Key words

Relativity 相对性

Postulates 假设

Inertial reference frame 惯性参考系

Event 事件

Time Dilation 时间膨胀

Length Contraction 长度收缩

Spatial separation 空间间隔

Temporal separation 时间间隔

Lorentz Transformation 洛仑兹变换

Doppler Effect 多普勒效应

Simultaneity 同时

Proper time (Δt_0) 固有时

Proper frequency (f_0) 固有频率

Proper length (l_0) 固有长度

Speed parameter (β) 速度参数

Lorentz factor (γ) 洛仑兹因子

Two postulates

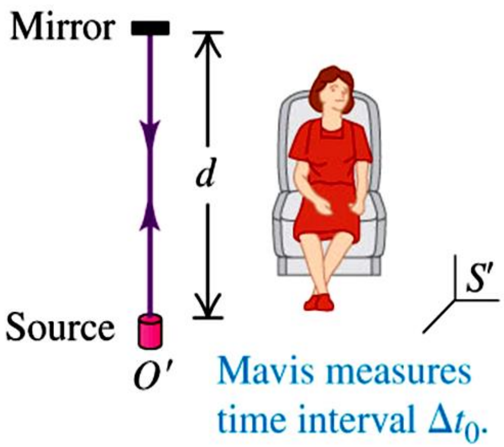
First postulate :

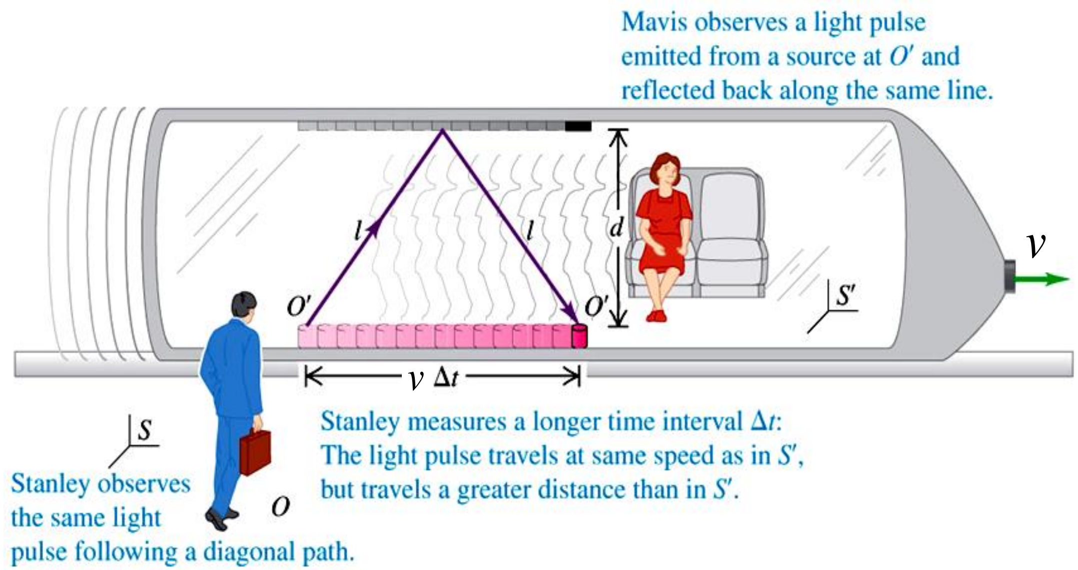
The laws of physics are the same for observers in all inertial reference frames (no acceleration). No one frame is preferred over any other

Second postulate :

The speed of light in vacuum has the same value c in all directions and in all inertial reference frames (is independent of the motion of the source)

Relativity of time intervals





$$\Delta t_0 = \frac{2d}{c}$$

$$l = \sqrt{d^2 + \left(\frac{v\Delta t}{2}\right)^2}$$

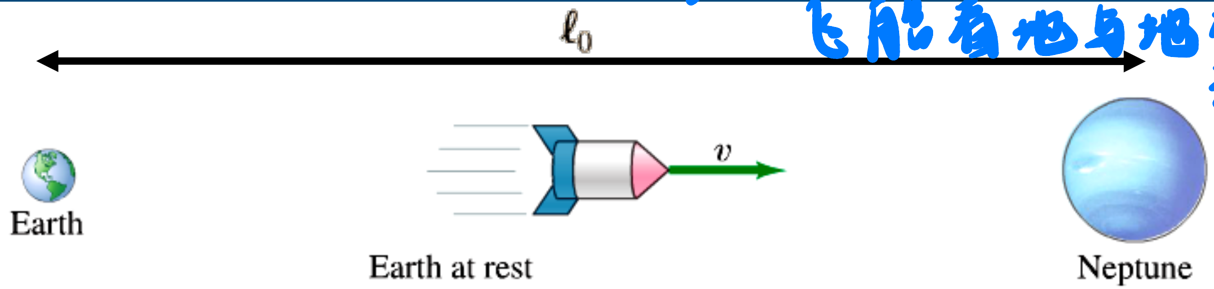
$$\Delta t = \frac{2l}{c} = \frac{2}{c}\sqrt{d^2 + \left(\frac{v\Delta t}{2}\right)^2}$$

Combine the 2 equations : $\boxed{\Delta t = \dfrac{\Delta t_0}{\sqrt{1 - v^2/c^2}}}$ Time dilation

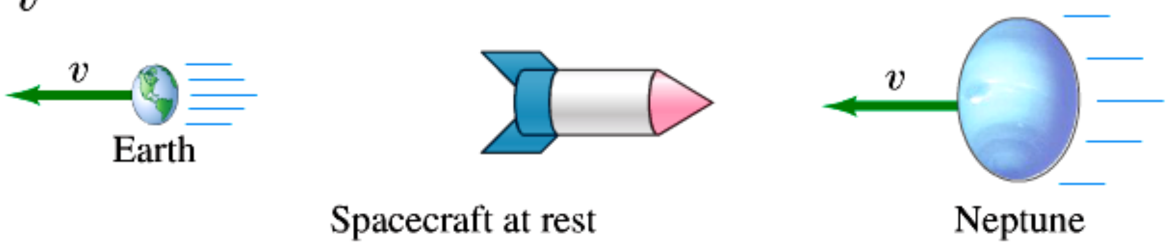
Proper time: When two events occur **at the same location** in an inertial reference frame, the time interval between them, measured in that frame, is called the proper time. Measurements of the time interval between the same two events from any other inertial reference frame are always greater.

Length contraction

孪生子佯谬：两个参考系对等，
飞船看地与地看飞船
都有时缓效应



$$\Delta t = \frac{\ell_0}{v}$$ Measured on earth



If I am in the spacecraft, the earth is leaving me at a speed of v and Neptune is coming into me at v

proper time: $$\Delta t_0 = \Delta t\sqrt{1 - v^2/c^2} = \Delta t/\gamma$$

the length I experience : $$\ell = v\,\Delta t_0 = v\,\Delta t\sqrt{1 - v^2/c^2} = \ell_0\sqrt{1 - v^2/c^2}$$

Proper length: The length of an object measured in the rest frame of the object is its proper length or rest length. Measurements of the length from any reference frame that is in relative motion parallel to that length are always less than the proper length

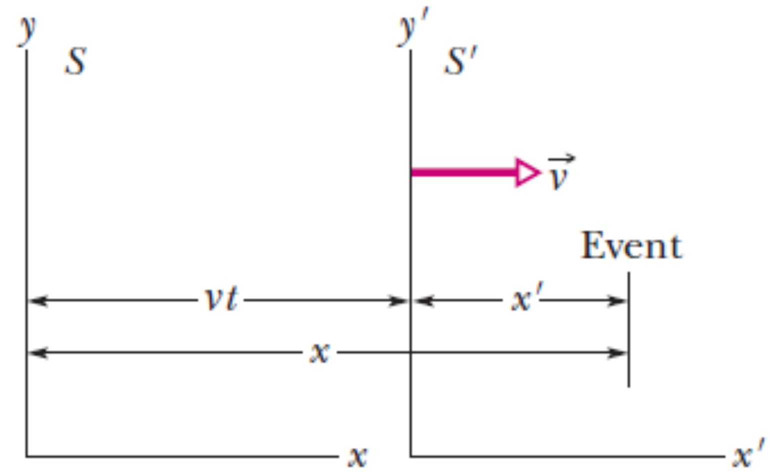
Galilean transformations VS Lorentz transformations

Galilean transformations:

$$x = x' + vt'$$

$$t = t'$$

$$u = u' + v$$



Lorentz transformations :

$$x = \gamma(x' + vt') \Rightarrow \Delta x = \gamma(\Delta x' + v\Delta t')$$

$$t = \gamma\left(t' + \frac{vx'}{c^2}\right) \Rightarrow \Delta t = \gamma\left(\Delta t' + \frac{v\Delta x'}{c^2}\right)$$

$$u = \frac{u' + v}{1 + vu'/c^2} \qquad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$x' = \gamma(x - vt); \quad t' = \gamma\left(t - \frac{vx}{c^2}\right); \quad u' = \frac{u - v}{1 - vu/c^2}$$

Consequences of the Lorentz transformation

Simultaneity

Two events happen at the same time in two different places **in the *S*' frame**. $\Delta x' \neq 0$, $\Delta t' = 0$.

$$\Delta t = \gamma\left(\Delta t' + \frac{v\Delta x'}{c^2}\right) \Longrightarrow \Delta t = \gamma\frac{v\Delta x'}{c^2} \neq 0$$

Time Dilation

Two events happen at different times at the same place **in the *S*' frame**. $\Delta x' = 0$, $\Delta t' \neq 0$ (proper time). In this case $\Delta t > \Delta t'$.

$$\Delta t = \gamma\left(\Delta t' + \frac{v\Delta x'}{c^2}\right) \Longrightarrow \Delta t = \gamma\Delta t'$$

Length Contraction

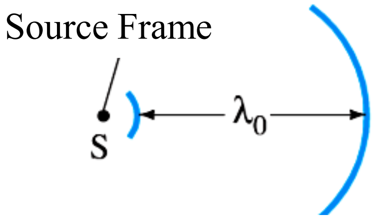
Suppose that the frame *S*' moves with the length. Then $\Delta x'$ is the proper length. Now Δx is the same length measured in Frame *S* given that the measurement is simultaneous in Frame *S*, $\Delta t = 0$.

$$\Delta x' = \gamma(\Delta x - v\Delta t) \Longrightarrow \Delta x' = \gamma\Delta x,\ L_0 = \gamma L.$$

Doppler shift for light

$$\Delta t = \Delta t_0 / \sqrt{1 - v^2/c^2} \qquad \boxed{\Delta t_0 = \frac{\lambda_0}{c}}$$

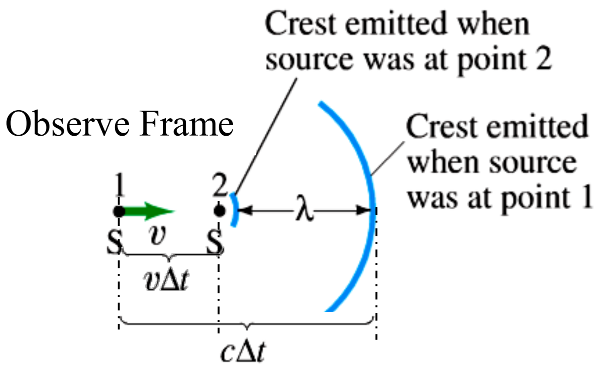
period in the source stationary reference frame



$$\lambda = c\,\Delta t - v\,\Delta t = (c - v)\,\Delta t$$

$$= (c - v)\frac{\Delta t_0}{\sqrt{1 - v^2/c^2}}$$

$$= \frac{(c - v)}{\sqrt{c^2 - v^2}}\lambda_0$$



$$\lambda = \lambda_0 \sqrt{\frac{1-\beta}{1+\beta}}$$

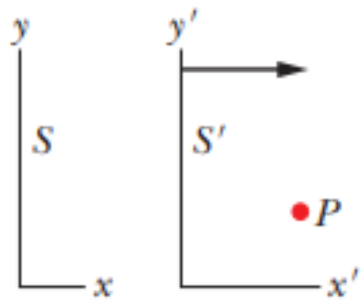
(source and observer moving toward each other)

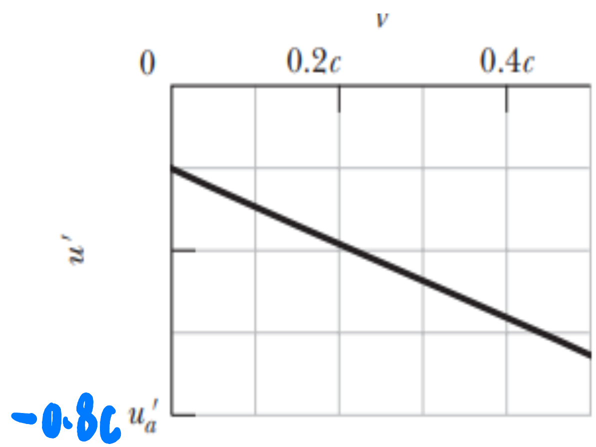
$$\lambda = \lambda_0 \sqrt{\frac{1+\beta}{1-\beta}}$$

(source and observer moving away from each other)

Problem 1

Particle P is to move parallel to the x and x' axes of reference frames S and S', at **a certain velocity** relative to frame S. Frame S' is to move parallel to the x axis of frame S at velocity v. Figure gives the velocity u' of the particle relative to frame S' for a range of values for v. The vertical axis scale is set by $u'_a = -0.800c$. What value will u' have if (a) $v = 0.90c$ and (b) $v \rightarrow c$?





Answer 1

By examining the value of u' when $v = 0$ on the graph, we infer $u = -0.20c$. Solving Eq. 37-29 for u' and inserting this value for u, we obtain

$$u' = \frac{u - v}{1 - uv/c^2} = \frac{-0.20c - v}{1 + 0.20v/c}$$

for the equation of the curve shown in the figure.

(a) With $v = 0.90c$, the above expression yields $u' = -0.93c$.

(b) As expected, setting $v = c$ in this expression leads to $u' = -c$.

Problem 2

An armada (舰队 a large group of war ships) of spaceships that is 1.00 ly long (in its rest frame) S_a moves with speed 0.800c relative to a ground station in frame S. A messenger travels from the rear of the armada to the front with a speed of 0.950c relative to S. How long does the trip take as measured

(a) in the armada's rest frame

(b) in the messenger's rest frame

(c) In frame S

(a) $u' = \dfrac{u - v_a}{1 - \frac{u v_a}{c^2}} = 0.625c$

$\Delta t = \dfrac{L_0}{u'} = 1.6y$

(b) $\Delta t_0 = \dfrac{\Delta t}{\gamma} = 1.25y$

(c) $L_{对地} = \dfrac{1\,ly}{\sqrt{1 - 0.8^2}}$

Answer 2

(a) In the armada's rest frame (called S_a), the velocity of the messenger is:

$$u' = \frac{u - v_a}{1 - uv_a / c^2} = \frac{0.95c - 0.8c}{1-(0.95c)(0.8c)/c^2} = 0.625c \ .$$

The length of the trip is proper length, and the time interval of the trip is

$$\Delta t = \frac{L_0}{|u'|} = \frac{1.0\,\mathrm{ly}}{0.625\mathrm{c}} = 1.6\ \mathrm{y}\ .$$

(b) In the messenger's rest system (called S_m), the time interval of the trip is proper time:

$$\Delta t_0 = \frac{\Delta t}{\gamma} = 1.6\,\mathrm{y}\sqrt{1-0.625^2} = 1.25\ \mathrm{y}\ .$$

Answer 2

(b) In the messenger's rest system (called S_m), the velocity of the armada is

$$u' = \frac{u - v_m}{1 - uv_m / c^2} = \frac{0.80c - 0.95c}{1 - (0.80c)(0.95c)/c^2} = -0.625c\ .$$

The length of the armada as measured in S_m is

$$L_1 = \frac{L_0}{\gamma} = 1.00y\sqrt{1 - (-0.625)^2} = 0.781ly$$

Thus, the length of the trip is:

$$\Delta t' = \frac{L_1}{|u'|} = \frac{0.781\text{ly}}{0.625\text{c}} = 1.25\ \text{y}\ .$$

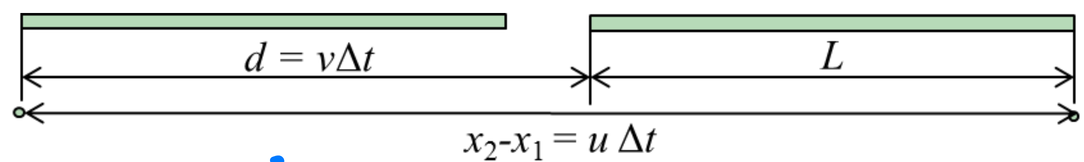
Answer 2

(c) in frame S

Event 1: messenger leaves the rear of the armada (x_1);
Event 2: messenger arrives at the front of the armada (x_2)

From the figure, we have:

$$v\Delta t + L = u\Delta t \Rightarrow \Delta t = \frac{L}{u-v}$$



追及运动

The length of the armada in the S frame is:

$$L = \frac{L_0}{\gamma}$$ Where $\gamma = 1/\sqrt{1-0.8^2}$, relates to the S frame and armada's frame

So we can get the time interval in the frame S:

S中长度

$$\Delta t = \frac{L}{\underbrace{0.950c - 0.800c}_{\Delta V}} = \frac{L_0\sqrt{1-0.800^2}}{0.950c-0.800c} = 4.00y$$

这里是一个参考系中两速度差，可以超光速

(不是两参考系的相对速度)

Answer 2

(c) in frame S

Event 1: messenger leaves the rear of the armada (x_1);
Event 2: messenger arrives at the front of the armada (x_2)

Alternatively, we also can get the time interval in S frame from the Lorentz transformation:

$$\Delta t = \gamma(\Delta t' + \frac{v\Delta x'}{c^2})$$

γ 在不同参考系间
可能不一样

For the frame S and messenger's frame:

$$\Delta t = \gamma(\Delta t_M + \frac{0.95c \times 0}{c^2}) = \frac{1.25y}{\sqrt{1-0.95^2}} = 4.0 \text{ y}$$

For the frame S and armada's frame:

$$\Delta t = \gamma(\Delta t_a + \frac{v_a \times (l_0)}{c^2}) = \frac{1}{\sqrt{1-0.8^2}}(1.6 + \frac{0.8cl_0}{c^2}) = 4.0 \text{ y}$$