Multi Instance Multi Label

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Overview

Binary classification

Goal: To produce a classifier able to decide whether an object belongs to one or more classes.

Idea: Supervised Learning: Given a dataset of already classified examples, the classifier *learns* a function that solves classification problem.

Notation

- Example: A vector $x \in \mathbb{R}^f$. It represent an object using f *relevant* features.
- Example label: A vector $y \in \{-1, +1\}^l$ indicating wether the example belong to each of the l classes.

Input of a classification problem is a dataset $D = \{X, Y\}$ where X is a set of examples and Y is a set of labels. $|X| = |Y| = n$.
While learning the target function, dataset is divided in *training set* and *test set*.

SVM idea

- For 1-class problems compute the *maximum-margin hyperplane* $w^T x + b$ which best separates positive examples from negative examples.

Optimization problem is:

$$argmin_w \frac{1}{2} ||w||^2$$

$$y^{(i)}(w^T x^{(i)} + b) \geq 1 \; \forall i \in [1, n]$$

SVM idea

- Examples may not be linearly serparable, so we introduce slack variables ξ

Optimization problem becames:

$$argmin_{w,\xi}\frac{1}{2}||w||^2 + C\sum_{i=1} n\xi^{(i)}$$

$$y^{(i)}(w^Tx^{(i)} + b) \geq 1 - \xi^{(i)} \ \forall i \in [1, n]$$

$$\xi^{(i)} \geq 0 \ \forall i \in [1, n]$$

Multi instance classification

Motivation:

- Sometimes a complex item can be well represented by a set of *instances*
- A single instance may belong or not to a class
- An example is positive if at least one of its instances is positive, it's negative otherwise
- Dataset labels are assigned to examples, not to instances
- We have a *semi-supervised learning* problem

Notation

Dataset is now a set of bags, where each bag is a set of instances:

$$D = \{(X^{(i)}, Y^{(i)}) | i \in [1 \ldots n]\}$$

$$X^{(i)} = \{x_k^{(i)} | k \in [1 \ldots k_i], x_k \in \mathbb{R}^f\}$$

Notice that each bag can be made of any number of instances, but every instance has a fixed number of features f.

mi-SVM

Instances label assignment:

- If an instance belongs to a negative bag we can say that its label is -1
- If an instance belongs to a positive bag we don't know its label

This leads to 2 new constraints in SVM problem:

$$y_k^{(i)} = -1 \ \textit{if} \ \ Y^{(i)} = -1$$

$$\sum_{k=1}^{k_i} \frac{y_k^{(i)} + 1}{2} \geq 1 \ \textit{if} \ \ Y^{(i)} = +1$$

mi-SVM

Our SVM problem becames the following:

$$min_Y min_{w,\xi} \frac{1}{2}||w||^2 + C \sum_{i=1} n\xi^{(i)}$$

$$y_k^{(i)}(w^T x_k^{(i)} + b) \geq 1 - \xi^{(i)} \; \forall i \in [1, n], k \in [1, k_i]$$

$$\xi^{(i)} \geq 0 \; \forall i \in [1, n]$$

$$y_k^{(i)} = -1 \; if \;\; Y^{(i)} = -1$$

$$\sum_{k=1}^{k_i} \frac{y_k^{(i)} + 1}{2} \geq 1 \; if \;\; Y^{(i)} = +1$$

That is an intractable mixed optimization problem

Algorithm

A feasible algorithm that finds a non optimal solution is the following:

1. $y_k^{(i)} = -1 \ if \ \ Y^{(i)} = -1$
2. $y_k^{(i)} = +1 \ if \ \ Y^{(i)} = +1$
3. do
    1. Solve regular SVM finding w, b
    2. $y_k^{(i)} = sign(w^T x_k^{(i)} + b) \ if \ \ Y^{(i)} = +1$
    3. Adjust each positive bag to satisfy constraints
4. while $y_k^{(i)}$ change

MI-SVM

This approach uses directly the dataset in its bag form:

$$argmin_{w,\xi} \frac{1}{2}||w||^2 + C \sum_{i=1} n\xi^{(i)}$$

$$y^{(i)}(max_k w^T x_k^{(i)} + b) \geq 1 - \xi^{(i)} \ \forall i \in [1, n]$$

$$\xi^{(i)} \geq 0 \ \forall i \in [1, n]$$

This is possible by selecting a *witness* from each bag instances.

Algorithm

A feasible algorithm that finds a solution is the following:

1. $x_s^{(i)} = avg(x_k^{(i)})\ \forall i \in [1, n]$
2. do
    1. Solve regular SVM finding w, b, balancing lagrange multipliers
    2. Find new $x_s^{(i)}$ by selecting the best one for each positive bag
3. while witnesses change

Multi label classification

Motivation:

- Sometimes a complex item can be well represented by a set of *labels*
- Helps single label classification when the concept is more complicated or general

Solutions:

- Problem transformation
- Algorithm adaptation

Notation

A set of labels $L = \{y_1, y_2, ...y_l\}$ is given.
Each object contained in the dataset is associated with a set of labels:

$$D = \{(X_i, Y_i | i \in [1, n]\}$$

$$X_i \in \mathbb{R}^f$$

$$Y_i = \{y_{i,h} | h \in [1, h_i], y_{i,h} \in L, h_i \leq l\}$$

Problem transformation

Attempt to convert the multilabel problem in a regular binary task.
Two lossy methods:

- Randomly discard each label information except one from each instance
- Remove instances that have actually more than one label

Other solutions:

- Train a binary classifier for each existing combination of labels
- Train a binary classifier for each label (used in this work)

Algorithm adaptation

Regular algorithms are modified to support multi-label tasks.
Sometimes they use problem transformation at the core.
An example using SVM-related approach based on ranking and label set size prediction.

Introduction to MIML

MIML problems combine motivations of multi instance and multi label ones.
Given a set of labels $L = \{y_1, y_2, ...y_l\}$

$$D = \{(X_i, Y_i) | i \in [1, n]\}$$

$$X_i = \{x_{i,k} | k \in [1, k_i], x_{i,k} \in \mathbb{R}^f\}$$

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