Key words

Electric current

Current density J

Resistance

Resistivity

Ohm's law

Power

Resistive dissipation

Semiconductors

Superconductors

Capacitive time constant
(RC time constant)

Electromotive force
(emf ε)

Voltmeter

Ammeter

Terminal Voltage

Kirchhoff's rules

Farad (Faraday)

Ampere (Ampère)

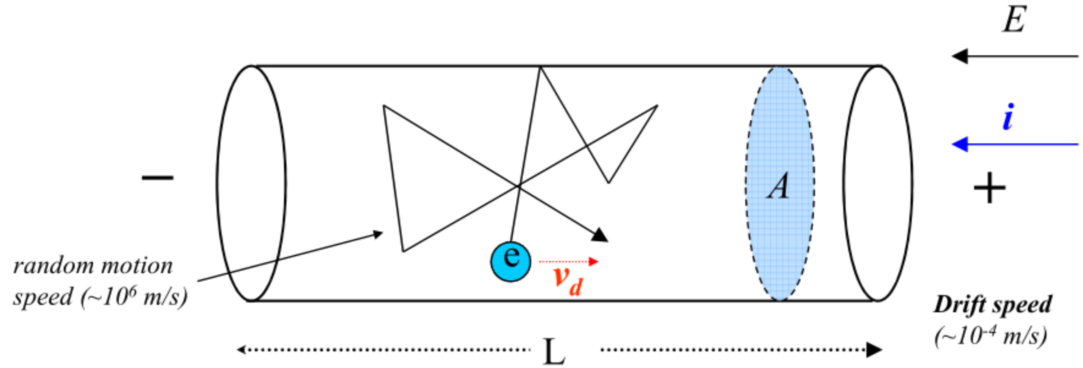
Electric current

(definition of current)

$$\boxed{i = \frac{dq}{dt}}$$

ampere(A)
1 A=1 C/S

Drift Speed of the Charge Carriers



$$t = \frac{L}{v_d} \qquad q = (nAL)e$$

Current density

$$\boxed{i = \int \vec{J} \cdot d\vec{A}}$$

$$i = \frac{q}{t} = \frac{nALe}{L/v_d} = nAev_d$$

$$\boxed{\vec{J} = (ne)\vec{v}_d}$$

Resistance and resistivity

Definition of Resistance

$$\boxed{R = \frac{V}{i}}$$

Relationship between resistance and resistivity

$$\boxed{R = \rho \frac{L}{A}}$$

Ohm(Ω)
1 Ω =1 V/A

$$R = \frac{V}{i} = \frac{EL}{J \cdot A} = \frac{\rho l}{A}$$

Definition of resistivity(ρ) and conductivity(σ)

$$\boxed{\rho = \frac{1}{\sigma} = \frac{E}{J}}$$

ρ is only related to the material properties

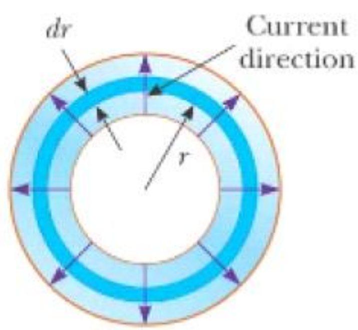
$$\boxed{\rho = \frac{m}{e^2 n \tau}}$$

$$\frac{unit(E)}{unit(J)} = \frac{V/m}{A/m^2} = \frac{V}{A} m = \Omega \cdot m$$

Resistance depends on the way of connection

The resistance of a resistor depends on how it is connected to the circuit：

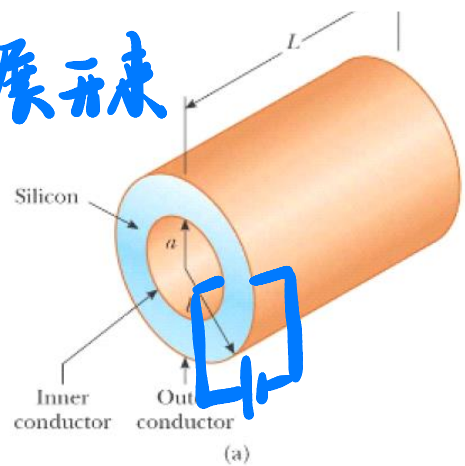
$$dR = \frac{dV}{i} = \frac{\vec{E}\cdot d\vec{L}}{\vec{J}\cdot\vec{A}}$$



$$dR = \rho\frac{dr}{A}$$

横面展开来

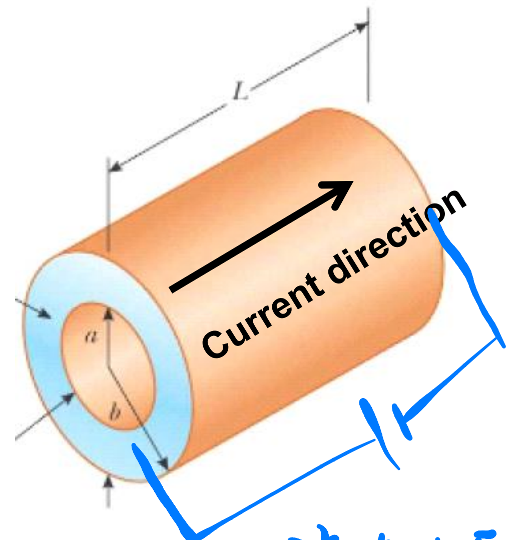
But $A = 2\pi rL$,



$$\therefore dR = \frac{\rho}{2\pi rL}dr$$

并联面积

$$R = \int_a^b dR = \frac{\rho}{2\pi L}\int_a^b \frac{dr}{r} = \frac{\rho}{2\pi L}\ln\left(\frac{b}{a}\right)$$



施加电压不同

$$dR = \rho\frac{dl}{A} \Rightarrow R = \rho\frac{L}{\pi(b^2 - a^2)}$$

Ohm's law

Microscopic and macroscopic quantities in ohm's law

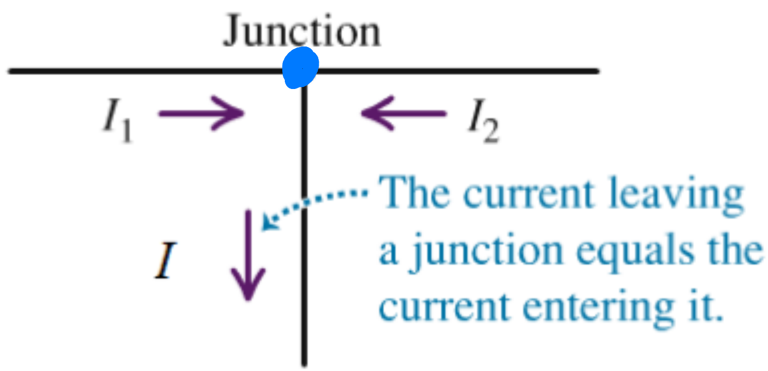
Microscopic	Macroscopic	Relation
$\mathbf{E}$	V	$V = \int \mathbf{E} \cdot d\mathbf{r} \quad \rightarrow \quad E\,L$
$\mathbf{J}$	I	$I = \int \mathbf{J} \cdot d\mathbf{A} \quad \rightarrow \quad J\,A$
ρ	R	$R = \dfrac{\rho\,L}{A}$
$\mathbf{J} = \dfrac{1}{\rho}\mathbf{E}$	$I = \dfrac{V}{R}$	

Power: $P = iV \quad P = i^2 R = \dfrac{V^2}{R}$

Kirchhoff's rules

1) Kirchhoff's junction rule（基尔霍夫結点定则）：

The algebraic sum of the currents into any junction is zero, i.e. at any junction : $\sum I = 0$



$$I_1 + I_2 + (-I) = 0$$

$$i.e. \quad I_1 + I_2 = I$$

$\because$ charge is conserved

Kirchhoff's rules

2) Kirchhoff's loop rule (基尔霍夫回路定则) :

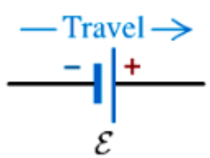
The algebraic sum of the potential difference of any loop is zero, including those associated with emfs or resistive elements, i.e. $\sum V = 0$

先设i方向再走

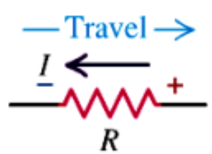
First, we choose a direction, either clockwise or counter-clockwise

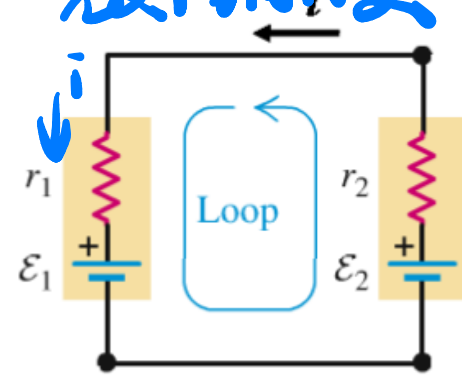
From lower potential to higher potential → +ve

$+\mathcal{E}$: Travel direction from – to +:



$+IR$: Travel *opposite to* current direction:

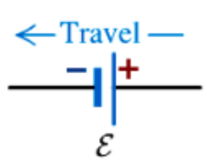




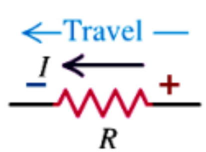
$$\varepsilon_2 - ir_2 - ir_1 - \varepsilon_1 = 0$$

From higher potential to lower potential → -ve

$-\mathcal{E}$: Travel direction from + to –:



$-IR$: Travel *in* current direction:



$\oint \vec{E} \cdot d\vec{l} = 0$

条件是：电场为保守场

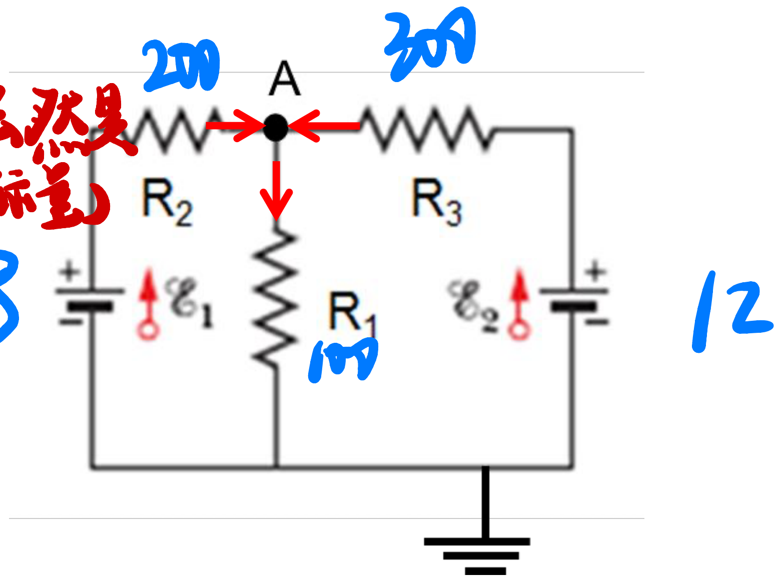
(静电场是，感生电场不是)

Problem 1

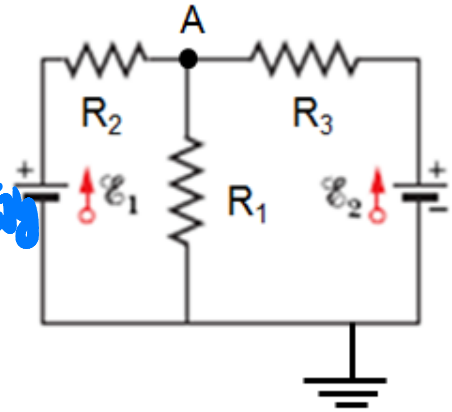
$\varepsilon_1 = 8.00$ V , $\varepsilon_2 = 12.0$ V, $R_1 = 100\ \Omega$， $R_2 = 200\ \Omega$ and $R_3 = 300\ \Omega$， one point of the circuit is grounded ($V = 0$).

What are (a) The size and (b) direction (up or down)of the current through resistor 1, (c) the size and (d)direction (left or right) of the current through resistor 2, the (e) size and (f) direction of the current through resistor 3? (g) what is the electric potential at point A？

* 问电流
也要答方向(虽然是标量)



Answer 1



R 乘对应流过的 i是

(a) Using the junction rule ($i_1 = i_2 + i_3$) we write two loop rule equations:

$$\varepsilon_1 - i_2 R_2 - (i_2 + i_3) R_1 = 0$$

$$\varepsilon_2 - i_3 R_3 - (i_2 + i_3) R_1 = 0$$

Solving, we find $i_2 = 0.0182$ A (rightward, as was assumed in writing the equations as we did), $i_3 = 0.02545$ A (leftward), and $i_1 = i_2 + i_3 = 0.04365$ A (downward).

(b) The direction is downward. See the results in part (a).

(c) $I_2 = 0.0182$ A . See the results in part (a).

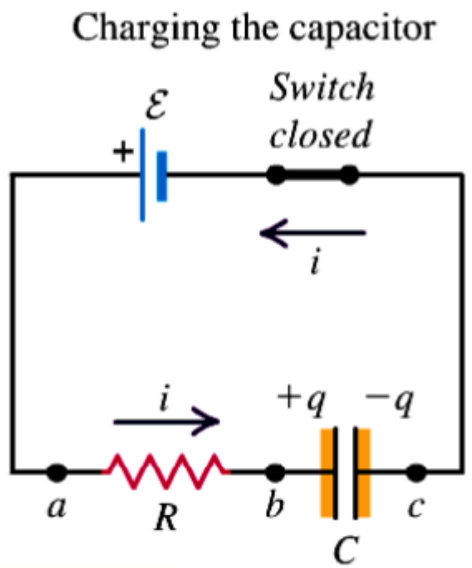
(d) The direction is rightward. See the results in part (a).

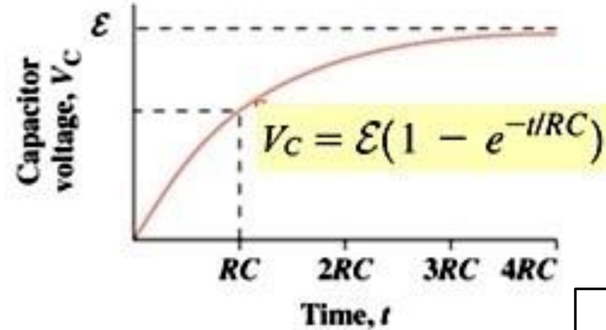
(e) $i_3 = 0.0254$ A. See the results in part (a).

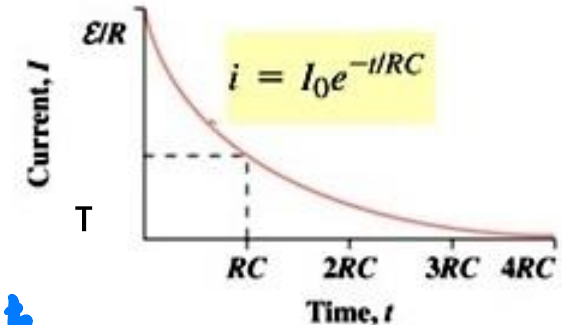
(f) The direction is leftward. See the results in part (a).

(g) The voltage across R_1 equals V_A: (0.0437)(100 Ω) = +4.37 V.

RC circuits







$\tau = RC$
capacitive time constant

$$\mathcal{E} - iR - \frac{q}{C} = 0$$

分离变量法

$$\frac{dq}{dt} = -\frac{1}{RC}(q - C\mathcal{E})$$

$$\int_0^q \frac{dq}{q - C\mathcal{E}} = -\int_0^t \frac{dt}{RC}$$

积分上下限要对应

$$\frac{q - C\mathcal{E}}{-C\mathcal{E}} = e^{-t/RC}$$

$$i = \frac{dq}{dt} = \frac{\mathcal{E}}{R} e^{-t/RC} = I_0 e^{-t/RC}$$

$$q = C\mathcal{E}(1 - e^{-t/RC})$$

RC circuit – charging

$$\mathcal{E} - iR - \frac{q}{C} = 0 \quad \rightarrow \quad \mathcal{E} = iR + \frac{q}{C}$$

$$i = I_0 e^{-t/RC}$$

$$V_C = \mathcal{E}(1 - e^{-t/RC})$$

Energy supplied by the **power** : $\mathcal{E}i = i^2R + \frac{iq}{C}$

流入的q全在C中

The energy stored in the capacitor when it is fully charged:

$$U_C = \int_0^\infty V_C i_c dt = \int_0^\infty \frac{iq}{C} dt = \int_0^\infty \varepsilon(1 - e^{-t/RC})(\frac{\varepsilon}{R} e^{-t/RC})dt$$

$\int \mathcal{E} i dt = \int i^2 R dt + \int i V_C dt$

$$= \int_0^\infty \frac{\varepsilon^2}{R} RC(e^{-t/RC} - e^{-2t/RC})d(t/RC) = \varepsilon^2 C(1 - 1/2) = \frac{C\varepsilon^2}{2}$$

一定50%能量 dissipation rate 具 power

The energy dissipated by the resistor during this process

跑不到C中

$$U_R = \int_0^\infty i^2 R dt = \int_0^\infty (\frac{\varepsilon}{R} e^{-2t/RC})dt = \int_0^\infty \varepsilon^2 C(e^{-2t/RC})d(t/RC) = \frac{C\varepsilon^2}{2}$$

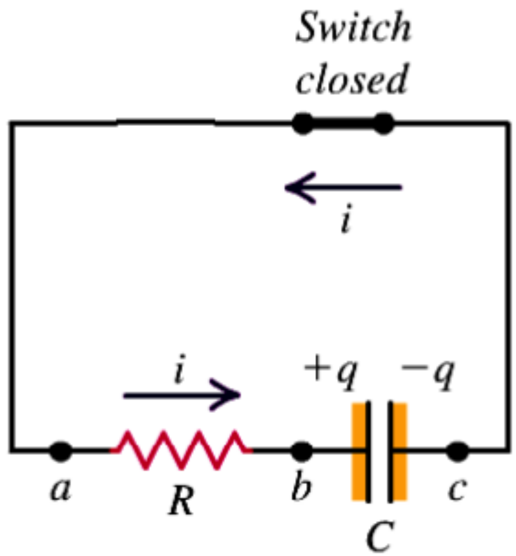
R越大 充电慢
R越小 充电快

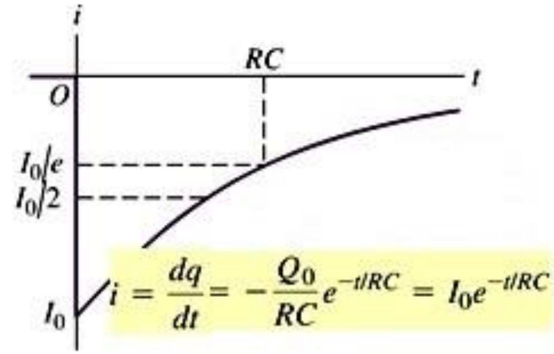
整个充电过程中耗散能量与R无关

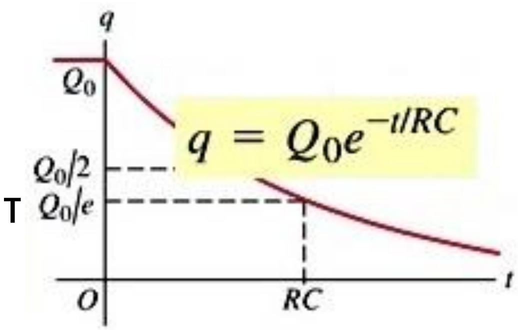
The total energy dissipated by ths resistor is independ of its resistance

RC circuits

Discharging the capacitor







T

$$\cancel{\mathcal{E}} - iR - \frac{q}{C} = 0$$

$$i = \frac{dq}{dt} = -\frac{q}{RC}$$

$$i = \frac{dq}{dt} = -\frac{Q_0}{RC}e^{-t/RC} = I_0e^{-t/RC}$$

$$\int_{Q_0}^{q}\frac{dq}{q} = -\frac{1}{RC}\int_0^t dt$$

$$\ln\frac{q}{Q_0} = -\frac{t}{RC}$$

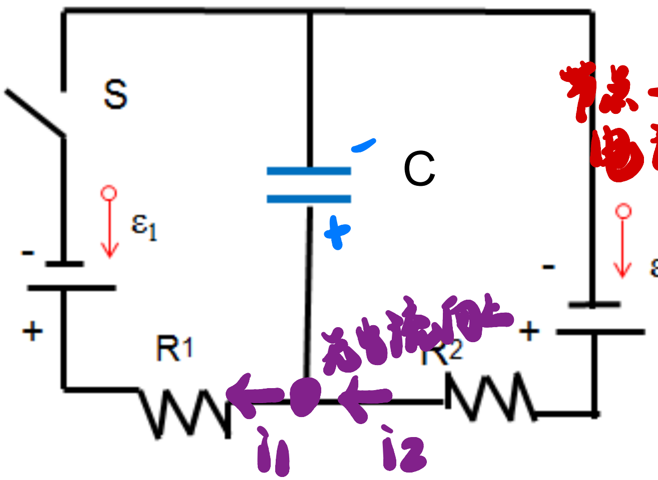
$$q = Q_0e^{-t/RC}$$

Problem 2

The circuit below shows a capacitor, two ideal batteries, two resistors, and a switch S. initially S has been open for a long time. If it is then closed for a long time , what is **the change in the charge on the capacitor**? Assume C=10μF , ε_1 =1.0 V , ε_2 =3.0 V, R_1 =0.20 Ω and R_2 =0.40 Ω

+电荷量不变 $\frac{dq}{dt}=0$

节点一个方向没有电流



$i_1 = i_2$

$V_C = \varepsilon_2$

$\varepsilon_2 - iR_2 - iR_1 - \varepsilon_1 = 0$

$i = \frac{\varepsilon_2 - \varepsilon_1}{R_2 + R_1}$

$V = \varepsilon_2 - iR_2$

$\Delta q = C\Delta V$

$= C(\overset{末}{V} - \overset{初}{\varepsilon_2})$

放电时

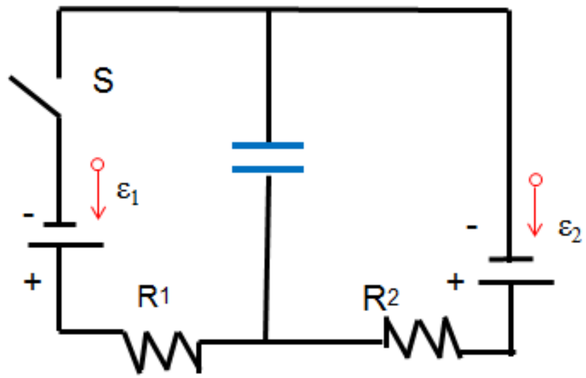
$i_2 = i_1 + i_C$ (节点)

$\varepsilon_1 + i_1R_1 - \frac{q}{C} = 0$ (环路)

$\varepsilon_2 - i_2R_2 - \frac{q}{C} = 0$

可写出 q 关 $\left(\frac{dq}{dt}\right)$

Answer 2



When S is open for a long time, the charge on C is $q_i = \varepsilon_2 C$. When S is closed for a long time, the current i in R_1 and R_2 is

$$i = (\varepsilon_2 - \varepsilon_1)/(R_1 + R_2) = (3.0\ \text{V} - 1.0\ \text{V})/(0.20\ \Omega + 0.40\ \Omega) = 3.33\ \text{A}.$$

The voltage difference V across the capacitor is then

$$V = \varepsilon_2 - iR_2 = 3.0\ \text{V} - (3.33\ \text{A})\ (0.40\ \Omega) = 1.67\ \text{V}.$$

Thus the final charge on C is $q_f = VC$. So the change in the charge on the capacitor is

$$\Delta q = q_f - q_i = (V - \varepsilon_2)C = (1.67\ \text{V} - 3.0\ \text{V})\ (10\ \mu\text{F}) = -13\ \mu\text{C}.$$

$$i_2 = i_C + i_1 \Rightarrow i_2 - i_1 = i_C$$

$$\left.\begin{aligned} &\varepsilon_2 - i_2 R_2 - \frac{q}{C} = 0 \Rightarrow i_2 = \left(\varepsilon_2 - \frac{q}{C}\right)\cdot\frac{1}{R_2} \\ &\varepsilon_1 + i_1 R_1 - \frac{q}{C} = 0 \Rightarrow -i_1 = \left(\varepsilon_1 - \frac{q}{C}\right)\cdot\frac{1}{R_1} \end{aligned}\right\}$$

$$\Rightarrow i_2 - i_1 = i_C = \left(\varepsilon_2 - \frac{q}{C}\right)\frac{1}{R_2} + \left(\varepsilon_1 - \frac{q}{C}\right)\frac{1}{R_1}$$

$$\Rightarrow \frac{\varepsilon_2}{R_2} + \frac{\varepsilon_1}{R_1} - \frac{q}{C}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) - \frac{dq}{dt} = 0$$

$$\Rightarrow \varepsilon_{eq} - \frac{q}{C} - R_{eq}\frac{dq}{dt} = 0$$

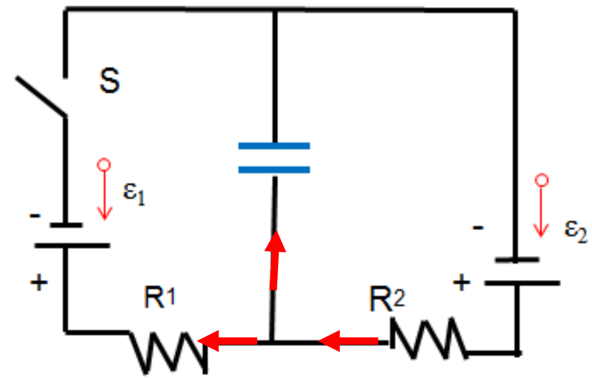
where $\varepsilon_{eq} = \left(\frac{\varepsilon_2}{R_1} + \frac{\varepsilon_1}{R_2}\right)\cdot R_{eq}$.

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

$$\Rightarrow \int_{q_i}^{q} \frac{dq}{q - C\varepsilon_{eq}} = -\int \frac{dt}{R_{eq} C}$$

$$\Rightarrow \frac{q - C\varepsilon_{eq}}{q_i - C\varepsilon_{eq}} = e^{-t/R_{eq} C}$$

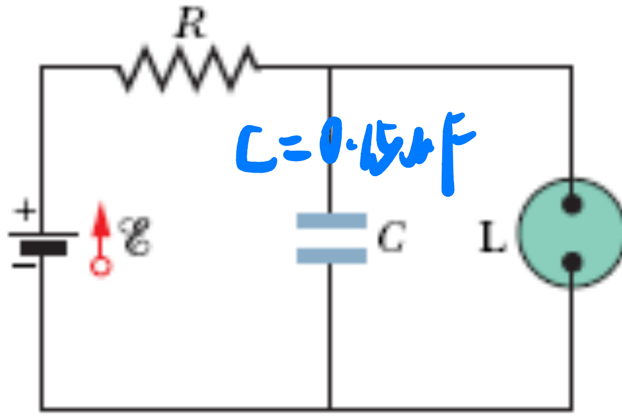
$$\Rightarrow q = C\varepsilon_{eq} + (q_i - C\varepsilon_{eq})e^{-t/R_{eq} C}.$$

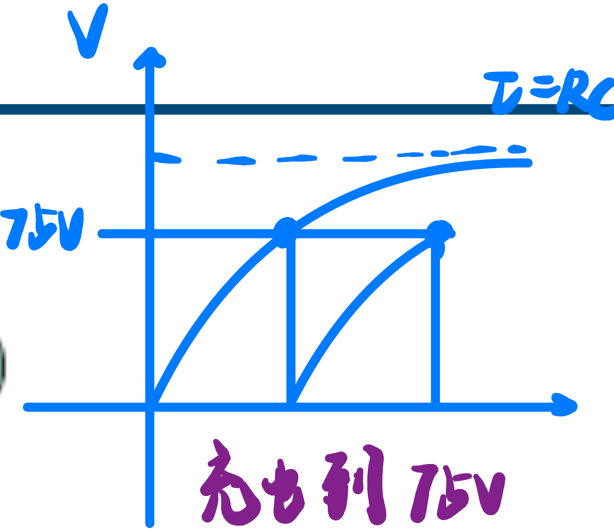


Problem 3

95V

C=0.15μF





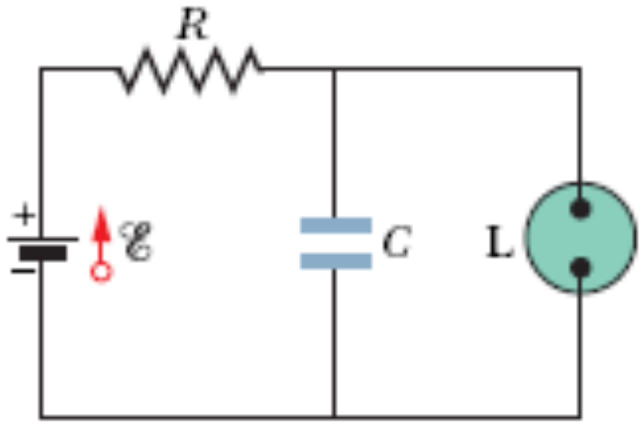
充电到75V
用0.5s

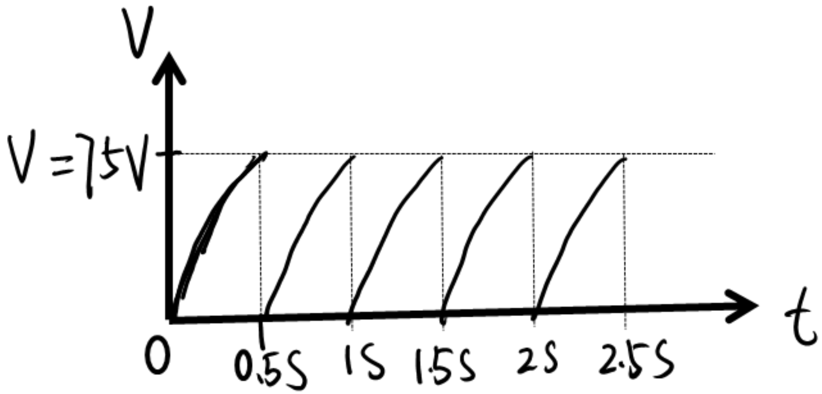
Figure shows the circuit of a flashing lamp, like those attached to barrels at highway construction sites. The fluorescent lamp *L* (of negligible capacitance) is connected in parallel across the capacitor C of an RC circuit. There is a current through the lamp only when the potential difference across it reaches the breakdown voltage V_L; Then the capacitor discharges completely through the lamp and the lamp flashes briefly. For a lamp with breakdown voltage $V_L = 75$ V, wired to a 95.0 V ideal battery and a 0.15 μF capacitor, what resistance *R* is needed for two flashes per second?

击穿电压

击穿后Rc极小 放电时间短

Answer 3





The time it takes for the voltage difference across the capacitor to reach V_L is given by: $V_L = \varepsilon(1 - e^{-t/RC})$

$$R = \frac{t}{C\ln(\varepsilon/(\varepsilon - V_L))} = \frac{0.5s}{(0.15\mu F)\ln(95.0/(95.0-75.0))} = 2.14\times10^6\,\Omega$$

Where we used t = 0.5s given (implicitly) in the problem.

Problem 4

A 3.00M Ω resistor and a 1.00μF uncharged capacitor are connected in series with an ideal battery of emf ε=4.0V . At 1.00s after the connection is made, what is the rate at which

(a) the **charge** of the capacitor is increasing ,

(b) **energy** is being stored in the capacitor,

(c) **thermal energy** is appearing in the resistor ,

(d) **energy** is being **delivered** by the battery?

Answer 4

(a) The charge on the positive plate of the capacitor is given by

$$q = C\varepsilon(1 - e^{-t/RC})$$

where ε is the emf of the battery, C is the capacitance, and τ is the time constant. The value of τ is

$$\tau = RC = (3.00 \times 10^{6}\ \Omega)(1.00 \times 10^{-6}\ \text{F}) = 3.00\ \text{s}.$$

At $t = 1.00$ s, $t/\tau = (1.00\ \text{s})/(3.00\ \text{s}) = 0.333$ and the rate at which the charge is increasing is

$$\frac{dq}{dt} = \frac{C\varepsilon}{\tau}e^{-t/\tau} = \frac{\left(1.00\times10^{-6}\ \text{F}\right)\left(4.00\ \text{V}\right)}{3.00\ \text{s}}e^{-0.333} = 9.55\times10^{-7}\ \text{C/s}.$$

(b) The energy stored in the capacitor is given by $U_C = \dfrac{q^2}{2C}$, and its rate of change is

$$\frac{dU_C}{dt} = \frac{q}{C}\frac{dq}{dt}.$$

Answer 4

Now

$$q = C\varepsilon(1 - e^{-t/RC}) = (1.00\times 10^{-6}F)(4.00V)(1 - e^{-0.333}) = 1.13\times 10^{-6}C$$

so

$$\frac{dU_C}{dt} = \frac{q}{C}\frac{dq}{dt} = \left(\frac{1.13\times 10^{-6}\,\mathrm{C}}{1.00\times 10^{-6}\,\mathrm{F}}\right)\left(9.55\times 10^{-7}\,\mathrm{C/s}\right) = 1.08\times 10^{-6}\,\mathrm{W}.$$

(c) The rate at which energy is being dissipated in the resistor is given by $P = i^2R$. The current is 9.55×10^{-7} A, so

$$P = (9.55\times 10^{-7}A)^2 \times 3.00\times 10^{6}\Omega = 2.74\times 10^{-6}W$$

(d) The rate at which energy is delivered by the battery is

$$i\varepsilon = (9.55\times 10^{-7}A)\times(4.00V) = 3.82\times 10^{-6}W$$

The energy delivered by the battery is either stored in the capacitor or dissipated in the resistor. Conservation of energy requires that $i\varepsilon = (q/C)(dq/dt) + i^2R$. Except for some round-off error the numerical results support the conservation principle.