Review of Ch32-Ch33 Key words

Displacement current 位移电流

Electromagnetic waves 电磁波

Transverse waves 横波 光是横波

Wavefront 波前

Plane wave 平面波

Spectrum 谱线

Infrared红外线

Ultraviolet紫外线

The index of refraction折射率

Energy transport 能量传输

Poynting vector 坡印廷矢量

Intensity光强

Transmitted intensity 透射光强

Radiation pressure 光压

Total absorption 全吸收

Polarization （偏振）

Unpolarized light 非偏振光

Polarizing filters 偏振片

Polarizer / Polaroid 偏振片/板

Reflection （反射） Refraction （折射）

Snell's Law 斯涅尔定律（折射定律）

Chromatic dispersion 色散

Brewster Angle （布鲁斯特角）

Total internal reflection （全反射）

Critical angle （临界角）

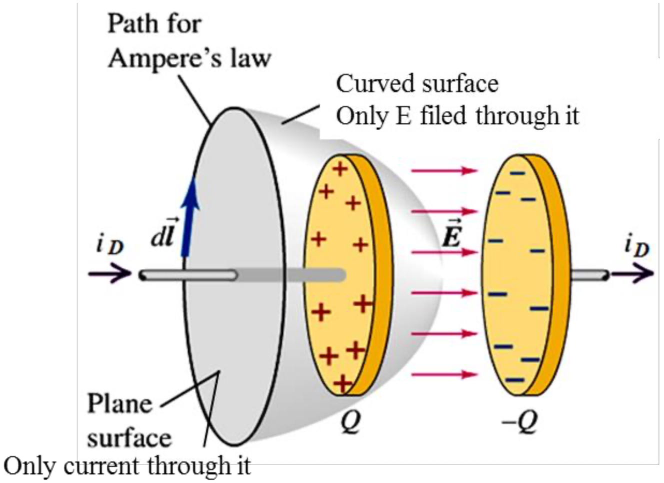
Chromatic dispersion (色散）

Review of Ch32

Displacement current: $i_D = \varepsilon_0 \dfrac{d\Phi_E}{dt}$

We can express the displacement current throgh **the region between the two plates** by the rate of flux Φ_E in terms of charge q on the plate:

$$i_D = \epsilon \frac{d\Phi_E}{dt} = \frac{dq}{dt}$$

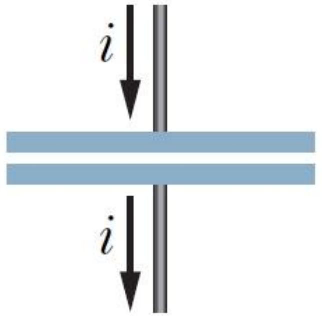


$$\oint \vec{B} \cdot d\vec{l} = \mu_0 (i + i_D)_{\text{encl}} \quad \text{(generalized Ampere's law)}$$

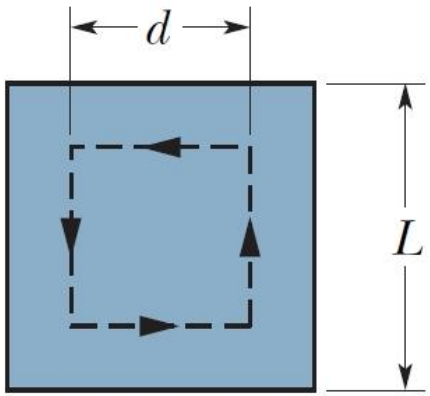
In some special case, the two terms can be present in the same region (32-45)

Review of Ch32

A problem for chapter 32: a parallel-plate capacitor has square plates of edge length $L = 1.0$ m. A current of 2.0 A charges the capacitor, producing a uniform electric field $\vec{E}$ between the plates, with $\vec{E}$ perpendicular to the plates. (a) What is the displacement current i_d through the region between the plates? (b) What is dE/dt in this region? (c) What is the displacement current encircled by the square dashed path of edge length $d = 0.50$ m? (d) What is $\oint \vec{B} \cdot d\vec{s}$ around this square dashed path?



Edge view



Top view

Solution:

The electric field between the plates in a parallel-plate capacitor is changing, so there is a nonzero displacement current $i_d = \varepsilon_0 (d\Phi_E / dt)$ between the plates.

Let A be the area of a plate and E be the magnitude of the electric field between the plates. The field between the plates is uniform, so $E = V/d$, where V is the potential difference across the plates and d is the plate separation. The current into the positive plate of the capacitor is

$$i = \frac{dq}{dt} = \frac{d}{dt}(CV) = C\frac{dV}{dt} = \frac{\varepsilon_0 A}{d}\frac{d(Ed)}{dt} = \varepsilon_0 A \frac{dE}{dt} = \varepsilon_0 \frac{d\Phi_E}{dt},$$

which is the same as the displacement current.

Review of Ch32

(a) At any instant the displacement current i_d in the gap between the plates equals the conduction current i in the wires. Thus $i_d = i = 2.0$ A.

(b) The rate of change of the electric field is

$$\frac{dE}{dt} = \frac{1}{\varepsilon_0 A}\left(\varepsilon_0 \frac{d\Phi_E}{dt}\right) = \frac{i_d}{\varepsilon_0 A} = \frac{2.0\,\text{A}}{\left(8.85\times10^{-12}\,\text{F/m}\right)\left(1.0\,\text{m}\right)^2} = 2.3\times10^{11}\,\frac{\text{V}}{\text{m}\cdot\text{s}}.$$

(c) The displacement current through the indicated path is

$$i_d' = i_d\left(\frac{d^2}{L^2}\right) = (2.0\text{A})\left(\frac{0.50\text{m}}{1.0\text{m}}\right)^2 = 0.50\text{A}.$$

(d) The integral of the field around the indicated path is

$$\int \vec{B}\cdot d\vec{s} = \mu_0 i_d' = (4\pi\times10^{-7}\,\text{H/m})(0.5\text{A}) = 6.3\times10^{-7}\,\text{T}\cdot\text{m}$$

Question

If the square plates are replaced by circular plates of radius R, What is the magnitude of the magnetic field that is induced at radial distances $r < R$ and $r > R$

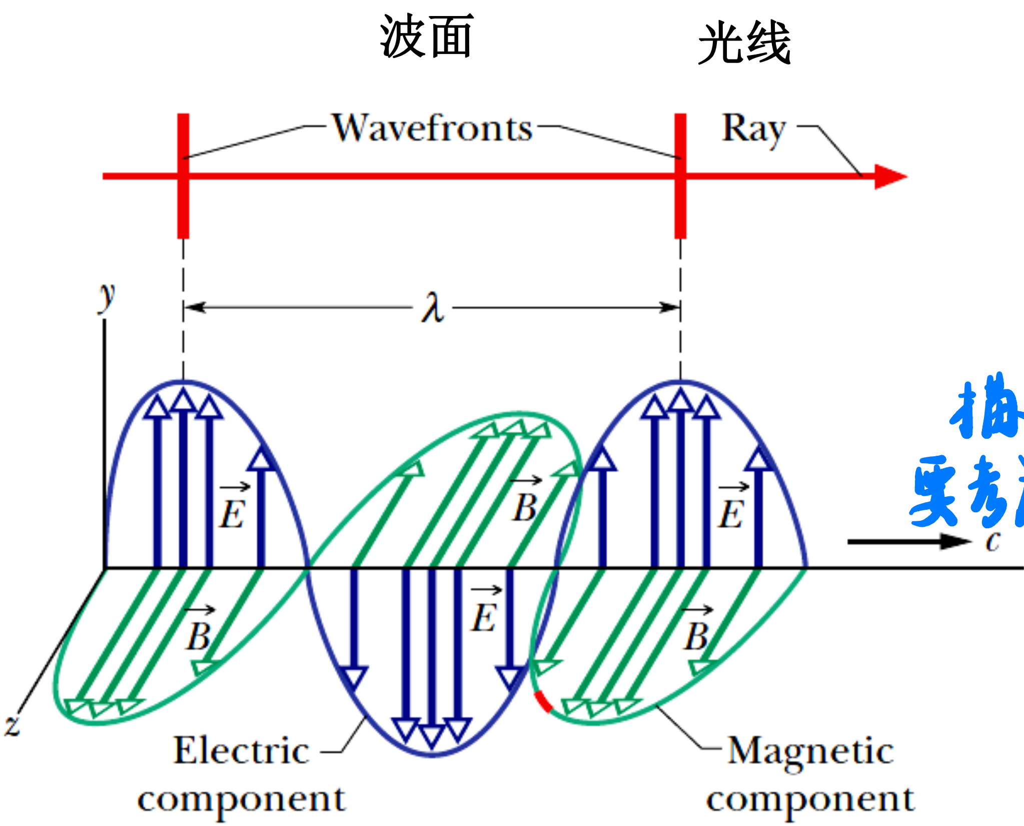
$$\oint \vec{B}\cdot d\vec{l} = \mu_0 i_d = \mu_0 \varepsilon_0 \frac{d}{dt}\int_A \vec{E}\cdot d\vec{A}$$

The B field induced have the same symmetry as that of the E field between two plates. The E field is zero out of the plates. So we can choose a concentric circle as our loop:

$$r < R:\ B(2\pi r) = \mu_0\varepsilon_0 A' \frac{d}{dt}E = \mu_0\varepsilon_0 \pi r^2 \frac{d}{dt}\left(\frac{q}{\pi R^2 \varepsilon_0}\right) = \mu_0 \frac{r^2}{R^2}\frac{dq}{dt} = \mu_0 \frac{r^2}{R^2} i \Rightarrow B = \frac{\mu_0 i}{2\pi}\frac{r}{R^2}$$

$$r > R:\ B(2\pi r) = \mu_0\varepsilon_0 A \frac{d}{dt}E = \mu_0\varepsilon_0 \pi R^2 \frac{d}{dt}\left(\frac{q}{\pi R^2 \varepsilon_0}\right) = \mu_0 \frac{dq}{dt} = \mu_0 i \Rightarrow B = \frac{\mu_0 i}{2\pi r}$$

EM waves → Transverse waves



$$
\begin{cases}
\oint_{\mathbf{A}=\partial V} \mathbf{E}\cdot \mathrm{d}\mathbf{A} = \dfrac{q_{\mathrm{enc}}}{\varepsilon_0} \\
\oint_{\mathbf{A}=\partial V} \mathbf{B}\cdot \mathrm{d}\mathbf{A} = 0 \\
\oint_{\mathbf{l}=\partial \mathbf{A}} \mathbf{E}\cdot \mathrm{d}\mathbf{l} = -\dfrac{\mathrm{d}}{\mathrm{d}t}\int_{\mathbf{A}} \mathbf{B}\cdot \mathrm{d}\mathbf{A} \\
\oint_{\mathbf{l}=\partial \mathbf{A}} \mathbf{B}\cdot \mathrm{d}\mathbf{l} = \mu_0\left(\int_{\mathbf{A}} \mathbf{J}\cdot \mathrm{d}\mathbf{A} + \varepsilon_0 \dfrac{\mathrm{d}}{\mathrm{d}t}\int_{\mathbf{A}} \mathbf{E}\cdot \mathrm{d}\mathbf{A}\right)
\end{cases}
$$

$q = 0$ & $\mathrm{J} = 0$

描述时只
要考虑E

$$
\begin{cases}
E = E_m \sin(kx - \omega t) \\
B = B_m \sin(kx - \omega t)
\end{cases}
$$

1, E and B are in phase

2, E and B are perpendicular

3, $\hat{E}\times\hat{B} = \hat{c}$ → 能量传播方向

4, $E_m = cB_m$

5, $c = 1/\sqrt{\mu_0\varepsilon_0}$

6, $u_E = u_B$ E,B能量密度一样

Quick question

A plane electromagnetic wave travels along $+x$ axis and the magnetic field of that wave is along the $+y$ axis and its magnitude is given by $B_m \sin(kx - \omega t)$ in SI units, then the electric field is

与E同相位

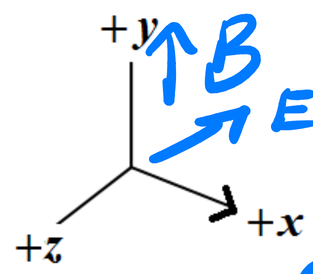
A) along the $+z$ axis and its magnitude is given by $(cB_m)\cos(kx - \omega t)$

B) along the $-z$ axis and its magnitude is given by $-(cB_m)\cos(kx - \omega t)$

C) along the $+z$ axis and its magnitude is given by $(cB_m)\sin(kx - \omega t)$

$\boxed{\text{D) along the -z axis and its magnitude is given by } (cB_m)\sin(kx - \omega t)}$

E) along the $+z$ axis and its magnitude is given by $B_m \sin(kx - \omega t)$



C

只要 c 与 $\vec{E}$ 就可描述

Poynting vector

$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B} \Rightarrow S = \frac{1}{\mu_0}EB = \frac{1}{c\mu_0}E^2 = \frac{c}{\mu_0}B^2$$

Since ***E*** and ***B*** are changing, it is more meaningful to use its average value, which is called **intensity *I* of the EM waves:**

$$I = \langle S\rangle = \frac{1}{c\mu_0}\langle E^2\rangle = \frac{1}{2c\mu_0}E_m^2 = \frac{1}{c\mu_0}E_{\mathrm{rms}}^2.$$

$$\boxed{\left(\frac{\text{power}}{\text{area}}\right)_{avg}}$$

The **intensity *I*** at a distance *r* from a point source :

$$\frac{\text{average power}}{4\pi r^2} \propto \frac{1}{r^2}$$

Radiation pressure

We can also understand light as particles, which we called ***photons*** (光子).

Photons have zero mass at rest, but they can carry energy and they have **momentum:**

$$\boxed{P = \frac{U}{c} \qquad (\,U \text{ is energy of the photon})}$$

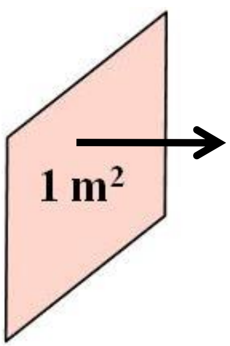
Change of momentum

$$\boxed{\Delta P = \frac{U}{c} = \frac{IA\Delta t}{c} \qquad \text{(totally absorbed)}}$$

$$\boxed{\Delta P = \frac{2U}{c} = \frac{2IA\Delta t}{c} \qquad \text{(totally reflected)}}$$

Therefore, light or EM waves can exert force / pressure on an object.

$$\boxed{F = \frac{\Delta P}{\Delta t} = \frac{IA}{c} \;\text{ or }\; F = \frac{2IA}{c}} \qquad \boxed{p = \frac{\text{Force}}{\text{Area}} = \frac{F}{A} = \frac{I}{c} \;\text{ or }\; p = \frac{2I}{c}}$$



$$\frac{I}{c} = \frac{S_{avg}}{c} = \frac{\text{energy}}{c\ \text{m}^2\ \text{sec}} = \textit{pressure}$$

momentum

平面波 A不变

球面波 $A = 4\pi r^2$

$I \propto \frac{1}{r^2}$, $E/B \propto \frac{1}{r}$
(振幅)

Problem 1

大圆柱平面 (平均功率) P

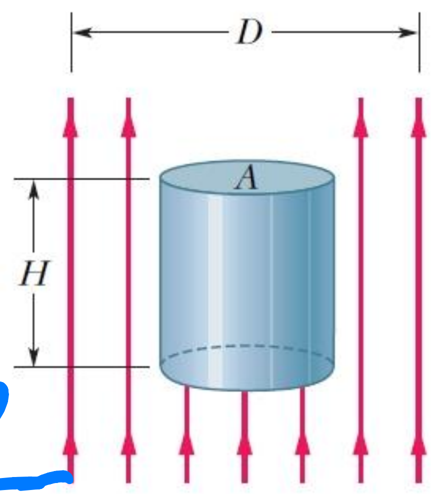
In figure, a laser beam of power 4.60 W and diameter D = 3.0mm is directed upward at one circular face (of diameter d < 3.0mm) of a perfectly reflecting cylinder. The cylinder is levitated because the upward radiation force matches the downward gravitational force. If the cylinder's density is 1.2 g/cm^3, what is its height *H*?

悬浮

$$G = \rho H A g$$

$$p = \frac{2I}{c}$$

$$F = \frac{2IA}{c}$$



$$I = \frac{P}{\pi\left(\frac{D}{2}\right)^2}$$

Problem 1

The mass of the cylinder is $m = \rho(\pi d^2/4)H$,

where d is the diameter of the cylinder. Since it is in equilibrium

$$F_{\text{net}} = mg - F_r = \cancel{\left(\frac{\pi d^2}{4}\right)} Hg\rho - \cancel{\left(\frac{\pi d^2}{4}\right)}\left(\frac{2I}{c}\right) = 0.$$

We solve for H:

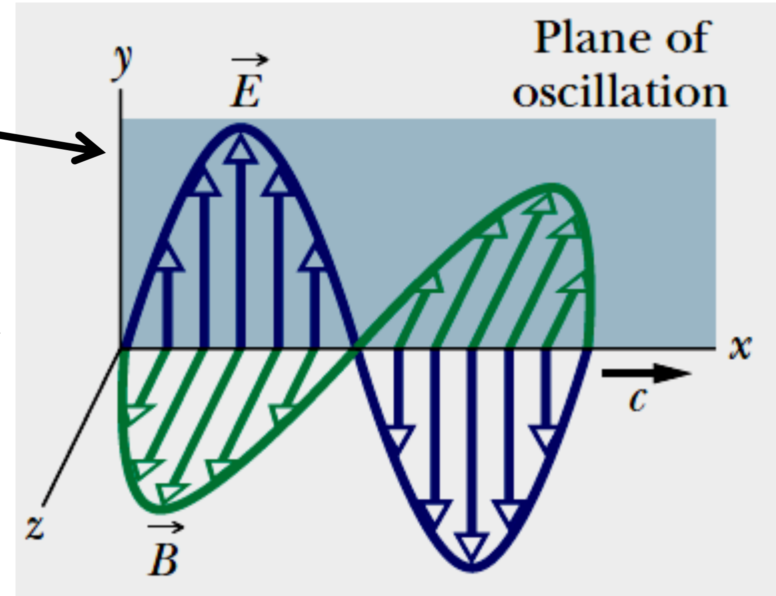
$$H = \frac{2I}{gc\rho} = \left(\frac{2P}{\pi D^2/4}\right)\frac{1}{gc\rho}$$

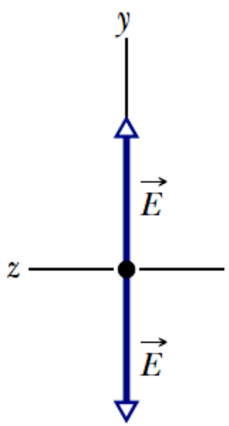
$$= \frac{2(4.60\,\text{W})}{[\pi(3.00\times10^{-3}\,\text{m})^2/4](9.8\,\text{m/s}^2)(3.0\times10^{8}\,\text{m/s})(1.20\times10^{3}\,\text{kg/m}^3)}$$

$$= 3.69\times10^{-7}\,\text{m}.$$

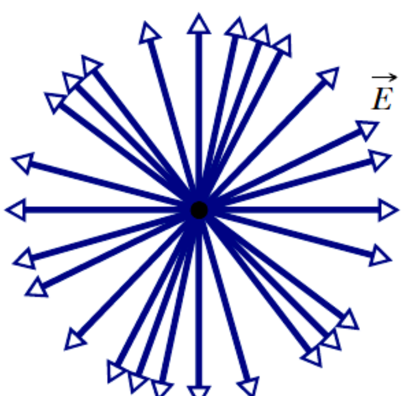
Polarization

We normally use the plane of ***E-field*** oscillation as ***the plane of oscillation*** of the wave

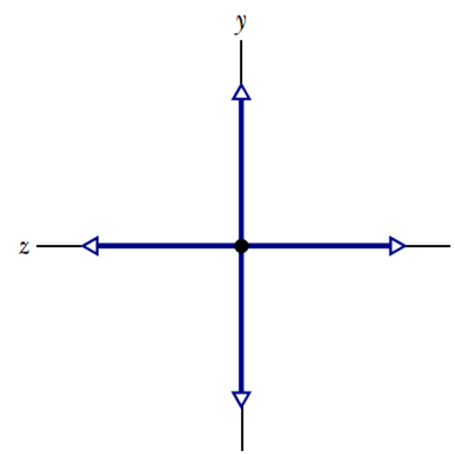




Linearly polarized light



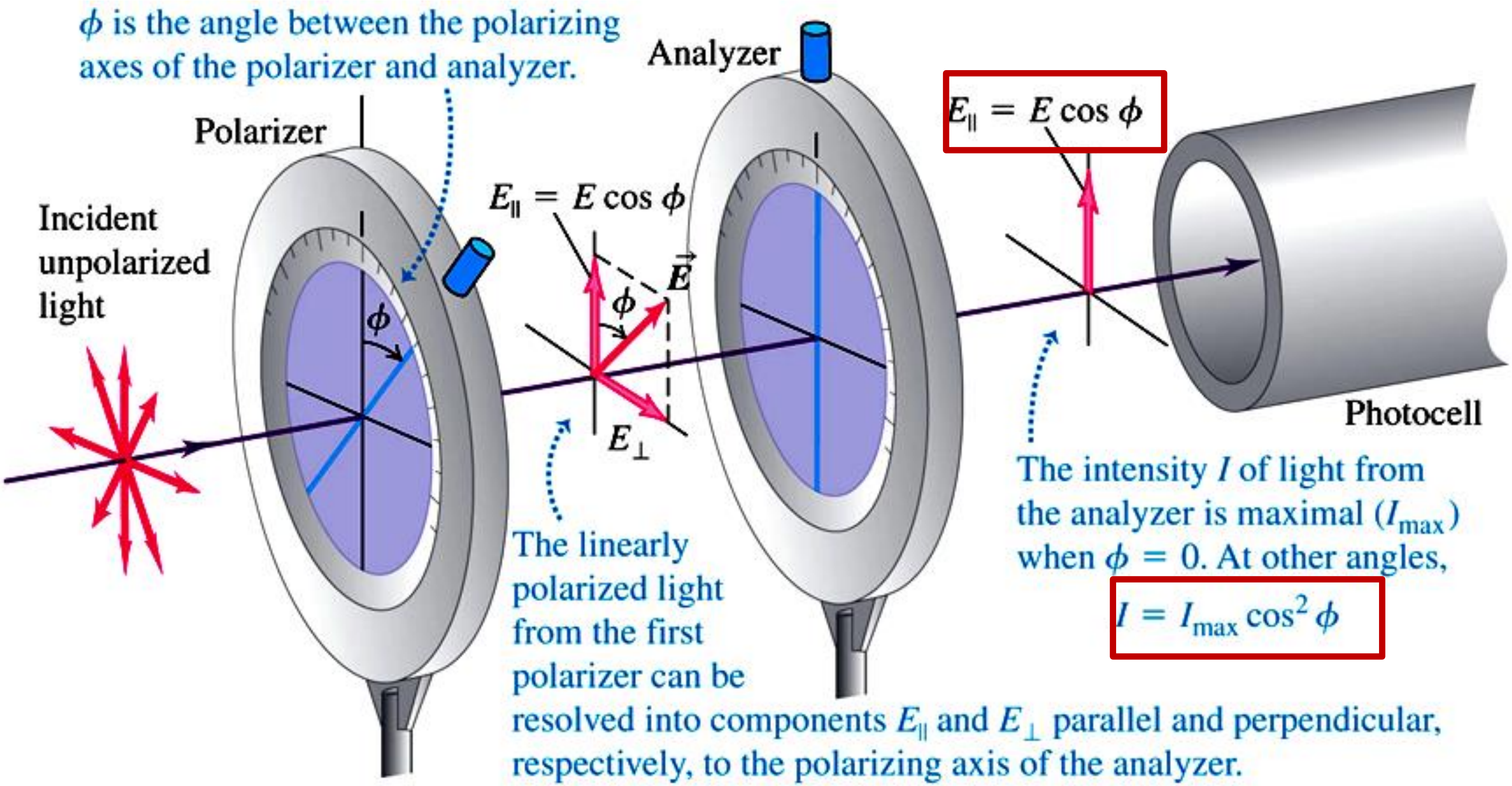
Another view: a superposition of two perpendicular polarized components



考试只考线偏振

Randomly polarized or unpolarized light (natural light)

Polarization



$I = I_0$

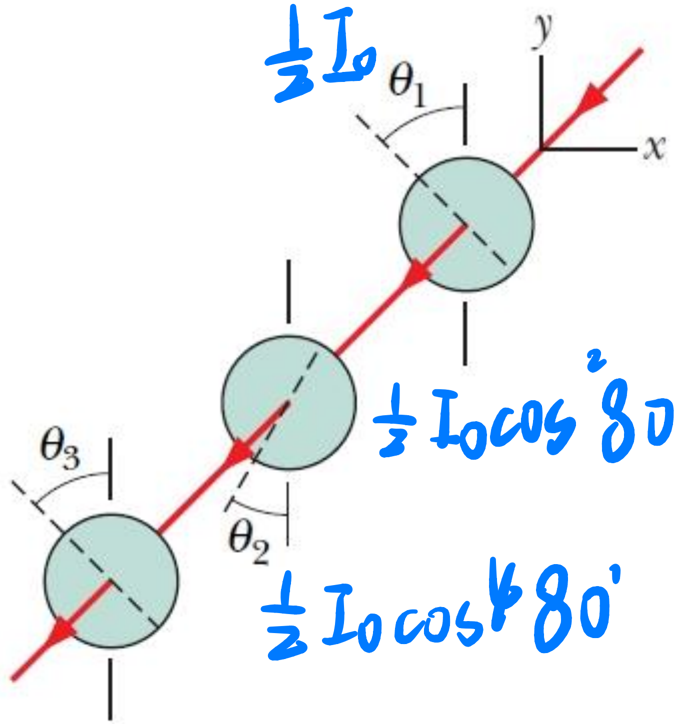
$I_1 = I_0 / 2$

$I_2 = I_1 \cos^2\phi$

$I = S_{avg} = \dfrac{E_{rms}^2}{\mu_0 c}$

Problem 2

In Fig. 33-40, initially unpolarized light is sent into a system of three polarizing sheets whose polarizing directions make angles of $\theta_1 = \theta_2 = \theta_3 = 50°$ with the direction of the y axis. What percentage of the initial intensity is transmitted by the system? (*Hint:* Be careful with the angles.)



$\frac{1}{2}I_0$

$\frac{1}{2}I_0\cos^2 80°$

$\frac{1}{2}I_0\cos^4 80°$

$\cos^2\alpha$

↓

两个偏振方向夹角

Problem 2

After passing through the first polarizer : $I_1 = I_0/2$

After passing through the second one it is further reduced by a factor of $\cos^2(\theta_1 + \theta_2)$. Finally, after passing through the third one it is again reduced by a factor of $\cos^2(\theta_2 + \theta_3)$. Therefore,

$$\frac{I_f}{I_0} = \frac{1}{2}\cos^2(\theta_1 + \theta_2)\cos^2(\theta_2 + \theta_3) = \frac{1}{2}\cos^2(40^\circ + 40^\circ)\cos^2(40^\circ + 40^\circ)$$
$$= 4.5 \times 10^{-4}.$$

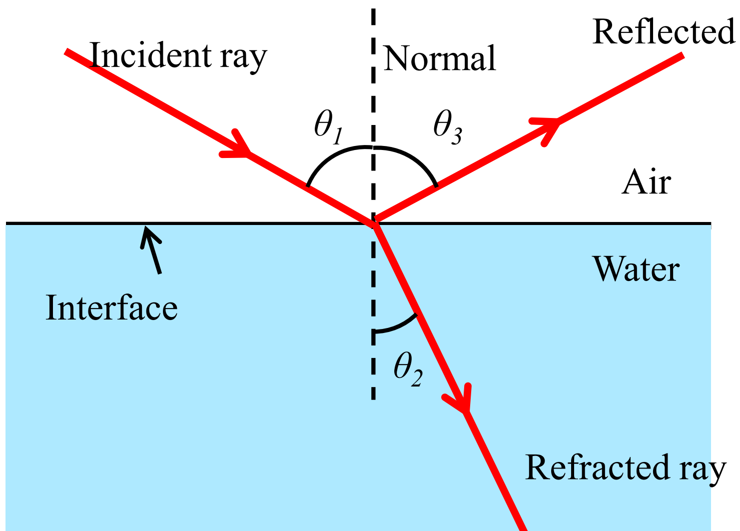
Thus, 0.045% of the light's initial intensity is transmitted.

Reflection (反射) and Refraction (折射)

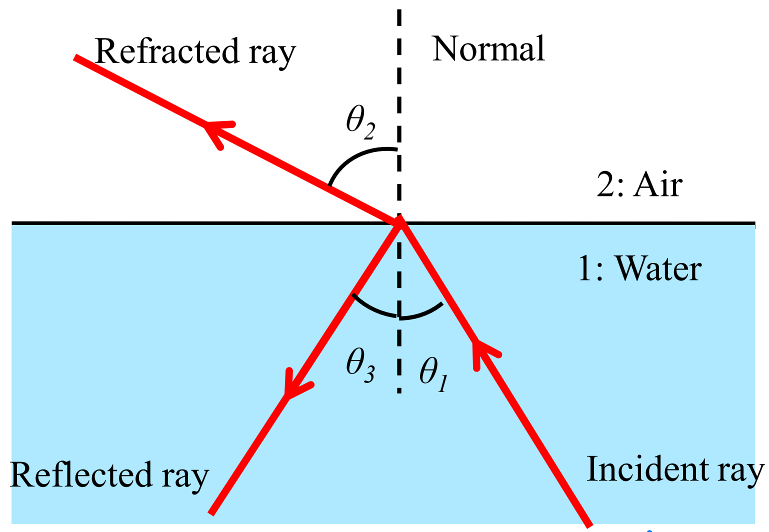
Wave speeds differ in different media.

Index of refraction: $n = \dfrac{c}{v}$ $\quad \lambda = \dfrac{v}{f} = \dfrac{c}{n\,f} = \dfrac{\lambda_{air}}{n}$

波长 ↓



$\boxed{\theta_1 = \theta_3 \qquad \text{the law of reflection}}$



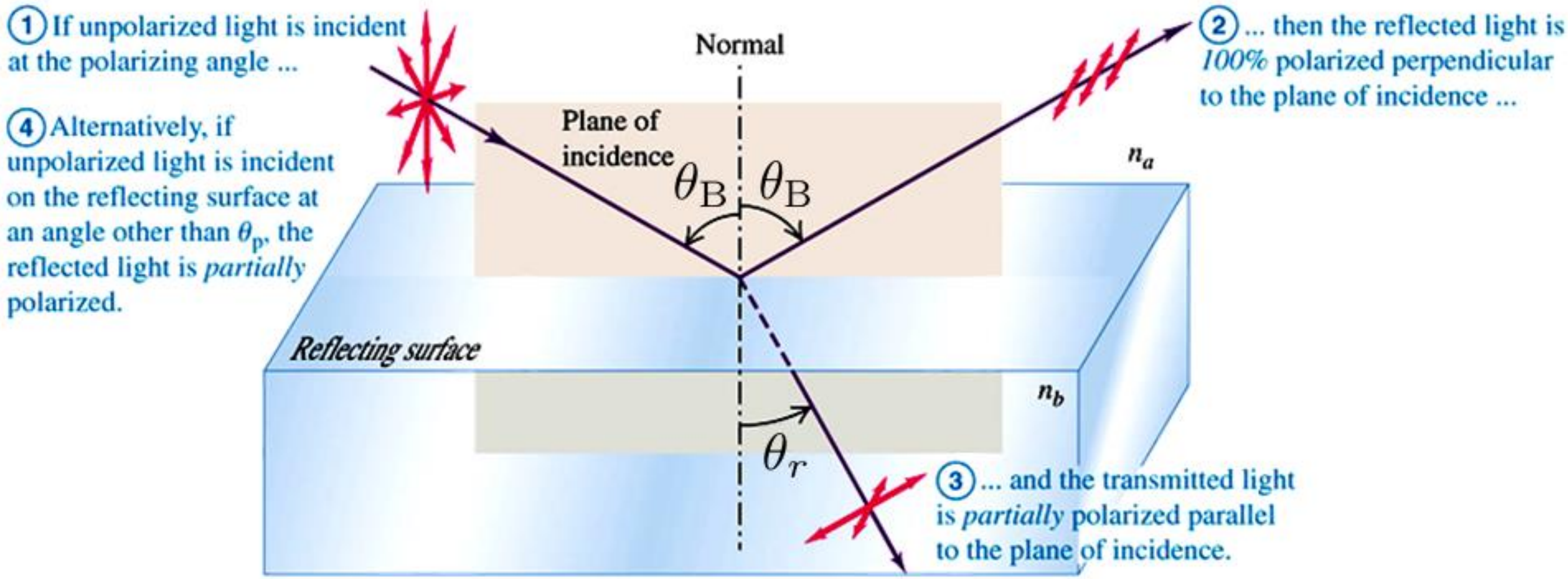
$\boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2 \quad \text{Snell's law}}$ $\quad \dfrac{v_1}{v_2} = \dfrac{\sin\theta_1}{\sin\theta_2}$

Critical angle: $n_1 \sin\theta_c = n_2 \sin 90° \Rightarrow \theta_c = \sin^{-1}(\dfrac{n_2}{n_1})$

Total internal reflection

Fermat's Principle: Light travels between two points along the path that requires the least time, as compared to other nearby paths

Polarization by reflection—Brewster's angle



$$\theta_B + \theta_r = 90°$$

Brewster's angle

$$\theta_B = \tan^{-1}\left(\frac{n_b}{n_a}\right)$$

P-polarization : E-field parallel to the plane of incidence

S-polarization : E-field perpendicular to the plane of incidence

Problem 3

The critical angle for an air-glass interface is 29.6˚. When a light (nature light) ray in air is incident on the interface, the reflected ray can be fully absorbed by a polarizer. What is the angle of refraction (折射角) of that ray?

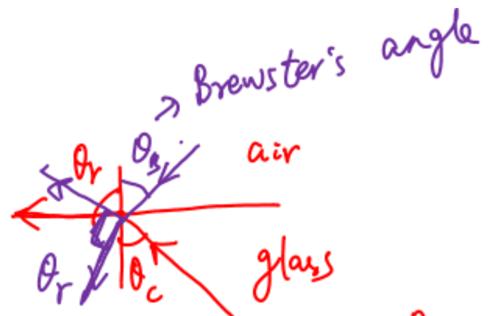
A) 23.9°

B) 25.7°

C) 25.1°

D) 24.5°

$\boxed{\text{E) 26.3°}}$



(1) $n_g \sin\theta_c = n_a \sin 90^\circ \Rightarrow \frac{n_a}{n_g} = \sin\theta_c$

(2) $n_a \sin\theta_B = n_g \sin\theta_r$

$\Rightarrow n_a \cos\theta_r = n_g \sin\theta_r \Rightarrow \tan\theta_r = \frac{n_a}{n_g} = \sin\theta_c$

$\Rightarrow \theta_2 = \tan^{-1}(\sin\theta_c)$