Khyati Naik: Data 605 - HW13

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1. Use integration by substitution to solve the integral below

$$\int 4e^{-7x}\,dx$$

Step 1: Choose a Substitution

Let $u = -7x$, then $du = -7\,dx$.

Step 2: Substitute and Simplify

$$\int 4e^{-7x}\,dx = \int 4e^{u}\left(-\frac{1}{7}\right)\,du$$

Step 3: Integrate and Back-Substitute

$$= -\frac{4}{7}\int e^{u}\,du$$

Now, integrate $\int e^{u}\,du$ and back-substitute. The result obtained after integration is:

$$-\frac{4}{7}e^{-7x} + C$$

where C is the constant of integration.

So, the solution to the integral $\int 4e^{-7x}\,dx$ is $-\frac{4}{7}e^{-7x} + C$, where C is the constant of integration.

2. Biologists are treating a pond contaminated with bacteria. The level of contamination is changing at a rate of dN/dt = (-3150/t^4) - 220dt bacteria per cubic centimeter per day, where t is the number of days since treatment began. Find a function N(t) to estimate the level of contamination if the level after 1 day was 6530 bacteria per cubic centimeter.

The level of contamination is changing at a rate of:

$$\frac{dN}{dt} = -\frac{3150}{t^4} - 220$$

Integrate the Equation

Assuming N is a function of t ($N(t)$), integrate the equation:

$$\int \frac{1}{N}\, dN = \int \left(-\frac{3150}{t^4} - 220 \right)\, dt$$

$$N(t) = \frac{1050}{t^3} - 220t + C$$

When t=1 and N(t)=6530 N(1) = 1050 -220 +C=6530 Therefore, C=5700

$$N(t) = \frac{1050}{t^3} - 220t + 5700$$

3. Find the total area of the red rectangles in the figure below, where the equation of the line is f(x) = 2x - 9.

```
# Define the function
function_integ <- function(x) {
    2 * x - 9
}

# Integrate the function from x = 4.5 to x = 8.5
area <- integrate(function_integ, 4.5, 8.5)

# Display the total area
area
```

```
## 16 with absolute error < 1.8e-13
```

The total area of the red rectangles in the figure is 16.

4. Find the area of the region bounded by the graphs of the given equations.

y = x^2 - 2x - 2, y = x + 2

Solution:

x^2 - 2x - 2=x+2 x2−3x−4=0 (x−4)(x+1)=0 x=4,x=−1

Subtract the two equations. x+2−(x^2−2x−2) x+2−x^2+2x+2 −x^2+3x+4 Integrate this equation using intersection points as interval.

```
function_integ <- function(x)
{
  -x^2 + 3*x + 4
}
integrate(function_integ, lower = -1, upper = 4)
```

```
## 20.83333 with absolute error < 2.3e-13
```

Thus, the area of the region bounded by the graphs of the given equations is 20.833.

5. A beauty supply store expects to sell 110 flat irons during the next year. It costs $3.75 to store one flat iron for one year. There is a fixed cost of $8.25 for each order. Find the lot size and the number of orders per year that will minimize inventory costs.

```
# Given values
D <- 110  # Demand per year
S <- 8.25  # Setup cost per order
H <- 3.75  # Holding cost per unit per year

# Economic Order Quantity (EOQ) formula
Q_star <- sqrt((2 * D * S) / H)

# Number of orders per year
Num_Orders <- D / Q_star

Q_star
```

```
## [1] 22
```

```
Num_Orders
```

```
## [1] 5
```

The lot size of 22 and the number of orders per year of 5 will minimize inventory costs.

6. Use integration by parts to solve the integral below.

$$\int \ln(9x) \cdot x^6 \, dx$$

Let:

$$u = \ln(9x)$$

$$dv = x^6 \, dx$$

Now, calculate du and v:

$$du = \frac{1}{\ln(9x)} \cdot \frac{d}{dx}(9x) \, dx = \frac{\ln(9x)}{9} \cdot 9 \, dx = \ln(9x) \, dx$$

$$v = \frac{1}{7}x^7$$

Now, apply the integration by parts formula:

$$\int \ln(9x) \cdot x^6 \, dx = uv - \int v \, du$$

$$= \ln(9x) \cdot \frac{1}{7}x^7 - \int \frac{1}{x}\frac{1}{7}x^7 dx$$

$$= \ln(9x) \cdot \frac{1}{7}x^7 - \frac{1}{49}x^7 + C$$

7. Determine whether f(x) is a probability density function on the interval [1,e^6]. If not, determine the value of the definite integral.

f(x) = 1/6x

Determining if f(x) is a Probability Density Function

To check if $f(x) = \frac{1}{6x}$ is a probability density function (pdf) on the interval $[1, e^6]$, we need to verify two conditions:

1. Non-negativity: $f(x)$ is non-negative for all x in the interval.
2. Normalization: The integral of $f(x)$ over the entire range is equal to 1.

Non-negativity

For $f(x) = \frac{1}{6x}$, $f(x)$ is non-negative as long as $x > 0$. In the given interval $[1, e^6]$, x is always greater than 0, so this condition is satisfied.

Normalization

We need to check whether the integral of $f(x)$ over the interval $[1, e^6]$ is equal to 1.

$$\int_1^{e^6} \frac{1}{6x}\,dx = \frac{1}{6}\int_1^{e^6} \frac{1}{x}\,dx$$

The integral $\int \frac{1}{x}\,dx$ is equal to $\ln(x)$, so:

$$\frac{1}{6}\int_1^{e^6} \frac{1}{x}\,dx = \frac{1}{6}[\ln(x)]_1^{e^6}$$

Evaluate this:

$$\frac{1}{6}[\ln(e^6) - \ln(1)] = \frac{1}{6}[6 - 0] = 1$$

So, the integral is equal to 1, and the second condition is satisfied.

Therefore, $f(x) = \frac{1}{6x}$ is a valid probability density function on the interval $[1, e^6]$.