Matriz Inversa

$\textcircled{1}$

$$\begin{bmatrix} x & 1 \\ 5 & 3 \end{bmatrix} * \begin{bmatrix} 3 & -1 \\ y & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow \begin{matrix} 3x + y = 1 \ (\mathrm{I}) \\ -x + 2 = 0 \ (\mathrm{II}) \\ 15 + 3y = 0 \ (\mathrm{III}) \\ -5 + 6 = 1 \ (\mathrm{IV}) \end{matrix}$$

$\mathrm{II} \Rightarrow x = 2$

$\mathrm{III} \Rightarrow y = -5$

$\mathrm{I} \Rightarrow 3 * 2 - 5 = 1$

$\Downarrow$

confirmação

$x + y = -3$

Ⓒ

②

Se Det = 0 $\Rightarrow M^{-1} \nexists$

$$\begin{array}{c}
 \\
A = \left[\begin{array}{ccc}1 & 0 & 1\\ k & 1 & 3\\ 1 & k & 3\end{array}\right] \\
\begin{array}{ccc}1 & 0 & 1\\ k & 1 & 3\end{array}
\end{array}
\begin{array}{l}
1 \\ 3k \\ 0 \\ = 0 \\ 3 \\ k^2 \\ 0
\end{array}$$

$$\left\{\begin{array}{l}
(k^2+3)-(3k+1)=0 \\
k^2-3k+2=0 \\
\Rightarrow \dfrac{1}{} + \dfrac{2}{} = 3 \\
\dfrac{1}{} \cdot \dfrac{2}{} = 2
\end{array}\right.$$

$k' = 1 \;/\; k'' = 2$

ⓒ

(3)

$A = \begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix} \Rightarrow \det A = 12 - 10 = 2$

$\Rightarrow$ inversa da matriz:

$A' = \begin{bmatrix} 4 & -5 \\ -2 & 3 \end{bmatrix} \div 2 \Rightarrow$ então $B = \begin{bmatrix} 2 & -5/2 \\ -1 & 3/2 \end{bmatrix}$ (C)

(4)

$$\begin{array}{cccc} & & & 20 \\ & & & 2x \\ \left[\begin{array}{ccc} x & 1 & 2 \\ 3 & 1 & 2 \\ 10 & 1 & x \end{array}\right] & & = 0 & \\ & & & 3x \\ x & 1 & 2 & x^2 \\ 3 & 1 & 2 & 6 \\ & & & 20 \end{array}$$

$(x^2 + 20 + 6) - (3x + 2x + 20)$

$D = x^2 - 5x + 6 = 0$

$\Delta = -5^2 - 4.1.6$

$\Delta = 1$

$x = \dfrac{5 \pm \sqrt{1}}{2}$

$x' \neq 3$

$x'' \neq 2$

(A)

↳ se $\det \neq 0$ ↳ M^{-1} E

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$$\begin{array}{ccc|c} & & & 2 \\ \hline -1 & -1 & 2 & 2 \\ 2 & 1 & -2 & 2 \\ 1 & 1 & -1 & \\ \hline -1 & -1 & 2 & 1 \\ 2 & 1 & -2 & 4 \\ & & & 2 \end{array}$$

$$\left.\begin{aligned} det &= (1+4+2)-(2+2+2) \\ det &= 7-6 = 1 \end{aligned}\right.$$

$$A=\begin{bmatrix}-1&-1&2\\2&1&-2\\1&1&-1\end{bmatrix} \Rightarrow A'=\begin{bmatrix}1&0&1\\1&-1&0\\0&2&-1\end{bmatrix} \Rightarrow \underset{A_1}{\overset{\bar{A}}{\downarrow}} = \underbrace{\begin{bmatrix}1&1&0\\0&-1&2\\1&0&-1\end{bmatrix}}_{\text{troca de sinal direta}} \div 1$$

$$A+A^1=\begin{bmatrix}0&0&2\\2&0&0\\2&1&0\end{bmatrix}$$ B

(6)

(1) transpor os dois lados:

$$((X.A)^t)^t = B^t \Rightarrow X.A = B^t$$

(2) multiplicar a matriz direta pela inversa de A:

$$X.A.A^{-1} = B^t.A^{-1} \Rightarrow \underbrace{XI = B^t A^{-1}}$$

isolar a matriz X:

$$X = B^t.A^{-1}$$

(B)

7

25

$$A = \begin{bmatrix}4 & 5\\5 & 6\end{bmatrix} = 24-25 = 1 \qquad \begin{bmatrix}6 & -5\\-5 & 4\end{bmatrix} \div 1 \Big\} \qquad \begin{bmatrix}-6 & 5\\5 & -4\end{bmatrix}$$

24

D

8

Se $k \neq 0 \Rightarrow \text{Det}^{-1} \exists$

$A = \begin{bmatrix} 2 & k \\ -2 & 1 \end{bmatrix}$ $\det A = 2 + 2k$ $\Bigg\}$ $\det_A^{-1} = \dfrac{1}{\det A}$

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$\Rightarrow \det_A^2 = 1 \Rightarrow \det A = \pm 1$

$\Rightarrow 2 + 2k = 1$
$2k = 1 - 2$
$k' = -\frac{1}{2}$

$2 + 2k = -1$
$2k = -1 - 2$
$k = -\frac{3}{2}$

$k' + k'' = \left(\frac{-1}{2}\right) + \left(-\frac{3}{2}\right)$

$k' + k'' = -2$

B

⑨

a) $(A+B).(A-B) = A^2 - AB + AB - B^2$

b) $(A+B)^2 = (A+B).(A+B) = A^2 + AB + BA + B^2$

$\hookrightarrow A^2 + \underbrace{AB + BA}_{AB = BA} + B^2 = A^2 + 2AB + B^2$

c)

Se $A = 2 \times 2$, então: $\det(-A) = (-1)^2 . \det A$

$\det A \neq 0$

$$\frac{\det(A)}{\det(-A)} = \frac{\det(A)}{\det(A)} = 1$$

d) Se $B = A^{-1}$, então $\det(AB) = 1$

$\det(A) . \det(B) = 1$

$\det B = \dfrac{1}{\det A}$