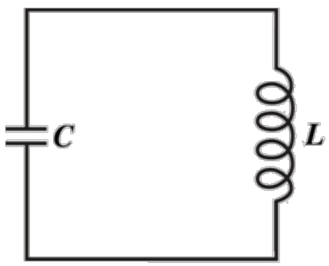
Review of Ch31 Key words

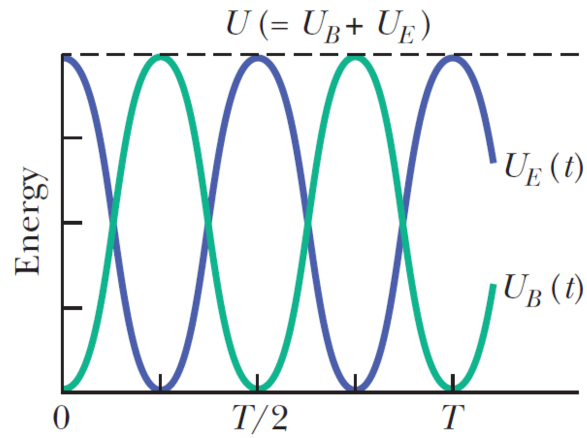
Electromagnetic Oscillations	**Phasor diagram**
LC Oscillations	**Capacitive reactance**
Damped oscillations	**Inductive reactance**
Driven/forced oscillations	**Impedance**
Natural angular frequency(ω)	**Phase constant**
Driving angular frequency(ω_d)	**Root-mean-square(rms)**
Electromagnetic energy	**Power factor($cos\phi$)**
AC alternating current	**Transformer**
DC direct current	**Primary winding(primary)**
Resonance	**Secondary winding(secondary)**

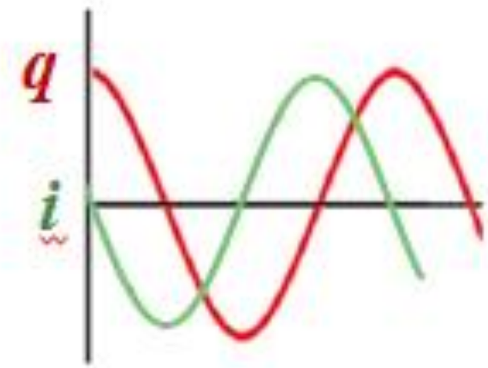
Review of Ch31

LC oscillator and Damped oscillator

$$\frac{Q^2}{2C} = \frac{1}{2}LI^2$$



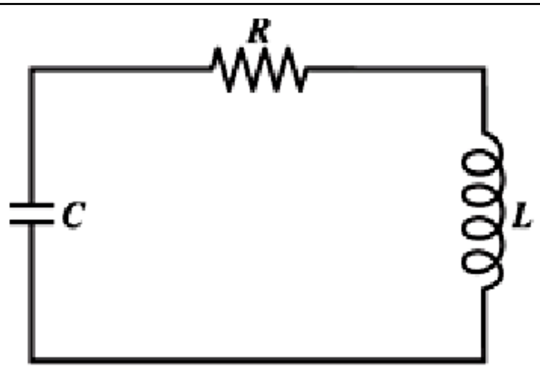


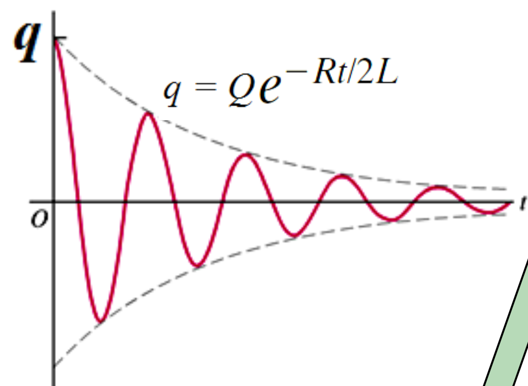


$$\frac{q^2}{2C} + \frac{1}{2}Li^2 = U \Rightarrow \frac{q}{C} + L\frac{d^2q}{dt^2} = 0 \quad \Rightarrow \quad q = Q\cos(\omega t + \phi) \qquad \omega = \frac{1}{\sqrt{LC}}$$

q : instantaneous value

Q : the amplitudes





$$q = Qe^{-Rt/2L}\cos(\omega' t + \phi)$$

$$\omega' = \sqrt{\omega^2 - (R/2L)^2}$$

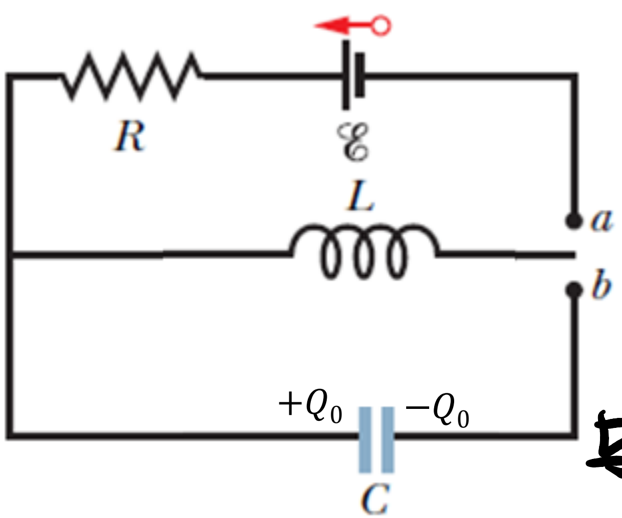
$$\omega = \frac{1}{\sqrt{LC}}$$

If R→0, ω’→ ω

$$\frac{q^2}{2C} + \frac{1}{2}Li^2 = U \Rightarrow \frac{dU}{dt} = \frac{q}{C}\frac{dq}{dt} + iL\frac{di}{dt} = -i^2R \Rightarrow \frac{q}{C} + L\frac{di}{dt} + iR = 0$$

Problem 1

In Fig, R $= 17.0\Omega, C = 6.20mF\ with\ initial\ charge\ Q_0 = 36.6mC,$ $L = 54.0mH,$ and the ideal battery has emf $\varepsilon = 34.0V$. The switch is kept at *a* for a long time and then thrown to position *b at t = 0* (Assuming this process has been finished instantaneously so that the current through the inductor keeps unchange at this moment). Find the expression of the charge q(t) on the right side of capacitor.



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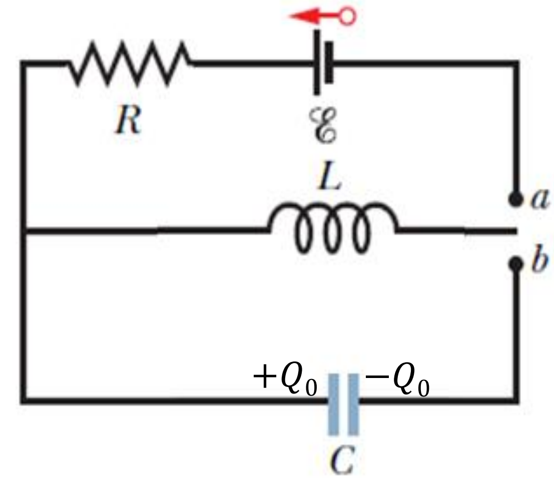
Answers

For the LC oscillating circuit, q on the right side of C is

$$q(t) = Q\cos(\omega t + \phi)$$

And the corresponding i (clockwise) is

$$i(t) = \frac{dq}{dt} = -\omega Q\sin(\omega t + \phi)$$



The switch is kept at a for a long time and then thrown to position b at $t = 0$, that means the initial current for the oscillating LC circuit is clockwise with magnitude:

$$i(t=0) = i_0 = \frac{\varepsilon}{R} = -\omega Q\sin(\phi) \qquad (1)$$

$C = 6.20mF$ $with$ $initial$ $charge$ $Q_0 = 36.6mC$ as shown in the figure, we can get the initial charge on the right side of C is:

$$q(t=0) = -Q_0 = Q\cos(\phi) \qquad (2)$$

Answers

The angular frequency is:

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{54\times10^{-3}\times6.2\times10^{-3}}} = 54\ s^{-1}$$

(1)+(2), we obtain:

$$\tan\phi = \frac{\varepsilon}{\omega R Q_0} = \sqrt{LC}\,\frac{\varepsilon}{RQ_0} = 1 \Rightarrow \phi = -135^\circ = -\frac{3\pi}{4} \quad (\sin\phi < 0\ \&\ \cos\phi < 0)$$

From the conservation of energy, we obtain:

$$U = U_C + U_L = \frac{q^2}{2C} + \frac{Li^2}{2} = \frac{Q^2}{2C} \Rightarrow Q = \sqrt{(-Q_0)^2 + LCi_0{}^2} = \sqrt{(-Q_0)^2 + LC(\frac{\varepsilon}{R})^2} = 51.8mC$$

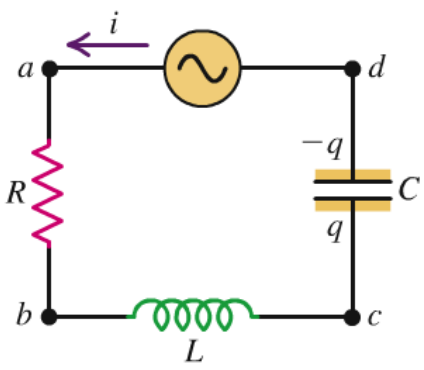
So the expression of the charge q(t) on the right side of capacitor:

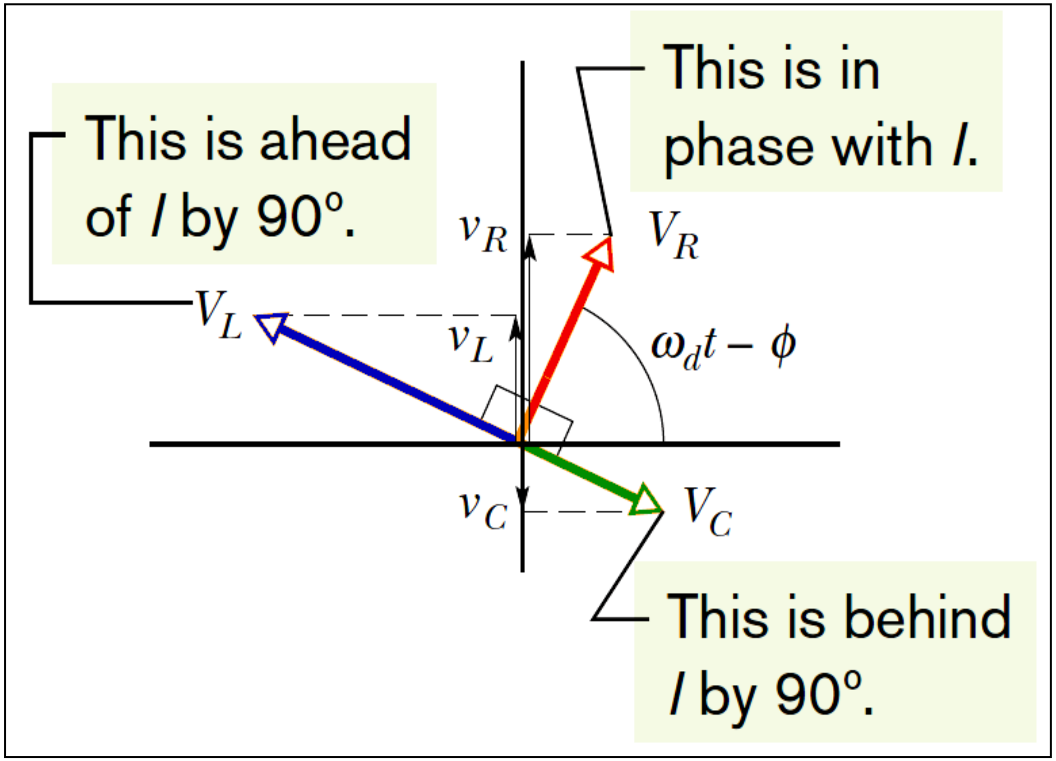
$$q(t) = 51.8mC\cos(54t - \frac{3\pi}{4})$$

Series RLC circuit

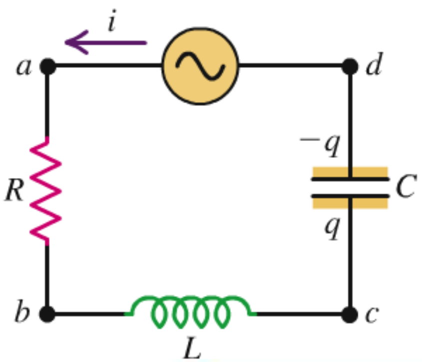
Circuit Elements with Alternating Current

Circuit Element	Amplitude Relationship	Circuit Quantity	Phase of v
Resistor	$V_R = IR$	R	In phase with i
Inductor	$V_L = IX_L$	$X_L = \omega L$	Leads i by 90°
Capacitor	$V_C = IX_C$	$X_C = 1/\omega C$	Lags i by 90°





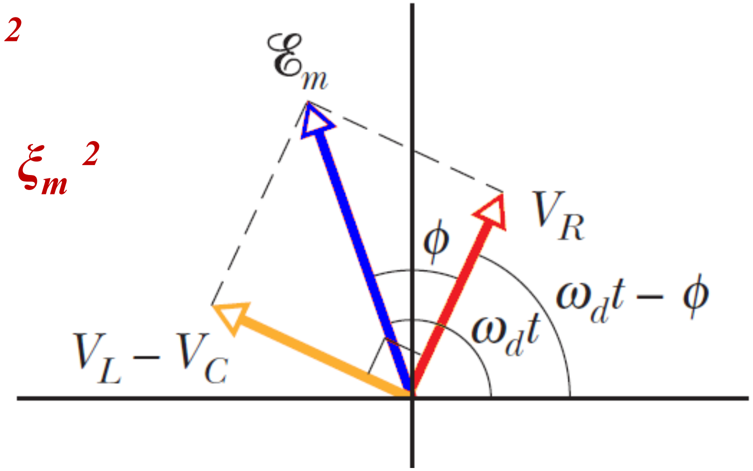
Review of Ch31 — Series RLC circuit—force oscillator



$$v_R + v_C + v_L = \varepsilon$$

$$\rightarrow V_R^2 + (V_L - V_C)^2 = \xi_m^2$$

$$(IR)^2 + (IX_L - IX_C)^2 = \xi_m^2$$



$$\boxed{I = \frac{\xi_m}{\sqrt{R^2 + (X_L - X_C)^2}}}$$

$$X_L = \omega L \qquad X_C = \frac{1}{\omega C}$$

***Z* : Impedance**

$$\boxed{Z = \sqrt{R^2 + (X_L - X_C)^2}}$$

Phase constant (angle) ϕ :

$$\tan\phi = \frac{V_L - V_C}{V_R} = \frac{IX_L - IX_C}{IR} = \frac{X_L - X_C}{R}$$

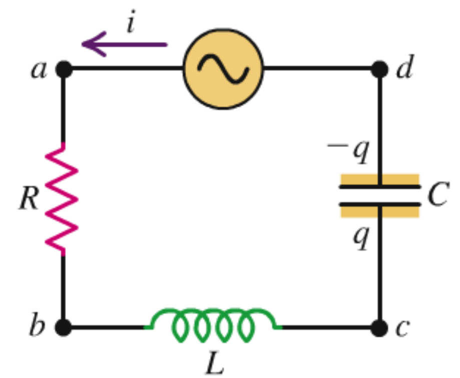
Resonance:

$$X_C = X_L$$

$$\boxed{\omega_d = \omega = \frac{1}{\sqrt{LC}} \qquad \text{(resonance)}}$$

Power in AC circuit

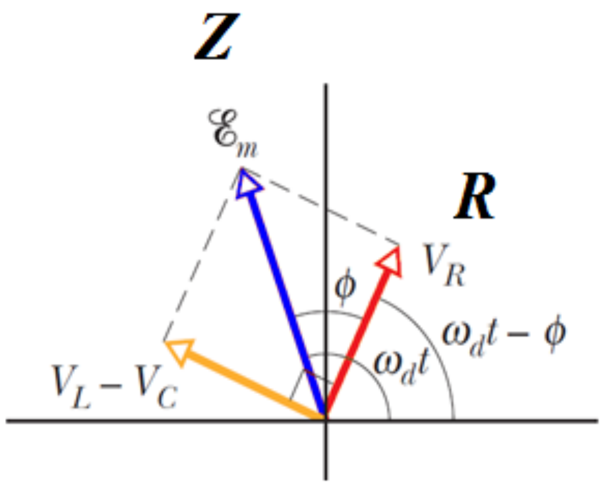
In DC circuit : *Power* = i^2R



In AC circuit : $P_{\text{avg}} = I_{\text{rms}}^2 R$ (average power)

$$I_{\text{rms}} = \frac{I}{\sqrt{2}} \qquad V_{\text{rms}} = \frac{V}{\sqrt{2}} \qquad \mathscr{E}_{\text{rms}} = \frac{\mathscr{E}_m}{\sqrt{2}}$$

The value shown in multimeter(万用表) is the rms value



$$I_{\text{rms}} = \frac{\mathscr{E}_{\text{rms}}}{Z} = \frac{\mathscr{E}_{\text{rms}}}{\sqrt{R^2 + (X_L - X_C)^2}}$$

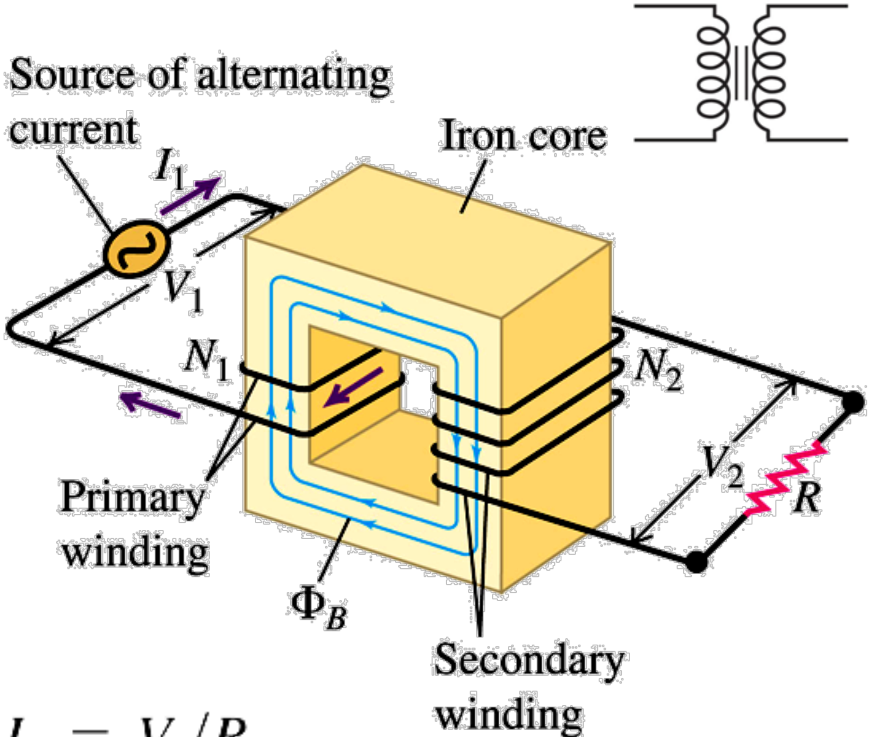
$$P_{\text{avg}} = I_{\text{rms}}^2 R = \frac{\mathscr{E}_{\text{rms}}}{Z} I_{\text{rms}} R = \mathscr{E}_{\text{rms}} I_{\text{rms}} \frac{R}{Z}$$

$$\boxed{P_{\text{avg}} = \mathscr{E}_{\text{rms}} I_{\text{rms}} \cos\phi}$$

Power factor: $\cos\phi = \frac{V_R}{\mathscr{E}_m} = \frac{IR}{IZ} = \frac{R}{Z}$

Transformers

Φ_B is the same for each turn of the primary and secondary windings



$$\boxed{\frac{V_2}{V_1} = \frac{N_2}{N_1}}$$

By conservation of energy : $\boxed{V_1 I_1 = V_2 I_2}$

put in: $V_2 = V_1 \dfrac{N_2}{N_1}$ **and** $I_2 = V_2/R$

we have $\dfrac{V_1}{I_1} = \dfrac{R}{(N_2/N_1)^2}$

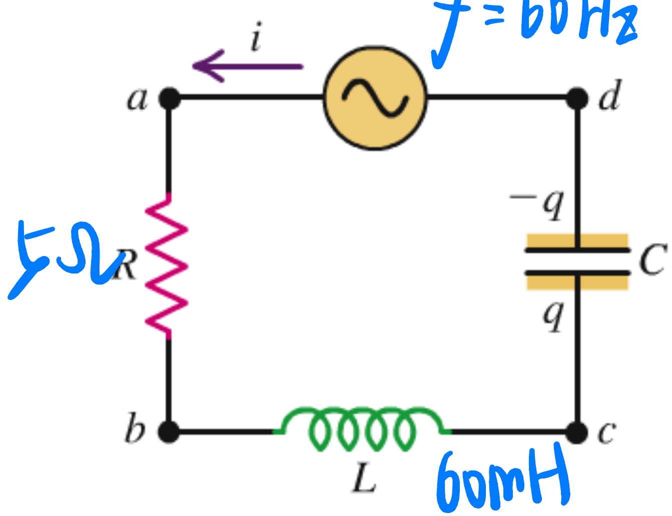
The value of load resistance seen by the generator

Problem 2

In an RLC circuit such as that of Fig. 31-7 assume that $R = 5.00\ \Omega$, $L = 60.0$ mH, $f_d = 60.0$ Hz, and $\mathscr{E}_m = 30.0$ V. For what values of the capacitance C would the average rate at which energy is dissipated in the resistance be (a) a maximum and (b) a minimum? What are (c) the maximum dissipation rate and the corresponding (d) phase angle and (e) power factor? What are (f) the minimum dissipation rate and the corresponding (g) phase angle and (h) power factor?

$\mathcal{E} = 30$ V

$f = 60$ Hz



Answers

We shall use $P_{\text{avg}} = \dfrac{\varepsilon_m^2 R}{2Z^2} = \dfrac{\varepsilon_m^2 R}{2\left[R^2 + (\omega_d L - 1/\omega_d C)^2\right]}$.

where $Z = \sqrt{R^2 + (\omega_d L - 1/\omega_d C)^2}$ is the impedance.

(a) Considered as a function of C, P_{avg} has its largest value when the factor $R^2 + (\omega_d L - 1/\omega_d C)^2$ has the smallest possible value. This occurs for $\omega_d L = 1/\omega_d C$, or

$$C = \frac{1}{\omega_d^2 L} = \frac{1}{(2\pi)^2 (60.0\,\text{Hz})^2 (60.0 \times 10^{-3}\,\text{H})} = 1.17 \times 10^{-4}\ \text{F}.$$

The circuit is then at resonance.

(b) In this case, we want Z^2 to be as large as possible. The impedance becomes large without bound as C becomes very small. Thus, the smallest average power occurs for $C = 0$ (which is not very different from a simple open switch).

(c) When $\omega_d L = 1/\omega_d C$, the expression for the average power becomes

$$P_{\text{avg}} = \frac{\varepsilon_m^2}{2R},$$

Answers

so the maximum average power is in the resonant case and is equal to

$$P_{avg} = \frac{(30.0\,\text{V})^2}{2(5.00\,\Omega)} = 90.0\,\text{W}.$$

(d) At maximum power, the reactances are equal: $X_L = X_C$. The phase angle ϕ in this case may be found from

$$\tan\phi = \frac{X_L - X_C}{R} = 0,$$

which implies $\phi = 0°$.

cos ϕ 为 power factor

(e) At maximum power, the power factor is cos $\phi = \cos 0° = 1$.

(f) The minimum average power is $P_{avg} = 0$ (as it would be for an open switch).

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(g) On the other hand, at minimum power $X_C \propto 1/C$ is infinite, which leads us to set $\tan\phi = -\infty$. In this case, we conclude that $\phi = -90°$.

(h) At minimum power, the power factor is cos $\phi = \cos(-90°) = 0$.