南方科技大学
SOUTHERN UNIVERSITY OF SCIENCE AND TECHNOLOGY

Course Name: College Physics II Dept.: Physics

Exam Duration: 2 hours Exam Paper Setter: Physics Teaching Team

Question No.	1	2	3	4	5	6	7	8	9	10
Score	36	10	12	12	12	12	6			

This exam paper contains __7__questions and the score is _100_ in total. (Please hand in your exam paper, answer sheet, and your scrap paper to the proctor when the exam ends.)

Q1. Multiple Choice Questions (3 points each and only one correct answer for each question.)

1~5 C A C C B 6~10 D C D C C 11~12 B D

Long Questions: (Please show the solving process in detail)

Q2. (10 points)

a) Consider a differential element having arc length ds, the (infinitesimal) charge on an element *ds* of the rod contains charge $dq = \lambda\, ds$. 1 point.

The element produces a differential electric field at the origin. From the symmetry analysis, we know the electric field sets up by the arc is along the symmetrical axis.

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda ds}{r^2}. \quad \text{1 point} \qquad dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda r d\theta}{r^2} \quad \text{1 point}$$

$$dE_x = \frac{1}{4\pi\varepsilon_0}\frac{\lambda r d\theta}{r^2}\cos\theta \quad \text{1 point}$$

$$E_{arc} = \int_{-90^\circ}^{90^\circ}\frac{\lambda R d\theta}{4\pi\varepsilon_0 R^2}\cos\theta \quad \text{1 point}$$

$$= \frac{\lambda}{4\pi\varepsilon_0 R}(2\sin 90^\circ) = \frac{\lambda}{2\pi\varepsilon_0 R} = \frac{Q}{2\pi^2\varepsilon_0 R^2} = \frac{2kQ}{\pi R^2} = 7.14\times 10^4 \text{ N/C} \quad \text{2 points}$$

b) $$V = \frac{Q}{4\pi\varepsilon_0 R} = 2.2\times 10^{3} \quad \text{3 points}$$

Q3. (12 points)

a) −5.00 μC (inner surface), −3.00 μC (outer surface) 4 points

b) $$\begin{cases} r > R_3 & E = \dfrac{q_{out}}{4\pi\varepsilon_0 r^2} \\ R_2 < r < R_3 & E = 0 \\ R_1 < r < R_2 & E = \dfrac{q}{4\pi\varepsilon_0 r^2} \\ r < R_1 & E = 0 \end{cases}$$

$$r = 2R_3 = 60\ \text{cm} \Rightarrow E = \frac{q_{out}}{4\pi\varepsilon_0 r^2} = \frac{-3\times10^{-6}}{4\pi\times8.85\times10^{-12}\times0.6^2} = -7.5\times10^{4}$$ 3 points

$$r = R_1/2 \Rightarrow V = \frac{q_{out}}{4\pi\varepsilon_0 R_3} + \frac{q}{4\pi\varepsilon_0 R_1} - \frac{q}{4\pi\varepsilon_0 R_2}$$

$$= \frac{-3\times10^{-6}}{4\pi\times8.85\times10^{-12}\times0.2} + \frac{5\times10^{-6}}{4\pi\times8.85\times10^{-12}\times0.05} - \frac{5\times10^{-6}}{4\pi\times8.85\times10^{-12}\times0.15} = 4.6\times10^{5}\ V$$

3 points

c) The location of the inner sphere does not affect the charge distribution on the outer surface of the conducting shell, so does the electric field for the region $r > R_3$. And the conducting shell is a equipotential body, so the potential at the inner surface is exactly the potential at out surface, so we have:

$$V = \frac{q_{out}}{4\pi\varepsilon_0 R_3} = \frac{-3\times10^{-6}}{4\pi\times8.85\times10^{-12}\times0.2} = -1.3\times10^{5}\ V$$ 2 points

Q4. (12 points)

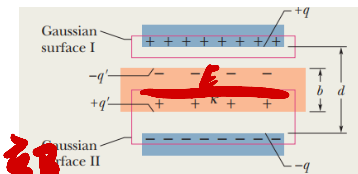
a) $$C_0 = \frac{\varepsilon_0 A}{d} = \frac{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(0.12\ \text{m})^2}{1.2\times10^{-3}\ \text{m}} = 0.885\ \text{nF}$$ 2 points

$q = C_0 V = (0.885\ \text{nF})(100\ \text{V}) = 88.5\ \text{nC}.$ 2 points

极板上储存是q

(b) Assume the charge on the positive plate is q and then the electric field in the region between the vacuum and the dielectric and the electric field inside the dielectric can be got from Gauss' law.

From the Gaussian surface II, we have



Gaussian surface II: $\kappa E A = \dfrac{q}{\varepsilon_0} \Rightarrow E_2 = \dfrac{q}{\kappa A \varepsilon_0}$ 3 points

只改变电介质内部电场

$$E_2 = \frac{q}{\kappa A \varepsilon_0} = \frac{88.5\times10^{-9}}{20\times0.12\times8.85\times10^{-12}} = 4.16\times10^3 \text{ V/m}$$

1 point

c) The potential difference between the two plates is:

$$V = E_1(d-b) + E_2 b$$

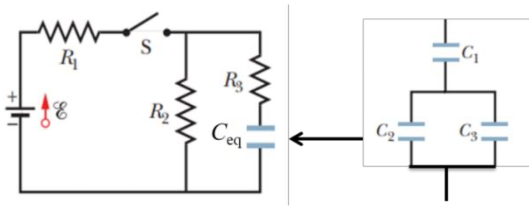
And the capacitance of the capacitor is given by the definition:

$$C = \frac{q}{V} = \frac{q}{E_1(d-b)+E_2 b} = \frac{q}{\frac{q}{A\varepsilon_0}(d-b)+\frac{q}{\kappa A\varepsilon_0}b} = \frac{\kappa A\varepsilon_0}{\kappa(d-b)+b}$$ 3 points

$$C = \frac{\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)(0.12\ \text{m}^2)(20)}{(20)(1.2-0.40)\times10^{-3}\,\text{m}+0.4\times10^{-3}\,\text{m}} = 1.30\text{nF}$$ 1 point

Q5. (12 points)

Solution:



a) $$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2+C_3} = \frac{1}{C} + \frac{1}{2C} \Rightarrow C_{eq} = \frac{2}{3}C = 4.0\ \mu\text{F}$$ 3 points

b) When the capacitors are fully charged, there is no current in equivalent capacitor, so the potential difference across C_{eq} is the potential difference across R_2

$$V_{eq} = R_2\frac{\varepsilon}{R_1+R_2} = \frac{\varepsilon}{2}$$ 2 points

The charge on C_{eq} is

$$Q_{eq} = C_{eq}V_{eq} = \left(\frac{2}{3}C\right)\left(\frac{\varepsilon}{2}\right) = \frac{C\varepsilon}{3} = 4.$$ 3 points

c) From the combination of the capacitors, we know $q_2 = q_{eq}/2$ 1 point

For the discharging circuit, we have:

$$q_2 = \frac{q_{eq}}{2} = \frac{1}{2}Q_{eq}e^{-t/2R\frac{2C}{3}} = \frac{C\varepsilon}{6}e^{-3t/4RC} = 2e^{-t/8}\ \text{mC}$$ 3 points

Q6. (12 points)

a) We choose the concentric circle as the Ampere's loop:

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc} \Rightarrow B2\pi r = \mu_0 I \frac{\pi r^2}{\pi R^2} \Rightarrow B = \frac{\mu_0 I r}{2\pi R^2} \quad r<R$$ 3 points

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc} \Rightarrow B2\pi r = \mu_0 I \Rightarrow B = \frac{\mu_0 I}{2\pi r} \quad r>R$$ 4 points

Ampere's loop　1 point

b) $$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc} \Rightarrow B2\pi r = \mu_0 (I + \int_a^r \frac{A}{r} 2\pi r dr)$$ 3 points

$$\Rightarrow B = \frac{\mu_0 I}{2\pi r} + \frac{\mu_0 2\pi A(r-a)}{2\pi r} = \text{constant} \Rightarrow A = \frac{I}{2\pi a}$$ 0.5 point

Positive A indicates that the current density is out of paper　0.5 point

Q7. Blank-filling Questions. (Complete each of the following questions according to your calculation)

1. (3 points).

$$V_{z=0} = \int_A dV = \int_A \frac{\sigma dA}{4\pi\varepsilon_0 R} = \frac{\sigma 2\pi R^2}{4\pi\varepsilon_0 R} = \frac{\sigma R}{2\varepsilon_0}$$

$$V_{z=R} = \int_A dV = \int_0^{\pi/2} \frac{2\pi R^2 \sigma \sin\theta d\theta}{4\pi\varepsilon_0 (R^2 + 2R^2\cos\theta + R^2)^{1/2}} = -\frac{2\pi\sigma d(2R^2 + 2R^2\cos\theta)}{8\pi\varepsilon_0 (2R^2 + 2R^2\cos\theta)^{1/2}}$$

$$= -\frac{\sigma}{2\varepsilon_0}(2R^2 + 2R^2\cos\theta)^{1/2}\Big|_0^{\pi/2} = \frac{\sigma R}{2\varepsilon_0}(2-\sqrt{2})$$

$$\Delta K = 0 \Rightarrow \Delta U = 0 \Rightarrow \Delta V = 0 \Rightarrow \frac{\sigma R}{2\varepsilon_0} - \frac{\rho d}{2\varepsilon_0}(R) = \frac{\sigma R}{2\varepsilon_0}(2-\sqrt{2})$$

$$\Rightarrow \sigma = \frac{\rho d(R)}{(\sqrt{2}-1)R} = \frac{R\rho}{4(\sqrt{2}-1)} = 0.60\rho R$$

3. (3 points).

$$\Delta V = V_{\text{outer}} - V_{\text{inner}} = -\frac{\lambda_{in}}{2\pi\varepsilon_0}\ln\frac{b}{a} \Rightarrow \lambda_{in}$$

$$\sigma_{in} = \frac{\lambda_{in}}{2\pi a} = -\frac{\varepsilon_0 \Delta V}{(\ln 2)(a)}$$

$$i = \frac{q}{T} = \frac{\sigma 2\pi r L}{2\pi / \omega} = \sigma\omega r L$$

$$BL = \mu_0 i_{enc} = \mu_0(\sigma_{in}\omega a L + \sigma_{out}\omega b L) \Rightarrow B = \mu_0(\sigma_{in}\omega a + \sigma_{out}\omega b) = \mu_0\omega(-\frac{\varepsilon_0 \Delta V}{\ln 2} + \sigma_{out} b)$$

$$= 6.5\times 10^{-11}\ \text{T}$$

$6.5 \pm 0.1 \times 10^{-11}$ T