Deliyannis, Theodore L. et al "A Sample of Filter Functions"
Continuous-Time Active Filter Design
Boca Raton: CRC Press LLC,1999

Appendix A

A Sample of Filter Functions

TABLE A.1

Butterworth Polynomials, $B_n(s)$ in Expanded and in Factored Form

n	Butterworth Polynomials
1	$s + 1$
2	$s^2 + 1.4142s + 1$
3	$s^3 + 2s^2 + 2s + 1 = (s + 1)(s - 2 + s + 1)$
4	$s^4 + 2.613s^3 + 3.414s^2 + 2.613s + 1 = (s^2 + 0.765s + 1)(s^2 + 1.848s + 1)$
5	$s^5 + 3.2361s^4 + 5.2361s^3 + 5.2361s^2 + 3.2361s + 1 = (s + 1.0000)(s^2 + 0.618s + 1)(s^2 + 1.618s + 1)$
6	$s^6 + 3.8637s^5 + 7.4641s^4 + 9.1416s^3 + 7.4641s^2 + 3.8637s + 1 = (s^2 + 0.5176s + 1)(s^2 + 1.4142s + 1)(s^2 + 1.9318s + 1)$
7	$s^7 + 4.4940s^6 + 10.0978s^5 + 14.5918s^4 + 14.59186s^3 + 10.0978s^2 + 4.4940s + 1 = (s + 1.0000)(s^2 + 0.445s + 1)(s^2 + 1.247s + 1)(s^2 + 1.802s + 1)$
8	$s^8 + 5.1258s^7 + 13.1317s^6 + 21.8462s^5 + 25.6884s^4 + 21.8462s^3 + 13.1371s^2 + 5.1258s + 1 = (s^2 + 0.3902s + 1)(s^2 + 1.1112s + 1)(s^2 + 1.663s + 1)(s^2 + 1.9616s + 1)$
9	$s^9 + 5.7588s^8 + 16.5817s^7 + 31.1634s^6 + 41.9864s^5 + 41.9864s^4 + 31.1634s^3 + 16.5817s^2 + 5.7588s + 1$ $= (s + 1.0000)(s^2 + 0.3474s + 1)(s^2 + 1.0000s + 1)(s^2 + 1.532s + 1)(s^2 + 1.8794s + 1)$
10	$s^{10} + 6.3925s^9 + 20.4317s^8 + 42.8021s^7 + 64.8824s^6 + 74.2334s^5 + 64.8824s^4 + 42.8021s^3 + 20.4317s^2 + 6.3925s + 1$ $= (s^2 + 0.3128s + 1)(s^2 + 0.908s + 1)(s^2 + 1.4142s + 1)(s^2 + 1.782s + 1)(s^2 + 1.9754s + 1)$

TABLE A.2

Denominator Polynomials in Expanded and in Factored Forms for Chebyshev Filters of Odd Order

n	**Polynomials**
	A_{max} = 0.1dB (ε = 0.15262)
1	$s + 6.55220$
3	$s^3 + 1.93881s^2 + 2.62950s + 1.63805 = (s^2 + 0.96941s + 1.68975)(s + 0.96941)$
5	$s^5 + 1.74396s^4 + 2.77070s^3 + 2.39696s^2 + 1.43556s + 0.40951 = (s^2 + 0.33307s + 1.19494)(s^2 + 0.87198s + 0.63592)(s + 0.53891)$
7	$s^7 + 1.69322s^6 + 3.18350s^5 + 3.16925s^4 + 2.70514s^3 + 1.48293s^2 + 0.56179s + 0.10238 = (s^2 + 0.16768s + 1.09245)(s^2 + 0.46983s + 0.75322)(s^2 + 0.67893s + 0.33022)(s + 0.37678)$
9	$s^9 + 1.67270s^8 + 3.64896s^7 + 3.96385s^6 + 4.19161s^5 + 2.93387s^4 + 1.73412s^3 + 0.69421s^2 + 0.19176s + 0.025595 = (s^2 + 0.10088s + 1.05421)(s^2 + 0.29046s + 0.83437)(s^2 + 0.44501s + 0.49754)(s^2 + 0.54589s + 0.20135)(s + 0.29046)$
	A_{max} = 0.5 dB (ε = 0.34931)
1	$s + 2.86278$
3	$s^3 + 1.25291s^2 + 1.53490s + 0.71569 = (s^2 + 0.62646s + 1.14245)(s + 0.62646)$
5	$s^5 + 1.17249s^4 + 1.93738s^3 + 1.30958s^2 + 0.75252s + 0.17892 = (s^2 + 0.22393s + 1.03578)(s^2 + 0.58625s + 0.47677)(s + 0.36232)$
7	$s^7 + 1.15122s^6 + 2.41265s^5 + 1.86941s^4 + 1.64790s^3 + 0.75565s^2 + 0.28207s + 0.04473 = (s^2 + 0.11401s + 1.01611)(s^2 + 0.31944s + 0.67688)(s^2 + 0.46160s + 0.25388)(s + 0.25617)$
9	$s^9 + 1.14257s^8 + 2.90273s^7 + 2.42933s^6 + 2.78150s^5 + 1.61139s^4 + 0.98362s^3 + 0.34082s^2 + 0.09412s + 0.01118 = (s^2 + 0.06891s + 1.00921)(s^2 + 0.19841s + 0.78937)(s^2 + 0.30398s + 0.45254)(s^2 + 0.37288s + 0.15634)(s + 0.19841)$
	A_{max} = 1 dB (ε = 0.50885)
1	$s + 1.96523$
3	$s^3 + 0.73782s^2 + 1.02219s + 0.32689 = (s^2 + 0.36891s + 0.88610)(s + 0.36891)$
5	$s^5 + 0.70646s^4 + 1.49954s^3 + 0.69348s^2 + 0.45935s + 0.08172 = (s^2 + 0.13492s + 0.95217)(s^2 + 0.35323s + 0.39315)(s + 0.21831)$

TABLE A.2

Denominator Polynomials in Expanded and in Factored Forms for Chebyshev Filters of Odd Order *(continued)*

n	**Polynomials**
7	$s^7 + 0.69809s^6 + 1.99367s^5 + 1.03955s^4 + 1.14460s^3 + 0.38264s^2 + 0.16613s + 0.02043 = (s^2 + 0.06913s + 0.97462)(s^2 + 0.19371s + 0.63539)(s^2 + 0.27991s + 0.21239)(s + 0.15534)$
9	$s^9 + 0.69468s^8 + 2.49129s^7 + 1.38375s^6 + 2.07675s^5 + 0.85687s^4 + 0.64447s^3 + 0.16845s^2 + 0.05438s + 0.00511 = (s^2 + 0.04189s + 0.98440)(s^2 + 0.12063s + 0.76455)(s^2 + 0.18482s + 0.42773)(s^2 + 0.22671s + 0.13153)(s + 0.12063)$
	$A_{max} = 2$ dB ($\varepsilon = 0.76478$)
1	$s + 1.30756$
3	$s^3 + 0.73782s^2 + 1.02219s + 0.32689 = (s^2 + 0.36891s + 0.88610)(s + 0.36891)$
5	$s^5 + 0.70646s^4 + 1.49954s^3 + 0.69348s^2 + 0.45935s + 0.08172 = (s^2 + 0.13492s + 0.95217)(s^2 + 0.35323s + 0.39315)(s + 0.21831)$
7	$s^7 + 0.69809s^6 + 1.99367s^5 + 1.03955s^4 + 1.14460s^3 + 0.38264s^2 + 0.16613s + 0.02043 = (s^2 + 0.06913s + 0.97462)(s^2 + 0.19371s + 0.63539)(s^2 + 0.27991s + 0.21239)(s + 0.15534)$
9	$s^9 + 0.69468s^8 + 2.49129s^7 + 1.38375s^6 + 2.07675s^5 + 0.85687s^4 + 0.64447s^3 + 0.16845s^2 + 0.05438s + 0.00511 = (s^2 + 0.04189s + 0.98440)(s^2 + 0.12063s + 0.76455)(s^2 + 0.18482s + 0.42773)(s^2 + 0.22671s + 0.13153)(s + 0.12063)$
	$A_{max} = 3$ dB ($\varepsilon = 0.99763$)
1	$s + 1.00238$
3	$s^3 + 0.59724s^2 + 0.92835s + 0.25059 = (s^2 + 0.29862s + 0.83917)(s + 0.29862)$
5	$s^5 + 0.57450s^4 + 1.41503s^3 + 0.54894s^2 + 0.40797s + 0.06265 = (s^2 + 0.10972s + 0.93603)(s^2 + 0.28725s + 0.37701)(s + 0.17753)$
7	$s^7 + 0.56842s^6 + 1.91155s^5 + 0.83144s^4 + 1.05185s^3 + 0.30002s^2 + 0.14615s + 0.01566 = (s^2 + 0.05629s + 0.96648)(s^2 + 0.15773s + 0.62726)(s^2 + 0.22792s + 0.20425)(s + 0.12649)$
9	$s^9 + 0.56594s^8 + 2.41014s^7 + 1.11232s^6 + 1.94386s^5 + 0.67893s^4 + 0.58351s^3 + 0.13139s^2 + 0.04759s + 0.00392 = (s^2 + 0.03413s + 0.97950)(s^2 + 0.09827s + 0.75966)(s^2 + 0.15057s + 0.42283)(s^2 + 0.18470s + 0.12664)(s + 0.09827)$

TABLE A.3

Elliptic Approximation Functions for A_{max} = 0.5 dB

n	A_{min}	Numerator constant K	Numerator of $F(s)$	Denominator of $F(s)$
			(a) $\Omega_s = 1.5$	
2	8.3	0.38540	$s^2 + 3.92705$	$s^2 + 1.03153s + 1.60319$
3	21.9	0.31410	$s^2 + 2.80601$	$(s^2 + 0.45286s + 1.14917)(s + 0.766952$
4	36.3	0.015397	$(s^2 + 2.53555)(s^2 + 12.09931)$	$(s^2 + 0.25496s + 1.06044)(s^2 + 0.92001s + 0.47183)$
5	50.6	0.019197	$(s^2 + 2.42551)(s^2 + 5.43764)$	$(s^2 + 0.16346s + 1.03189)(s^2 + 0.57023s + 0.57601)(s + 0.42597)$
			(b) $\Omega_s = 2.0$	
2	13.9	0.20133	$s^2 + 7.4641$	$s^2 + 1.24504s + 1.59179$
3	31.2	0.15424	$s^2 + 5.15321$	$(s^2 + 0.53787s + 1.14849)(s + 0.69212)$
4	48.6	0.0036987	$(s^2 + 4.59326)(s^2 + 24.22720)$	$(s^2 + 0.30116s + 1.06258)(s^2 + 0.88456s + 0.41032)$
5	66.1	0.0046205	$(s^2 + 4.36495)(s^2 + 10.56773)$	$(s^2 + 0.19255s + 1.03402)(s^2 + 0.58054s + 0.52500)(s + 0.392612)$
			(c) $\Omega_s = 3.0$	
2	21.5	0.083974	$s^2 + 17.48528$	$s^2 + 1.35715s + 1.55532$
3	42.8	0.063211	$s^2 + 11.82781$	$(s^2 + 0.58942s + 1.14559)(s + 0.65263)$
4	64.1	0.00062046	$(s^2 + 10.4554)(s^2 + 58.471)$	$(s^2 + 0.32979s + 1.063281)(s^2 + 0.86258s + 0.37787)$
5	85.5	0.00077547	$(s^2 + 9.8955)(s^2 + 25.0769)$	$(s^2 + 0.21066s + 1.0351)(s^2 + 0.58441s + 0.496388)(s + 0.37452$