Khyati Naik: Data 605 - HW7

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Contents

1. Let X1, X2, . . . , Xn be n mutually independent random variables, each of which is uniformly distributed on the integers from 1 to k. Let Y denote the minimum of the Xi's. Find the distribution of Y.

The distribution of Y, the minimum of the random variables X1, X2, …, Xn, can be found by considering the complementary event: P(Y > y), where y is a particular value.

For Y to be greater than y, all of the X1, X2, …, Xn must be greater than y. Since each Xi is uniformly distributed on the integers from 1 to k, the probability that any Xi is greater than y is (k - y)/k.

Since the random variables X1, X2, …, Xn are mutually independent, we can multiply their probabilities together:

P(Y > y) = P(X1 > y) * P(X2 > y) * … * P(Xn > y)

= [(k - y)/k] * [(k - y)/k] * … * [(k - y)/k]

= [(k - y)/k]^n

Now, to find the distribution of Y, we need to find P(Y = y), which is the probability that Y is exactly equal to y:

P(Y = y) = P(Y > y - 1) - P(Y > y)

= [(k - (y - 1))/k]^n - [(k - y)/k]^n

This gives you the probability mass function (PMF) of the distribution of Y for each value of y from 1 to k.

In summary, the distribution of Y, the minimum of the Xi's, follows a discrete probability distribution where the PMF is given by:

P(Y = y) = [(k - (y - 1))/k]^n - [(k - y)/k]^n

for y = 1 to k.

2. Your organization owns a copier (future lawyers, etc.) or MRI (future doctors). This machine has a manufacturer's expected lifetime of 10 years. This means that we expect one failure every ten years. (Include the probability statements and R Code for each part.)

a. What is the probability that the machine will fail after 8 years?. Provide also the expected value and standard deviation. Model as a geometric. (Hint: the probability is equivalent to not failing during the first 8 years.

```r
# Probability of not failing in 8 years
probability_survive_8_years <- (9/10)^8

# Probability of failing after 8 years (complement of surviving for 8 years)
probability_fail_after_8_years <- probability_survive_8_years

# Expected value (mean) of the geometric distribution
expected_value <- 1 / (1/10)

# Standard deviation of the geometric distribution
standard_deviation <- sqrt((1 - 1/10) / (1/10)^2)

# Print the results
cat("Probability of failing after 8 years:", probability_fail_after_8_years, "\n")
```

```
## Probability of failing after 8 years: 0.4304672
```

```r
cat("Expected value (mean):", expected_value, "\n")
```

```
## Expected value (mean): 10
```

```r
cat("Standard deviation:", standard_deviation, "\n")
```

```
## Standard deviation: 9.486833
```

b. What is the probability that the machine will fail after 8 years?. Provide also the expected value and standard deviation. Model as an exponential.

```r
# Rate parameter
lambda <- 1/10

# Time (in years) after which we want to find the probability
x <- 8

# Probability that the machine will not fail before 8 years
probability_survive_8_years <- exp(-lambda * x)

# Probability of failing after 8 years (complement of surviving for 8 years)
probability_fail_after_8_years <- probability_survive_8_years
```

```r
# Expected value (mean) of the exponential distribution
expected_value <- 1 / lambda

# Standard deviation of the exponential distribution
standard_deviation <- 1 / lambda

# Print the results
cat("Probability of failing after 8 years (Exponential):", probability_fail_after_8_years, "\n")
```

```
## Probability of failing after 8 years (Exponential): 0.449329
```

```r
cat("Expected value (mean):", expected_value, "\n")
```

```
## Expected value (mean): 10
```

```r
cat("Standard deviation:", standard_deviation, "\n")
```

```
## Standard deviation: 10
```

c. What is the probability that the machine will fail after 8 years?. Provide also the expected value and standard deviation. Model as a binomial. (Hint: 0 success in 8 years)

```r
# Probability of failure in a single year
p_failure <- 1/10  # Since the machine fails on average once every 10 years

# Number of trials (years)
n_years <- 8

# Number of failures (successes for our binomial)
x_failures <- 0

# Probability of 0 failures in 8 years (binomial)
probability_0_failures <- dbinom(x_failures, size = n_years, prob = p_failure)

# Expected value (mean) of the binomial distribution
expected_value_binomial <- n_years * p_failure

# Standard deviation of the binomial distribution
standard_deviation_binomial <- sqrt(n_years * p_failure * (1 - p_failure))

# Print the results
cat("Probability of failing after 8 years (Binomial):", 1 - probability_0_failures, "\n")
```

```
## Probability of failing after 8 years (Binomial): 0.5695328
```

```r
cat("Expected value (mean):", expected_value_binomial, "\n")
```

```
## Expected value (mean): 0.8
```

```r
cat("Standard deviation:", standard_deviation_binomial, "\n")
```

```
## Standard deviation: 0.8485281
```

d. What is the probability that the machine will fail after 8 years?. Provide also the expected value and standard deviation. Model as a Poisson.

```r
# Rate parameter for Poisson distribution
lambda <- 8 * (1/10)  # Average number of failures in 8 years

# Number of failures (successes for Poisson)
x_failures <- 0

# Probability of 0 failures in 8 years (Poisson)
probability_0_failures <- dpois(x_failures, lambda)

# Expected value (mean) of the Poisson distribution
expected_value_poisson <- lambda

# Standard deviation of the Poisson distribution
standard_deviation_poisson <- sqrt(lambda)

# Print the results
cat("Probability of failing after 8 years (Poisson):", probability_0_failures, "\n")
```

```
## Probability of failing after 8 years (Poisson): 0.449329
```

```r
cat("Expected value (mean):", expected_value_poisson, "\n")
```

```
## Expected value (mean): 0.8
```

```r
cat("Standard deviation:", standard_deviation_poisson, "\n")
```

```
## Standard deviation: 0.8944272
```