



3D Geometric Shape Assembly via Efficient Point Cloud Matching

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Geometric Shape Assembly?

Given a set of fractured parts, our goal is to predict the **6-DoF pose in SE(3)** for each part to restore the underlying object.

Motivation and Overview

★ 1 Assembly requires to find **reliable** correspondences.

→ Use **high-order conv**: a trending approach for correlation analysis! [NCNet, HSNet, etc.]

★ 2 However, high-order conv has **quadratic** complexity.

High-order conv: Quadratic complexity

Proxy Match Transform: Sub-quadratic complexity

Propose **PMT**: effective approximation of existing high-order conv with only sub-quadratic complexity!

Main Contributions

- Proxy Match Transform (PMT)**: a novel high-order feature transform layer with only sub-quadratic complexity.
- Proxy Match TransformerR (PMTR)**: a coarse-to-fine framework with a series of PMTs, achieving SoTA on Breaking Bad.

Proxy Match Transform: Efficient High-order Feature Transform

We propose **low-complexity high-order** feature transform layer, Proxy Match Transform (PMT), for analyzing **6D correlation** between two 3D point clouds.

$$\text{PMT}(\mathbf{F}_x) := \sum_{h \in [N_h]} \mathbf{A}_x^{(h)} \mathbf{F}_x \mathbf{P}^{(h)\top} w_x^{(h)}$$

$$\text{PMT}(\mathbf{F}_y) := \sum_{h \in [N_h]} \mathbf{A}_y^{(h)} \mathbf{F}_y \mathbf{P}^{(h)\top} w_y^{(h)}$$

Information exchange by sharing a proxy tensor!

Dot product of PMTs can **approximate high-order conv.** with two constraints (complete proof in paper):

$$(\text{PMT}(\mathbf{F}_x) \cdot \text{PMT}(\mathbf{F}_y)^\top)_{(\mathbf{x}, \mathbf{y})} \approx \text{Conv}(\mathbf{F}_x, \mathbf{F}_y)_{(\mathbf{x}, \mathbf{y})}$$

Effectively express high-order convolution with **sub-quadratic complexity**!

Proxy Match TransformerR (PMTR) for Geometric Assembly

Step 1. Coarse-level Matching: PMT processes the coarse feature pair, evaluating **potential** local correspondences between the point features.

Step 2. Fine-level Matching: Two PMT matchers refine the high-resolution feature pairs, facilitating the capture of more **detailed** fine-level geometric feature correlations.

Step 3. Transformation Estimation: Given fine-level correspondences, we utilize Optimal Transport to estimate the relative transformation between a pair of input parts.

Experiments and Analysis

Improving SOTA for Geometric Assembly

(top: pairwise assembly, bottom: multi-part assembly)

We evaluate our PMTR on the tasks of (1) pairwise assembly and (2) multi-part assembly. Our results outperform state-of-the-art baselines while remaining highly efficient.

| Method | CRD ↓ (10 ⁻²) | CD ↓ (10 ⁻³) | RMSE (R) ↓ (°) | RMSE (T) ↓ (10 ⁻²) |
|-----------------------|---------------------------|--------------------------|----------------|--------------------------------|
| everyday | | | | |
| GeoTransformer (2022) | 0.61 | 0.51 | 22.81 | 7.28 |
| Jigsaw (2023) | 5.48 | 1.34 | 38.73 | 2.73 |
| PMTR (Ours) | 0.39 | 0.25 | 17.14 | 5.53 |

| Method | CRD ↓ (10 ⁻²) | CD ↓ (10 ⁻³) | RMSE (R) ↓ (°) | RMSE (T) ↓ (10 ⁻²) | PA _{CRD} ↑ (%) | PA _{CD} ↑ (%) |
|--------------------|---------------------------|--------------------------|----------------|--------------------------------|-------------------------|------------------------|
| everyday | | | | | | |
| Wu et al. (2023b) | 28.18 | 19.70 | 54.98 | 15.59 | 35.66 | 36.28 |
| Jigsaw (2023) | 14.13 | 11.82 | 41.12 | 11.74 | 52.48 | 60.26 |
| PMTR (Ours) | 6.51 | 5.56 | 31.57 | 9.95 | 66.95 | 70.56 |

t-SNE Visualization of Proxy Tensor

The visualization indicates that point features on each mating surface are closer to the proxy, implying **strong correlations** in the feature space.

Legend: proxy (purple), source (non-mating) (red), source (mating) (orange), target (non-mating) (blue), target (mating) (light blue)

Ablation Analysis

| proxy | shared proxy | CRD ↓ (10 ⁻²) | CD ↓ (10 ⁻³) | RMSE (R) ↓ (°) | RMSE (T) ↓ (10 ⁻²) |
|-------|--------------|---------------------------|--------------------------|----------------|--------------------------------|
| ✗ | ✗ | 0.53 | 0.47 | 21.04 | 6.93 |
| ✓ | ✗ | 0.44 | 0.31 | 18.66 | 5.97 |
| ✓ | ✓ | 0.39 | 0.25 | 17.14 | 5.53 |

By sharing a proxy tensor in each PMT, two **independent** feature transforms exchange relevant information for effective matching.

Qualitative Comparison

(top: pairwise assembly, bottom: multi-part assembly)

GeoTr, Jigsaw, Ours, GT, Wu et al., Jigsaw, Ours, GT

Efficiency of Proxy Match Transform (PMT)

| Method | Coarse-level Matcher | Fine-level Matcher | FLOPS ↓ (G) | # Param. ↓ (K) | Mem. train ↓ (GB) | Mem. test ↓ (GB) | Train time ↓ (ms) | Inference time ↓ (ms) |
|-----------------------|----------------------|--------------------|-------------|----------------|-------------------|------------------|-------------------|-----------------------|
| GeoTransformer (2022) | GeoTr | None | 9.67 | 926.85 | 6.96 | 3.10 | 8.93 | 8.04 |
| PMTR (Coarse-only) | PMT | None | 0.45 | 273.85 | 2.12 | 0.28 | 4.06 | 3.23 |
| PMTR (Coarse + Fine) | PMT | PMT | 0.78 | 296.15 | 3.78 | 0.88 | 5.35 | 3.75 |

PMT is approximately **x21.5** more efficient in FLOPS with **x3.4** less number of parameters and **x3.28 / x11.07** less memory required for training / inference phases compared to the previous SoTA matching module, e.g., GeoTr.