Key words

Capacitance

Free charge

Potential difference

Charging a capacitor

Discharging

In parallel

In series

Equivalent

Energy density

Polar dielectrics

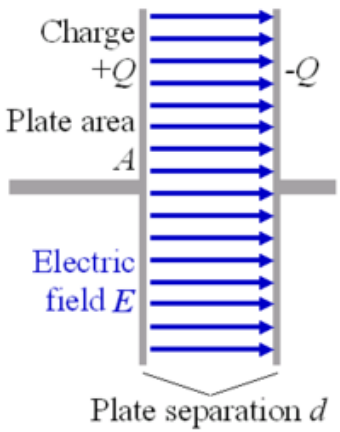
Nonpolar dielectrics

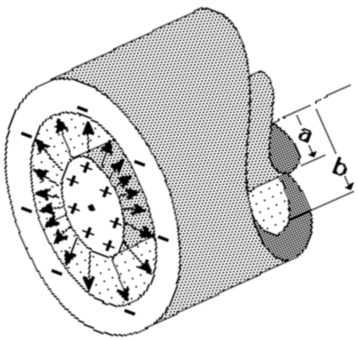
Induced surface charges

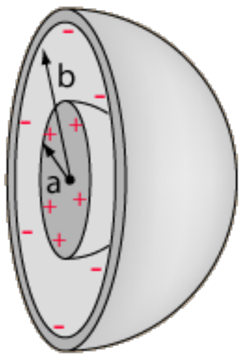
Capacitance

$$\boxed{q = CV}$$

q is the free charge
V is the potential difference between two plates
C is only dependent on the geometry of the capacitor







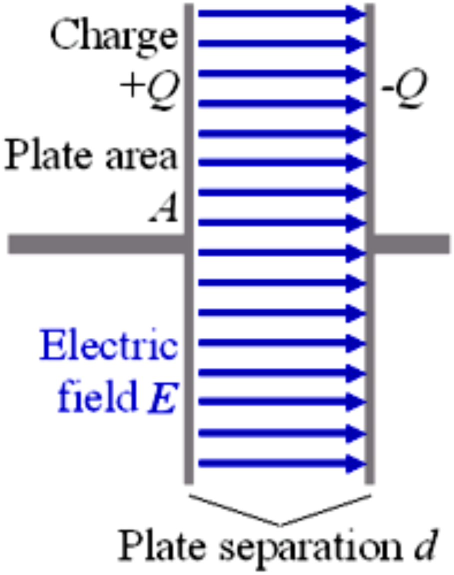
$$C = \frac{\varepsilon_0 A}{d}$$

$$C = 2\pi\varepsilon_0 \frac{L}{\ln(b/a)}$$

$$C = 4\pi\varepsilon_0 \frac{ab}{b-a}$$

1, Assume charge q on the positive plate
2, Calculate the E field between the plates
3, Calculate potential difference V between the plates
4, $C = q/V$ to find C

Electric field energy / Electric energy density



$$U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

u = Energy density

$$u = \frac{1}{2}\epsilon_0 E^2$$

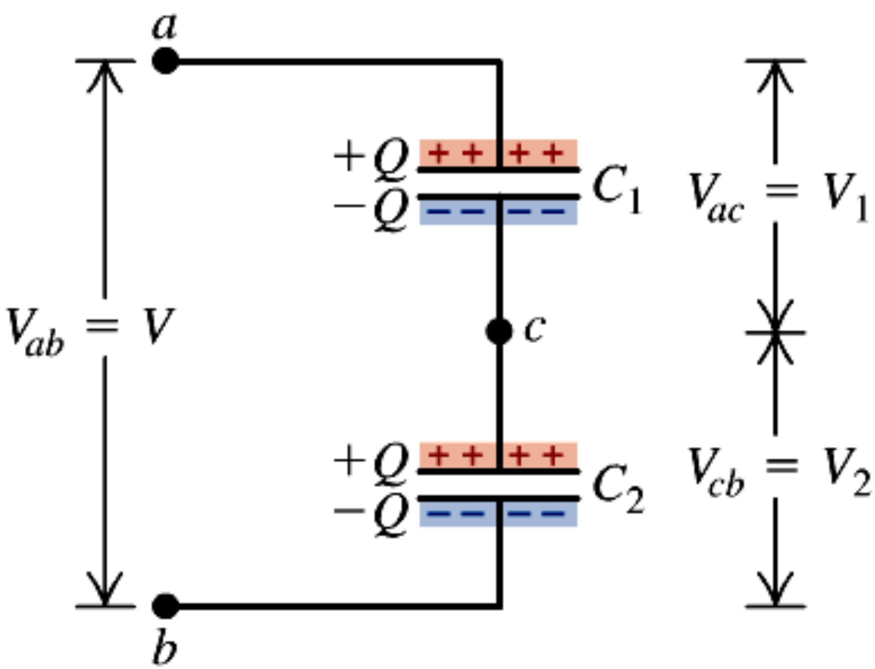
(electric energy density in a vacuum)

Capacitors in series / parallel connection

Capacitors in series:

- The capacitors have the same charge Q.
- Their potential differences add:

  $V_{ac} + V_{cb} = V_{ab}$.



$$V_{ab} = V = V_1 + V_2$$

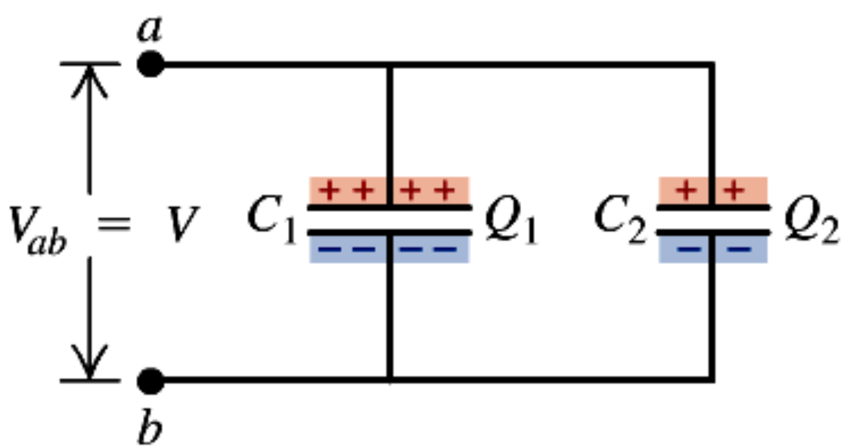
$$Q_1 = Q_2 = Q$$

$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots \qquad \text{(capacitors in series)}}$$

Capacitors in series / parallel connection

Capacitors in parallel:

- The capacitors have the same potential V.
- The charge on each capacitor depends on its capacitance: $Q_1 = C_1V$, $Q_2 = C_2V$.



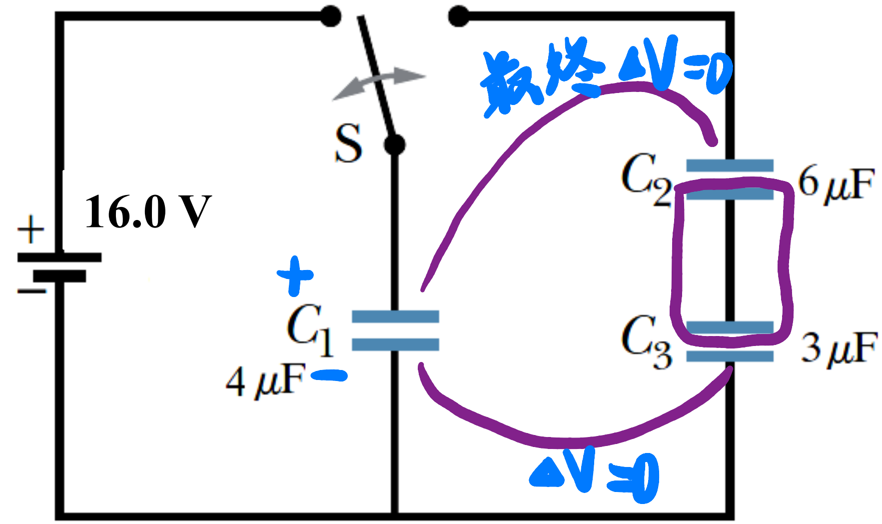
$$Q = Q_1 + Q_2$$

$$V_1 = V_2 = V$$

$$\boxed{C_{eq} = C_1 + C_2 + C_3 + \cdots \qquad \text{(capacitors in parallel)}}$$

Question 1

Three uncharged capacitors are connected as shown in the figure. Switch is thrown to the left side until C_1 is fully charged. Then the switch is thrown to the right.



$Q = C_1 V$

否则电荷仍会移动

What is the final charge on C_1 , C_2 and C_3?

$\left\{ \begin{array}{l} C_2\ C_3\ 串 \\ C_1\ 并 \end{array} \right.$

$\begin{array}{c} ++ \\ -- \end{array}$ 保持电势相同

Answer:

(a) Solving the two equations

$$\frac{q_1}{C_1} = \frac{q_2}{C_{eq}}$$ 等势

$$q_1 + q_2 = C_1 V_0$$

for q_1 and q_2, we obtain: $$q_1 = \frac{C_1^2 V_0}{C_{eq} + C_1} = \frac{C_1^2 V_0}{\dfrac{C_2 C_3}{C_2 + C_3} + C_1} = \frac{C_1^2 (C_2 + C_3) V_0}{C_1 C_2 + C_1 C_3 + C_2 C_3}.$$

$C_{eq} = 2.00\ \mu F$ $\qquad$ $q_1 = 42.7 \mu C$

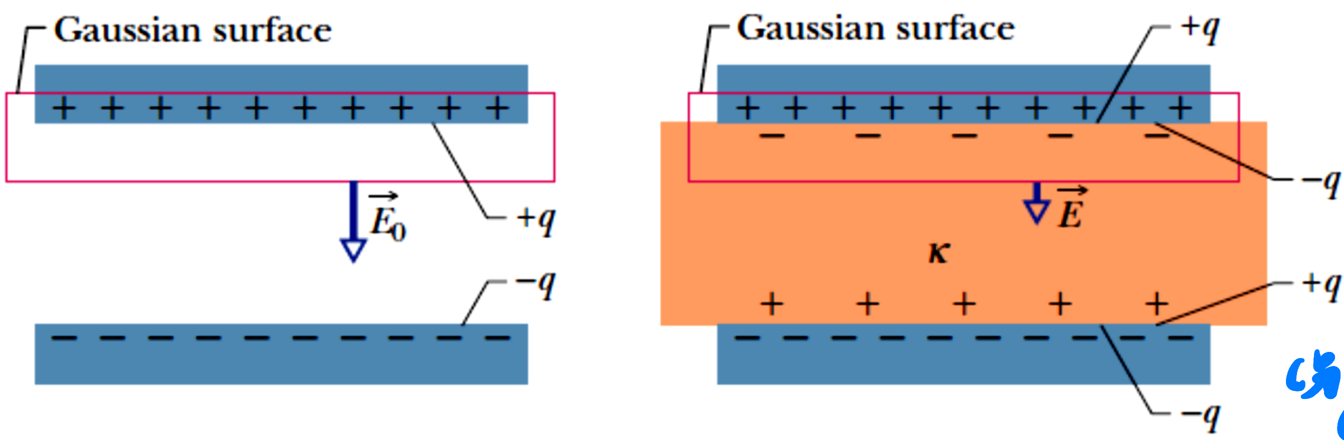
(b) The charge on capacitors 2 is

$$q_2 = C_1 V_0 - q_1 = (4.00 \mu\text{F})(16.0\ \text{V}) - 42.7\ \mu\text{C} = 21.3\ \mu\text{C}$$

(c) The charge on capacitor 3 is the same as that on capacitor 2:

$$q_3 = C_1 V_0 - q_1 = (4.00\ \mu\text{F})(16.0\ \text{V}) - 42.7\ \mu\text{C} = 21.3\ \mu\text{C}$$

Dielectric and Gauss' Law



极化

微观解释：

电偶极子趋向与E同向

$$C \longrightarrow \kappa C$$

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = \varepsilon_0 EA = q_{enc} = q_f$$

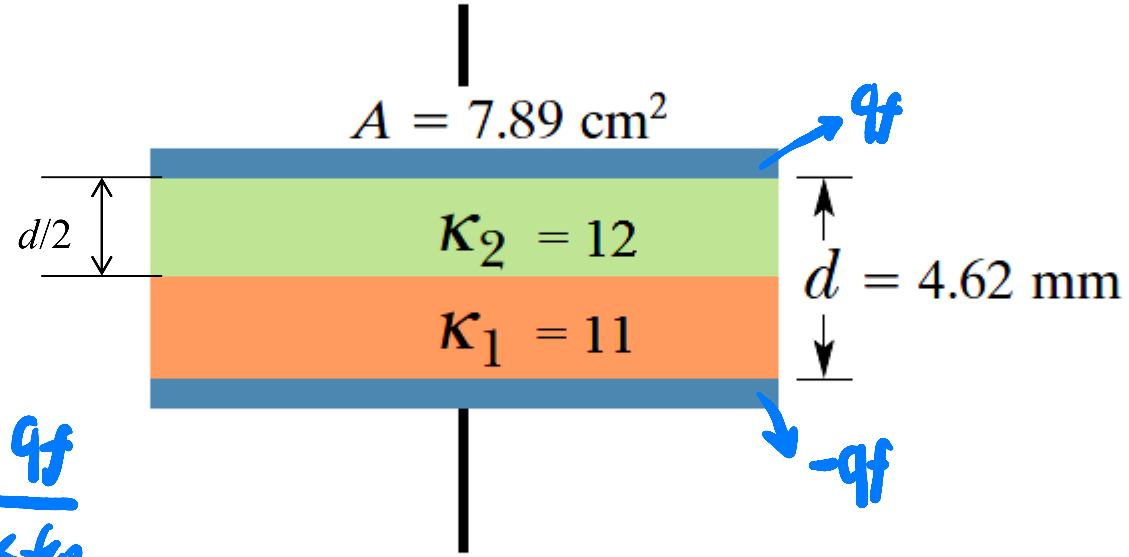
$C = \frac{q}{V}$ → 极板上自由电荷

$$\left.\begin{array}{l} \varepsilon_0 \oint \vec{E} \cdot d\vec{A} = \varepsilon_0 EA = q_{enc} = q_f - q' \\ \varepsilon_0 \oint \kappa \vec{E} \cdot d\vec{A} = \varepsilon_0 \kappa EA = q_f \end{array}\right\}$$

$$\Rightarrow q_f - q' = \frac{q_f}{\kappa}$$

Question

Capacitance?



$$C = \frac{q}{V}$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_f}{\kappa_2 \varepsilon_0}$$

$$E_2 = \frac{q_f}{\kappa_2 \varepsilon_0 A}$$

$$E_1 = \frac{q_f}{\kappa_1 \varepsilon_0 A}$$

$$V = E_1 \frac{d}{2} + E_2 \frac{d}{2}$$

$$C = \frac{q_f}{\left(\frac{q_f}{\kappa_1 \varepsilon_0 A} + \frac{q_f}{\kappa_2 \varepsilon_0 A}\right)\frac{d}{2}} \Rightarrow \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

Answer:

We assume there is charge q on one plate and charge $-q$ on the other. The electric field in the lower half of the region between the plates is $E_1 = \dfrac{q}{\kappa_1 \varepsilon_0 A}$,

where A is the plate area. The electric field in the upper half is $E_2 = \dfrac{q}{\kappa_2 \varepsilon_0 A}$.

Let $d/2$ be the thickness of each dielectric. Since the field is uniform in each region, the potential difference between the plates is

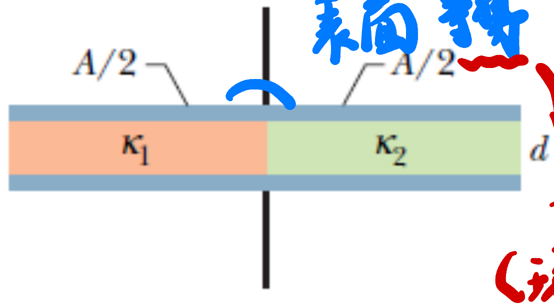
$$V = \frac{E_1 d}{2} + \frac{E_2 d}{2} = \frac{qd}{2\varepsilon_0 A}\left[\frac{1}{\kappa_1} + \frac{1}{\kappa_2}\right] = \frac{qd}{2\varepsilon_0 A}\frac{\kappa_1 + \kappa_2}{\kappa_1 \kappa_2},$$

$$C = \frac{q}{V} = \frac{2\varepsilon_0 A}{d}\frac{\kappa_1 \kappa_2}{\kappa_1 + \kappa_2}.$$

This expression is exactly the same as that for C_{eq} of two capacitors in series, one with dielectric constant κ_1 and the other with dielectric constant κ_2. Each has plate area A and plate separation $d/2$. Also we note that if $\kappa_1 = \kappa_2$, the expression reduces to $C = \kappa_1 \varepsilon_0 A/d$, the correct result for a parallel-plate capacitor with plate area A, plate separation d, and dielectric constant κ_1.

$$C = \frac{2(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2})(7.89\times10^{-4}\ \mathrm{m^2})}{4.62\times10^{-3}\ \mathrm{m}}\frac{(11.0)(12.0)}{11.0+12.0} = 1.73\times10^{-11}\,\mathrm{F}.$$

Example of Series and parallel for capacitors

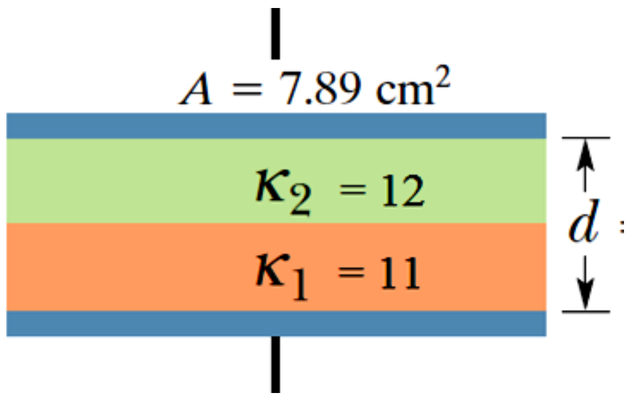


表面等势

并联 (球壳)

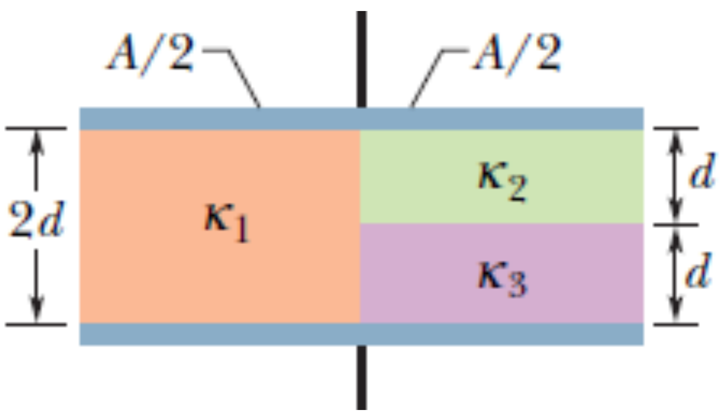
the plates of the capacitors are conductor, so two capacitors in parallel:

$$C_{eq} = C_1 + C_2 = \kappa_1 \frac{\varepsilon_0 A/2}{d} + \kappa_2 \frac{\varepsilon_0 A/2}{d} = (\kappa_1 + \kappa_2)\frac{\varepsilon_0 A}{2d}$$



It can be considered as two capacitors, having same charge q, in series:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{d/2}{\kappa_1 \varepsilon_0 A} + \frac{d/2}{\kappa_2 \varepsilon_0 A} \Rightarrow C_{eq} = \frac{\kappa_1 \kappa_2}{(\kappa_1 + \kappa_2)} \frac{\varepsilon_0 A}{d/2}$$

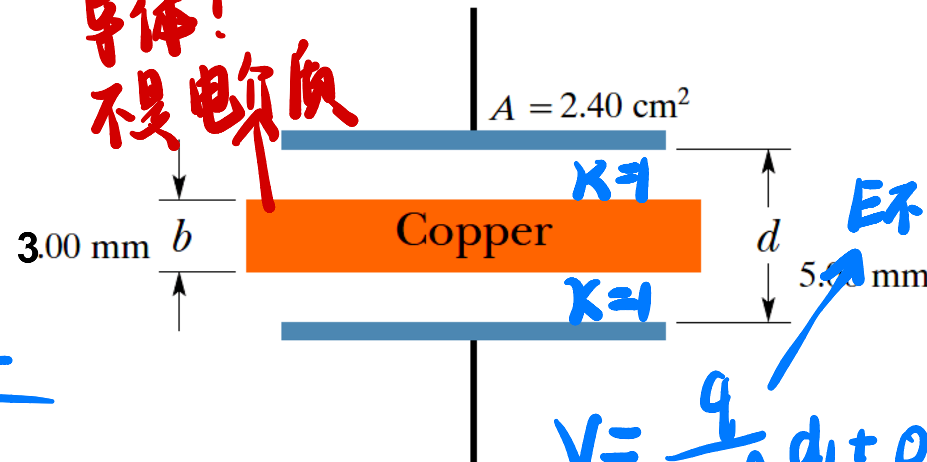


2 and 3 are in series and then parallel with 1, so:

$$C_{eq} = C_1 + C_{23} = \kappa_1 \frac{\varepsilon_0 A}{4d} + \frac{\kappa_2 \kappa_3}{(\kappa_2 + \kappa_3)} \frac{\varepsilon_0 A/2}{d}$$

Question

Copper is introduced to the isolated parallel capacitor



导体！不是电介质

E不改变

$$V = \frac{q}{\varepsilon_0 A} d_1 + 0 + \frac{q}{\varepsilon_0 A} d_2$$

$$C = \frac{q}{V} = \frac{q}{\frac{q}{\varepsilon_0 A}(d-b)} = \frac{\varepsilon_0 A}{d-b}$$

感应出电荷 得有F_{apply}拉，对系统做负功

(a) Capacitance with the copper introduced?

(b) If $q = 3.4\ \mu C$ is on the plates, what is the ratio of the stored energy before to that after the copper is inserted? (i.e. $U_{before} : U_{after}$)

(c) How much work is done by applied force on the copper as it is inserted?

(d) Is the copper sucked in or pushed in?

Answer:

(a) The length d is effectively shortened by b so $C' = \varepsilon_0 A/(d - b) = 1.06$ pF.

(b) The energy before, divided by the energy after inserting the slab is

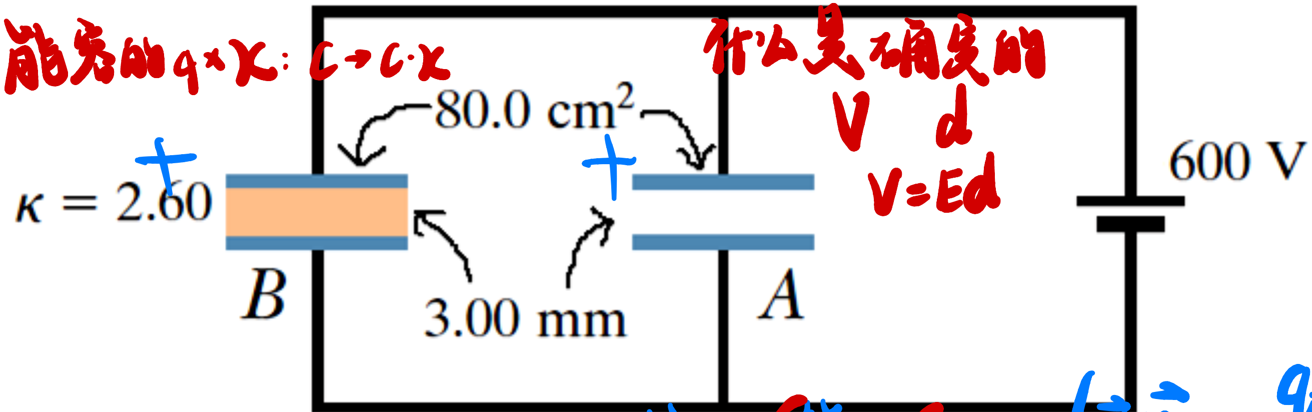
$$\frac{U}{U'} = \frac{q^2/2C}{q^2/2C'} = \frac{C'}{C} = \frac{\varepsilon_0 A/(d-b)}{\varepsilon_0 A/d} = \frac{d}{d-b} = \frac{5.00}{5.00-3.00} = 2.50.$$

(c) The work done is

$$W = \Delta U = U' - U = \frac{q^2}{2}\left(\frac{1}{C'} - \frac{1}{C}\right) = \frac{q^2}{2\varepsilon_0 A}(d-b-d) = -\frac{q^2 b}{2\varepsilon_0 A} = -\ 8.16\text{J}$$

(d) Since $W < 0$, the slab is sucked in.

Question 3



能容的q×κ: C→C·κ

什么是确定的 V d

V=Ed

Find

AB 等V、等E ⇒ $\oint \vec{E} \cdot d\vec{A} = \frac{q_f}{\varepsilon_0} = \frac{q_f}{\kappa \varepsilon_0}$

(a) E-field in the dielectric of capacitor B

(b) E-field in capacitor A

$Q = CV$ $\sigma_f = \frac{Q}{A}$

(c) Free charge density σ_f on the higher potential plate of capacitor A and capacitor B

(d) What is the induced charge density σ' on the top surface of the dielectric?

Answer:

(a) Ed = V, since V and d are constant, E is the same.

Since the field is constant and the capacitors are in parallel (each with 600 V across them) with identical distances ($d = 0.00300$ m) between the plates, then the field in A is equal to the field in B:

$$|\vec{E}| = \frac{V}{d} = 2.00 \times 10^5 \text{ V/m} .$$

(b) $|\vec{E}| = 2.00 \times 10^5 \text{ V/m}$. See the note in part (a).

(c) For the air-filled capacitor,

$$\sigma = \frac{q}{A} = \varepsilon_0 |\vec{E}| = 1.77 \times 10^{-6} \text{ C/m}^2 .$$

For the dielectric-filled capacitor,: 净荷数相等

↓ $\kappa EA = \frac{q_{in}}{\varepsilon_0}$ induced 是抵消原电场的

$$\sigma = \kappa\varepsilon_0 |\vec{E}| = 4.60 \times 10^{-6} \text{ C/m}^2 .$$

Answer:

(d) Capacitor B has a relatively large charge but only produces the field that A produces (with its smaller charge). We see that the difference in charge densities between parts (c) and (d) is due to the (negative) layer of charge at the top surface of the dielectric; consequently,

$$\sigma' = (1.77\times10^{-6}) - (4.60\times10^{-6}) = -2.83\times10^{-6}\ \mathrm{C/m^2}\ .$$

Or we can use the following equation:

$$\sigma_{fB} - \sigma' = \frac{\sigma_{fB}}{\kappa} \Longrightarrow \sigma' = \left(1 - \frac{1}{\kappa}\right)\sigma_{fB} = 2.83\times10^{-6}\ \mathrm{C/m^2}$$

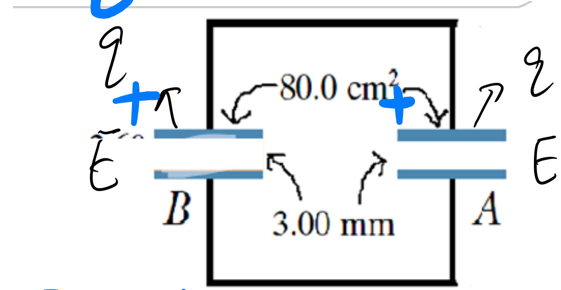
So the induced charge density σ’ on the top surface of the dielectric is:

$$-\sigma' = -2.83\times10^{-6}\ \mathrm{C/m^2}$$

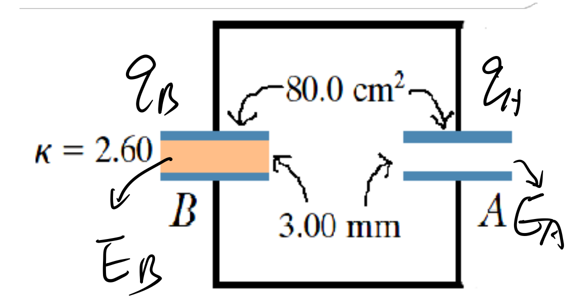
A similar example

Two identical capacitors, each with charge q and E between the plates, are shown in figure. If a dielectric slab is inserted into one of the capacitors. After reaching to a new equilibrium state， what is the free charge on the two capacitors?

$V = \frac{Q}{C}$



总电荷量守恒



A中q会到B
直至等势
总体 V ↓

Explanation

$$q_{Af} + q_{Bf} = 2q$$

$$q_{Af} = CV$$

$$q_{Bf} = KC \cdot V$$

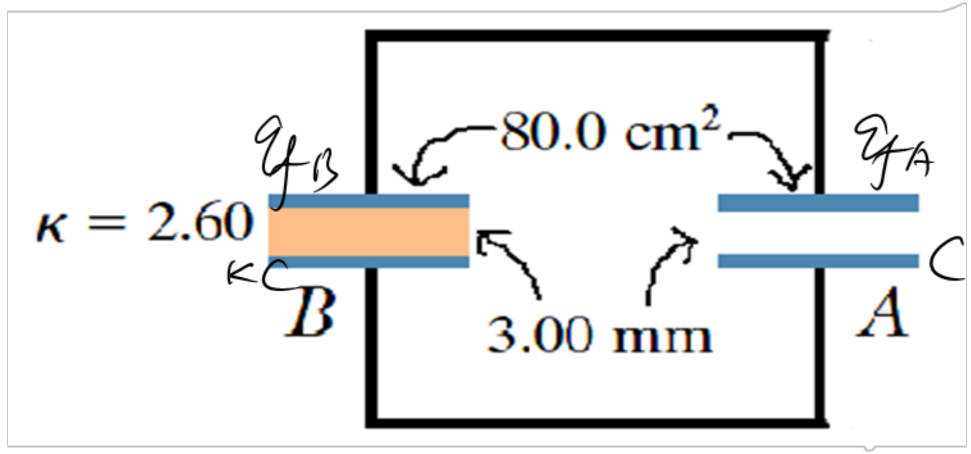
$$\Rightarrow \begin{cases} q_{Bf} = K q_{Af} \\ q_{Bf} + q_{Af} = (1+K) \cdot CV = 2q \end{cases}$$

V等

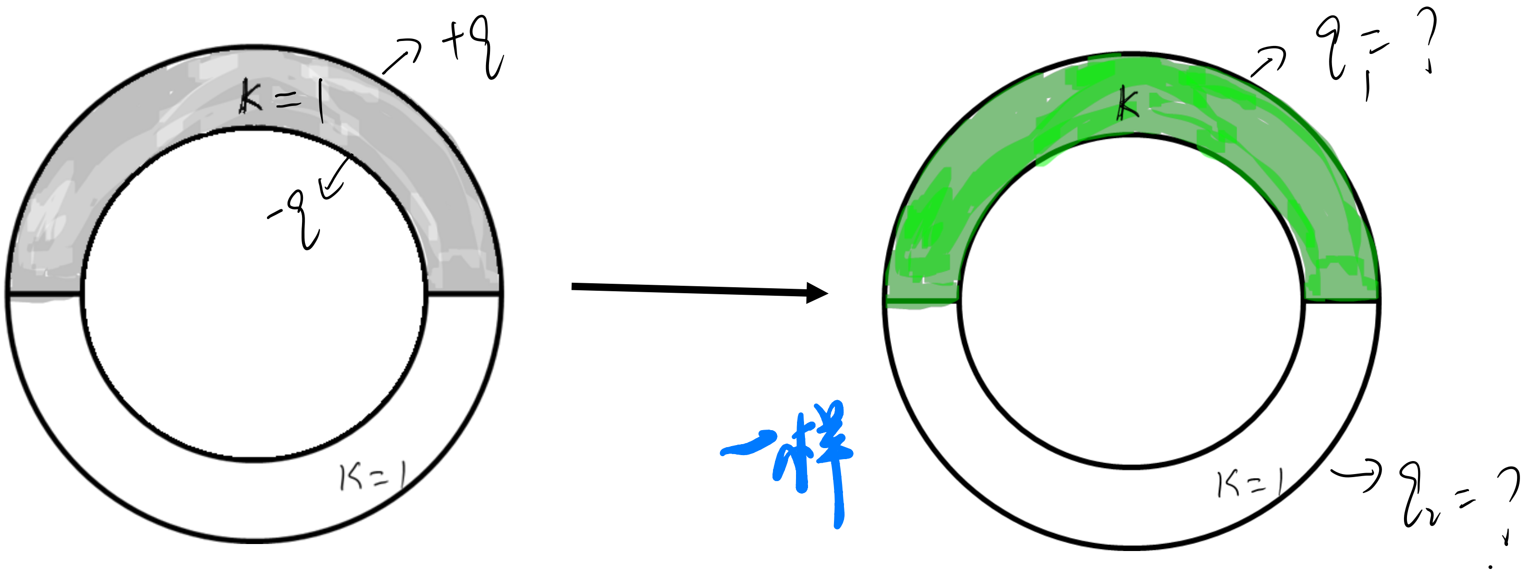
q不

$$\Rightarrow V = \frac{2q}{(1+K)C} \Rightarrow E = \frac{2q}{(1+K)\cdot C \cdot d} = \frac{2q}{(1+K)A\varepsilon_0}$$

$$q_{Af} + Kq_{Af} = 2q \Rightarrow q_{Af} = \frac{2q}{1+K}, \quad q_{Bf} = \frac{K}{1+K} 2q$$



Try it after class



The charge on an isolated spherical capacitor shown in left is q. If a dielectric is filled in the upper half of the capacitor, what are the charge q_1 on the upper half and q_2 on the lower half surface of the capacitor after reaching a new equilibrium state.