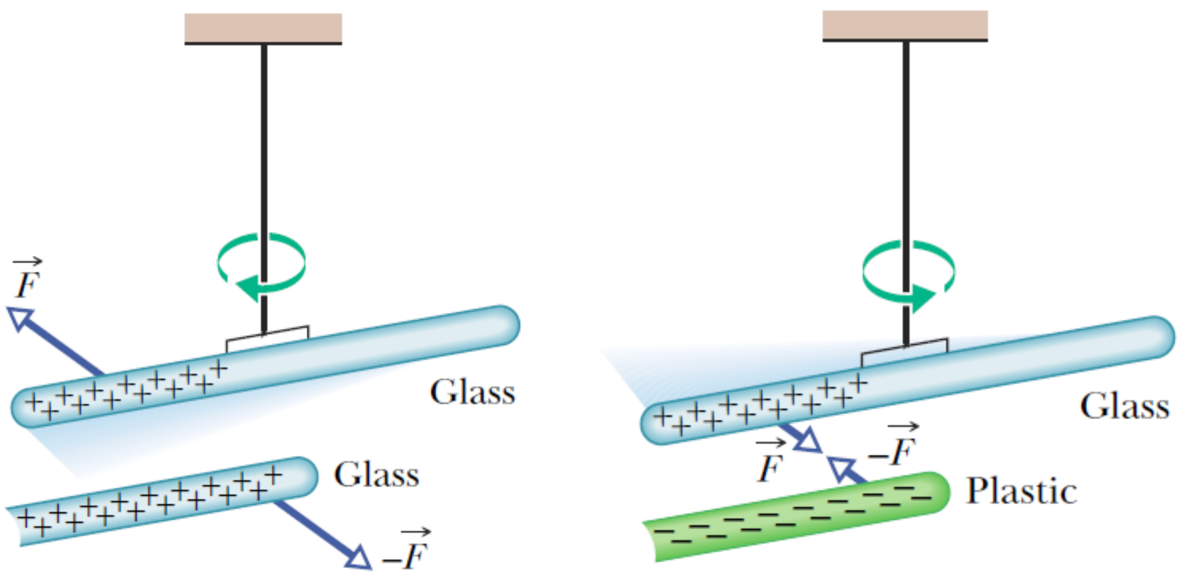
Chapter 21: Coulomb's Law

Electric Charge

- There are two types of electric charge, positive (+) and negative (-), as we know it from high school physics.
- Particles with the same sign of electric charge *repel* each other, whereas particles with the opposite signs *attract* each other.



Conductors, Semiconductors and Insulators

- **Conductors** are materials through which charge can move rather freely.
- **Nonconductors**—also called **insulators**—are materials through which charge cannot move freely.
- **Semiconductors** are materials that are intermediate between conductors and insulators; examples include silicon and germanium in computer chips.

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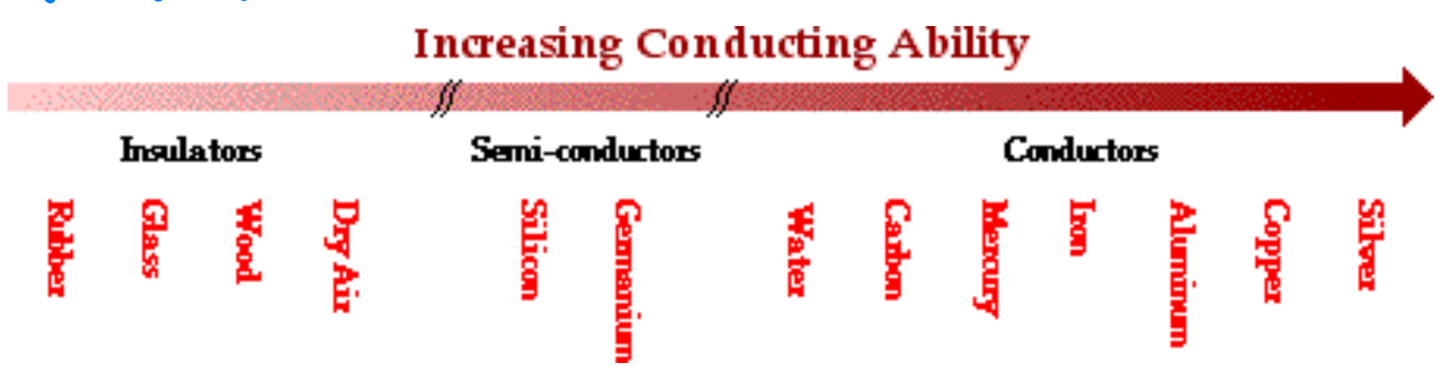


Coulomb's Law

For two particles with charges q_1 and q_2 and relative distance $\mathbf{r} = r\mathbf{e}_r$, the electric force between them measures

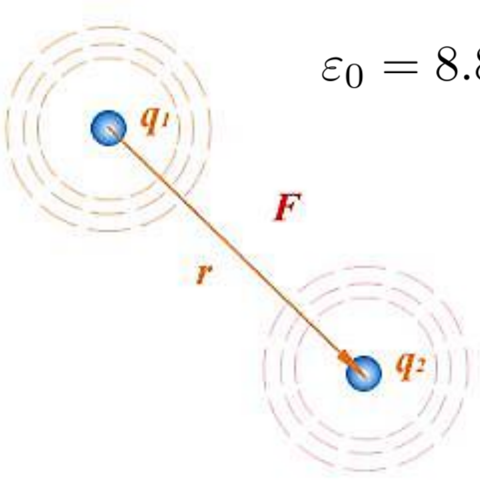
$$\mathbf{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^3}\mathbf{r} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\mathbf{e}_r \equiv k\frac{q_1 q_2}{r^2}\mathbf{e}_r,$$

where boldface characters **F**, **r** and $\mathbf{e}_r$ are vectors,

$$k \equiv \frac{1}{4\pi\varepsilon_0} = 8.99 \times 10^9 \mathrm{N}\cdot\mathrm{m}^2/\mathrm{C}^2,$$

and the **vacuum permittivity constant**

$$\varepsilon_0 = 8.85 \times 10^{-12}\mathrm{C}^2/\mathrm{N}\cdot\mathrm{m}^2.$$



$$\boxed{\mathbf{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\mathbf{e}_r}$$

Inverse Square Law

1. The SI unit of charge is the **coulomb**.

$$1\mathrm{C} = 1\mathrm{A}\cdot\mathrm{s}$$

2. Electric force follows the **inverse square law** (force proportional to the inverse square of the distance, $F \propto 1/r^2$).

The only other force known to follow the same law is the gravitational force $\mathbf{F}_g$, where

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$$\mathbf{F}_g = -G\frac{m_1 m_2}{r^2}\mathbf{e}_r,$$

with m_1, m_2 the mass of the two particles, and $G = 6.673 \times 10^{-11}\mathrm{N}\cdot\mathrm{m}^2/\mathrm{kg}^2$ the gravitational constant.

Ask yourself: What will happen to earth's orbit over a long period of time, if the gravitational force is proportional to, say, $1/r2.5$? Will earth crash into the sun?

(For explanation, see http://www.conservapedia.com/Debate:What_is_the_exponent_of_r_in_Newtonian_gravity%3F)

Superposition Principle; Example 1

3. The electric (Coulomb) force obeys the superposition principle:

$$\mathbf{F}_{\text{tot}} = \sum_i \mathbf{F}_i$$

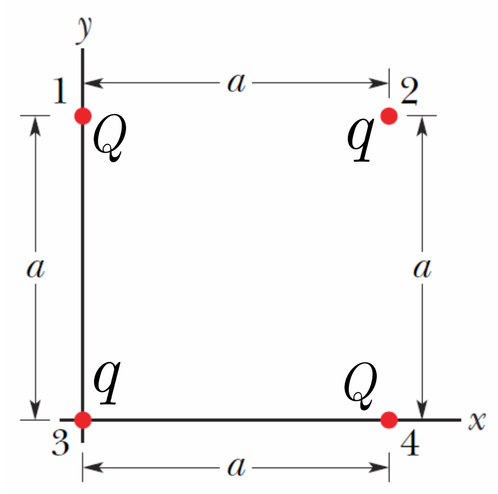
Vector Sum!!

Example 1 (Problem 21-2 of the textbook):

Four particles form a square. The charges are $q1 = q4 = Q$, and $q2 = q3 = q$.

(a) What is Q/q if the net electrostatic force on particles 2 and 3 is zero?

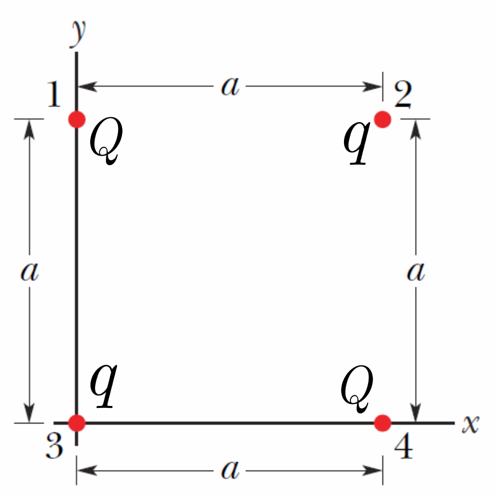
(b) Is there any value of q that makes the electrostatic force on each of the four particles zero?



Example 1

Solution:

(a) We calculate the net force on particle 2 (particle 3 would be the same) by:



$$\mathbf{F}_2 = \sum_i \mathbf{F}_{i2} = \mathbf{F}_{12} + \mathbf{F}_{32} + \mathbf{F}_{42}$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{Qq}{a^2}\mathbf{e}_x + \frac{q^2}{(\sqrt{2}a)^2}\mathbf{e}_{32} + \frac{Qq}{a^2}\mathbf{e}_y\right)$$

Since $\mathbf{e}_{32} = \frac{\sqrt{2}}{2}\mathbf{e}_x + \frac{\sqrt{2}}{2}\mathbf{e}_y,$

$$\mathbf{F}_2 = \frac{1}{4\pi\varepsilon_0}\left[\left(\frac{Qq}{a^2} + \frac{\sqrt{2}q^2}{4a^2}\right)\mathbf{e}_x + \left(\frac{Qq}{a^2} + \frac{\sqrt{2}q^2}{4a^2}\right)\mathbf{e}_y\right].$$

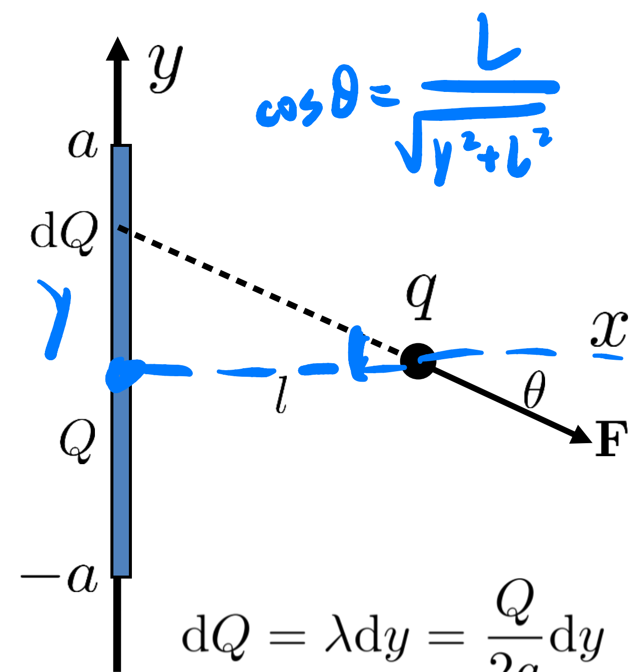
In order that **F**2 = 0, we must have

$$\frac{Qq}{a^2} + \frac{\sqrt{2}q^2}{4a^2} = 0, \quad \text{i.e.,} \quad q = 0 \quad \text{or} \quad \frac{Q}{q} = -\frac{\sqrt{2}}{4}.$$

Example 2

A rod with length $2a$ and uniformly distributed charge Q is placed on the y-axis, spanning the region $\{(x,y)|x=0,-a<y<a\}$. Find the electric force felt by a charge q at point $(l,0)$.

$$
\begin{aligned}
F_x &= \int \frac{1}{4\pi\varepsilon_0}\frac{q\mathrm{d}Q}{l^2+y^2}\cos\theta \\
&= \frac{1}{4\pi\varepsilon_0}\frac{Qq}{2a}\int_{-a}^{a}\frac{\mathrm{d}y}{l^2+y^2}\frac{l}{\sqrt{l^2+y^2}} \\
&= \frac{Qql}{8\pi\varepsilon_0 a}\frac{2a}{l^2\sqrt{a^2+l^2}} \\
&= \frac{Qq}{4\pi\varepsilon_0 l\sqrt{a^2+l^2}}; \\
F_y &= \int_{-a}^{a}\frac{1}{4\pi\varepsilon_0}\frac{q\mathrm{d}Q}{l^2+y^2}\sin\theta = 0
\end{aligned}
$$



$$\mathrm{d}Q = \lambda\mathrm{d}y = \frac{Q}{2a}\mathrm{d}y$$

So,

$$\mathbf{F} = \frac{Qq}{4\pi\varepsilon_0 l\sqrt{a^2+l^2}}\mathbf{e}_x$$

*The **Static** Electric Force is a Conservative Force

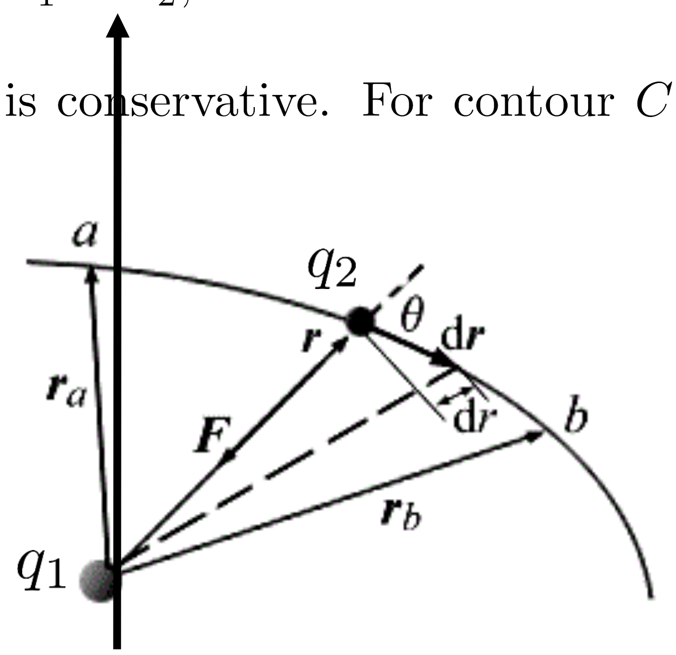
If the work done by a force on a particle does not depend on the trajectory of the particle, i.e.,

$$W = \int_{C_1} \mathbf{F} \cdot \mathrm{d}\mathbf{r} = \int_{C_2} \mathbf{F} \cdot \mathrm{d}\mathbf{r}$$

where C_1 and C_2 are different paths going from $\mathbf{r}_1$ to $\mathbf{r}_2$, the force $\mathbf{F}$ is called a **conservative force**.

Now we prove that the static electric force is conservative. For contour C that begins at $\mathbf{r}_a$ and ends at $\mathbf{r}_b$,

$$\begin{aligned} W &= \int_{r_a}^{r_b} \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^3} \mathbf{r} \cdot \mathrm{d}\mathbf{r} \\ &= \frac{q_1 q_2}{4\pi\varepsilon_0} \int_{r_a}^{r_b} \frac{r\cos\theta |\mathrm{d}\mathbf{r}|}{r^3} = \frac{q_1 q_2}{4\pi\varepsilon_0} \int_{r_a}^{r_b} \frac{\mathrm{d}r}{r^2} \\ &= -\frac{q_1 q_2}{4\pi\varepsilon_0} \left(\frac{1}{r_b} - \frac{1}{r_a}\right). \end{aligned}$$



Therefore, the work done by the Coulomb force depends only on the magnitude r but not the trajectory of the path. The Coulomb force is conservative.

Conservation of Angular Momentum

We knew from last semester that **angular momentum is conserved for a system with no external torque**. Since torque

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F},$$

so for any centripetal force $\mathbf{F} = F\mathbf{e}_r,$

$$\boldsymbol{\tau} = F\mathbf{r} \times \mathbf{e}_r \equiv \mathbf{0}.$$

只有静电力的系统角动量守恒

Therefore, **angular momentum is conserved for any system with only centripetal force, including the electric force exerted by a single particle**.

$$\mathbf{L} = \mathbf{r} \times \mathbf{p} = m\mathbf{r} \times \mathbf{v} = \text{const.}$$

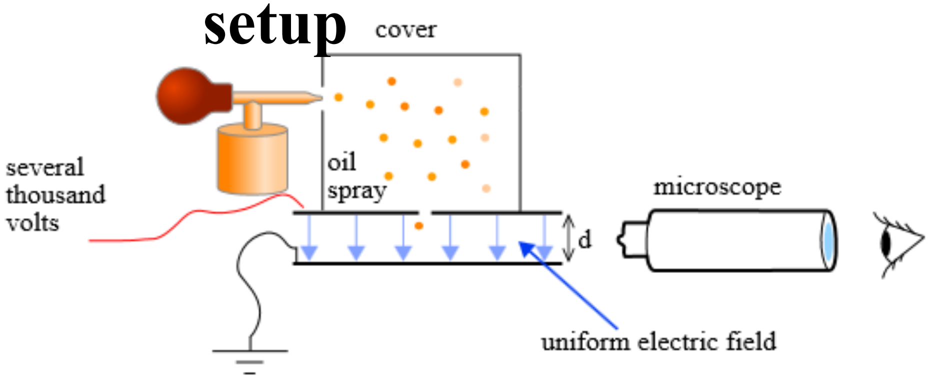
The magnitude of angular momentum (scalar) follows

$$mr_1v_1 \sin\phi_1 = mr_2v_2 \sin\phi_2$$

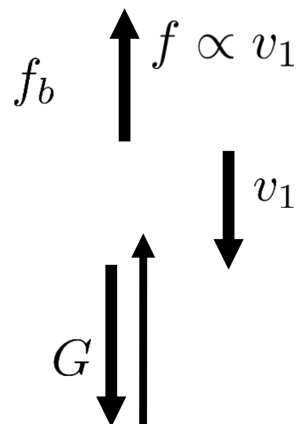
where *f* is the angle measured from **e***r* to **e***v*.

Millikan Oil Drop Experiment - **Charge is quantized**

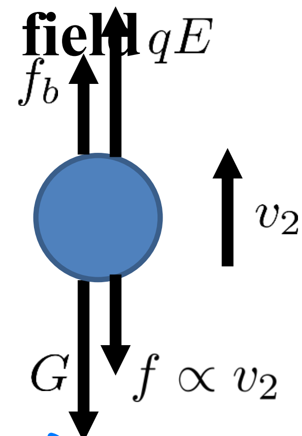
Experimental setup



No E-field



E-field



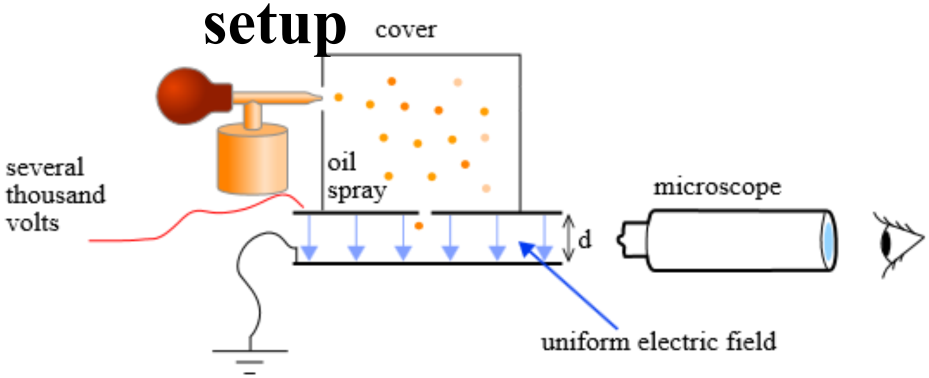
Millikan oil drop experiment measures the electric charge of tiny oil drops in order to confirm that every amount of charge is an integer multiple of some fundamental value.

When no electric field is present, the oil drops travel at a constant downward velocity $v1$ due to the force equilibrium between the gravity $G = mg$, air friction $f = 6\pi r\eta v_1$, and air buoyant force $f_b = (4\pi/3)r^3\rho_{\rm air}g$,

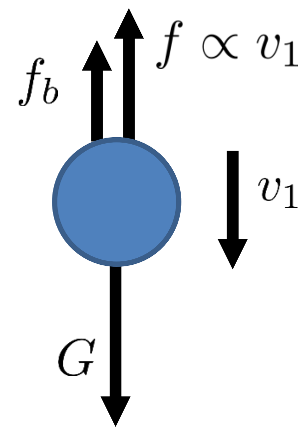
$$f = mg - f_b \Rightarrow 6\pi r\eta v_1 = \frac{4\pi}{3}r^3(\rho_{\rm oil} - \rho_{\rm air})g \Rightarrow r^2 = \frac{9\eta v_1}{2(\rho_{\rm oil} - \rho_{\rm air})g}.$$

Millikan Oil Drop Experiment - **Charge is quantized**

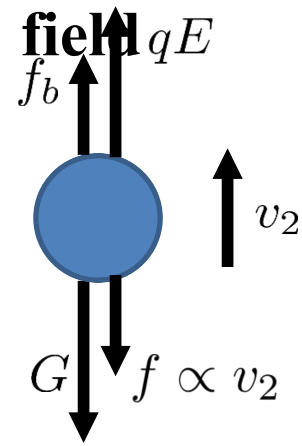
Experimental setup



No E-field



E-field



When a downward electric field is present, the oil drops travel at a constant upward velocity *v*2 due to the force equilibrium between the electric force *qE*, gravity *G*, air friction $f = 6\pi r\eta v_2$, and air buoyant force *fb*. now,

$$6\pi r\eta v_2 = qE - \frac{4\pi}{3}r^3(\rho_{\rm oil} - \rho_{\rm air})g.$$

Combining the two equations, we obtain the magnitude of the charge:

$$q = (1 + \frac{v_2}{v_1})\frac{9\sqrt{2}\pi\eta v_1}{E}\sqrt{\frac{\eta v_1}{(\rho_{\rm oil} - \rho_{\rm air})g}}.$$

Millikan Oil Drop Experiment - **Charge is quantized**

Example of raw data by Millikan

Drop No. 14.

t_g	t_F	$\frac{1}{t_F}$	n'	$\frac{1}{n'}\left(\frac{1}{t'_F}-\frac{1}{t_F}\right)$	n	$\frac{1}{n}\left(\frac{1}{t_g}+\frac{1}{t_F}\right)$	
18.606							
18.732							
18.784							
18.700	46.172	.02163			11	.006820	
18.730	17.896		5	.006874			V_i = 5077
18.652	17.818	.05600			16	.006833	V_f = 5073
18.656	46.328		5	.006886			t = 23.09° C.
18.730	46.258	.02157			11	.006815	p = 75.28
18.760	46.266		1	.006803			v_1 = .05451
18.708	67.473						a = .0002185
18.658	67.148	.01484			10	.006823	l/a = .04348
18.668	67.148		6	.006840			e_1 = **5.064**
18.826	17.896	.05588			16	.006831	
18.710	15.868	.06305			17	.006850	
18.802	15.854		9	.006853			
18.778	730.0	.001370			8	.006845	
18.790	23.376	.04266	6	.006882	14	.006861	
18.846	23.504		4	.006850			
18.804	65.416	.01526			10	.006865	
18.662	118.970	.008389	1	.006871	9	.006864	
18.704	622.8	.001605	1	.006784	8	.006874	
18.730				**.006850**		**.006844**	

R. A. Millikan, *Phys. Rev.* **2**, 109 (1913)

In Millikan's paper, a total of 58 drops are very carefully studied. Based on the data, he concluded that the elementary charge constant is

e = 4.774(5) ×10-10 esu
= 1.5924(17)×10−19 C

In 2019, the elementary charge is set to be the *exact* number of

$\boldsymbol{e}$ **= 1.602176634×10−19 C**

This is the charge of an electron.

Charge is Conserved

Electric charge is conserved: the net charge of any *isolated* system can not change.

Mathematically, we can state the law as a continuity equation:

$$Q(t_2) = Q(t_1) + Q_{\mathrm{IN}} - Q_{\mathrm{OUT}}.$$

where $Q(t)$ is the quantity of electric charge in a specific volume at time t, Q_{IN} (Q_{OUT}) is the amount of charge flowing into (out of) the volume between time t_1 and t_2.

Experimental evidences

Charge non-conserving processes has never been found:

$e^- \to \nu + \gamma$ mean lifetime is greater than 4.6×10^{26} years;
$n \to p^+ + \nu + \bar{\nu}$ chances less than 8×10^{-27}.

Homework

1. **Problems 21-5, 21-10, 21-24**.
2. **Bonus problem**:

A *semi-infinite* chain of evenly separated charged particles contain alterative amount of charges $+q$, $-q$, $+q$, $-q$, … The separation between any two neighboring charges is a (see Figure). Find the net electric force felt by Charge 1 at the left end of the chain. (Hint: use known algebraic sums)