Khyati Naik: Data 605 - HW14

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1. $f(x) = \frac{1}{1-x}$

The Taylor Series expansion for $f(x) = \frac{1}{1-x}$ is given by the geometric series:

$$f(x) = \sum_{n=0}^{\infty} x^n$$

This is a geometric series with a common ratio x, converging when $|x| < 1$. The sum is:

$$f(x) = 1 + x + x^2 + x^3 + \dots$$

2. $f(x) = e^x$

The Taylor Series expansion for $f(x) = e^x$ is the exponential series:

$$f(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$$

The general equation for this series is:

$$\sum_{k=0}^{\infty} \frac{x^k}{k!}$$

3. $f(x) = \ln(1 + x)$

The Taylor Series expansion for $f(x) = \ln(1 + x)$ is obtained using the series expansion for the natural logarithm:

$$f(x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$$

The general equation for this series is:

$$\sum_{k=1}^{\infty} \frac{(-1)^{k+1} x^k}{k}$$

4. $f(x) = x^{1/2}$

The Taylor Series expansion for $f(x) = x^{1/2}$ (square root function) is found using the binomial series expansion:

$$f(x) = (1+x)^{1/2} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 - \dots$$

The general equation for this series is:

$$f(x) = \sum_{k=0}^{\infty} \frac{(-1)^k (2k-1)!!}{2^k k!} x^k$$