# SML HW1 Zhao Nailin 1003003

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1. $\theta^T x = \theta_1 X_1 + \dots + \theta_d X_d$

$L(x,\lambda) = (x-y)^T(x-y) + \lambda(\theta^T x + \theta_0)$

1) $\frac{\partial L(x,\lambda)}{\partial x} = \frac{\partial[(x-y)^T(x-y) + \lambda(\theta^T x + \theta_0)]}{\partial x}$

$= \frac{\partial[(x-y)^T(x-y)]}{\partial x} + \frac{\partial[\lambda(\theta^T x + \theta_0)]}{\partial x}$

$= 2(x-y)^T \cdot \frac{\partial(x-y)}{\partial x} + \lambda\theta^T + 0$

$= 2(x-y)^T + \lambda\theta^T$ #

$\frac{\partial L(x,\lambda)}{\partial\lambda} = \frac{\partial[(x-y)^T(x-y)]}{\partial\lambda} + \frac{\partial[\lambda(\theta^T x + \theta_0)]}{\partial x}$

$= 0 + \theta^T x + \theta_0$

$= \theta^T x + \theta_0$ #

2) Let $\frac{\partial L(x,\lambda)}{\partial x} = 2(\tilde{x}-y)^T + \lambda\theta^T = 0$

$\therefore 2(\tilde{x}-y)^T + \lambda\theta^T = 0$

$2\tilde{x}^T - 2y^T + \lambda\theta^T = 0$

$2\tilde{x} = 2y^T - \lambda\theta^T$

$\tilde{x} = y^T - \frac{\lambda\theta^T}{2}$

$\therefore x = y - \frac{\lambda\theta}{2}$

let $\frac{\partial L(x,\lambda)}{\partial\lambda} = \theta^T x + \theta_0 = 0$

$\therefore \theta^T x + \theta_0 = 0$ ②

Sub ① into ②: $\theta^T(y - \frac{\lambda\theta}{2}) + \theta_0 = 0$

$\theta^T y - \frac{\theta^T\lambda\theta}{2} + \theta_0 = 0$

$\frac{\theta^T\lambda\theta}{2} = \theta^T y + \theta_0$

$\theta^T\lambda\theta = 2\theta^T y + 2\theta_0$

$\lambda = \frac{2\theta^T y}{\|\theta\|^2} + \frac{2\theta_0}{\theta^T\theta}$

$\lambda = \frac{2(\theta^T y + \theta_0)}{\|\theta\|^2}$

$= \frac{2(\theta^T y + \theta_0)}{\|\theta\|^2}$

$\therefore \tilde{x} = y - \frac{2(\theta^T y + \theta_0)}{\|\theta\|^2}\theta \cdot (\frac{1}{2})$

$\tilde{x} = y - \frac{(\theta^T y + \theta_0)}{\|\theta\|^2}\theta$ #

3) Minimise $(x-y)^T(x-y)$

Subject to $\theta^T x + \theta_0 = 0$

Required Distance $= \|y - \tilde{x}\|$

$= \|y - (y - \frac{(\theta^T y + \theta_0)\theta}{\|\theta\|^2})\|$

$= \|\cancel{y} - \cancel{y} + \frac{(\theta^T y + \theta_0)\theta}{\|\theta\|^2}\|$

$= \|\frac{(\theta^T y + \theta_0)\theta}{\|\theta\|^2}\|$

$= \frac{(\theta^T y + \theta_0)\|\theta\|}{\|\theta\|^2} = \frac{(\theta^T y + \theta_0)}{\|\theta\|}$ #

2、1) $\mathbb{P}(X=x)=\frac{\alpha^x e^{-\alpha}}{x!}$

$\mathbb{P}(Y=y)=\frac{\beta^y e^{-\beta}}{y!}$

$\mathbb{P}(Z=z)=\mathbb{P}\left(\frac{\alpha^x e^{-\alpha}}{x!}+\frac{\beta^y e^{-\beta}}{y!}\right)$ *

2) Simplify $\mathbb{P}_X$ and $\mathbb{P}_Y$ to $X\sim\mathbb{P}(\alpha)$ and $Y\sim\mathbb{P}(\beta)$

Show $Z\sim\mathbb{P}(\alpha+\beta)$ is a Poisson Distribution

$$\begin{aligned}
\mathbb{P}_X[t] &= E[t^X] \\
&= \sum_{x=0}^{\infty} t^x e^{-\alpha}\left(\frac{\alpha^x}{x!}\right) \\
&= \sum_{x=0}^{\infty} e^{-\alpha}\cdot\left(\frac{(\alpha t)^x}{x!}\right) \\
&= e^{-\alpha}e^{\alpha t} \\
&= e^{-\alpha(1-t)}
\end{aligned}$$

$$\begin{aligned}
\mathbb{P}_Y[t] &= E[t^Y] \\
&= \sum_{y=0}^{\infty} t^y e^{-\beta}\left(\frac{\beta^y}{y!}\right) \\
&= \sum_{y=0}^{\infty} e^{-\beta}\cdot\frac{(\beta t)^y}{y!} \\
&= e^{-\beta(1-t)}
\end{aligned}$$

$$\begin{aligned}
\therefore P_Z(t) &= P_X(t)P_Y(t) \\
&= E[t^X]E[t^Y] \\
&= E[t^{X+Y}] \\
&= E[t^X t^Y] \\
&= e^{-\alpha(1-t)}e^{-\beta(1-t)} \\
&= e^{-(\alpha+\beta)(1-t)} \quad *
\end{aligned}$$

$\therefore Z=X+Y$ follows $P_o(\alpha+\beta)$

3.