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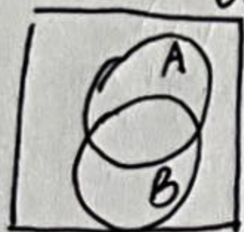
T1. let event A: the computer has problems with MB

event B: the computer has problems with HD



$P(A) = 0.4 \quad P(B) = 0.3$

$P(AB) = 0.15$

$P(A \cup B) = P(A) + P(B) - P(AB)$

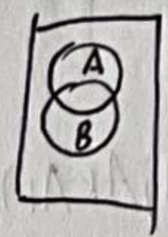
$= 0.55$

$P(\bar{A}\bar{B}) = P(\overline{A \cup B}) = 1 - 0.55 = 0.45$

so: 45% of a 10-year computer still has fully functioning MB and HD

T2. (1) let event A: the programmer knows Java

event B: the programmer knows Python



$P(A) = 0.7 \quad P(B) = 0.6$

$P(AB) = 0.5$

$P(A \cup B) = P(A) + P(B) - P(AB)$

$= 0.8$

$P(\bar{A}\bar{B}) = P(\overline{A \cup B}) = 0.2$

so: 20% he/she does not know Python and does not know Java

(2) $P(A\bar{B}) = P(A) - P(AB) = 0.2$

so: 20% he/she knows Java but not Python

(3) $P(A|B) = \frac{P(AB)}{P(B)} = \frac{0.5}{0.6} = \frac{5}{6} = 83.3\%$

so: 83.3% he/she knows Java given that he/she knows Python.

T3. We assume that we have n different elements in total, and we choose k elements for permutation and combination.

Permutation: for we can select with replacements, we have $\underbrace{n \cdot n \cdots n}_{k} = n^k$

Combination: we consider how many times every element is choosed. x_i means that the ith element is choosed for x_i times.

$x_1 + x_2 + \cdots + x_n = k \quad (x_i \geqslant 0, 1 \leq i \leq n)$

$(x_1+1) + (x_2+1) + \cdots + (x_n+1) = k+n \quad (x_i+1 \geqslant 1, 1 \leq i \leq n)$

we have C_{k+n-1}^{n-1}

T4. (1) event A: exactly k pairs are formed among the $2k$ shoes picked

$P(A) = \frac{C_n^k \cdot C_k^k}{C_{2n}^{2k}} = \frac{C_n^k}{C_{2n}^{2k}}$

(2) event B: no pair is formed among the $2k$ shoes picked

$P(B) = \frac{C_n^{2k} \cdot 2^{2k}}{C_{2n}^{2k}}$

(3) event C: exactly one pair is formed among the $2k$ shoes picked

$P(C) = \frac{C_n^1 C_{n-1}^{2k-2} \cdot 2^{2k-2}}{C_{2n}^{2k}}$

$= \frac{n \cdot 2^{2k-2} C_{n-1}^{2k-2}}{C_{2n}^{2k}}$

T5. We consider that

event A_i: couple A_i is paired, $1 \le i \le 4$

$$P(A_i) = \frac{3!}{4!} = \frac{1}{4}$$

$$P(A_iA_j) = \frac{2!}{4!} = \frac{1}{12} \quad 1 \le i < j \le 4$$

$$P(A_iA_jA_k) = \frac{1!}{4!} = \frac{1}{24} \quad P(A_iA_jA_kA_l) = \frac{0!}{4!} = \frac{1}{24}$$

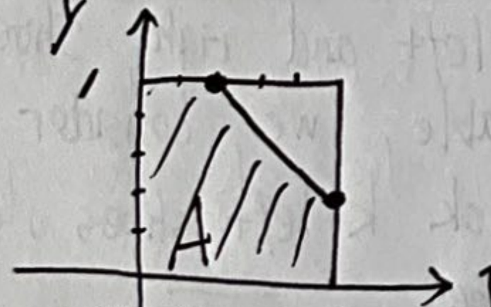
$$P(A_1 \cup A_2 \cup A_3 \cup A_4) = \sum_{i=1}^{4} P(A_i) - \sum_{1 \le i < j \le 4} P(A_iA_j) + \sum_{1 \le i < j < k \le 4} P(A_iA_jA_k) - P(A_iA_jA_kA_l)$$

$$= \frac{1}{4} \times 4 - \frac{1}{12} \times C_4^2 + \frac{1}{24} \times 4 - \frac{1}{24}$$

$$= \frac{5}{8}$$

so the probability is $\frac{5}{8}$

T6.



let event A: the two numbers' sum is less than $\frac{7}{5}$

the sample space is $\Omega = \{(x,y) \in [0,1] \times [0,1]\}$,

$$x + y \le \frac{7}{5} \quad x, y \in [0,1]$$

$$P(A) = \frac{\text{the area of } A}{\text{total area of } \Omega} = 1 - \frac{3}{5} \times \frac{3}{5} \times \frac{1}{2} = \frac{41}{50}$$

so the probability is $\frac{41}{50}$.

T7. let event A: test reports a positive result for a randomly chosen person

event B: a randomly chosen person who has a certain symptom has the disease

$$P(B) = \frac{1}{1000}$$

$$P(A|B) = 0.95 \quad P(A|\bar{B}) = 0.001$$

$$P(\bar{A}|B) = 0.05$$

$$P(B|A) = \frac{P(AB)}{P(A)} = \frac{P(A|B)P(B)}{P(A|B)P(B) + P(A|\bar{B})P(\bar{B})}$$

$$= \frac{0.95 \times \frac{1}{1000}}{0.95 \times \frac{1}{1000} + 0.001 \times \frac{999}{1000}}$$

$$\approx 0.487$$

so the probability is 0.487

it's higher than that in example 1.13

T8. opinion (1) is correct

let event A: the child you meet is a boy

event B: the other child is a boy

$P(B) = \frac{1}{2}$ event C_i: there are i boys

$P(A) = P(A|C_0) + P(A|C_1) + P(A|C_2)$

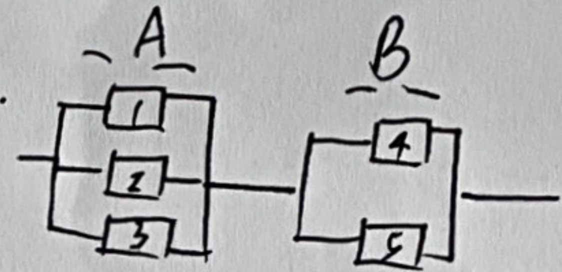
$= 0 + \frac{1}{2} \times \frac{2}{4} + 1 \times \frac{1}{4} = \frac{1}{2}$

$P(AB) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

$P(B|A) = \frac{P(AB)}{P(A)} = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$

so opinion (1) is correct

T9.



event A: part A works properly

event B: part B works properly

$P(A) = 1 - P(\bar{A}) = 1 - 0.3^3 = 0.973$

$P(B) = 1 - P(\bar{B}) = 1 - 0.3^2 = 0.91$

$P(AB) = 0.88543$

the probability is 0.88543