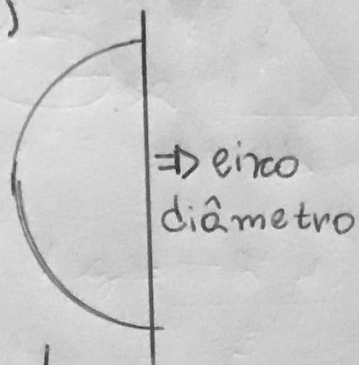


*Esfera*

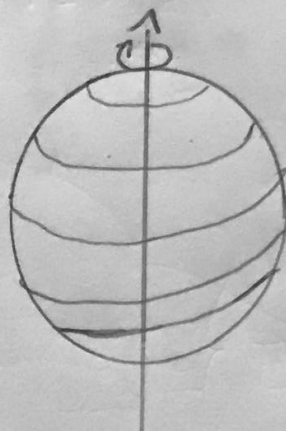
(1)

(I)



↳ desenhar  
uma semicircunferência

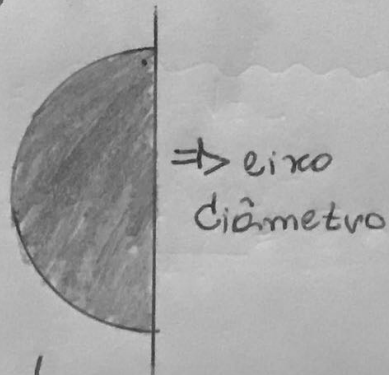
(II)



⇒ girar a semi-c.  
em  $360^\circ$  sob seu  
eixo

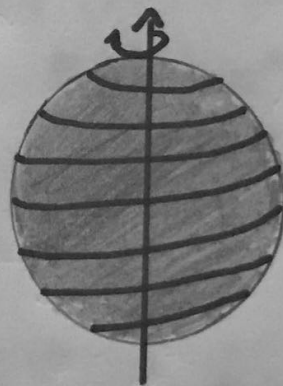
⇒ criar uma "casca",  
a superfície esférica

(III)



↳ desenhar uma  
semi-círculo

(IV)



⇒ girar em  
 $360^\circ$  sob seu eixo

⇒ criação do  
sólido, a esfera

(C) Rotação de um  
semi-círculo em torno  
de seu diâmetro.

$$(2) \quad V_1 = V_2$$

$$V_1 = \frac{4}{3} \pi \cdot 1^3 = \frac{4}{3} \pi$$

$$V_2 = \frac{4}{3} \pi \cdot 1.000.000$$

$$V = \frac{4}{3} \pi \cdot R^3$$

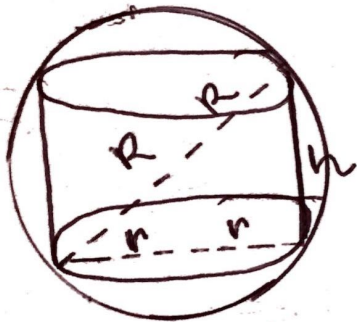
$$\left. \begin{array}{l} \frac{4}{3} \pi \cdot 1^3 = \frac{4}{3} \pi \\ \frac{4}{3} \pi \cdot R^3 = 1000000 \cdot \frac{4}{3} \pi \end{array} \right\}$$

$$R^3 = 10^6$$

$$R = \sqrt[3]{10^6}$$

$$R = 10^2 \Rightarrow R = 100$$

(3)



$$\begin{array}{c} 2R \\ \triangle \\ 2r \end{array} h \Rightarrow (2r)^2 = h^2 + (2R)^2 \Rightarrow h = 2r$$

$$(2r)^2 + (2r)^2 = (2R)^2$$

$$8r^2 = 4R^2$$

$$2r^2 = R^2$$

$$r\sqrt{2} = R$$

$$V_{\text{sfera}} = \frac{4\pi R^3}{3} \Rightarrow \frac{4\pi (R\sqrt{2})^3}{3} = \frac{8\pi R^3 \sqrt{2}}{3}$$

$$V_{\text{cilindro}} = \pi r^2 \cdot h$$

$$V_{\text{cilindro}} = \pi \cdot r^2 \cdot 2r$$

$$V_{\text{cilindro}} = \pi \cdot r^3 \cdot 2$$

$$\left. \begin{array}{l} V_{\text{sfera}} \\ V_{\text{cilindro}} \end{array} \right\} = \frac{8\pi r^3 \sqrt{2}}{\frac{3}{2\pi r^3}} = \frac{4\sqrt{2}}{3}$$

4)



o volume das duas esferas  
é igual ao cilindro

$$\hookrightarrow \frac{4\pi 1^3}{3} + \frac{4\pi 2^3}{3} = \pi r^2 \cdot 3 \Rightarrow 9r^2 = 36$$

$$r = \sqrt{\frac{36}{9}} = \frac{6}{3}$$

$$r = 2 \text{ cm}$$

5

Vesfera = V. que subiu



1 cm  
cilindro  
de  $h = 1 \text{ cm}$

$$V_{\text{cilindro}} = \pi \cdot 6^2 \cdot 1 = 36\pi$$

$$V_{\text{esfera}} = \frac{4}{3}\pi \cdot r^3$$

$$\left\{ \begin{array}{l} \frac{4}{3}\pi r^3 = 36\pi \end{array} \right.$$

$$36 \cdot 3\pi = 4\pi r^3$$

$$108\pi = 4\pi r^3$$

$$\frac{108}{4} = r^3 \Rightarrow r = \sqrt[3]{27} = 3$$

6)

(I) O diâmetro da esfera deve ter o tamanho de uma aresta.

$$V_{\text{esfera}} = \frac{4}{3} \pi r^3 \Rightarrow 288\pi = \frac{4\pi r^3}{3}$$

$$288 \cdot 3 = 4r^3$$

$$864 = 4r^3$$

$$\frac{864}{4} = r^3$$

$$216 = r^3$$

$$\sqrt[3]{216} = r$$

$$r = 6 \text{ cm}$$

$$D = A = 2r$$

$$D = A = 2 \cdot 6$$

$$A = 12 \text{ cm}$$

7

$$boli = \frac{V_{cilindro}}{V_{doce}} \Rightarrow boli = \frac{1600\pi}{\frac{\pi}{3}} = \frac{1600\pi}{\pi} \cdot \frac{3}{1} = 1600 \cdot 3 = 4800$$

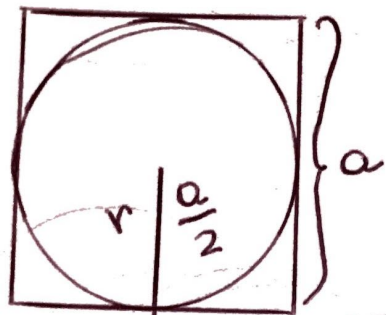
$V_{cilindro} = \pi r^2 \cdot h$   
 $r = \frac{20}{2}$   
 $r = 10cm$   
 $V_{cilindro} = \pi \cdot 10^2 \cdot 16$   
 $V_{cilindro} = 100 \cdot 16 \cdot \pi$   
 $V_{cilindro} = 1600\pi$

$V_{doce} = \frac{4}{3} \cdot \pi r^3$   
 $V_{doce} = \frac{4}{3} \cdot \pi \cdot 2^3$   
 $V_{doce} = \frac{8 \cdot 4 \pi}{3}$   
 $V_{doce} = \frac{32 \pi}{3}$



# *Inserção e Circunscrição de Sólidos*

②  $A_{\text{superficial (cubo)}} = 6 \cdot a^2$



$A_{\text{superficial (esfera)}} = 4\pi r^2$

$4\pi \left(\frac{a}{2}\right)^2$

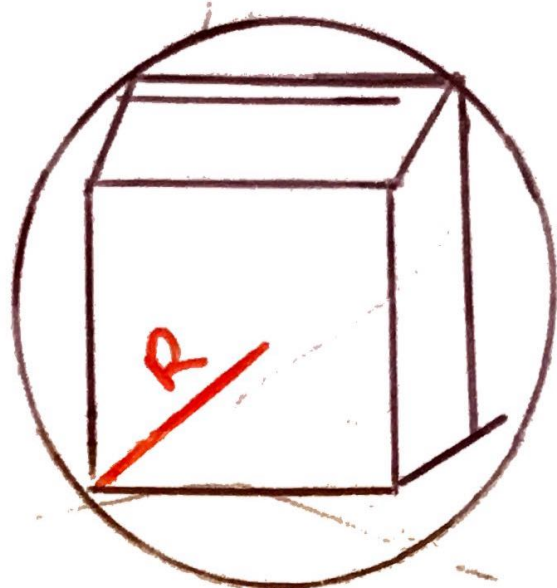
$\cancel{4}\pi \cdot \frac{a^2}{\cancel{4}} = \pi \cdot a^2$

↳ 4 circunferências nas  
4 faces do cubo

$$\frac{A_{\text{superficial (esfera)}}}{A_{\text{superficial (cubo)}}} = \frac{\pi \cdot a^2}{6 \cdot a^2} = \frac{\pi}{6}$$

3)

$$V_{\text{esfera}} = \frac{4}{3} \pi \cdot R^3$$



$$V_{\text{cubo}} = a^3$$

$$d = a\sqrt{3}$$

$$d = 2R$$

$$2R = a\sqrt{3}$$

$$a = \frac{2R}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2R\sqrt{3}}{3}$$

$$V_{\text{cubo}} = \left( \frac{2R\sqrt{3}}{3} \right)^3$$

$$\frac{V_{\text{esfera}}}{V_{\text{cubo}}} = \frac{\frac{4}{3} \pi R^3}{\left( \frac{2R\sqrt{3}}{3} \right)^3} = \frac{\frac{4\pi R^3}{3}}{\frac{8R^3 \cdot \sqrt{3}}{9}}$$

$$\frac{\cancel{4\pi R^3}}{3} \cdot \frac{9}{8\pi \cancel{R^3} \sqrt{3}} = \frac{4 \cdot 9}{3 \cdot 8 \cdot \sqrt{3}}$$

$$\frac{4\pi \cdot R^3}{3} \cdot \frac{9}{8\sqrt{3} \cdot R^3} = \frac{\cancel{4}\pi \cancel{R^3} \cdot \cancel{9}^3}{2\cancel{8}\sqrt{3} \cdot \cancel{R^3} \cdot \cancel{3}} = \frac{\pi \cdot 3}{2\sqrt{3}} = \frac{3\pi}{2\sqrt{3}}$$

$$\frac{3\pi}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\cancel{3}\pi \sqrt{3}}{2 \cdot \cancel{3}} = \frac{\pi \sqrt{3}}{2} = \boxed{\frac{\sqrt{3}}{2} \pi}$$

4)



↳ o volume das duas esferas  
é igual ao cilindro

$$\hookrightarrow \frac{4\pi 1^3}{3} + \frac{4\pi 2^3}{3} = \pi r^2 \cdot 3 \Rightarrow 9r^2 = 36$$

$$r = \sqrt{\frac{36}{9}} = \frac{6}{3}$$

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