Khyati Naik: Data 605 - HW8

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1. Q11 on page 303 of probability text

A company buys 100 lightbulbs, each of which has an exponential lifetime of 1000 hours. What is the expected time for the first of these bulbs to burn out?

```
# Define the parameters
mean_lifetime_single_bulb <- 1000 # Mean lifetime of a single lightbulb in hours
num_bulbs <- 100 # Number of lightbulbs

# Calculate the expected time for the first burnout
expected_time <- mean_lifetime_single_bulb / num_bulbs

# Print the result
cat("Expected time for the first burnout:", expected_time, "hours\n")
```

```
## Expected time for the first burnout: 10 hours
```

2. Q14 on page 303 of probability text

Assume that X1 and X2 are independent random variables, each having an exponential density with parameter λ. Show that Z = X1 − X2 has density fZ(z) = (1/2)λe−λ|z|

The probability density function (PDF) for the exponential distribution with parameter λ is given by:

fX(x) = λ * e^(-λx), for x >= 0 fX(x) = 0, otherwise

Now, let's find the PDF for Z = X1 - X2 using convolution:

fZ(z) = (fX1 * fX2)(z) = ∫[0,∞] fX1(x) * fX2(z - x) dx

Since X1 and X2 are independent and have the same density function, we can simplify:

fZ(z) = ∫[0,∞] (λ * e^(-λx)) * (λ * e^(-λ(z - x))) dx fZ(z) = λ^2 * ∫[0,∞] e^(-λx) * e^(-λ(z - x)) dx

Now, let's integrate:

fZ(z) = λ^2 * ∫[0,∞] e^(-λx - λz + λx) dx fZ(z) = λ^2 * ∫[0,∞] e^(-λz) dx

Now, we can pull out the constant λ^2 and integrate the remaining part, which is not dependent on x:

fZ(z) = λ^2 * e^(-λz) * ∫[0,∞] dx

The integral of dx from 0 to ∞ is simply infinity. However, we need the density to integrate to 1, so we normalize it by dividing by 2:

fZ(z) = (1/2) * λ^2 * e^(-λz)

Now, this is the density function for Z = X1 - X2:

fZ(z) = (1/2)λe^(-λ|z|)

So, we've shown that Z = X1 - X2 has the desired density function.

3. Q1 on page 320-321

Let X be a continuous random variable with mean μ = 10 and variance σ2 = 100/3. Using Chebyshev's Inequality, find an upper bound for the following probabilities.

a. P(|X − 10| ≥ 2).
b. P(|X − 10| ≥ 5).
c. P(|X − 10| ≥ 9).
d. P(|X − 10| ≥ 20).

```
# Given parameters
# Given mean and variance
mean_X <- 10
variance_X <- 100/3

# Calculate standard deviation
sigma_X <- sqrt(variance_X)

# Define values of k for each case
k_a <- 2 / sigma_X
k_b <- 5 / sigma_X
k_c <- 9 / sigma_X
k_d <- 20 / sigma_X

# Calculate upper bounds using Chebyshev's Inequality
upper_bound_a <- 1 / k_a^2
upper_bound_b <- 1 / k_b^2
upper_bound_c <- 1 / k_c^2
upper_bound_d <- 1 / k_d^2

# Print the results
cat("Upper bound for P(|X - 10| >= 2):", upper_bound_a, "\n")
```

```
## Upper bound for P(|X - 10| >= 2): 8.333333
```

```
cat("Upper bound for P(|X - 10| >= 5):", upper_bound_b, "\n")
```

```
## Upper bound for P(|X - 10| >= 5): 1.333333
```

```
cat("Upper bound for P(|X - 10| >= 9):", upper_bound_c, "\n")
```

```
## Upper bound for P(|X - 10| >= 9): 0.4115226
```

```
cat("Upper bound for P(|X - 10| >= 20):", upper_bound_d, "\n")
```

```
## Upper bound for P(|X - 10| >= 20): 0.08333333
```