Key words of chapter 28

Magnetic dipole 磁偶极子

Crossed field 混合场

Hall effect 霍尔效应

Number density 数密度

Electromagnet 电磁铁

Cyclotron 回旋加速器

Synchrotron 同步加速器

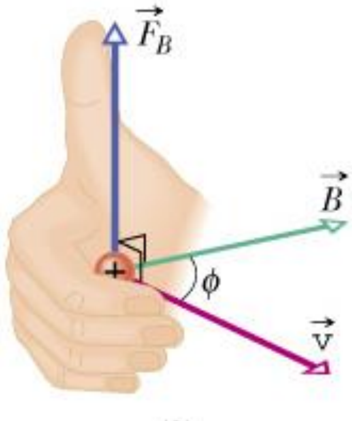
Helical paths 螺旋路径

Magnetic dipole moment 磁偶极矩

Coil 线圈

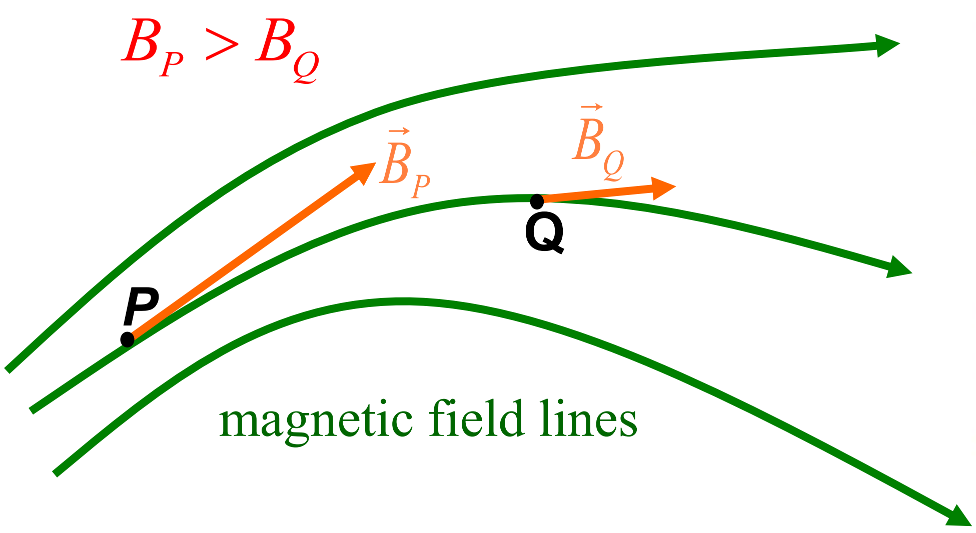
Orientation energy 方向能

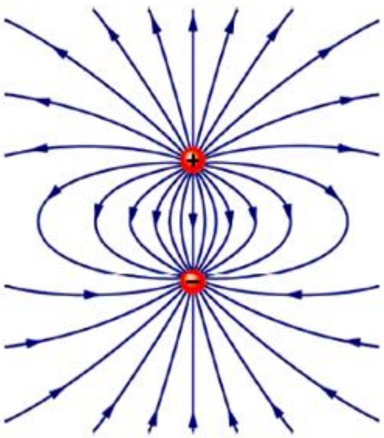
Magnetic force and Magnetic field lines

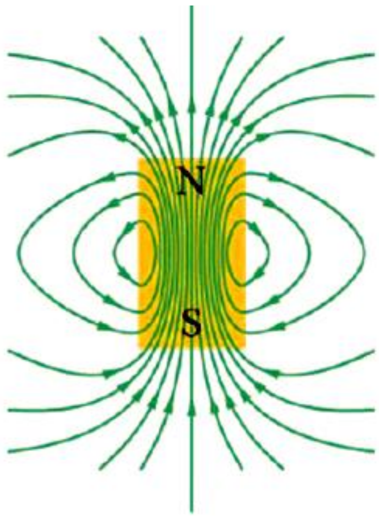


$$\boxed{\vec{F}_B = q\vec{v} \times \vec{B}}$$

$$\boxed{F_B = |q| vB \sin\phi}$$

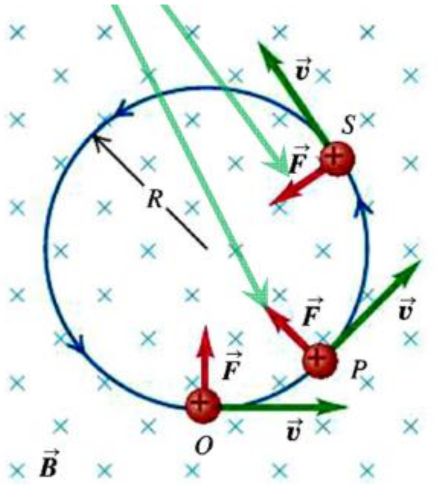






Moving charge in B-Field

Circular motion

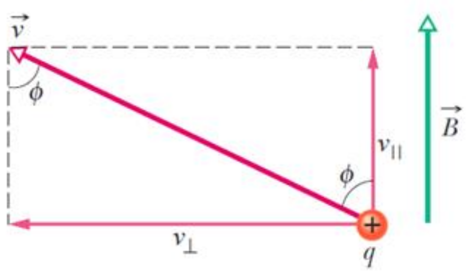


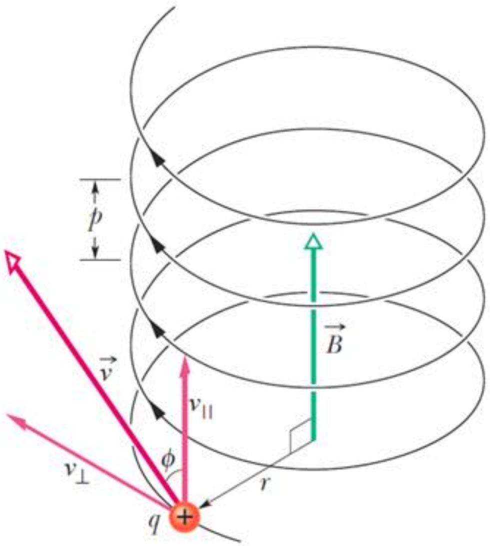
$$r = \frac{m\,v}{q\,B}$$

$$T = \frac{2\pi r}{v} = \frac{2\pi}{v}\,\frac{mv}{q\,B} = \frac{2\pi m}{q\,B}$$

Helical motion

The velocity component perpendicular to the field causes circling, which is stretched upward by the parallel component.

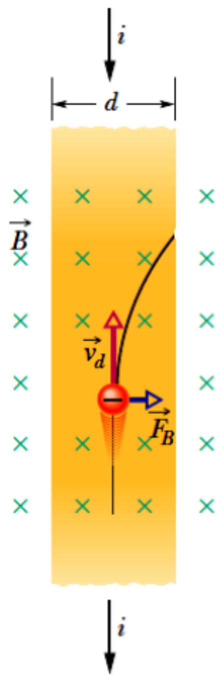


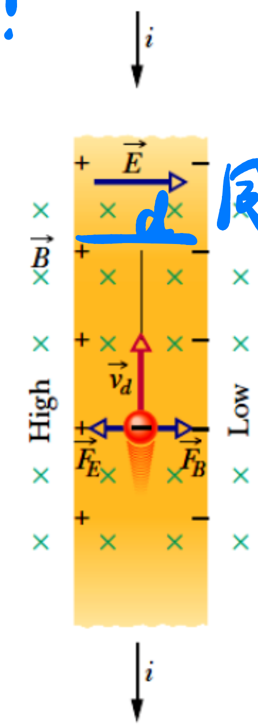


Moving in Crossed field (E-field and B-field):

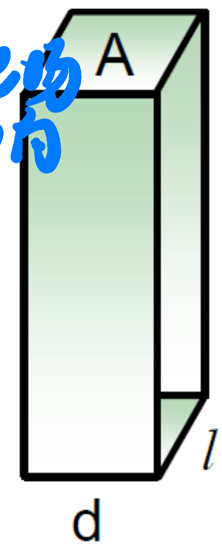
对i!

$F = iBL$

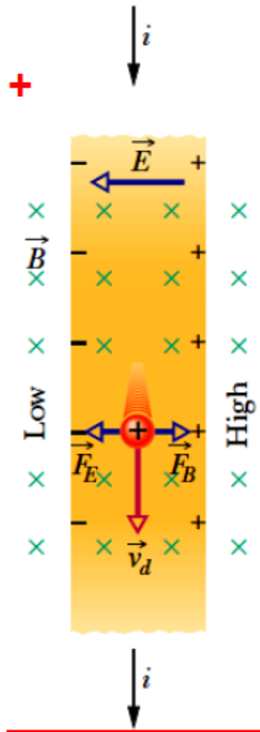




$\perp$ 同场方向



If the carrier is +



$$eE = ev_dB.$$

By a voltmeter, we can get ***V*** and ***V= Ed***

$$\boxed{v_d = \frac{J}{ne} = \frac{i}{neA}}$$

$$n = \frac{Bi}{Vle}$$

l = thickness of the copper strip

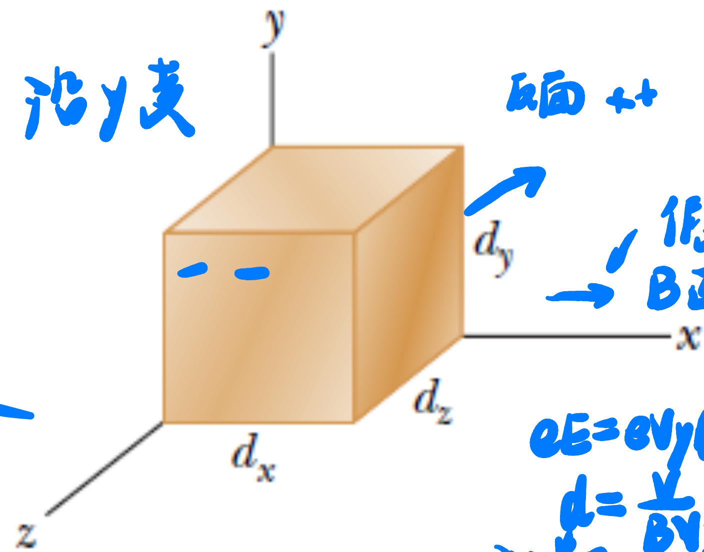
n = number of carrier per volume

Problem 1

正负电荷分离

A metallic block, with its faces parallel to coordinate axes. The block is in a uniform magnetic field of magnitude B=0.020 T. One edge length of the block is 25 cm; the block is *not* drawn to scale.

沿y走



反面 ++

假设 B正向

$qE = qV_yB$

$d = \frac{V}{BV_y}$

x方向的

The block is moved at **3.0 m/s parallel** to each axis, in turn, and the resulting potential difference V that appears across the block is measured. With the motion **parallel to the y axis, V=12 mV;** with the motion **parallel to the z axis, V=18 mV;** with the motion parallel to **the x axis, V=0.** What are the block lengths (a) d_x, (b) d_y, and (c) d_z?

$\vec{F} = q\vec{v} \times \vec{B}$

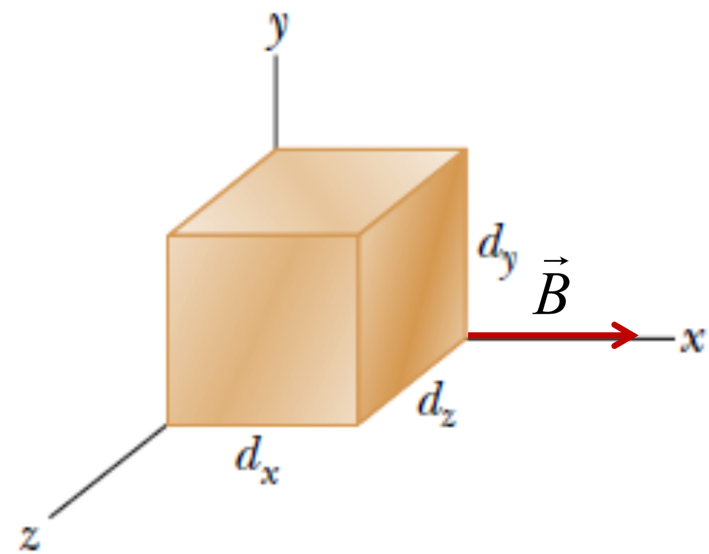
(自由电子才能[illegible])

with the motion parallel to the *x* axis, *V*=0, So B must be along the *x* axis

$$d = \frac{V}{E} = \frac{V}{vB}$$

$$d = d_z = \frac{0.012\text{ V}}{(3.0\text{ m/s})(0.020\text{ T})} = 0.2\text{ m}$$

$$d = d_y = \frac{0.018\text{ V}}{(3.0\text{ m/s})(0.020\text{ T})} = 0.30\text{ m}$$



Thus, our answers are

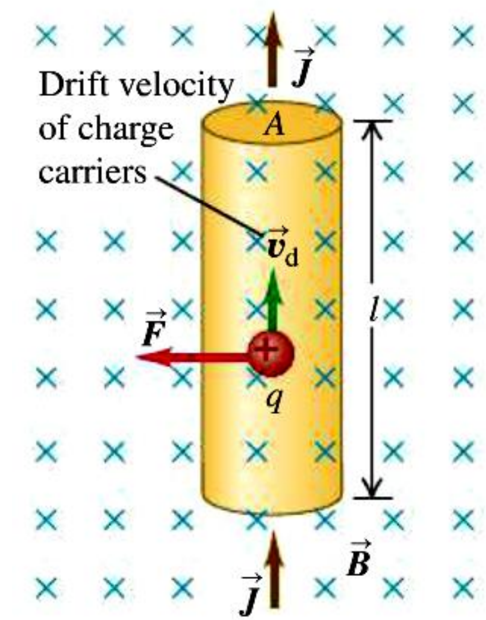
(*a*) d_x = 25 cm

which we arrive at "by elimination," since we already have figured out d_y and d_z

(b) d_y = 30 cm

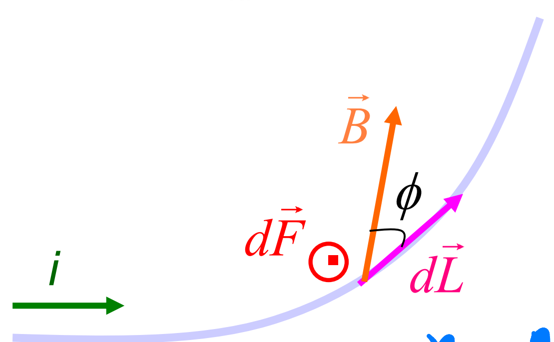
(c) d_z = 20 cm.

Magnetic force



$$\vec{J} = nq\vec{v}_d$$

$$\boxed{\vec{F}_B = i\vec{L} \times \vec{B}}$$



$$\boxed{d\vec{F}_B = i d\vec{L} \times \vec{B}}$$

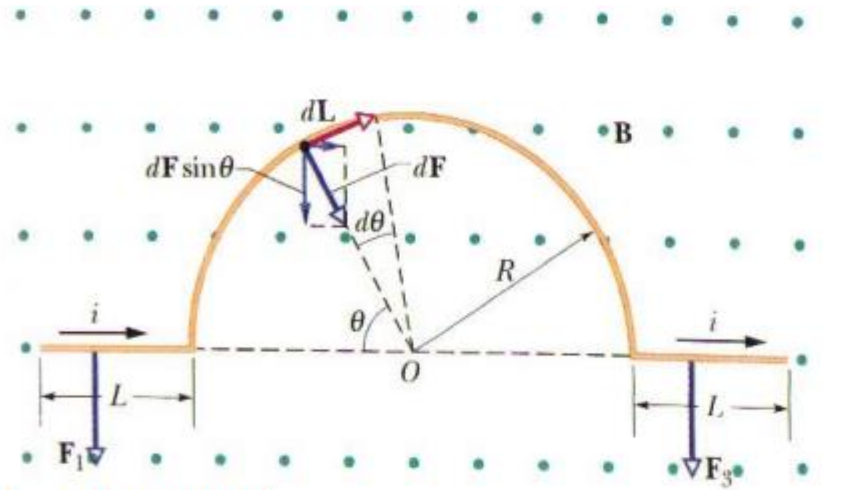
$$\boxed{\vec{F}_B = i\int d\vec{L} \times \vec{B}}$$

闭合电路
在均匀B中受力=0

如果B是均匀的
可等效L于导线首尾
闭合导线 $F_B=0$

Example 1

Wire with current i.
Magnetic field out of page.
What is net force on wire?



$$F_1 = F_3 = iLB$$

$$dF = iBdL = iBRd\theta$$

By symmetry, F_2 will only have a vertical component,

$$F_2 = \int_0^{\pi} \sin(\theta) dF = iBR \int_0^{\pi} \sin(\theta) d\theta = 2iBR$$

$$F_{\text{total}} = F_1 + F_2 + F_3 = iLB + 2iRB + iLB = 2iB(L+R)$$

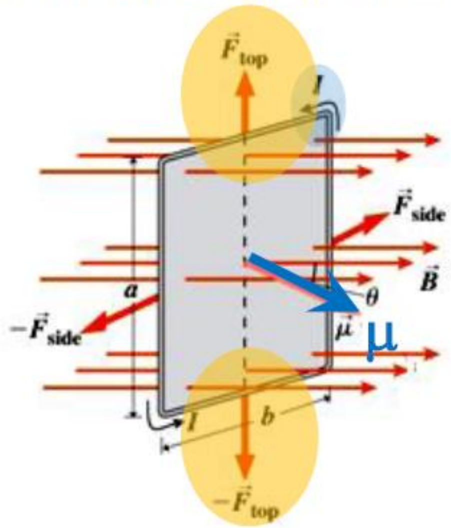
The net force acting on a curved current perpendicular to a uniform magnetic field doesn't depend on the shape

Example 2

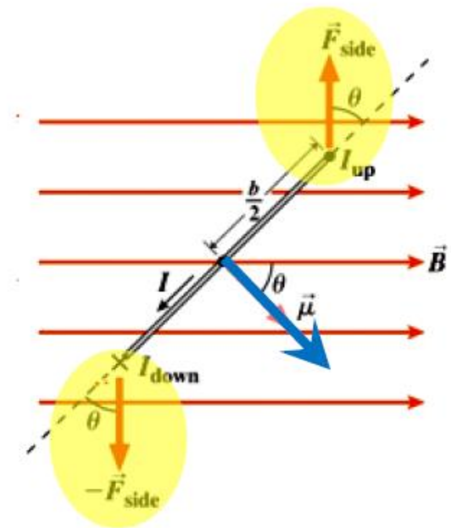
30 A long, rigid conductor, lying along an x axis, carries a current of 7.0 A in the negative x direction. A magnetic field $\vec{B}$ is present, given by $\vec{B} = 3.0\hat{i} + 8.0x^2\hat{j}$, with x in meters and $\vec{B}$ in milliteslas. Find, in unit-vector notation, the force on the 2.0 m segment of the conductor that lies between $x = 1.0$ m and $x = 3.0$ m.

The right hand rule determines the direction of the magnetic moment

Forces on top & bottom cancel out



Forces on sides also cancel; but give net torque.



$$F_{side} = a\, I\, B \qquad \tau_{side} = \frac{1}{2} b\, F_{side} \sin\theta = \frac{1}{2} b\, a\, I\, B \sin\theta$$

$$\tau = 2\tau_{side} = b\, a\, I\, B \sin\theta = \mu\, B \sin\theta$$

$$\mu = area \bullet current$$

magnetic dipole moment
direction ⊥ coil

Torque on dipole: $\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}$

The right hand rule determines the direction of the magnetic moment

The magnitude of the magnetic dipole moment is $\mu = NiA$.

In vector form: $\vec{\tau} = \vec{\mu} \times \vec{B}$.

The orientation energy of the coil is: $U = -\mu B \cos\theta = -\vec{\mu} \cdot \vec{B}$.

U has a minimum value of $-\mu B$ for $\theta = 0$ (position of **stable** equilibrium).

U has a maximum value of μB for $\theta = 180°$ (position of **unstable** equilibrium).

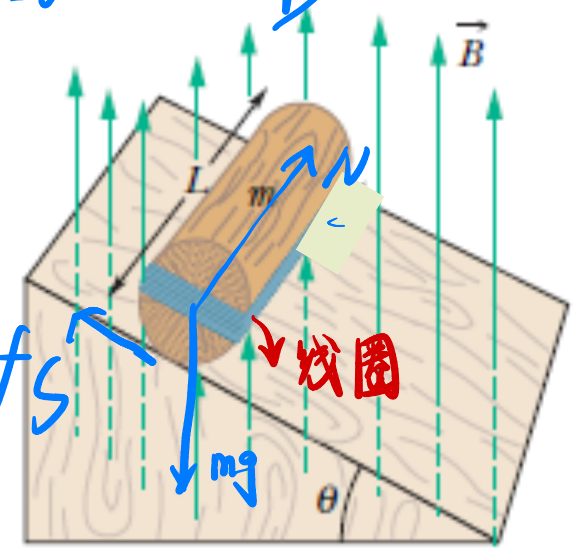
Note: For both positions the net torque is $\tau = 0$.

$\vec{\tau} = \int \vec{r_i} \times (i\,d\vec{L} \times \vec{B})$

Problem 2

$\begin{cases} F_{合} = 0 \\ \tau_{合} = 0 \end{cases}$

A wood cylinder of mass $\boldsymbol{m}$ **= 0.250 kg** and **length** $\boldsymbol{L}$**=0.100 m**, with N**=10.0 turns** of wire wrapped around it longitudinally, so that the plane of the wire coil contains the long central axis of the cylinder.



(切向B)
磁场力合外力为0

The cylinder is released on a plane inclined at an angle θ to the horizontal, with the plane of the coil parallel to the incline plane. If there is a vertical uniform magnetic field of magnitude $B = 0.500T$, what is the current through the coil that keeps the cylinder stay at rest on the plane?

$f = mg\sin\theta$

$r f_s = r mg\cos\theta$

$\vec{\mu} \times \vec{B} = N i L 2 r B \sin\theta$

$i = \frac{mg}{2NLB}$ 5.0 无关

Problem 2

Since the plane of the loop is parallel to the incline the dipole moment is normal to the incline. The forces acting on the cylinder are the force of gravity mg, acting downward from the center of mass, the normal force of the incline F_N, acting perpendicularly to the incline through the center of mass, and the force of friction f, acting up the incline at the point of contact. We take the x axis to be positive down the incline. Then the x component of Newton's second law for the center of mass yields

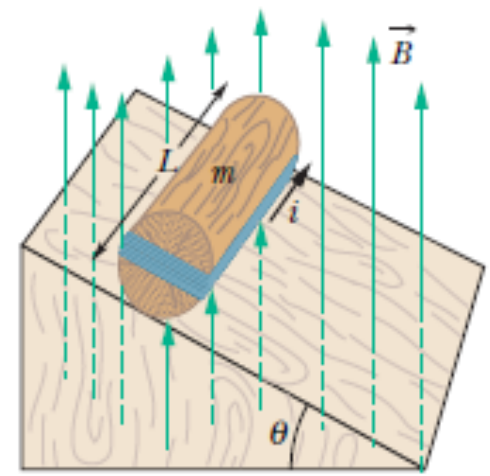
$$mg\sin\theta - f = ma.$$

Problem 2

For purposes of calculating the torque, we take the axis of the cylinder to be the axis of rotation. The magnetic field produces a torque with magnitude $\mu B \sin\theta$, and the force of friction produces a torque with magnitude fr, where r is the radius of the cylinder. The first tends to produce an angular acceleration in the counterclockwise direction, and the second tends to produce an angular acceleration in the clockwise direction. Newton's second law for rotation about the center of the cylinder, $\tau = I\alpha$, gives

$$fr - \mu B \sin\theta = I\alpha.$$

Since we want the current that holds the cylinder in place, we set $a = 0$ and $\alpha = 0$, and use one equation to eliminate f from the other. The result is The loop is rectangular with two sides of length L and two of length $2r$, so its area is $A = 2rL$ and the dipole moment is Thus, and



$$i = \frac{mg}{2NLB} = \frac{(0.250\,\mathrm{kg})(9.8\,\mathrm{m/s^2})}{2(10.0)(0.100\,\mathrm{m})(0.500\,\mathrm{T})} = 2.45\,\mathrm{A}.$$