Khyati Naik: Data 605 - HW9

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1. Problem 11 page 363

The price of one share of stock in the Pilsdorff Beer Company (see Exer□cise 8.2.12) is given by Yn on the nth day of the year. Finn observes that the differences Xn = Yn+1 − Yn appear to be independent random variables with a common distribution having mean μ = 0 and variance σ 2 = 1/4. If Y1 = 100, estimate the probability that Y365 is (a) ≥ 100. (b) ≥ 110. (c) ≥ 120.

```
# Given parameters
mean_X <- 0
variance_X <- 1/4
n <- 364  # Number of days
initial_price <- 100

# Calculate the standard deviation of Sn
std_dev_Sn <- sqrt(n * variance_X)

# Calculate z-scores for different values
z_score_a <- (100 - initial_price) / std_dev_Sn
z_score_b <- (110 - initial_price) / std_dev_Sn
z_score_c <- (120 - initial_price) / std_dev_Sn

# Calculate probabilities
prob_a <- pnorm(z_score_a)
prob_b <- 1 - pnorm(z_score_b)
prob_c <- 1 - pnorm(z_score_c)

# Print the probabilities
cat("(a) P(Y365 >= 100):", round(prob_a, 4), "\n")
```

```
## (a) P(Y365 >= 100): 0.5
```

```
cat("(b) P(Y365 >= 110):", round(prob_b, 4), "\n")
```

```
## (b) P(Y365 >= 110): 0.1473
```

```
cat("(c) P(Y365 >= 120):", round(prob_c, 4), "\n")
```

```
## (c) P(Y365 >= 120): 0.018
```

2. Calculate the expected value and variance of the binomial distribution using the moment generating function.

The probability mass function of the binomial distribution is given by:

$$f(x) = \binom{n}{x} p^x (1-p)^{n-x}$$

Here, n represents the number of trials, x is the number of successes, and p is the probability of success.

The MGF of the binomial distribution is found by taking the sum of the probabilities of all possible outcomes, each weighted by e^{tx}, where t is a real-valued parameter:

$$M(t) = \sum_{x=0}^{n} e^{tx} \binom{n}{x} p^x (1-p)^{n-x}$$

Simplifying further:

$$M(t) = (pe^t + 1 - p)^n$$

Where: - n is the number of trials. - p is the probability of success in each trial.

Expected Value ($E[X]$)

To find the expected value ($E[X]$) of the binomial distribution, we take the derivative of the MGF with respect to t and evaluate it at $t = 0$:

Step 1: Calculate the First Derivative

Let's calculate the first derivative of the MGF:

$$M'(t) = \frac{d}{dt}\left[(pe^t + 1 - p)^n\right]$$

Step 2: Evaluate at $t = 0$

Now, evaluate the first derivative at $t = 0$:

$$E[X] = M'(0)$$

Variance ($Var[X]$)

To find the variance ($Var[X]$), we need to take the second derivative of the MGF, subtract the square of the first derivative, and evaluate it at $t = 0$.

Step 1: Calculate the Second Derivative

Let's calculate the second derivative of the MGF:

$$M''(t) = \frac{d^2}{dt^2}\left[(pe^t + 1 - p)^n\right]$$

Step 2: Subtract Square of First Derivative

Subtract the square of the first derivative:

$$Var[X] = M''(0) - [M'(0)]^2$$

The variance is $np(1-p)$, indicating the spread or variability of the number of successes.

3. Calculate the expected value and variance of the exponential distribution using the momentgenerating function.

Moment Generating Function (MGF)

The moment generating function (MGF) of the exponential distribution is defined as:

$$M(t) = \frac{1}{1 - \lambda t}$$

Where: - λ is the rate parameter.

Expected Value ($E[X]$)

To find the expected value ($E[X]$), we need to take the first derivative of the MGF with respect to t and evaluate it at $t = 0$.

Step 1: Calculate the First Derivative

Let's calculate the first derivative of the MGF:

$$M'(t) = \frac{d}{dt}\left[\frac{1}{1 - \lambda t}\right]$$

Step 2: Evaluate at $t = 0$

Now, evaluate the first derivative at $t = 0$:

$$E[X] = M'(0)$$

The expected value is $\frac{1}{\lambda}$, representing the mean time between events.

Variance ($Var[X]$)

To find the variance ($Var[X]$), we need to take the second derivative of the MGF, subtract the square of the first derivative, and evaluate it at $t = 0$.

Step 1: Calculate the Second Derivative

Let's calculate the second derivative of the MGF:

$$M''(t) = \frac{d^2}{dt^2}\left[\frac{1}{1-\lambda t}\right]$$

Step 2: Subtract Square of First Derivative

Subtract the square of the first derivative:

$$Var[X] = M''(0) - [M'(0)]^2$$

This results in:

$$Var[X] = \frac{1}{\lambda^2}$$

The variance of the exponential distribution using the moment generating function is $\frac{1}{\lambda^2}$.