Khyati Naik: Data 605 - HW15

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1. Find the equation of the regression line for the given points. Round any final values to the nearest hundredth, if necessary.

(5.6, 8.8), (6.3, 12.4), (7, 14.8), (7.7, 18.2), (8.4, 20.8)

To find the equation of the regression line, we can use the least squares method. The equation of a straight line is given by y=mx+b,where m is the slope and b is the y-intercept.

Let’s calculate the slope (m) and y-intercept (b) using the given points:

```
# Given data points
x <- c(5.6, 6.3, 7, 7.7, 8.4)
y <- c(8.8, 12.4, 14.8, 18.2, 20.8)

# Calculate the slope (m) and y-intercept (b)
fit <- lm(y ~ x)
m <- coef(fit)[2]
b <- coef(fit)[1]

# Print the equation of the regression line
cat("The equation of the regression line is: y =", round(m, 2), "x +", round(b, 2), "\n")
```

```
## The equation of the regression line is: y = 4.26 x + -14.8
```

2. Find all local maxima, local minima, and saddle points for the function given below. Write your answer(s) in the form (x, y, z). Separate multiple points with a comma.

f (x, y) = 24x - 6xy^2 - 8y^3

Solution:

1. Critical Points

Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$:

$$\frac{\partial f}{\partial x} = 24 - 6y^2$$

$$\frac{\partial f}{\partial y} = -y(12x + 24y)$$

Set $\frac{\partial f}{\partial x} = 0$ to find $y = \pm 2$. - For $y = 2$, substitute into $\frac{\partial f}{\partial y} = 0$ to find $x = -4$. - For $y = -2$, substitute into $\frac{\partial f}{\partial y} = 0$ to find $x = 4$.

Therefore, the critical points are $(-4, 2)$ and $(4, -2)$.

2. Second Derivative Test

Find $\frac{\partial^2 f}{\partial x^2}$, $\frac{\partial^2 f}{\partial y^2}$, and $\frac{\partial^2 f}{\partial x \partial y}$:

$$\frac{\partial^2 f}{\partial x^2} = 0$$

$$\frac{\partial^2 f}{\partial y^2} = -(12x + 48y)$$

$$\frac{\partial^2 f}{\partial x \partial y} = -12y$$

Evaluate $\Delta = \frac{\partial^2 f}{\partial x^2} \frac{\partial^2 f}{\partial y^2} - \left(\frac{\partial^2 f}{\partial x \partial y} \right)^2$ at $(-4, 2)$ and $(4, -2)$:

$$\Delta(-4, 2) = 0$$

$$\Delta(4, -2) = 0$$

Since $\Delta = 0$, the test is inconclusive.

3. Conclusion

- The critical points are $(-4, 2)$ and $(4, -2)$.
- The Second Derivative Test is inconclusive.
- Additional analysis is needed to determine the nature of these points.

3. A grocery store sells two brands of a product, the “house” brand and a “name” brand. The manager estimates that if she sells the “house” brand for x dollars and the “name” brand for y dollars, she will be able to sell 81 - 21x + 17y units of the “house” brand and 40 + 11x - 23y units of the “name” brand.

Step 1. Find the revenue function R (x, y).

The revenue function R(x,y) is given by the product of the quantity sold and the price for each brand. So, for the “house” brand, the revenue is x(81−21x+17y), and for the “name” brand, the revenue is y(40+11x−23y). Therefore, the total revenue function is:

R(x,y)=x(81−21x+17y)+y(40+11x−23y)

Step 2. What is the revenue if she sells the “house” brand for $2.30 and the “name” brand for $4.10?

```
# Step 1: Define the revenue function
revenue_function <- function(x, y) {
  return(x * (81 - 21 * x + 17 * y) + y * (40 + 11 * x - 23 * y))
}

# Step 2: Find the revenue when x = 2.30 and y = 4.10
revenue_at_given_prices <- revenue_function(2.30, 4.10)

# Display the result
print(paste("The revenue when selling the 'house' brand for $2.30 and the 'name' brand for $4.10
is $", round(revenue_at_given_prices, 2)))
```

```
## [1] "The revenue when selling the 'house' brand for $2.30 and the 'name' brand for $4.10 is $
116.62"
```

4. A company has a plant in Los Angeles and a plant in Denver. The firm is committed to produce a total of 96 units of a product each week. The total weekly cost is given by

C(x, y) = (x^2/6) + (y^2/6) +7x+25y+700

, where x is the number of units produced in Los Angeles and y is the number of units produced in Denver. How many units should be produced in each plant to minimize the total weekly cost?

Solution:

Cost Function: The cost function is given by:

$$C(x, y) = \frac{1}{6}x^2 + \frac{1}{6}y^2 + 7x + 25y + 700$$

Constraint: The constraint is $x + y = 96$, which can be rewritten as $x = 96 - y$.

Objective: Minimize the cost function $C(x, y)$.

Optimization Steps: 1. Substitute the expression $x = 96 - y$ into the cost function. 2. Take the first derivative of the modified cost function with respect to y. 3. Set the first derivative equal to zero and solve for y. 4. Substitute the optimal y back into the constraint to find x.

Result: The optimal solution is $x = 75$ (units produced in Los Angeles) and $y = 21$ (units produced in Denver).

5. Evaluate the double integral on the given region.Write your answer in exact form without decimals.

Solution:

To evaluate the double integral

$$\int_2^4 \int_2^4 e^{8x+3y}\, dy\, dx$$

we'll integrate with respect to y first, and then with respect to x.

1. Integrate with respect to y

$$\int e^{8x+3y}\, dy$$

The result is $\frac{1}{3}e^{8x+3y}$. Evaluate this from $y = 2$ to $y = 4$.

2. Integrate with respect to x

$$\int_2^4 \frac{1}{3}e^{8x+3y}\, dx$$

The result is $\frac{1}{3} \cdot \frac{1}{8}e^{8x+3y}$. Evaluate this from $x = 2$ to $x = 4$.

Substitute the upper and lower limits:

$$\frac{1}{3} \cdot \frac{1}{8}\left(e^{8(4)+3y} - e^{8(2)+3y}\right)$$