Key words

Electric Potential

Electric potential energy

Equipotential surface

Electron-volts

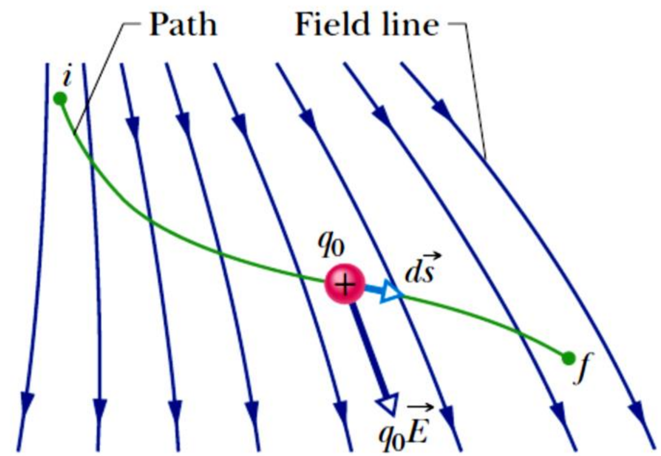
Isolated conductor

Non-conductor

Insulator

Electric Potential

$$\Delta U = U_f - U_i = -W = -q_0 \int_i^f \vec{E} \cdot d\vec{s}$$



$$\boxed{\text{Define V} = \frac{U}{q_0} \Rightarrow V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}}$$

无穷大平面无穷远处不可当V=0

无穷远处看就不是无穷大 还是场强有

等一段E积分

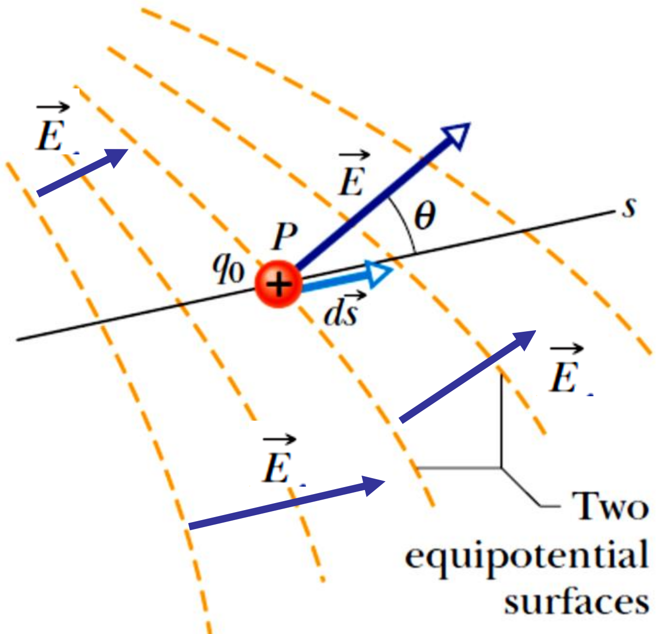
If i is at ∞ where U is 0, V_i *is 0.*

$$V = -\int_\infty^f \vec{E} \cdot d\vec{s} \qquad \text{For point charge:} \quad V = \frac{1}{4\pi\varepsilon_0} \frac{q}{r}$$

Calculating E from V

$$\Delta V = -\int_i^f \vec{E} \cdot d\vec{s}$$

$$dV = -\vec{E} \cdot d\vec{s} = -E_s ds$$

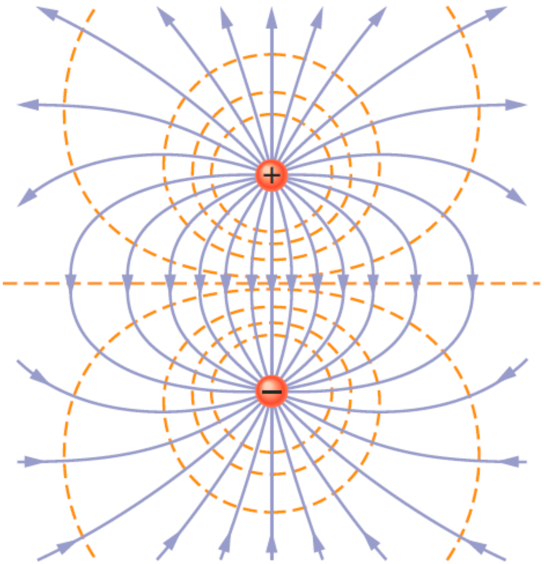


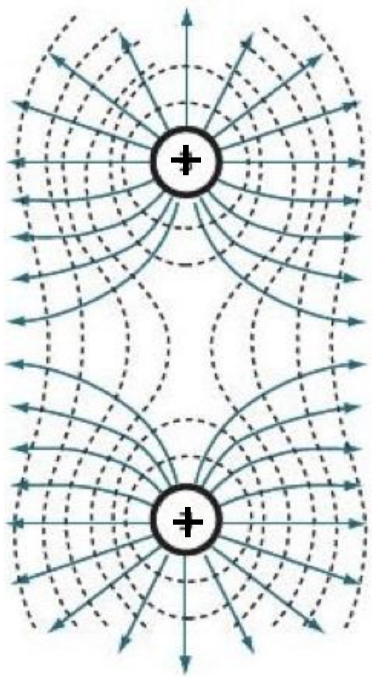
$$E_s = -\frac{\partial V}{\partial s}$$ **The s component of E vector**

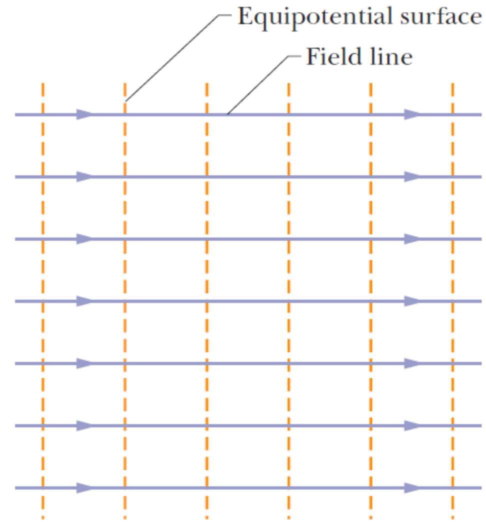
E_s is the negative rate of potential along s direction

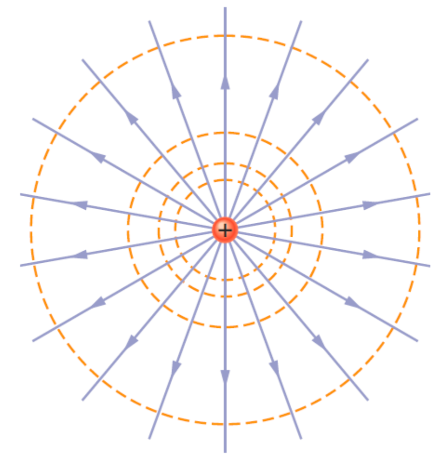
$$E_x = -\frac{\partial V}{\partial x} \qquad E_y = -\frac{\partial V}{\partial y} \qquad E_z = -\frac{\partial V}{\partial z} \qquad \text{(components of } \vec{E} \text{ in terms of } V\text{)}$$

Equipotential Surfaces and the Electron Field









E field lines must be always along the normal direction of the Equipotential surfaces， pointing from higher potential to lower potential

Methods to calculate V due to given charge distribution

Method 1: Using the principle of superposition:

Net potential for discrete charged particles is the **scalar sum** of individual potential

$$V_{\text{net}} = \sum_i V_i = \frac{1}{4\pi\varepsilon_0} \sum_i \frac{q_i}{r_i}$$

When charges are distributed continuously, the sum changes to an integral.
Usually, for charge with regular shape:

$$V = \int \mathrm{d}V = \frac{1}{4\pi\varepsilon_0} \int \frac{\mathrm{d}q}{r}$$

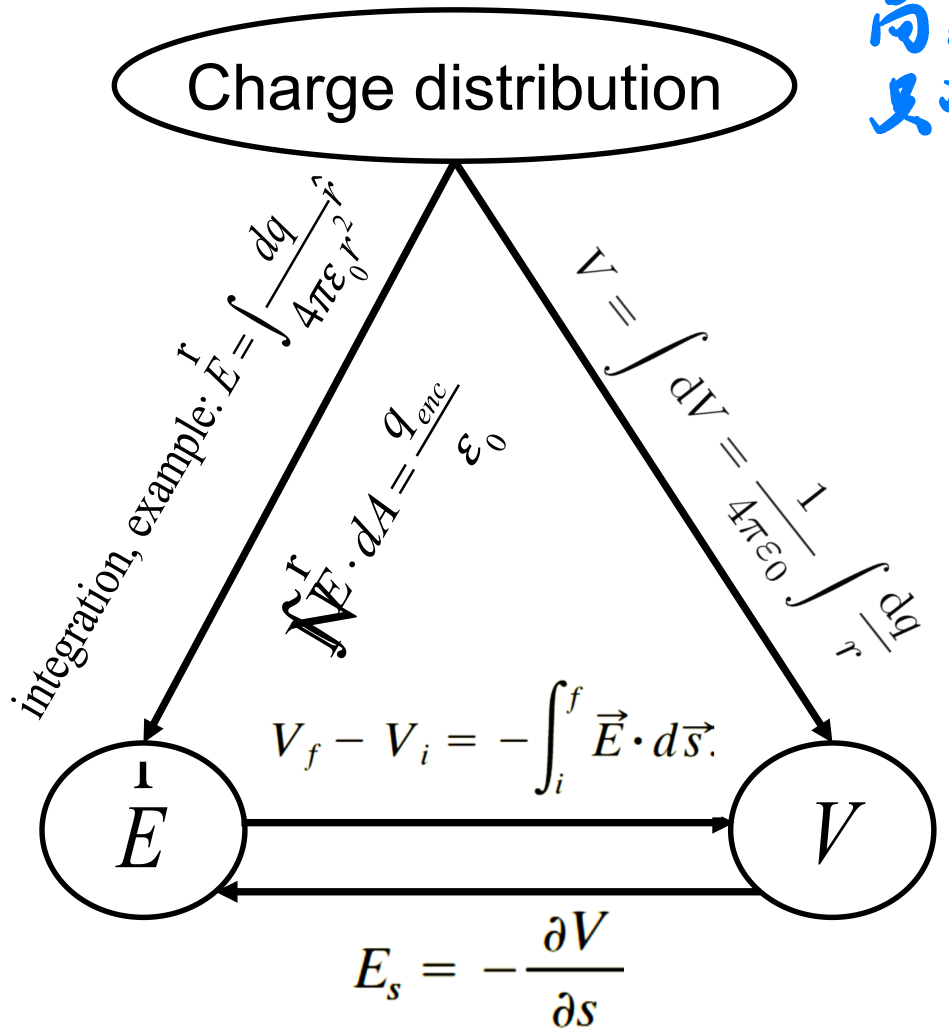
Method 2: Calculate V from E field:

Usually for the charge with special symmetry such that it's easy to use gauss's law to calculate the E field

有时E可高斯定律

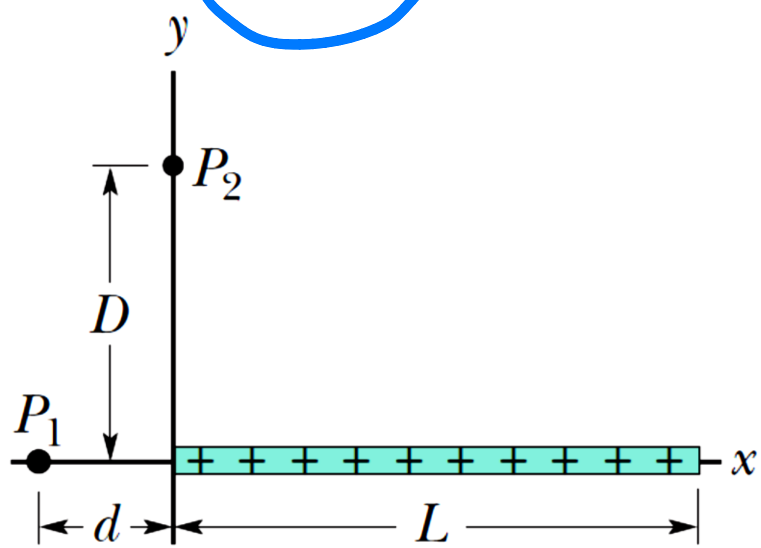
$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}. \qquad V = -\int_\infty^f \vec{E} \cdot d\vec{s}$$

高斯定律特殊：
只可3种对称性及其叠加



Problem 1

The thin plastic rod of length L = 12.0 cm has a **nonuniform linear charge density $\lambda = cx$**, where c = 49.9 pC/m^2.



$dq = \lambda dx$

(a) With V = 0 at infinity, find the electric potential at point P_2 on the y axis at $y = D$ = 3.56 cm.

(b) Find the electric field component E_y at P_2.

E可求偏导

$E_y = -\dfrac{\partial V}{\partial y}$

Problem 1

(a) Consider an charge element of the rod from x to $x + dx$. Its contribution to the potential at point P_2 is

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{\lambda(x)dx}{\sqrt{x^2+y^2}} = \frac{1}{4\pi\varepsilon_0}\frac{cx}{\sqrt{x^2+y^2}}dx.$$

$$V = \int_{\text{rod}} dV_P = \frac{c}{4\pi\varepsilon_0}\int_0^L \frac{x}{\sqrt{x^2+y^2}}dx = \frac{c}{4\pi\varepsilon_0}\left(\sqrt{L^2+y^2}-y\right)$$

$$= (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(49.9\times10^{-12}\,\text{C/m}^2)\left(\sqrt{(0.120\ \text{m})^2+(0.0356\ \text{m})^2}-0.0356\ \text{m}\right)$$

$$= 4.02\times10^{-2}\ \text{V}.$$

(b) The y component of the field is 公式仍适用

$$E_y = -\frac{\partial V_P}{\partial y} = -\frac{c}{4\pi\varepsilon_0}\frac{d}{dy}\left(\sqrt{L^2+y^2}-y\right) = \frac{c}{4\pi\varepsilon_0}\left(1-\frac{y}{\sqrt{L^2+y^2}}\right)$$

$$= (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(49.9\times10^{-12}\,\text{C/m}^2)\left(1-\frac{0.0356\ \text{m}}{\sqrt{(0.120\ \text{m})^2+(0.0356\ \text{m})^2}}\right)$$

$$= 0.321\ \text{N/C}.$$

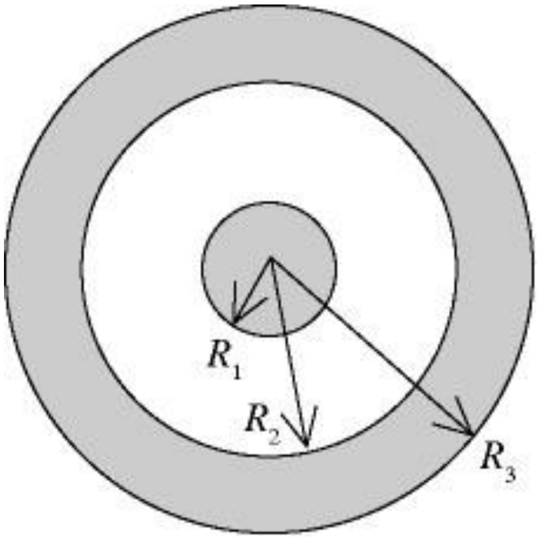
Problem 2

Two concentric spheres are shown in the figure. The inner sphere is a solid **nonconductor** and has a positive uniform volume charge density $15/2\pi\ \mu\text{C/m}^3$. The outer sphere is a **conducting shell** that carries a net charge of $Q = -5.00\ nC$. No other charges are present. The radii shown in the figure have the values $R_1 =$ 10.0 cm, $R_2 = 20.0$ cm, and $R_3 = 30.0$ cm. ($k = 1/4\pi\varepsilon_0 = 8.99 \times 10^9\ \text{N} \cdot \text{m}^2/\text{C}^2$)

(a) Find the total excess charge on the inner nonconductor, inner and outer surfaces of the conducting sphere.

(b) Find electric field magnitude and direction (pointing outward or inward) of the electric field E and the potential V (With $V = 0$ at infinity) at the following distances r from the center of the inner sphere: (i) $r = 60cm$, (ii) $r=15cm$, (iii) $r=5cm$.

(c) Sketch $E(r)$ and $V(r)$.



Problem 2

Solution: a) Inner nonconducting sphere: $q = \rho V = 10nC$

Inner conducting surface: $q_{in} = -q = -10nC$

Outer conducting surface: $q_{out} = Q - q_{in} = Q + q = 5nC$

b) According to Gauss' law: $\boxed{\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{\text{enc}} \quad \text{(Gauss' law)}}$

$$
\begin{cases}
r > R_3 & E = \dfrac{q + q_{in} + q_{out}}{4\pi\varepsilon_0 r^2} = \dfrac{q_{out}}{4\pi\varepsilon_0 r^2} \\[2ex]
R_2 < r < R_3 & E = \dfrac{q + q_{in}}{4\pi\varepsilon_0 r^2} = 0 \\[2ex]
R_1 < r < R_2 & E = \dfrac{q}{4\pi\varepsilon_0 r^2} \\[2ex]
r < R_1 & E 4\pi r^2 = \dfrac{\rho 4\pi r^3}{3\varepsilon_0} \Rightarrow E = \dfrac{\rho r}{3\varepsilon_0}
\end{cases}
$$

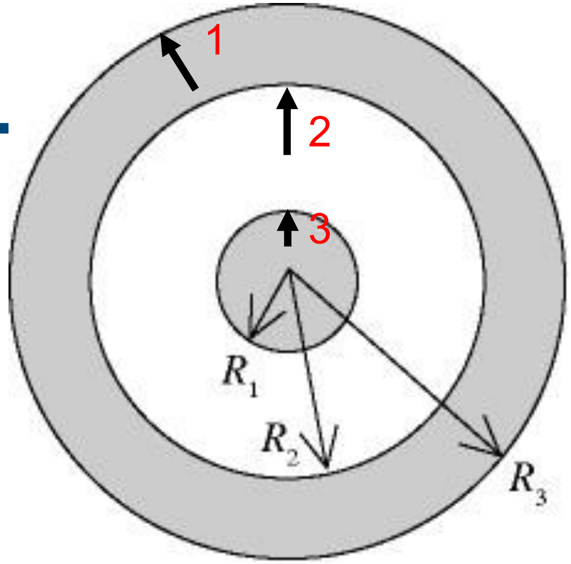
Problem 2

$$V_r = -\int_\infty^r E\cdot dr = -\int_\infty^r E\cdot dr$$

$$\begin{cases}
r > R_3 & V_r = -\int_\infty^r \frac{q+q_{in}+q_{out}}{4\pi\varepsilon_0 r^2}dr = \frac{q+q_{in}+q_{out}}{4\pi\varepsilon_0 r} = \frac{q_{out}}{4\pi\varepsilon_0 r} \\
R_2 < r < R_3 & V_r = -\left(\int_\infty^{R_3} \frac{q+q_{in}+q_{out}}{4\pi\varepsilon_0 r^2}dr + \int_{R_3}^r (0)dr\right) = \frac{q_{out}}{4\pi\varepsilon_0 R_3} \\
R_1 < r < R_2 & V_r = -\left(\int_\infty^{R_3} \frac{q_{out}}{4\pi\varepsilon_0 r^2}dr + \int_{R_3}^{R_2}(0)dr + \int_{R_2}^r \frac{q}{4\pi\varepsilon_0 r^2}dr\right) \\
 & = \frac{q_{out}}{4\pi\varepsilon_0 R_3} + \frac{q}{4\pi\varepsilon_0 r} - \frac{q}{4\pi\varepsilon_0 R_2} \\
r < R_1 & V_r = -\left(\int_\infty^{R_3} \frac{q_{out}}{4\pi\varepsilon_0 r^2}dr + \int_{R_3}^{R_2}(0)dr + \int_{R_2}^{R_1} \frac{q}{4\pi\varepsilon_0 r^2}dr + \int_{R_1}^r \frac{\rho r'}{3\varepsilon_0}dr'\right) \\
 & = \frac{q_{out}}{4\pi\varepsilon_0 R_3} + \frac{q}{4\pi\varepsilon_0 R_1} - \frac{q}{4\pi\varepsilon_0 R_2} - \int_{R_1}^r \frac{\rho r'}{3\varepsilon_0}dr' \\
 & = \frac{q_{out}}{4\pi\varepsilon_0 R_3} + \frac{q}{4\pi\varepsilon_0 R_1} - \frac{q}{4\pi\varepsilon_0 R_2} + \frac{1}{6\varepsilon_0}\rho(R_1^2 - r^2)
\end{cases}$$

分层

Problem 2



$$V_r = -\int_\infty^r E \cdot dr = -\int_\infty^r E \cdot dr$$

$r > R_3$: $V_r = -\int_\infty^r \frac{q + q_{in} + q_{out}}{4\pi\varepsilon_0 r^2} dr = \frac{q_{out}}{4\pi\varepsilon_0 r}$

$R_2 < r < R_3$: for path 1: $V_f - V_i = -\int_i^f \vec{E}\cdot d\vec{s} \Rightarrow V_{R_3} - V_r = -\int_r^{R_3} 0 \cdot ds \Rightarrow V_r = V_{R_3} = \frac{q_{out}}{4\pi\varepsilon_0 R_3}$

$R_1 < r < R_2$: for path 2: $V_{R_2} - V_r = -\int_r^{R_2} \frac{q}{4\pi\varepsilon_0 r^2} dr \Rightarrow V_r = V_{R_2} + \int_r^{R_2} \frac{q}{4\pi\varepsilon_0 r^2} dr \Rightarrow V_r = \frac{q_{out}}{4\pi\varepsilon_0 R_3} + \frac{q_{in}}{4\pi\varepsilon_0 R_2} + \frac{q}{4\pi\varepsilon_0 r}$

$r < R_1$: for path 3: $V_{R_1} - V_r = -\int_r^{R_1} \frac{\rho r}{3\varepsilon_0} dr \Rightarrow V_r = V_{R_1} + \int_r^{R_1} \frac{\rho r}{3\varepsilon_0}$

$$= \frac{q_{out}}{4\pi\varepsilon_0 R_3} + \frac{q_{in}}{4\pi\varepsilon_0 R_2} + \frac{q}{4\pi\varepsilon_0 R_1} + \frac{\rho r^2}{6\varepsilon_0}(R_1^2 - r^2)$$

Problem 2

$$r=60cm \quad E=\frac{q+Q}{4\pi\varepsilon_0 r^2}=45\frac{1}{r^2}=125(N/C), \quad V=\frac{q+Q}{4\pi\varepsilon_0 r}=45\frac{1}{r}=75(V)$$

$$r=15cm \quad E=\frac{q}{4\pi\varepsilon_0 r^2}=90\frac{1}{r^2}=4000\ (N/C),$$

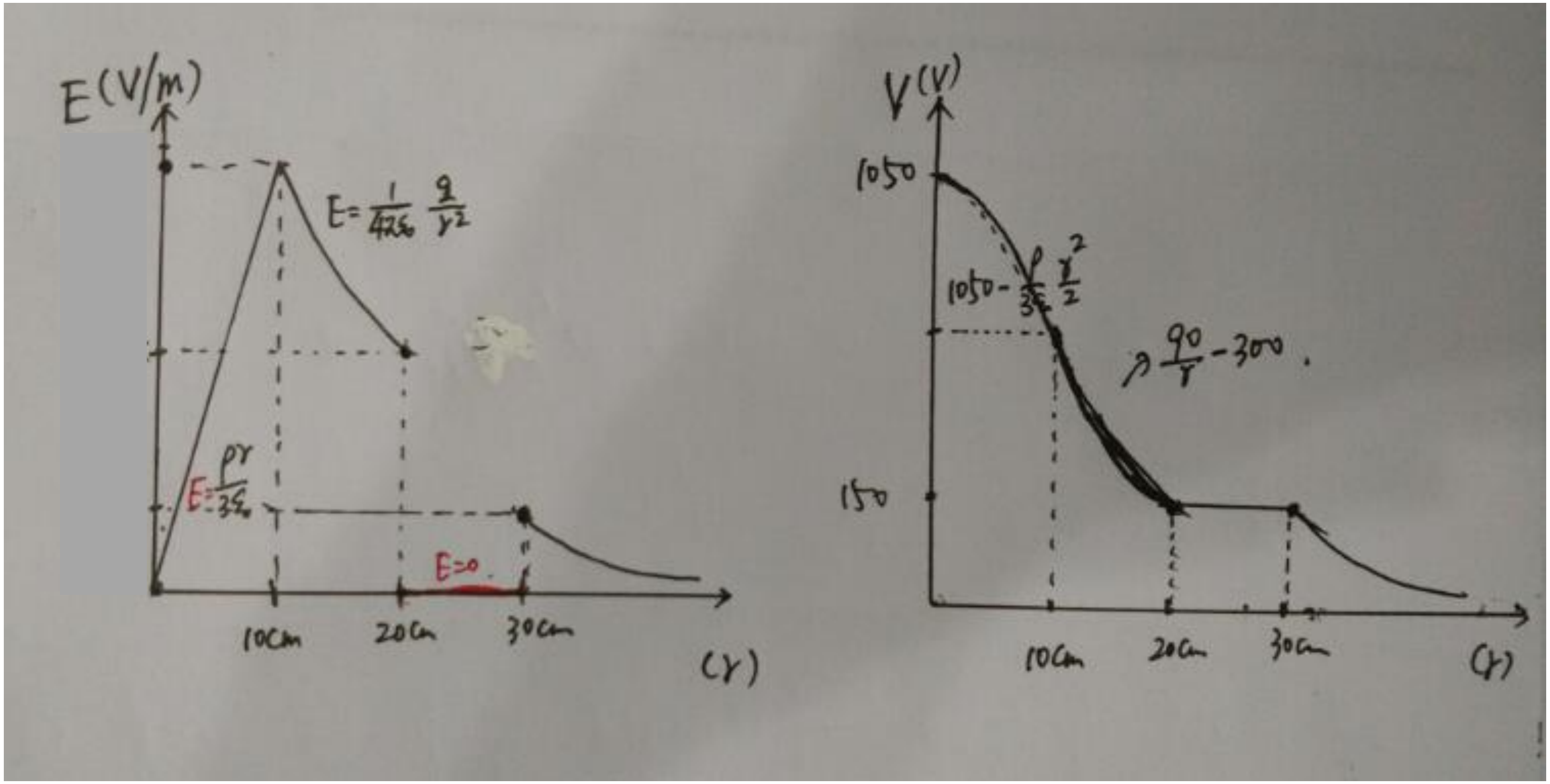
$$V=\frac{q}{4\pi\varepsilon_0 r}-\frac{q}{4\pi\varepsilon_0 R_2}+\frac{q_{out}}{4\pi\varepsilon_0 R_3}=90\frac{1}{r}-90\frac{1}{R_2}+45\frac{1}{R_3}=300V$$

$$r=5cm \quad 4\pi r^2 E=\frac{\rho 4\pi r^3}{3\varepsilon_0}\Rightarrow E=\frac{\rho r}{3\varepsilon_0}=8.99\times10^4 r=4495(N/C)$$

$$V=90\frac{1}{R_1}-90\frac{1}{R_2}+45\frac{1}{R_3}+4.45\times10^4(R_1^2-r^2)=930V$$

Problem 2

c)

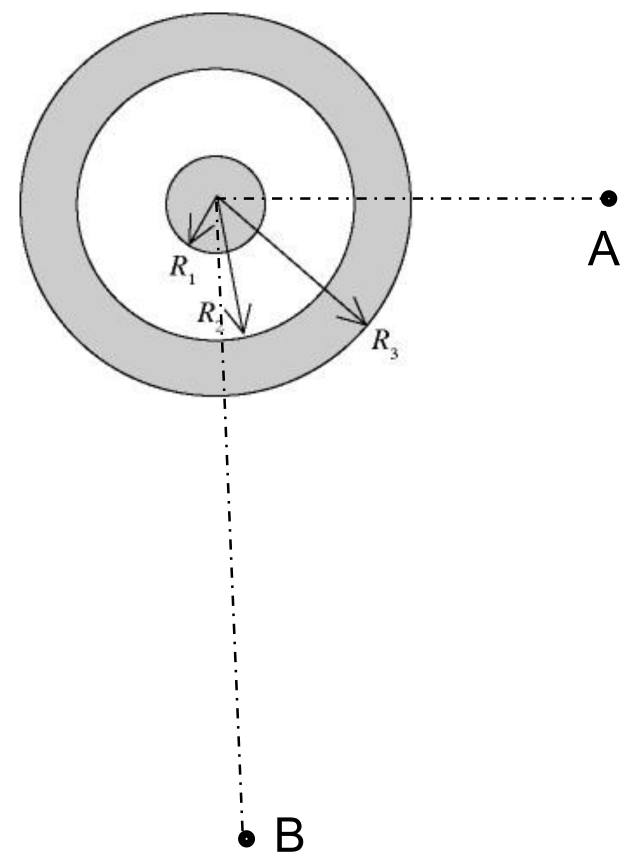


E(V/m)
$E=\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$
$E=\frac{\rho r}{3\varepsilon_0}$
E=0
10cm
20cm
30cm
(r)
V(V)
1050
$1050-\frac{\rho}{3\varepsilon_0}\frac{r^2}{2}$
$\frac{90}{r}-300$
150
10cm
20cm
30cm
(r)

Problem 2

d) As shown in figure, a particle of elementary charge +e is initially at a distance $r = 2R_3$ from the center of the spheres. The particle is then moved to a point B, at a distance $r = 4R_3$. What is the work done by the force moving the particle from point A to point B?

The electric field is a conservative field, so the work done by the field on the charge particle does not depend on the path, so does the external force, we apply the work-energy theorem, we have:



$$W_{ext} = U_f - U_i = eV_f - eV_i = e\left(\frac{q_{out}}{4\pi\varepsilon_0 R_B} - \frac{q_{out}}{4\pi\varepsilon_0 R_B}\right)$$

$$W_{ext} = \frac{eq_{out}}{4\pi\varepsilon_0}\left(\frac{1}{4R_3} - \frac{1}{2R_3}\right) = -\frac{eq_{out}}{16\pi\varepsilon_0 R_3}$$