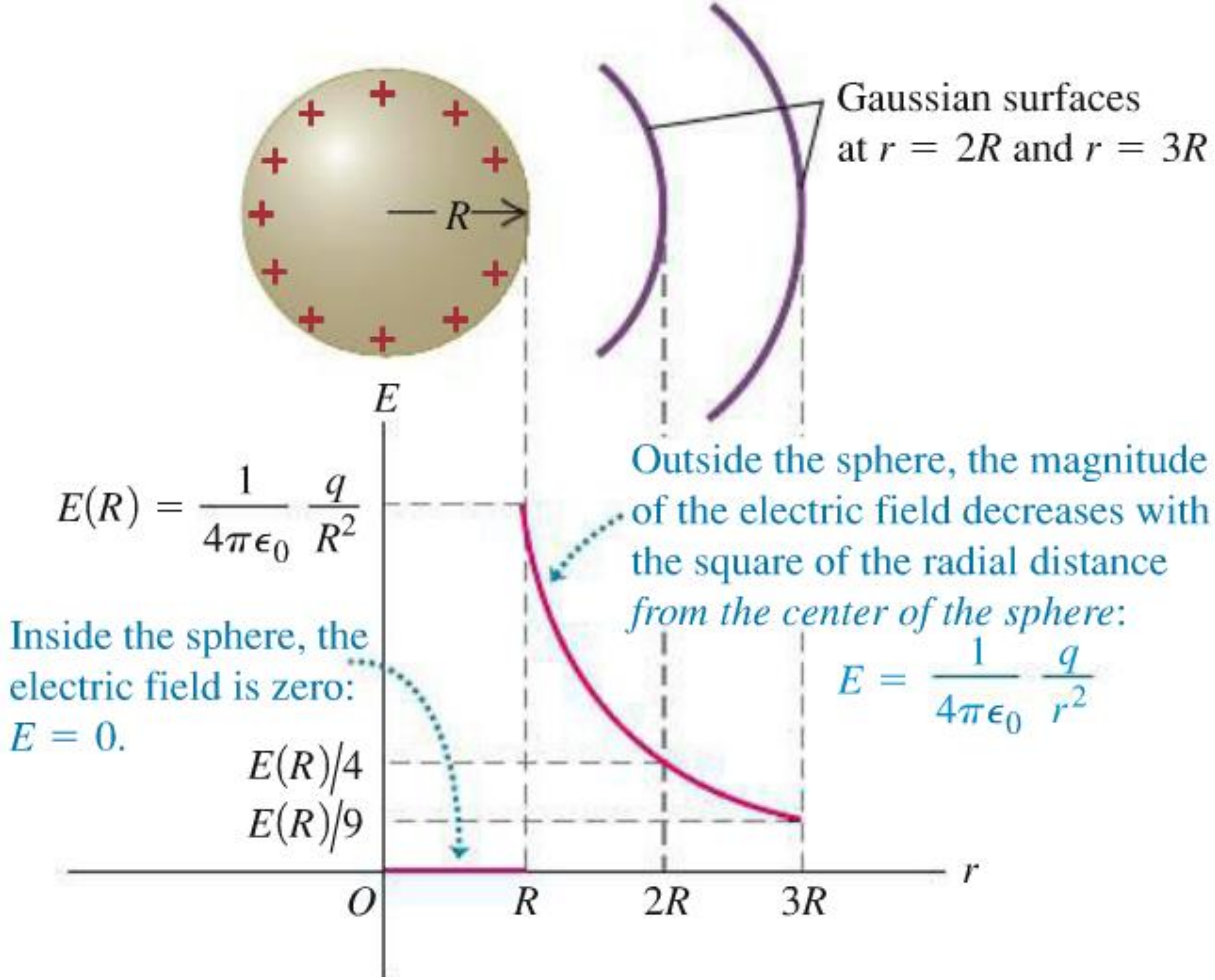
$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl}}}{\epsilon_0}$$







Uniform + charged conducting sphere



Uniform + charge insulating sphere

绝缘的

Positive charge Q is distributed uniformly

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\mathrm{encl}}}{\epsilon_0}$$

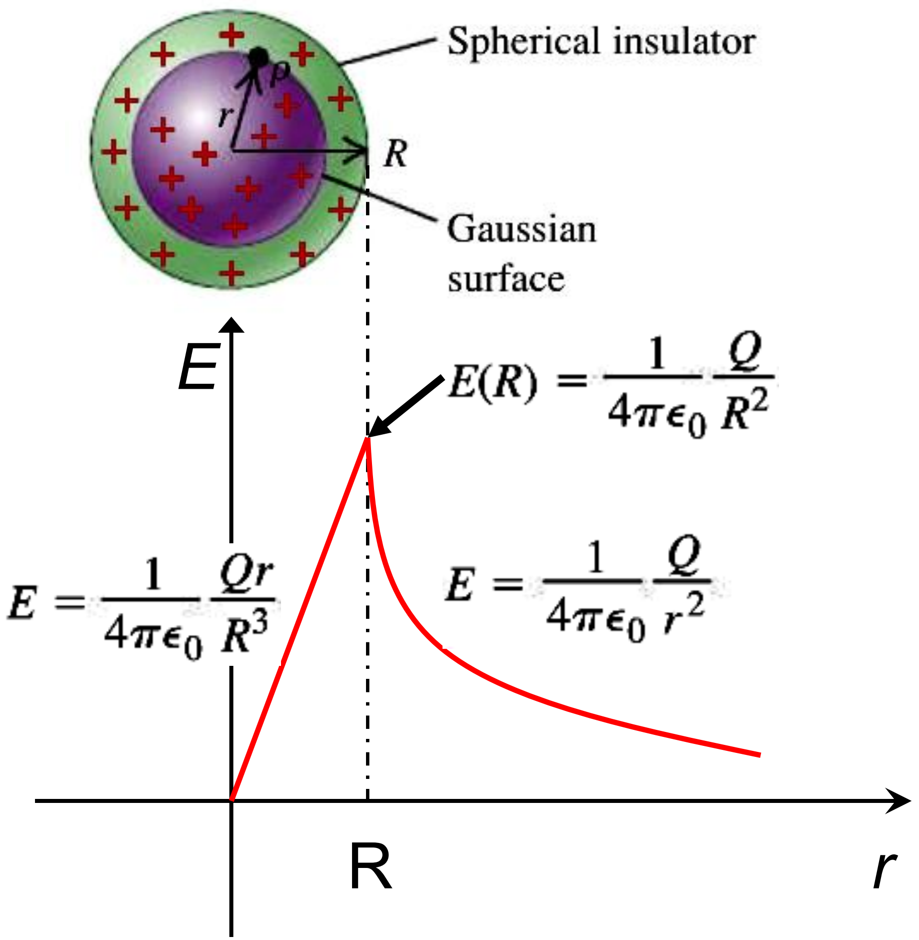
$$\rho = \frac{Q}{4\pi R^3/3}$$

For *r* < *R*

$$\boxed{E = \frac{1}{4\pi\epsilon_0}\frac{Qr}{R^3}}$$

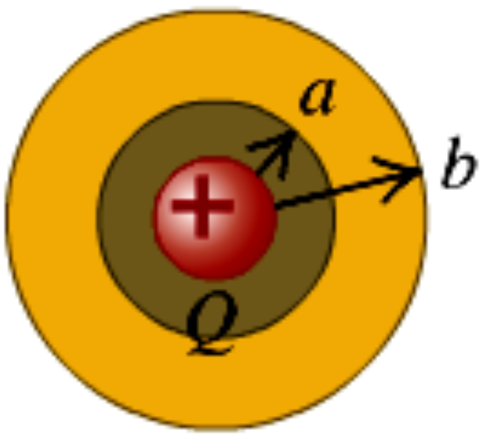
For *r* > *R*

$$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$$



Problem 1

A conducting spherical shell with inner radius a and outer radius b has a positive point charge Q located at its center. The total charge on the shell is $-3Q$, and it is insulated from its surroundings.



1. Derive expressions for the E-field magnitude in terms of the distance r from the center for the regions $r < a$, $a < r < b$ and $r >$ b
2. What is the surface charge density on the inner surface of the conducting shell?
3. What is the surface charge density on the outer surface of the conducting shell?

Problem 1

1. a spherical Gaussian surface with radius r

(a) $r < a$, Gauss' law yields

$$4\pi\varepsilon_0 r^2 E = Q \to E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

(b) $a < r < b$, since these points are within the conducting material.

E=0

(c) r > b, Gauss' law yields

$$4\pi\varepsilon_0 r^2 E = -3Q + Q \to E = \frac{-2Q}{4\pi\varepsilon_0 r^2}$$

The magnitude is: $$E = \frac{2Q}{4\pi\varepsilon_0 r^2}$$

Problem 1

2. Since a Gaussian surface with radius r, for a<r<b, must enclose zero net charge, the total charge on the inner surface is –Q, so

$$\sigma = \frac{-Q}{4\pi a^2}$$

3. Since the net charge on the shell is –3Q and there is –Q on the inner surface , there must be –2Q on the outer surface, so

$$\sigma = \frac{-2Q}{4\pi b^2}$$

Problem 2

Charge is distributed uniformly throughout the volume of an infinitely long solid cylinder of radius R.

(a) Show that, at a distance $r < R$ from the cylinder axis,

$$E = \frac{\rho r}{2\varepsilon_0},$$ where ρ is the volume charge density.

(b) Write an expression for E when $r > R$.

Problem 2

(a) The Gaussian surface:

a cylinder with radius *r* and length L, coaxial with the charged cylinder.

The charge enclosed by it:

$$q = \rho V = \pi r^2 L \rho$$

If ρ is positive, the electric field lines are radially outward, normal to the Gaussian surface and distributed uniformly along it.

$$\Phi = EA_{cylinder} = E(2\pi rL)$$

Gauss' law leads to

$$2\pi\varepsilon_0 rLE = \pi r^2 L\rho \quad \Rightarrow \quad E = \frac{\rho r}{2\varepsilon_0}.$$

Problem 2

(b) We consider a cylindrical Gaussian surface of radius $r > R$.

If the external field is E_{ext} then the flux is

$$\Phi = 2\pi r L E_{\text{ext}}.$$

The charge enclosed is the total charge in a section of the charged cylinder with length L.

$$q = \pi R^2 L \rho$$

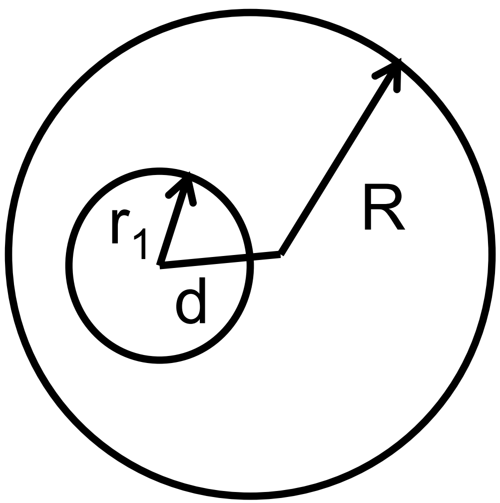
In this case, Gauss' law yields

$$2\pi\varepsilon_0 r L E_{\text{ext}} = \pi R^2 L \rho \quad \Rightarrow \quad E_{\text{ext}} = \frac{R^2 \rho}{2\varepsilon_0 r}.$$

Problem 2

(c) Now, If this infinity long solid cylinder contains a cylindrical cavity of radius r_1 and the central axes of the cylinder and cavity are parallel. The distance between two axes is $d = 2.00$ cm, as shown in the figure. What is the electric field in the cavity? If the volume density does not change.

E



The cross section of the cylinder and cavity

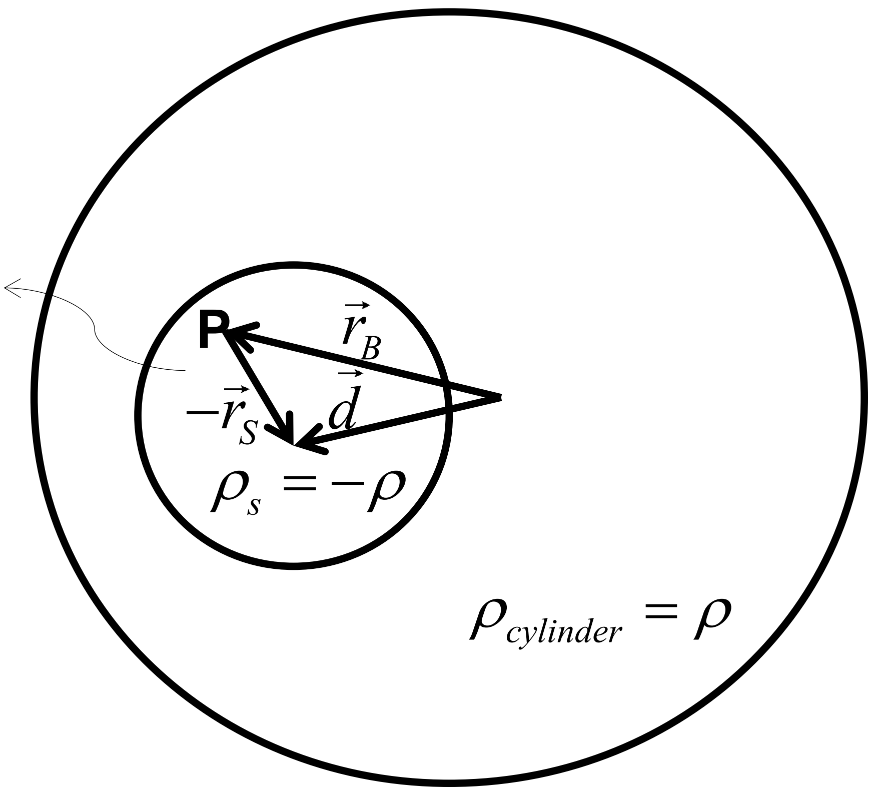
Problem 2

(c) The total charge in the region of the cavity is zero, so:

For the small cylinder $\rho_s = -\rho$

For the big cylinder $\rho_b = \rho$

For the cavity: $\rho_{cavity} = -\rho + \rho = 0$

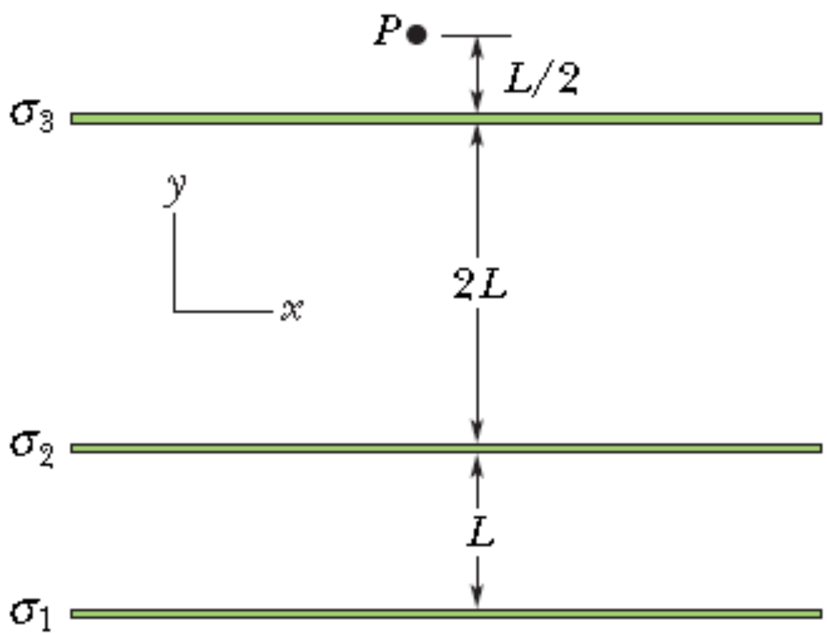


From (a), we know at point p in the cavity, the electrical field due to the small cylinder and big cylinder are:

$$\left.\begin{aligned} \vec{E}_S &= -\frac{\rho \vec{r}_S}{2\varepsilon_0} \\ \vec{E}_B &= \frac{\rho \vec{r}_B}{2\varepsilon_0} \end{aligned}\right\} \Rightarrow \vec{E} = \vec{E}_S + \vec{E}_B = \frac{\rho \vec{r}_B}{2\varepsilon_0} - \frac{\rho \vec{r}_S}{2\varepsilon_0} = \frac{\rho}{2\varepsilon_0}(\vec{r}_B - \vec{r}_S) = \frac{\rho}{2\varepsilon_0}\vec{d}$$

Problem 3

The figure shows, in cross section, three infinitely large nonconducting sheets on which charge is uniformly spread. The surface charge densities are $\sigma_1 = +2.00\ \mu C/m^2$, $\sigma_2 = +4.00\ \mu C/m^2$, and $\sigma_3 = -5.00\ \mu C/m^2$, and distance $L = 1.50$ cm. In unit-vector notation, what is the net electric field at point P?



Problem 3

The fields involved are uniform, the precise location of P is not relevant;

The positively charged sheets contributing upward fields;

The negatively charged sheet contributing a downward field;

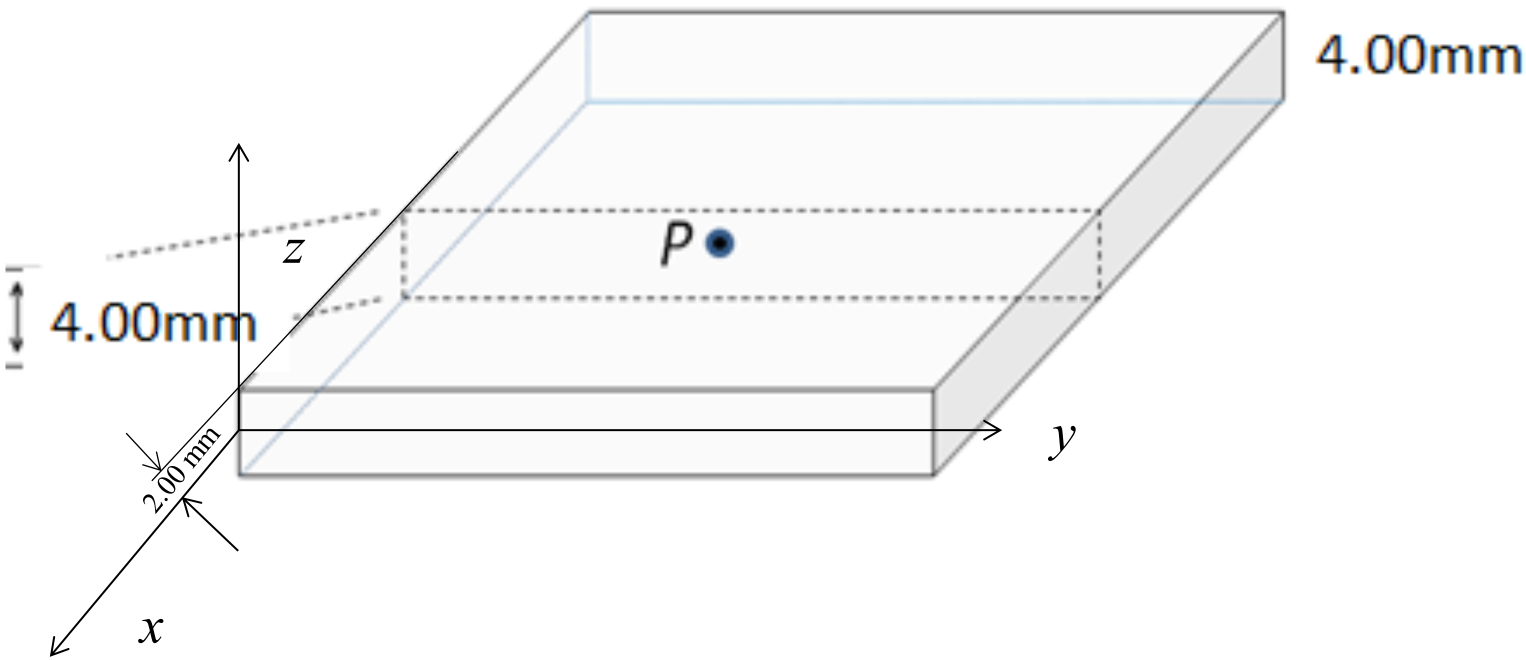
$$|\vec{E}| = \frac{\sigma_1}{2\varepsilon_0} + \frac{\sigma_2}{2\varepsilon_0} + \frac{\sigma_3}{2\varepsilon_0} = \frac{1.0\times10^{-6}\,\text{C/m}^2}{2(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = 5.65\times10^4\ \text{N/C}.$$

The net field is directed upward $(+\hat{\text{j}})$

In unit-vector notation: $\vec{E} = (5.65\times10^4\ \text{N/C})\hat{\text{j}}$

Problem 4

As shown in the figure, a square **insulating slab** 4.0 mm thick measuring 2.0 m × 2.0 m has a uniform volume charge density of ρ. Determine the magnitude of electric field as a function of z, both inside and outside (z<< 2.0m) the slab

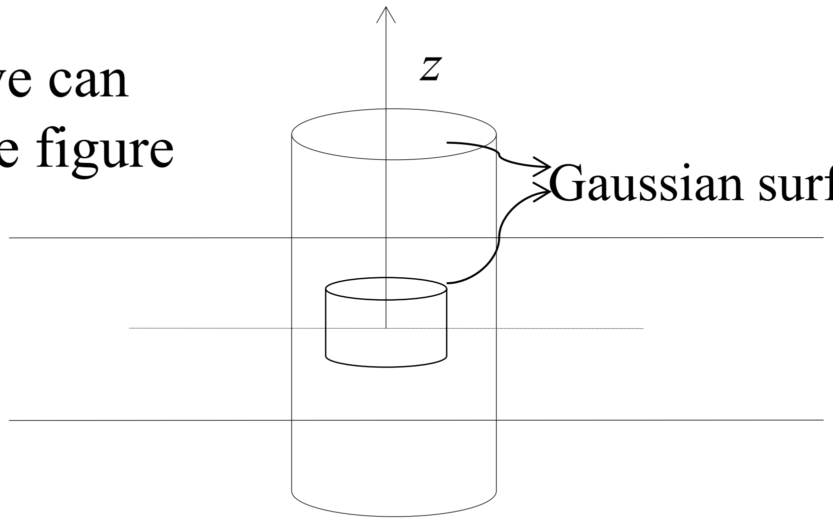


Problem 4

Inside the slab:

According to the planar symmetry, we can Choose the Gaussian surface as in the figure Shown:

$$\Phi = EA + EA + 0 = 2EA = \frac{\rho A 2z}{\varepsilon_0} \Rightarrow E = \frac{\rho z}{\varepsilon_0}$$



Side view of the slab

outside the slab:

$$\Phi = EA + EA + 0 = 2EA = \frac{\rho A d}{\varepsilon_0} \Rightarrow E = \frac{\rho d}{2\varepsilon_0} (d = 4mm)$$