PARTE 1

parte 1

1

$$\begin{cases} ax + 4y = 1 \\ x + 2y = b \end{cases}$$

$$D = \begin{vmatrix} a & 4 \\ 1 & 2 \end{vmatrix} = 0$$

caso $D = 0$, é indeterminado

$4 \qquad 2a$

$$2a - 4 = 0$$

$$2a = 4$$

$$\frac{4}{2} = a$$

$$a = 2$$

B

a) INCORRETO. Caso $b = \frac{1}{2}$, não há apenas uma solução, se a = 2.

$$D_x = \begin{vmatrix} 1 & 4 \\ b & 2 \end{vmatrix} \quad \left.\begin{array}{l} 2 - 4b = 0 \\ b = \frac{2}{4} \rightarrow \frac{1}{2} \end{array}\right\}$$ a determinante seria 0.

c) INCORRETO, pois apresenta solução única para mais de um valor.

d) INCORRETO. Pois há soluções que fiquem indeterminados

2)

$$\begin{cases} x + ky = 1 \\ kx + y = 1 - k \end{cases}$$

$$\begin{array}{c} -k \\ \hookrightarrow \end{array}\begin{pmatrix} 1 & k & : 1 \\ \underbrace{k \quad 1}_{k=-1} & & : 1-k \end{pmatrix} \sim \begin{pmatrix} & & \\ 0 & -k^2+1 & : -2k+1 \end{pmatrix}$$

$\hookrightarrow y = \dfrac{-2k+1}{-k^2+1}$

↳ para ser determinado,

$k \neq 1$ ou $k \neq -1$

3

$$\begin{cases} x + 2y + cz = 1 \\ \quad\;\; y + z = 2 \\ 3x + 2y + 2z = -1 \end{cases}$$

$\therefore$

a)

$$A = \left|\begin{array}{ccc} 1 & 2 & c \\ 0 & 1 & 1 \\ 3 & 2 & 2 \end{array}\right| \begin{array}{l} 3c \\ 2 \\ 0 \end{array}$$

$$\begin{array}{ccc} 1 & 2 & c \\ 0 & 1 & 1 \end{array} \qquad \begin{array}{l} 2 \\ 0 \\ 6 \end{array}$$

$$(2+6)-(3c+2) = 6-3c$$

$$\boxed{\det = 6-36}$$

b) para solução única,

$D \neq 0$, então:

$$6 - 3c \neq 0$$

$$6 \neq 3c$$

$$\frac{6}{3} \neq c$$

$$c \neq 2$$

4)

$$\begin{cases} x - y = k \\ 12x - ky + z = 1 \\ 36x + kz = 2 \end{cases}$$

$$D = \begin{vmatrix} 1 & -1 & 0 \\ 12 & -k & 1 \\ 36 & 0 & k \end{vmatrix} \begin{array}{l} 0 \\ 0 \\ -12k \end{array}$$

$$\begin{array}{ccc} 1 & -1 & 0 \\ 12 & -k & 1 \end{array} \qquad \begin{array}{l} -k^2 \\ 0 \\ -36 \end{array}$$

$$\left.\begin{array}{c} -k-36-(-12k) \\ \neq 0 \end{array}\right.$$

$-k + 12k - 36 \neq 0$

$\underline{6} + \underline{6} = 12$

$\underline{6} \cdot \underline{6} = 36$

$S = \{6, 6\}$

$k \neq 6$

5

$$\begin{cases} x - y + z = 6 \\ 2x + y - z = -3 \\ x + 2y - z = -5 \end{cases}$$

$$\begin{array}{c} -1 \quad -2 \\ + \\ + \end{array} \begin{pmatrix} 1 & -1 & 1 & : 6 \\ 2 & 1 & -1 & : -3 \\ 1 & 2 & -1 & : -5 \end{pmatrix} \sim \begin{array}{c} \\ -3 \\ +3 \end{array} \begin{pmatrix} 0 & 3 & -3 & : -15 \\ 0 & 3 & -2 & : -11 \end{pmatrix}$$

$$\sim \begin{pmatrix} & & & \\ 0 & 0 & 3 & : 12 \end{pmatrix}$$

$3z = 12$

$z = 4$ ///

$3 \cdot y - 2z = -15$

$3y + 12 = -15$

$3y = -3$

$y = -1$ ///

$x - y + z = 6$

$x + 1 + 4 = 6$

$x = 6 - 1 - 4$

$x = 1$ //

possível e determinado

$x \cdot y \cdot z = -4$

6

$$\begin{cases} x + y + z = k \\ kx + y + z = 1 \\ x + y - z = k \end{cases} \quad \begin{array}{c} -1 \; -k \\ + \\ + \end{array} \left(\begin{array}{ccc|c} 1 & 1 & 1 & k \\ k & 1 & 1 & 1 \\ 1 & 1 & -1 & k \end{array}\right) \sim \left(\begin{array}{ccc|c} 0 & -k+1 & -k+1 & -k+1 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

Caso o valor for somente igual a 1, a segunda linha igualará a 0, admitindo uma $D=0$, possuindo infinitas soluções.

D

7)

$$\begin{cases} x + y + z = 1 \\ mx - 2y + 4z = 5 \\ m^2x + 4y + 16z = 25 \end{cases}
\qquad
\begin{array}{c} -m^2 \\ -m \end{array}
\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ m & -2 & 4 & 5 \\ m^2 & 4 & 16 & 25 \end{array}\right)$$

$$\begin{array}{ccc} 1 & 1 & 1 \\ m & -2 & 4 \\ m^2 & 4 & 16 \\ 1 & 1 & 1 \\ m & -2 & 4 \end{array}
\qquad
\begin{array}{l} -2m^2 \\ 16 \\ 16m \\ -32 \\ 4m \\ 4m^2 \end{array}$$

$$\sim \left(\begin{array}{ccc|c} 0 & -m-2 & -m+4 & -m+5 \\ 0 & -m^2+4 & -m^2+16 & -m^2+25 \end{array}\right)$$

$$(4m^2 + 4m + (-32)) - (-2m^2 + 16m + 16) =$$

$$6m^2 - 12m - 48 = 0 \Rightarrow \begin{array}{l} 4 + -2 = 2 \\ 4 \cdot -2 = -8 \\ S = \{4, -2\} \end{array} \Rightarrow m_1 + m_2 = 2$$

PARTE 2

parte 2

1

$$\begin{bmatrix} 1 & 7 \\ 1 & 7 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = k \cdot \begin{bmatrix} x \\ y \end{bmatrix} \qquad \begin{cases} x + 7y = kx \\ 7x + y = ky \end{cases}$$

$$D = \begin{vmatrix} 1 & 7 \\ 7 & 1 \end{vmatrix} = 1 - 49 = -48 \qquad Dx = \begin{vmatrix} k & 7 \\ k & 1 \end{vmatrix} = k - 7k = -6x$$

$$\begin{array}{c} 7 \\ \\ \end{array} \begin{pmatrix} 1 & 7 & k \\ 7 & 1 & k \end{pmatrix} \sim \begin{pmatrix} 0 & -48 & -6k \end{pmatrix} \rightarrow Z = -48 = 6k$$

$D \neq 0$ se:

$$48 \neq -6k$$

$$\frac{-48}{-6} = k$$

$$k = 8$$

02

$$\begin{cases} 3x + 4y - z = 0 \\ 2x - y + 3z = 0 \\ x + y = 0 \end{cases}$$

$$D = \begin{vmatrix} 3 & 4 & -1 \\ 2 & -1 & 3 \\ 1 & 1 & 0 \end{vmatrix} \qquad \begin{array}{ccc} 3 & 4 & -1 \\ 2 & -1 & 3 \end{array}$$

1, 9, 0

0, -2, 12

$(12-2+0) -$

$(1+9+0) =$

$10 - 10$

$D = 0$

$$Dx = \begin{vmatrix} 0 & 4 & -1 \\ 0 & -1 & 3 \\ 0 & 1 & 0 \end{vmatrix} \qquad \begin{array}{ccc} 0 & 4 & -1 \\ 0 & -1 & 3 \end{array}$$

0, 0, 0

0, 0, 0

$D=0$

$$Dy = \begin{vmatrix} 3 & 0 & -1 \\ 2 & 0 & 3 \\ 1 & 0 & 0 \end{vmatrix} \qquad \begin{array}{ccc} 3 & 0 & -1 \\ 2 & 0 & 3 \end{array}$$

0, 0, 0

0, 0, 0

$D=0$

$$Dz = \begin{vmatrix} 5 & 4 & 0 \\ 2 & -1 & 0 \\ 1 & 1 & 0 \end{vmatrix} \qquad \begin{array}{ccc} 5 & 4 & 0 \\ 2 & -1 & 0 \end{array}$$

0, 0, 0

0, 0, 0

$D=0$

(3)

$$\begin{cases} x + y + z = 0 \\ x + 3y + 4z = 0 \\ x + ky + 3z = 0 \end{cases}$$

$$D = \begin{vmatrix} 1 & 1 & 1 \\ k & 3 & 4 \\ 0 & k & 3 \end{vmatrix} = 0$$

$$\begin{array}{ccc} 1 & 1 & 1 \\ k & 3 & 4 \end{array}$$

0, 4K, 3K

9, k^2, 4

$$\begin{cases} 9 + k^2 - 4k - 3k = 0 \\ k^2 - 7k + 9 = 0 \\ S = k' + k'' = \dfrac{-}{a} \\ S = \dfrac{-(-7)}{1} = \dfrac{7}{1} \Rightarrow \boxed{S = 7} \end{cases}$$

(2)

$$\begin{cases} x + kz = 0 \\ kx + y = 0 \\ x + ky = 0 \end{cases}$$

$$D = \left|\begin{array}{ccc|cc} 1 & 0 & k & 1 & 0 \\ k & 1 & 0 & k & 1 \\ 1 & k & 0 & 1 & k \end{array}\right|$$

$k \quad 0 \quad 0$

$0 \quad 0 \quad k^3$

$D = k^3 - k$

$D = k^2$

$$D = \left|\begin{array}{ccc|cc} 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \end{array}\right|$$

$1 \quad 0 \quad 0$

$0 \quad 0 \quad 1$

$1 - 1 = 0$

$D = 0$

Com outro valor:

$$D = \left|\begin{array}{ccc} 1 & 0 & 2 \\ 2 & 1 & 0 \\ 1 & 2 & 0 \end{array}\right| \quad D = 6 \ \times$$

$1 \quad 0 \quad 2$

$2 \quad 1 \quad 0$

$2 \quad 0 \quad 0$

$0 \quad 8 \quad 0$

Para que apresente apenas uma solução: $k \neq 0$, $k \neq 1$ e $k \neq -1$

(A)

(5)

$$\begin{cases} -x + 2y - 3 = 0 \\ 3x - y + 3 = 0 \\ 2x - 4y + 6 = 0 \end{cases}$$

$$\begin{array}{c} \overbrace{6 \quad 12 \quad 18}^{54} \\ D = \left| \begin{array}{ccc|cc} -1 & 2 & -3 & -1 & 2 \\ 3 & -1 & 3 & 3 & -1 \\ 2 & -4 & 6 & 2 & -4 \end{array} \right| \\ \underbrace{6 \quad 12 \quad 36}_{54} \end{array} \quad D = 0$$

$$\left.\begin{array}{l} \to Dx = 0 \\ Dy = 0 \\ Dz = 0 \end{array}\right\} \frac{0}{0}$$

→ a solução é possível, mas indeterminada