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Chapter 3

Active Elements

3.1 Introduction

The ideal active elements are devices having one to three ports with properties that make them very useful in network synthesis. Some active elements are more useful than others, in the sense that their realizations are more practical than others.

The most important ideal active elements in network synthesis fall into the following groups:

- Ideal controlled sources
- Generalized impedance converters (GICs)
- Generalized impedance inverters (GIIs)
- Negative resistance
- Current conveyors

Although the GIIs, generally speaking, can be regarded as GICs, they are presented separately here for reasons of clarity.

The first three groups consist of two-port devices, and the fourth is one-port. The fifth is a three-port device. We present each of these groups separately below.

For all these ideal active elements, we give practical realizations using two active devices which are commercially available, namely, the operational amplifier (opamp) and the operational transconductance amplifier (OTA). The opamp and OTA are special cases of two ideal active elements, and their implementations in IC form make them indispensable today, both in discrete and fully integrated analog network design. Because of this exclusive use in active filter design, we introduce them here both as ideal and practical elements, giving emphasis on the imperfections of the practical realizations.

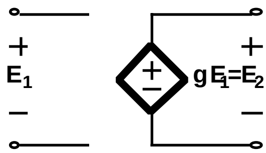
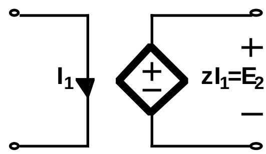
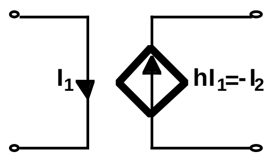
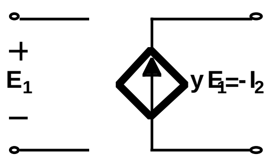
Before proceeding with the development of this chapter, one point should be clarified. Although the transistor, either the bipolar (BJT) or the unipolar (FET), is essentially the basic active element in the realization of all other active elements in practice, we prefer not to consider it as such here but rather to treat it as a type of a nonideal controlled current source.

3.2 Ideal Controlled Sources

An ideal controlled source is a source whose magnitude (voltage or current) is proportional to another quantity (voltage or current) in some part of the network. Table 3.1 lists the four

TABLE 3.1

Ideal Controlled Sources

	Description	Symbol	A Matrix	Reciprocal or Nonreciprocal
1	Voltage controlled voltage source (VCVS)		$\begin{bmatrix} 1/g & 0 \\ 0 & 0 \end{bmatrix}$	Nonreciprocal
2	Current controlled voltage source (CCVS)		$\begin{bmatrix} 0 & 0 \\ 1/z & 0 \end{bmatrix}$	Nonreciprocal
3	Current controlled current source (CCCS)		$\begin{bmatrix} 0 & 0 \\ 0 & 1/h \end{bmatrix}$	Nonreciprocal
4	Voltage controlled current source (VCCS)		$\begin{bmatrix} 0 & 1/y \\ 0 & 0 \end{bmatrix}$	Nonreciprocal

types of controlled sources. Their characteristic feature is that the transmission matrix has just one nonzero element. Among them, only the CCVS and the VCCS are basic devices. A VCCS followed by a CCVS gives a VCVS, and reversing the order gives a CCCS.

3.3 Impedance Transformation (Generalized Impedance Converters and Inverters) [1, 2]

Consider a two-port to be terminated at port 2 in an impedance Z_L as shown in Fig. 3.1(a). The input impedance Z_{i1} at port 1 expressed in terms of Z_L and the transmission matrix parameters a_{ij}, $i, j = 1, 2$ is as follows:

$$Z_{i1} \equiv \frac{V_1}{I_1} = \frac{a_{11}Z_L + a_{12}}{a_{21}Z_L + a_{22}} \tag{3.1}$$

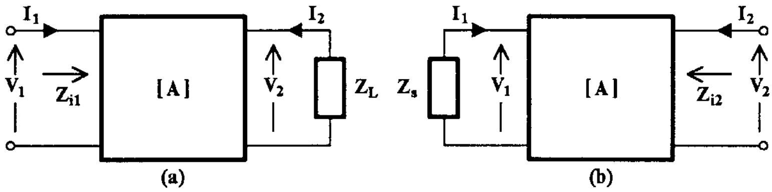


FIGURE 3.1
Two-port device terminated at port 2 or at port 1

where, in the general case a_{ij}, $i, j = 1, 2$, and Z_L are functions of the complex variable s. Thus, the action of the two-port is to transform the impedance Z_L to another Z_{i1} which, depending on the nature of a_{ij}, can produce very interesting and useful values.

Similarly, if the two-port is terminated at port 1 in an impedance Z_s, the input impedance Z_{i2} at port 2 will be as follows:

$$Z_{i2} \equiv \frac{V_2}{I_2} = \frac{a_{12} + a_{22}Z_s}{a_{11} + a_{21}Z_s} \tag{3.2}$$

Thus, again the two-port acts as an impedance transformer or converter.

We may consider now the following two specific cases:

- Case a: $a_{11}, a_{22} \neq 0$ while $a_{12} = a_{21} = 0$
- Case b: $a_{11} = a_{22} = 0$ while $a_{12}, a_{21} \neq 0$

Substituting in Eq. (3.1) we get for

Case a: $$Z_{i1} = \frac{a_{11}}{a_{22}}Z_L = G_aZ_L \qquad G_a = \frac{a_{11}}{a_{22}} \tag{3.3}$$

Case b: $$Z_{i1} = \frac{a_{12}}{a_{22}}\frac{1}{Z_L} = G_b\frac{1}{Z_L} \qquad G_b = \frac{a_{12}}{a_{22}} \tag{3.4}$$

Similarly, substituting in Eq. (3.2) we get for

Case a: $$Z_{i2} = \frac{a_{22}}{a_{11}}Z_s = \frac{1}{G_a}Z_s \tag{3.5}$$

Case b: $$Z_{i2} = \frac{a_{12}}{a_{21}}\frac{1}{Z_s} = G_b\frac{1}{Z_s} \tag{3.6}$$

Clearly, in Case a, when $a_{12} = a_{21} = 0$, the two-port is a generalized impedance converter (GIC), when G_a, the conversion constant, is a function of s. The conversion constant is not the same for port 1 and port 2.

On the other hand, in Case b, when $a_{11} = a_{22} = 0$, the two-port is a generalized impedance inverter (GII). In this case, the inversion constant G_b is the same for port 1 and port 2.

3.3.1 Generalized Impedance Converters [3]

Following the argument presented above, we can define the GIC as the two-port for which the transmission matrix parameters a_{12} and a_{21} are zero for all s, while $a_{11} = k$ and $a_{22} = k/f(s)$, i.e.,

$$[A] = \begin{bmatrix} k & 0 \\ 0 & k/f(s) \end{bmatrix} \tag{3.7}$$

where k is a positive constant.

Following this definition of the GIC and referring to Fig. 3.1, if the impedances Z_L and Z_s are connected across port 2 and port 1, respectively, then the input impedances Z_{i1} and Z_{i2} at ports 1 and 2 will be

$$Z_{i1} = f(s)Z_L \tag{3.8a}$$

$$Z_{i2} = \frac{1}{f(s)}Z_s \tag{3.8b}$$

The conversion function $f(s)$ can take any complex value realizable by an active RC two-port. However, some simple expressions of $f(s)$ have been proven to be of very high practical value in active network synthesis, as shown below.

3.3.1.1 *The Ideal Active Transformer*

Let $a_{11} = \pm 1/n_1$, and $a_{22} = \pm n_2$, with $n_1 \neq n_2$. Then,

$$\frac{a_{11}}{a_{22}} = \frac{1}{n_1 n_2} \tag{3.9}$$

and the converter transforms Z_L to

$$Z_i = \frac{1}{n_1 n_2} Z_L \tag{3.10}$$

If $n_1 = n_2 = n$, the two-port is the ideal transformer (Fig. 3.2) which, of course, is passive and reciprocal.

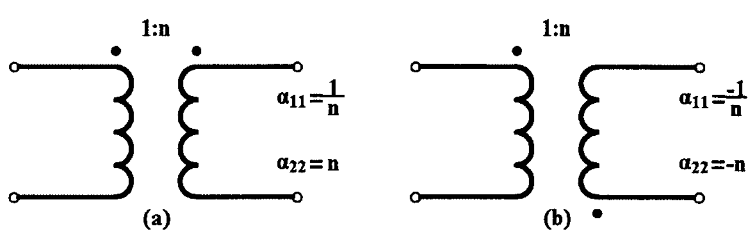


FIGURE 3.2
Ideal transformer, (a) normal and (b) reverse polarity.

3.3.1.2 The Ideal Negative Impedance Converter

Let $$a_{11} = \mp k \qquad a_{22} = \pm\frac{1}{k} \qquad [f(s)] = 1$$

In either case, we have

$$\frac{a_{11}}{a_{22}} = -k^2 \tag{3.11}$$

and

$$Z_i = -k^2 Z_L \tag{3.12}$$

Thus, an ideal negative impedance converter (NIC) is a two-port device that presents across one of its ports the negative of the impedance that is connected across the other port within a constant k^2.

Two types of NICs can be identified according to the signs of a_{11} and a_{22}. When

$$a_{11} = -k \qquad a_{22} = \frac{1}{k}$$

the voltage negative-impedance converter (VNIC) is obtained, since then the polarity of the voltage at port 1 is reversed with respect to the polarity of the voltage at port 2.

On the other hand, when

$$a_{11} = k \qquad a_{22} = -\frac{1}{k}$$

the current negative impedance converter (CNIC) is obtained, since then the direction of the output current with respect to that of the input current is reversed.

With $k = 1$, the VNIC and CNIC of unity gain are obtained. The concept of negative resistance is explained in Section 3.4.

3.3.1.3 The Positive Impedance Converter

Let $$f(s) = s \qquad \text{and} \qquad k = 1$$

Then, for $Z_L = R$, Eq. (3.8a) gives

$$Z_{i1} = sR \tag{3.13}$$

Thus, terminating this GIC with a resistor makes the input impedance look like that of a grounded inductor. This is very significant in filter design, as we shall see later (Chapters 4 and 6).

This GIC, when it was first introduced [4], was called the positive-immittance converter or the PIC (*immittance* from **im**pedance and ad**mittance**).

On the other hand, for $Z_L = 1/sC$,

$$Z_{i1} = \frac{1}{C} \tag{3.14}$$

i.e., a grounded resistance of $1/C$ ohms.

Looking now at port 2, if $Z_s = R$, Eq. (3.8b) gives [$f(s) = s$],

$$Z_{i2} = \frac{R}{s} \tag{3.15}$$

i.e., the impedance of a capacitor 1/R farads.

The use of the PIC in filter design is explained in Chapter 6.

3.3.1.4 The Frequency-Dependent Negative Resistor [5]

As a last case, consider that

$$f(s) = \frac{1}{s}$$

Then, with $Z_L = R$,

$$Z_{i1} = \frac{R}{s}$$

i.e., the impedance of a grounded capacitor of $1/R$ farads.

However, if $Z_L = 1/sC$, then

$$Z_{i1} = \frac{1}{s^2 C} \tag{3.16}$$

Substituting $j\omega$ for s gives

$$Z_{i1}(\omega) = -\frac{1}{\omega^2 C} \tag{3.17}$$

Clearly, this is a negative resistance dependent on frequency. For this reason it is called the frequency-dependent negative resistor (FDNR) of type D (D-FDNR).

Usually, the impedance of a D-FDNR is written as $1/s^2D$ with the unit of D being farad-second. The symbol of this in a circuit is similar to that of a capacitor but with four parallel lines instead of two. For this reason the D-FDNR is sometimes referred to as **supercapacitor**.

An E-type FDNR can be obtained if a PIC is terminated at port 2 by an inductor. Then, with $Z_L = sL$ and $f(s) = s$, Eq. (3.8a) gives

$$Z_{i1} = s^2 L \tag{3.18}$$

This is sometimes called the **superinductor**, but it is not so useful in active RC filter design as the supercapacitor, as we shall see in Chapter 6.

The GIC is a nonreciprocal two-port, as can be easily derived from its transmission matrix. If it is loaded by the same impedance at both its ports, Z_{i1} and Z_{i2} will be different, as can be seen through Eqs. (3.8a) and (3.8b) for $Z_s = Z_L$. Depending on $f(s)$ and Z_L or Z_s, either of the ports (port one or port two) can be used to represent the component that has been obtained by the impedance conversion.

The GIC is thus a very flexible device, which can be used to simulate the transformer, negative resistance, inductance, and the D-FDNR, all of importance in filter design.

3.3.2 Generalized Impedance Inverters

The generalized impedance inverter (GIV) can be defined as the two-port with transmission parameters a_{11}, $a_{22} = 0$, and a_{12}, $a_{21} \neq 0$ for all s.

If such a two-port is terminated at one port by an impedance Z_L, the impedance Z_i seen in the other port will be

$$Z_i = \frac{a_{12}}{a_{22}} \frac{1}{Z_L} = G_b \frac{1}{Z_L} \tag{3.19}$$

where $G_b = a_{12}/a_{21}$ can be defined as the inversion constant with units Ω^2.

G_b will be, in general, a function of s. However, in network synthesis, two cases have attracted the interest of the designers: the positive impedance inverter or gyrator and the negative impedance inverter. These are now considered.

3.3.2.1 The Gyrator

The gyrator, or positive impedance inverter, is a very attractive two-port, because it can be used to simulate inductance. Its symbol and transmission matrix are shown in Fig. 3.3.

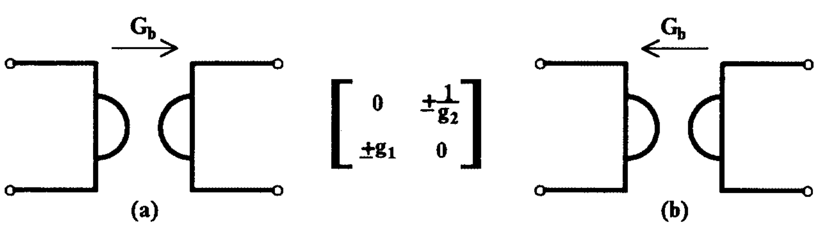
This definition through its transmission matrix, with $g_1 \neq g_2$ and positive, refers to the active gyrator. However, if $g_1 = g_2 = g$, the gyrator is a passive two-port.

Clearly, the gyrator is a nonreciprocal two-port, since

$$a_{11}a_{22} - a_{12}a_{21} \neq 1$$

Its importance in network synthesis stems from the fact that, if it is terminated at port 2 by a capacitance C_L, the impedance seen in port 1, according to Eq. (3.19), is

$$Z_i = G_b s C_L = s\frac{C_L}{g_1 g_2} = sL_{eq} \tag{3.20}$$



$$\begin{bmatrix} 0 & \pm\frac{1}{g_2} \\ \pm g_1 & 0 \end{bmatrix}$$

FIGURE 3.3
Symbols of gyrator: (a) a_{12}, $a_{21} > 0$, and (b) a_{12}, $a_{21} < 0$ with $G_b = (g_1 g_2)^{-1}$.

i.e., the impedance of an equivalent inductance

$$L_{eq} = \frac{C_L}{g_1 g_2} \tag{3.21}$$

The use of the gyrator in network synthesis is explained in Chapter 6, where this device is studied more rigorously.

3.3.2.2 *Negative Impedance Inverter*

A negative impedance inverter (NIV) is a two-port device whose input impedance at one-port is the negative reciprocal of the terminating impedance at the other port. This can be obtained, if G_b in Eq. (3.19) is equal to –1, i.e.,

$$G_b = -1$$

Then,

$$Z_i = -\frac{1}{Z_L} \tag{3.22}$$

Since, in this case, either

$$a_{12} = -1 \qquad a_{21} = 1$$

or

$$a_{12} = 1 \qquad a_{21} = -1$$

with a_{11}, $a_{22} = 0$, the NIV is a reciprocal two-port.

3.4 Negative Resistance

The concept of negative resistance is exciting from both theoretical and practical points of view. A negative resistance is a two-terminal device defined by the relationship between the voltage and the current in it, i.e.,

$$V = -RI \qquad R > 0 \tag{3.23}$$

Its physical meaning can be explained by the fact that it absorbs negative power; therefore, it acts as an energy source.

The defining Eq. (3.23) is valid in practice for a limited range of voltages and currents, over which it can behave linearly.

In practice, negative resistance can be seen at one port of an NIC or NIV when the other port is terminated in a positive resistance. Its presence can be detected by the simple experimental setup shown in Fig. 3.4. A positive resistance of value equal to the magnitude of

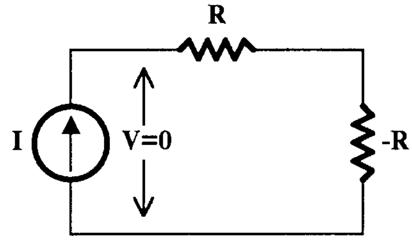


FIGURE 3.4
Demonstrating the action of a negative resistance.

the negative one is connected in series with $-R$. If a current is sent through this combination, the voltage measured across it is zero, in spite of the fact that the voltage drops across R and $-R$ are nonzero.

The concept of negative resistance can be also explained through the V-I characteristics of the tunnel diode and the unijunction transistor. In these cases, incremental negative resistance appears in the part of characteristics with negative slope. There are two types of such characteristics shown in Fig. 3.5. The S-type corresponding to the V-I characteristic of the unijunction transistor and the N-type corresponding to the V-I characteristic of the tunnel diode (or the tetrode electronic tube).

The negative resistance obtained by the means explained above is supposed to be independent of frequency, and indeed this is true in practice for a range of frequencies. It is, however, possible to obtain negative resistance dependent on frequency, and this has been exploited usefully in network synthesis.

Consider the case of a GIV with an inversion "constant" ks to be terminated at port 2 by a capacitor C_L. The input impedance seen at port 1 will be, from Eq. (3.4),

$$Z_i = G_b \frac{1}{Z_L} = s^2 k C_L$$

Substituting $j\omega$ for s in this equation, we obtain

$$Z_i = -\omega^2 k C_L$$

which is, in fact, a negative resistance dependent on ω^2. This is the frequency-dependent negative resistance type E (E-FDNR) that we saw in Section 3.3.1.

The second FDNR type, type D, can be obtained if a GIC with a conversion function $f(s) = 1/s$ is terminated at port 2 by a capacitor C_L. Then, the input impedance at port 1 using Eq. (3.3), will be

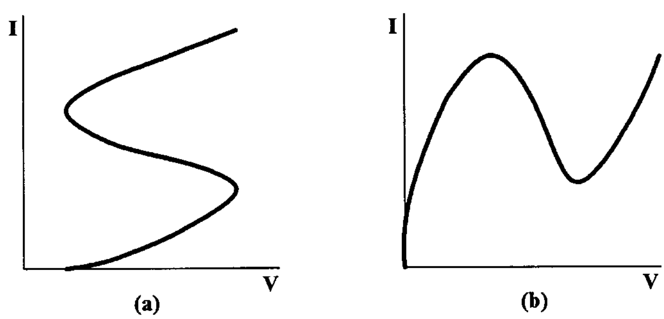


FIGURE 3.5
(a) S-type and (b) N-type V-I characteristics.

$$Z_i = G_b \frac{1}{Z_L} = s^2 k C_L$$

which for $s = j\omega$ gives

$$Z_i = -\frac{1}{\omega^2 C_L}$$

As mentioned earlier, the D-type FDNR will be realized and used in filter synthesis in Chapter 6.

3.5 Ideal Operational Amplifier

The operational amplifier, or opamp, is the most versatile active element. All active elements that have been used in active network synthesis in the past can be realized using the opamp.

The ideal opamp is an ideal differential voltage controlled voltage source (DVCVS) with infinite gain. It has infinite input impedance and zero output impedance. Its symbol and equivalent circuit are shown in Fig. 3.6. The ground connection in Fig. 3.6(a) is not generally shown.

By definition,

$$\upsilon_o = A(\upsilon_1 - \upsilon_2) \tag{3.24}$$

with υ_1 applied to the noninverting input and υ_2 to the inverting input.

Assuming finite υ_o, infinite A calls for

$$\upsilon_1 - \upsilon_2 = 0 \tag{3.25}$$

or equivalently

$$\upsilon_1 = \upsilon_2 \tag{3.26}$$

This equality holds approximately quite satisfactorily in practice also, since the input voltage difference $\upsilon_1 - \upsilon_2$ is A times ($A \approx 10^5$) smaller than υ_o, the maximum value of which can be, say, up to 10 V for IC opamps, depending on the power supply voltage.

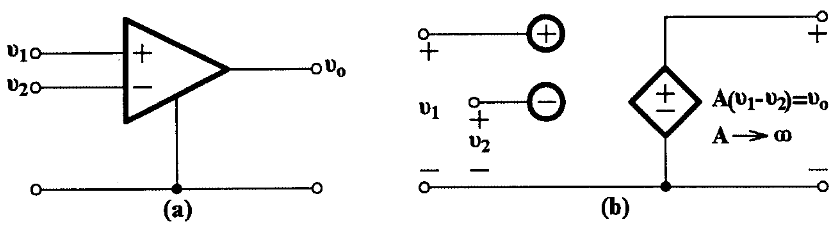


FIGURE 3.6
The ideal operational amplifier (a) symbol and (b) equivalent circuit.

In many cases, the noninverting input is grounded, which leads to the inverting input being at nearly earth potential, i.e.,

$$\upsilon_2 \cong 0$$

In such cases, the inverting input node of the opamp is called the virtual earth (VE) or virtual ground point.

3.5.1 Operations Using the Ideal Opamp

The infinite voltage gain of the ideal opamp, coupled with its infinite input resistance and zero output resistance, make it suitable for performing some useful mathematical operations on voltages. The most important of these operations are explained below.

3.5.1.1 Summation of Voltages

The circuit arrangement for such an operation is shown in Fig. 3.7, where the opamp is used in its single input mode.

Assuming a virtual ground at the inverting input (i.e., $V = 0$), we can write for this node

$$\frac{V_1}{R_1} + \frac{V_2}{R_2} + \dots + \frac{V_n}{R_n} + \frac{V_o}{R_f} = 0$$

which is obtained through Kirchhoff's current law. This leads to

$$V_o = -\left(\frac{R_f}{R_1}V_1 + \frac{R_f}{R_2}V_2 + \dots + \frac{R_f}{R_n}V_n\right) \tag{3.27}$$

If $R_f = R_1 = R_2 = \dots = R_n$, then

$$V_o = -(V_1 + V_2 + \dots + V_n) \tag{3.28}$$

Thus, the negative of the sum of voltages can be obtained. If the difference of two voltages is required, the arrangement in Fig. 3.8 can be used.

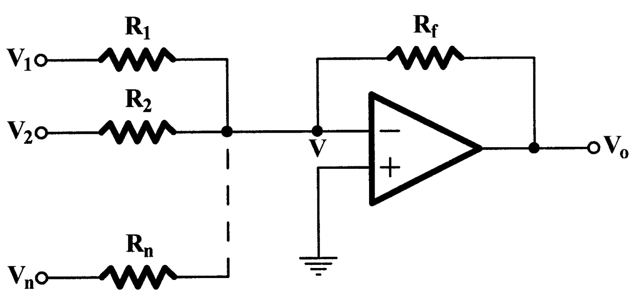


FIGURE 3.7
The opamp as a summer.

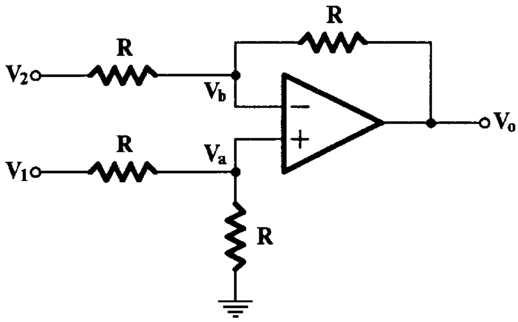


FIGURE 3.8
Circuit giving the difference of two voltages.

Clearly, we can write the following:

$$V_a = \frac{1}{2}V_1 \tag{3.29}$$

and

$$V_b = \frac{1}{2}(V_2 + V_o) \tag{3.30}$$

Since

$$V_a \cong V_b$$

using Eqs. (3.29) and (3.30), we get

$$V_o = V_1 - V_2 \tag{3.31}$$

In the case that some of the voltages in Eq. (3.28) have to be added with the opposite sign, a second opamp should be used to sum those voltages first in the manner shown in Fig. 3.7. Then this sum should be fed through the appropriate resistor to the input node of the main opamp.

3.5.1.2 Integration

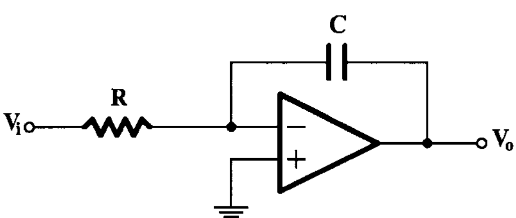


FIGURE 3.9
Integrator.

The arrangement to obtain the integration of a voltage is shown in Fig. 3.9 (if V_i and V_o are the Laplace transforms of voltages υ_i and $\upsilon_{i'}$, respectively.) Assuming zero initial conditions [i.e., $\upsilon_o(0) = 0$], we will have

$$\frac{\upsilon_i}{R} + C\frac{d\upsilon_o}{dt} = 0$$

from which we obtain the following:

$$\upsilon_o(t) = -\frac{1}{RC}\int_0^t \upsilon_i dt \tag{3.32}$$

Initial conditions can be introduced by charging the capacitor to the appropriate voltage before starting the integration.

In the complex frequency domain, Eq. (3.32) is written as

$$V_o = -\frac{1}{RCs}V_i \tag{3.33}$$

where V_i and V_o are the Laplace transforms of voltages υ_i and υ_o, respectively.

As an example, if υ_i is the unit step voltage $u(t)$, from Eq. (3.33), we obtain

$$\upsilon_o(t) = -\frac{1}{RC} \cdot t \tag{3.34}$$

It is seen that the slope of the ramp thus obtained is determined by the time constant RC.

If the position of the passive components in Fig. 3.9 is interchanged, the circuit of a differentiator results. However, in practice, such a circuit will not work properly because of excessive noise. Differentiation using an opamp in this configuration is never used. However, with a resistor of some low value connected in series with C, the noise can be reduced, but only approximate differentiation will be obtained.

3.5.2 Realization of Some Active Elements Using Opamps

The opamp can be "programmed" to realize other active elements that are useful in the synthesis of active networks. We include here some examples of such circuits, whereas others such as the gyrator, PIC, GIC, FDNR, and FDNC will be presented in Chapter 6, where they are also used in filter synthesis.

3.5.2.1 Realization of Controlled Sources

Clearly, the opamp, being a voltage-controlled voltage source in itself, is most suitable for realizing other controlled voltage sources of finite gain. In Fig. 3.10(a), the realization of a finite-gain K VCVS is shown, while in Fig. 3.10(b) that of a finite-gain CCVS is shown.

For the arrangement in Fig. 3.10(a), if V is the voltage at the inverting input of the opamp, we have

$$V = \frac{R_1}{R_1 + R_2}V_o$$

Since $V_i \cong V$ (because $A \cong \infty$), we easily deduce that

$$K \equiv \frac{V_o}{V_i} = 1 + \frac{R_2}{R_1} \tag{3.35}$$

Similarly, for the circuit in Fig. 3.10(b), since the inverting input of the opamp is at virtual earth, we get

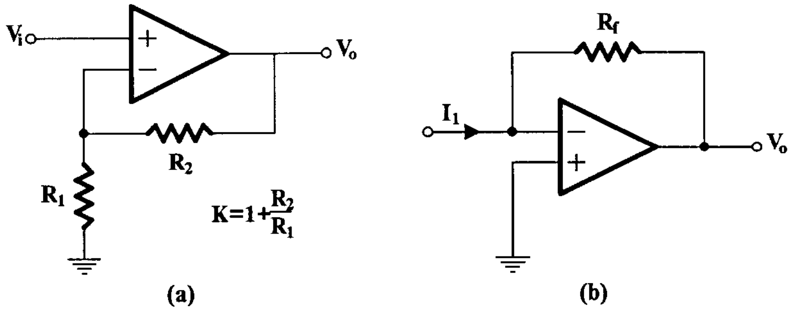


FIGURE 3.10
Realization of (a) a finite-gain VCVS and (b) a finite-gain CCVS.

$$I_1 + \frac{V_o}{R_f} = 0$$

or

$$V_o = -R_f I_1 \tag{3.36}$$

Clearly, the circuit in Fig. 3.10(b) acts as a current-to-voltage converter.

The opamp can also be used to translate voltage-to-current or current-to-current. An example of a voltage-to-current converter is shown in Fig. 3.11. The derivation of this follows the observation that

$$V_s = R_1 I$$

and consequently the current through Z is independent of Z.

3.5.2.2 Realization of Negative-Impedance Converters

As explained in Section 3.3.1, there are two types of negative-impedance converters: the current NIC and the voltage NIC. If their conversion ratio is unity, they possess the following A matrices:

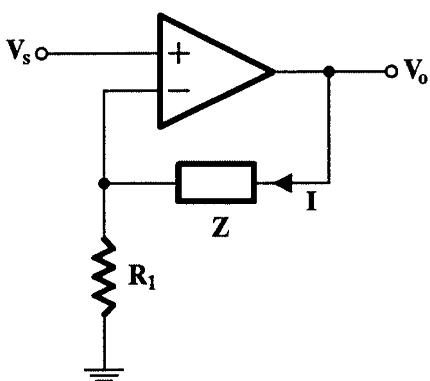


FIGURE 3.11
A voltage-to-current converter.

INIC: $$[A] = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \tag{3.37}$$

VNIC: $$[A] = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \tag{3.38}$$

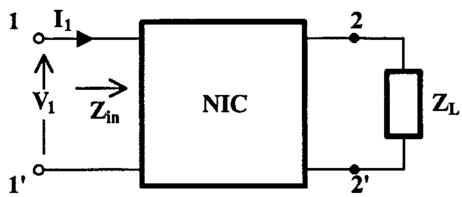


FIGURE 3.12
NIC terminated by impedance Z_L.

A NIC of either type terminated in an impedance Z_L at port 2,2´ (Fig. 3.12) has an input impedance Z_{in} at port 1,1´ given by

$$Z_{in} \equiv \frac{V_1}{I_1} = -Z_L \tag{3.39}$$

If Z_L is purely resistive, the resulting negative resistance can be used to compensate for a positive or dissipative resistance of equal magnitude, thus reducing power dissipation, e.g., the copper loss in a wire transmission system. The concept of negative resistance and its types are explained in more detail in Section 3.4.

The realization of both types of NIC using an opamp is shown in Fig. 3.13(a) for the INIC and Fig. 3.13(b) for the VNIC [6]. To prove this, consider the circuit in Fig. 3.13(a) first. Clearly,

$$V_1 = V_2 \qquad I_1 = \frac{V_1 - V_0}{R}$$

Therefore,

$$I_2 = \frac{V_2 - V_0}{R} = \frac{V_1 - V_0}{R}$$

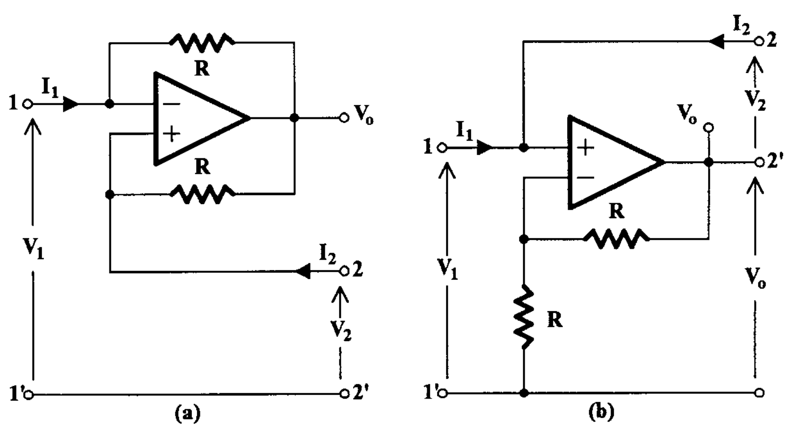


FIGURE 3.13
Opamp realization of (a) the INIC and (b) the VNIC.

or

$$I_1 = -(-I_2)$$

which is the case for an INIC.

Coming now to Fig. 3.13(b), it can be observed that

$$V_0 = 2V_1$$

$$V_2 = V_1 - V_0 = V_1 - 2V_1 = -V_1$$

$$I_1 = -I_2$$

which conforms to the transmission matrix of a VNIC.

3.5.2.3 Gyrator Realizations

A number of gyrator realizations using opamps have appeared in the literature. Some of these have been successfully used in practice [7, 8, 9]. The Orchard-Wilson gyrator [7] is a single-opamp active one ($g_1 \neq g_2$), whereas Riordan's [8] employs two opamps. Useful gyrator circuits have also been suggested by Antoniou [9].

In Fig. 3.14, the Riordan arrangement for inductance simulation is shown. Straightforward analysis, assuming ideal opamps, gives that the input impedance Z_{in} is

$$Z_{in} = \frac{V_1}{I_1} = sCR^2$$

Thus,

$$L_{eq} = CR^2$$

One of Antoniou's gyrator circuits is shown in Fig. 3.15. This is a four-terminal circuit, and it cannot be used when a three-terminal one is required. It is a very useful circuit though, because from this a useful generalized-immittance-converter circuit is obtained as shown below.

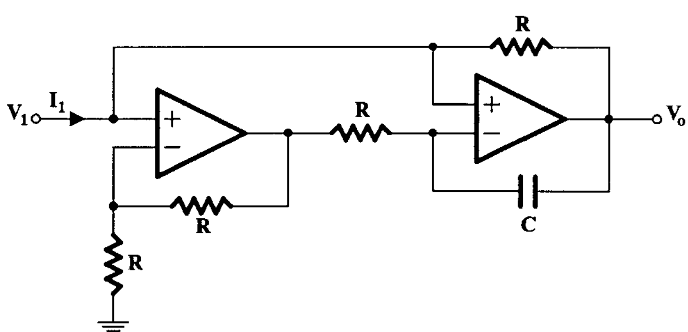


FIGURE 3.14
Riordan circuit for inductance simulation.

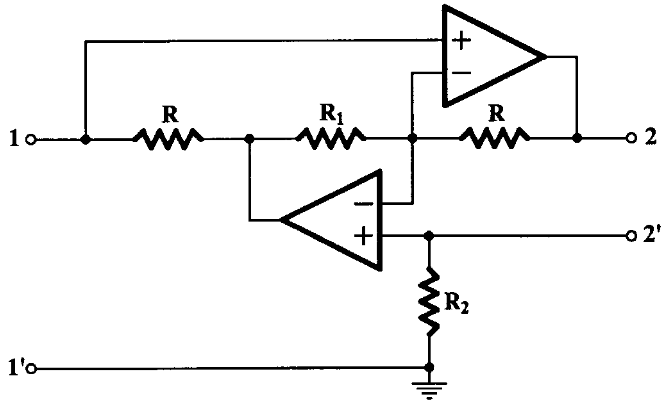


FIGURE 3.15
One of Antoniou's gyrator circuits.

Assuming $R_1 = R_2 = R$ and identical open-loop gains of the opamps, it can be shown [9] that this circuit is unconditionally (absolutely) stable, while that in Fig. 3.14 is conditionally stable.

3.5.2.4 GIC Circuit Using Opamps

The Antoniou gyrator circuit [9] that appears in Fig. 3.15 is redrawn, in its general form, in Fig. 3.16 (within the broken lines). All Y_i are admittances.

It can be seen that the voltages at nodes a, b, and c, assuming ideal opamps, are equal. Thus,

$$V_1 = V_b = V_2$$

To determine I_1 in terms of $-I_2$, we can write the following successively:

$$I_1 = Y_2 (V_1 - V_3) \tag{3.40a}$$

$$Y_3 (V_3 - V_1) = Y_4 (V_1 - V_4) \tag{3.40b}$$

$$Y_5 (V_4 - V_1) = -I_2 \tag{3.40c}$$

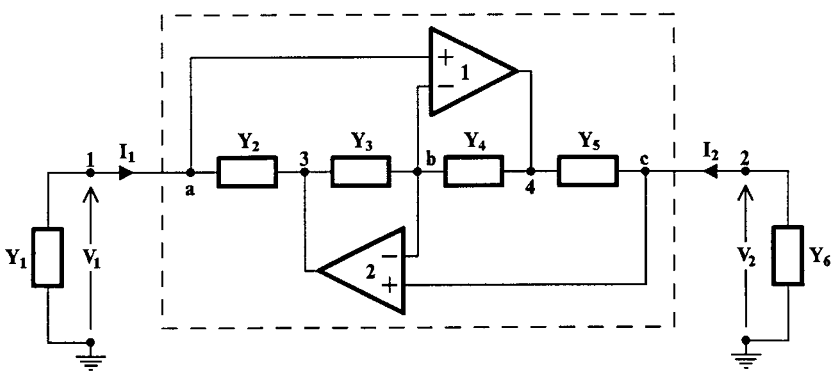


FIGURE 3.16
General GIC circuit using opamps.

From Eq. (3.40c), V_4 is obtained, which is then inserted in Eq. (3.40b) to give V_3. Next, the value of V_3 is inserted in Eq. (3.40a) to give

$$I_1 = \frac{Y_2 Y_4}{Y_3 Y_5}(-I_2) \tag{3.41}$$

Thus, the transmission matrix of the GIC is as follows:

$$[A] = \begin{bmatrix} 1 & 0 \\ 0 & \dfrac{Y_2 Y_4}{Y_3 Y_5} \end{bmatrix} \tag{3.42}$$

Consequently, the conversion function $f(s)$ and the constant take the following values:

$$k = 1 \qquad f(s) = \frac{Y_3 Y_5}{Y_2 Y_4} \tag{3.43}$$

Assuming that $Y_2 = Y_3 = Y_4 = Y_6 = R^{-1}$ and $Y_5 = sC$, the input impedance $Z_{i,1}$ at port 1 will be

$$Z_{i,1} \equiv \frac{V_1}{I_1} = f(s) \cdot \frac{1}{Y_6} = sCR^2 \tag{3.44}$$

The same result can be obtained if $Y_5 = R^{-1}$ and $Y_3 = sC$, whereas if $Y_6 = sC$, then $Z_{i,1} = R$.

If the admittance Y_1 is connected across port 1, the input impedance $Z_{i,2}$ at port 2 will be

$$Z_{i,2} \equiv \frac{V_2}{I_2} = \frac{1}{f(s)} \frac{1}{Y_1}$$

Then again, for $Y_1 = Y_2 = Y_3 = Y_4 = R^{-1}$ and $Y_5 = sC$,

$$Z_{i,2} = \frac{1}{sC} \tag{3.45}$$

while, if Y_1 is also equal to sC, then

$$Z_{i,2} = \frac{1}{s^2 C^2 R} \tag{3.46}$$

giving a supercapacitor or D-FDNR. The same results are obtained for $Z_{i,2}$ if $Y_5 = R^{-1}$ and $Y_3 = sC$.

One important point that should be noted is that, if $Y_1 = sC$ and $Y_6 = R^{-1}$ are both connected to the GIC circuit as shown in Fig. 3.16, the overall circuit will be a resonator, simulating a parallel LC circuit. Another observation concerns the connections of the opamps to the nodes of the Y-subnetwork. Inspection of Eqs. (3.40a) through (3.40c) reveals that it is immaterial which input terminal of opamp 1 is connected to node a and which to node b. The same is true for opamp 2. We will make use of this circuit in Chapters 4 and 6 to simulate inductance.

3.5.3 Characteristics of IC Opamps

Practical opamps have characteristics that differ from those of the ideal element used in previous sections. Apart from their open-loop voltage gain, which is noninfinite, their input impedance and output admittance are not infinite either. There are also some additional parameters associated with the operation of the practical opamp [10] which degrade its performance, and the designer should always keep them in mind. In spite of these, though, the nonideal behavior of the practical opamp does not prevent it from being the most versatile linear active element in use today.

3.5.3.1 Open-Loop Voltage Gain of Practical Opamps

The dc and very low frequency open-loop voltage gain of most IC bipolar opamps is of the order of 10^5 (100 dB), and for MOS opamps at least one order of magnitude lower. In most practical cases, the error introduced in circuits incorporating opamps is not very significant, and the operation at dc can still be considered ideal. This, however, is not true at frequencies above a few hertz.

For reasons of stability of the circuits in which the opamp is embedded, its magnitude response is shaped such that the falloff rate is 6 dB/octave, as shown in Fig. 3.17(a). The associated phase response is shown in Fig. 3.17(b). This behavior can be described mathematically as follows:

$$A(s) = \frac{A_o}{1 + s\tau} \tag{3.47}$$

where A_o is the dc gain and τ a time constant that creates a pole at $-1/\tau$. The cutoff frequency is

$$\omega_c = 2\pi f_c = \frac{1}{\tau}$$

with f_c equal to about 10 Hz for general-purpose bipolar IC opamps such as, e.g., the 741. The frequency f_T at which the magnitude of $A(j\omega)$ becomes unity is the most important characteristic of each opamp, since it actually denotes its gain-bandwidth (GB) product. We can explain this as follows.

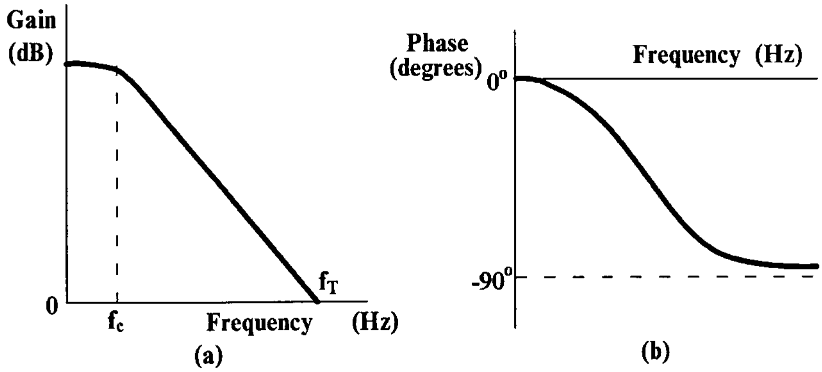


FIGURE 3.17
(a) Magnitude and (b) phase response of the opamp (741 type) open-loop voltage gain.

With $\omega_c = 1/\tau$, Eq. (3.47) gives

$$A(s) = \frac{A_o}{1 + s/\omega_c}$$

and so

$$|A(j\omega)| = \frac{|A_o|}{\left[1 + \left(\frac{\omega}{\omega_c}\right)^2\right]^{1/2}}$$

Thus, when $|A(j\omega)| = 1$,

$$\left[1 + \left(\frac{\omega_T}{\omega_c}\right)^2\right]^{1/2} = |A_o|$$

and since $(\omega_T/\omega_c)^2 >> 1$, we have

$$\frac{\omega_T}{\omega_c} \cong |A_o|$$

whence

$$\omega_T = |A_o|\omega_c$$

i.e., the gain-bandwidth product—the product of the dc gain and the 3-dB bandwidth. Note that if $\omega_T = 2\pi f_T$, then $f_T = |A_o| f_c$ also.

Clearly, the high gain of the opamp is not available at frequencies higher than about 10 Hz. It should also be mentioned that, as it can be easily obtained from Eq. (3.47), the maximum gain that can be obtained at the frequency f_x using the opamp is

$$|A(j\omega_x)| = \frac{f_T}{f_x} \tag{3.48}$$

This model will be taken into consideration whenever we examine the performance of various circuits using opamps throughout this book.

The effect of this single-pole model of the opamp on the performance of the VCVS realized using the opamp is examined in Section 3.5.4. In some cases, when we are interested in frequencies well above f_c, this single-pole model can be simplified by writing Eq. (3.47) in the following approximate form:

$$A(s) \cong \frac{\omega_T}{s} \tag{3.49}$$

3.5.3.2 Input and Output Impedances

The input impedance of the opamp can be defined when measured either between each input terminal and the ground, or, as differential, i.e., between the two input terminals.

Although a function of frequency, it is usually considered purely resistive, R_i. The value of R_i is around 150 kΩ for bipolar opamps, while for opamps using FETs as the input stage or MOSFETs throughout, it is very high indeed. However, even in the case of bipolar opamps, since they are always used with negative feedback, the error introduced due to its presence is insignificant and therefore can be neglected in practice. Similarly, the output impedance of a bipolar general-purpose IC opamp is of the order of 100 Ω, which when the opamp is used with negative voltage feedback introduces an insignificant error, usually ignored in practice.

To be sure that these impedances will not affect the performance of the circuit using bipolar IC opamps, the impedance level of the associated circuit should be chosen greater than 1 kΩ and smaller than 100 kΩ, with 10 kΩ being the most appropriate choice. The upper limit is set by other imperfections of the opamp, which are explained below.

3.5.3.3 Input Offset Voltage V_{IO}

If both inputs of the real opamp are grounded, the output voltage will not be zero in practice, as would be expected. This is a defect that causes the output voltage to be offset with respect to ground potential. For large ac input signals, the output voltage waveform will then be unsymmetrically clipped; that is, the opamp will display a different degree of nonlinear behavior for positive and negative excursions of the input signals. The input offset voltage V_{IO} is that voltage which must be applied between the input terminals to balance the opamp. In many opamps, this defect may be "trimmed" to zero by means of an external potentiometer connected to terminals provided for this reason.

3.5.3.4 Input Offset Current I_{IO}

This is defined as the difference between the currents entering the input terminals when the output voltage is zero. These currents are actually the base bias currents of the transistors at the input stage of the opamp (for bipolar opamps), and their effect is the appearance of an undesired dc voltage at the output. This defect of the opamp can be modeled by connecting two current generators at the input terminals of the ideal opamp. This is shown in Fig. 3.18 for the case of the circuit in Fig. 3.11, which is used to provide $1 + R_f/R_1$ voltage gain. R_2 is inserted to reduce the effect of the input bias currents as we show below and has no effect on the signal. If the input voltage V_s is zero, and assuming linear operation of the opamp, we may observe the following.

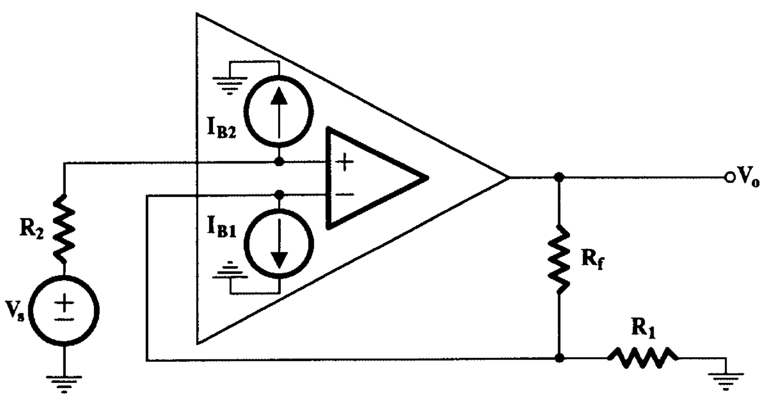


FIGURE 3.18
Current sources I_{B1}, I_{B2} represent the presence of input offset currents.

The action of I_{B1} causes the output voltage to be ($I_{B2} = 0$)

$$V_{01} = R_f I_{B1} \tag{3.50}$$

The action of I_{B2}, assuming $I_{B1} = 0$, will result in the output voltage

$$V_{02} = -\left(1 + \frac{R_f}{R_1}\right) I_{B2} R_2 \tag{3.51}$$

Then applying superposition when both I_{B1} and I_{B2} are present, we get the output voltage

$$V_0 = V_{01} + V_{02} = I_{B1} R_f - \left(1 + \frac{R_f}{R_1}\right) I_{B2} R_2 \tag{3.52}$$

For this voltage to be zero when $I_{B1} = I_{B2}$, which is the optimistic case, the following relationship between the resistor values should hold:

$$R_2 = \frac{R_1 R_f}{R_1 + R_f} \tag{3.53}$$

However, even under this condition, when $I_{B1} \neq I_{B2}$, the output voltage will be

$$V_0 = (I_{B1} - I_{B2}) R_f = I_{I0} R_f \tag{3.54}$$

i.e., nonzero. Note though that without R_2, $V_o = I_{B1} R_f$ and, since $I_{IO} << I_{B1}$ in practice, the output voltage arising from the input bias currents is reduced by including R_2.

3.5.3.5 Input Voltage Range V_I

Assuming that the imperfections of the opamp due to input offset voltage and input offset current have been corrected, the voltage transfer characteristic of the amplifier will be as shown in Fig. 3.19, where V_i represents the differential input voltage. It can be seen that the opamp behaves linearly only in the region of V_i.

$$-V_2 \le V_i \le V_1$$

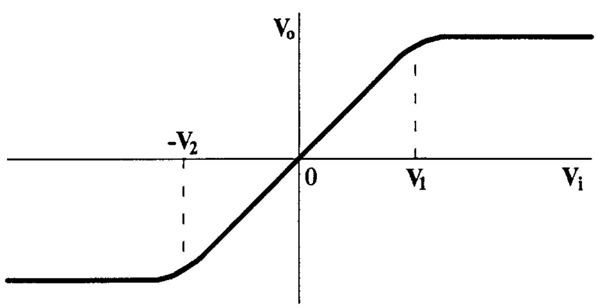


FIGURE 3.19
The saturation characteristics of the opamp.

i.e., only for this range of V_i can one get the benefit of the full voltage gain of the opamp. Beyond this voltage range, the amplifier goes to saturation.

Although the nonlinear behavior of the opamp is a cause of concern in the design of active RC filters, one can get advantage of the saturation characteristic to build analog voltage comparators, which are very useful in practice (for example, as zero crossing detectors).

3.5.3.6 Power Supply Sensitivity $\Delta V_{IO} / \Delta V_{GG}$

This is the ratio of the change of the input offset voltage ΔV_{IO} to the change in the power supply ΔV_{GG} that caused it. The change in the power supply is considered symmetrical.

3.5.3.7 Slew Rate SR

The rate of change of the output voltage cannot be infinite due to the various internal time constants of the opamp circuitry. The slew rate (SR) is defined as the maximum rate of change of the output voltage for a unit step input excitation. This is normally measured for unity gain at the zero voltage point of the output waveform.

The SR sets a serious limitation to the amplitude of the signal at high frequencies. This can be shown in the case of a sinewave as follows. Let

$$\upsilon_o = V_m \sin\omega t$$

Then,

$$\frac{d\upsilon_o}{dt} = V_m \omega \cos\omega t$$

which becomes maximal at the zero crossing points, i.e., when $\omega t = 0, \pi, 2\pi, \ldots$. Thus, at $\omega t = 0$

$$\left.\frac{d\upsilon_o}{dt}\right|_{\omega t = 0} = V_m \omega \tag{3.55}$$

Since this cannot be larger than the SR, i.e.,

$$SR \geq V_m \omega \tag{3.56}$$

it is clear, that for linear operation at a high frequency ω, the amplitude of the output voltage cannot be greater than SR/ω. Thus, at high frequencies, the opamp cannot work properly at its full input voltage swing, as it does at low frequencies.

3.5.3.8 Short-Circuit Output Current

This denotes the maximum available output current from the opamp, when its output terminal is short circuited with the ground or with one of its power supply rails.

3.5.3.9 Maximum Peak-to-Peak Output Voltage Swing V_{opp}

This is the maximum undistorted peak-to-peak output voltage, when the dc output voltage is zero.

3.5.3.10 Input Capacitance C_i

This is the capacitance between the input terminals with one of them grounded.

3.5.3.11 Common-Mode Rejection Ratio CMRR

Ideally, the opamp should reject completely all common-mode signals (i.e., the same signals applied to both inputs) and amplify the differential-mode ones. However, for reasons of circuit imperfections, the amplifier gain is not exactly the same for both of its inputs. The result of this is that common-mode signals are not rejected completely. A measure of this imperfection is the common-mode rejection ratio (CMRR). Expressed in dB, the CMRR is the ratio of open-loop differential gain to the corresponding common-mode gain of the opamp. Its value at low frequencies is typically better than 80 dB, but it decreases at higher frequencies.

3.5.3.12 Total Power Dissipation

This is the total dc power that the opamp absorbs from its power supplies, minus the power that the amplifier delivers to its load.

3.5.3.13 Rise Time t_r

This is the time required for the output voltage of the amplifier to increase from 10 to 90 percent of its final value for a step input voltage. It can be shown simply that $t_r \times f_c \cong 0.35$ (see Section 1.4.1).

3.5.3.14 Overshoot

This is the maximum deviation of the output voltage above its final value for a step input excitation.

3.5.4 Effect of the Single-Pole Compensation on the Finite Voltage Gain Controlled Sources

Consider the two circuits in Fig. 3.20. For an ideal opamp, the voltage gains of these circuits are the following:

Fig. 3.20(a) (noninverting) $$\alpha_n = \frac{R_1 + R_2}{R_1}$$

Fig. 3.20(b) (inverting) $$-\alpha_i = -\frac{R_f}{R_1}$$

Assuming that the open-loop gain A of the opamp is finite, the voltage gain $G_N(s)$ of the circuit in Fig. 3.20(a) is written as follows:

$$G_N \equiv \frac{V_o}{V_i} = \frac{A}{1 + \beta A} \tag{3.57}$$

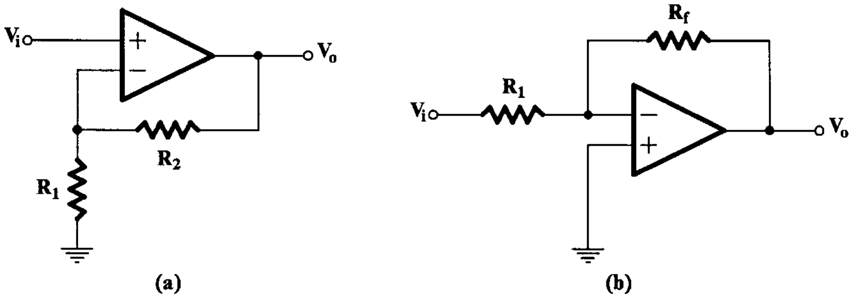


FIGURE 3.20
(a) Noninverting and (b) inverting voltage amplifiers using the opamp.

where β is the feedback ratio given by

$$\beta = \frac{R_1}{R_1 + R_2} \tag{3.58}$$

If A follows the single-pole model, i.e.,

$$A(s) = \frac{A_o\omega_c}{s + \omega_c} \tag{3.59}$$

substituting for A in Eq. (3.58), we get

$$G_N = \frac{A_o\omega_c}{s + \omega_c + \beta A_o\omega_c} \cong \frac{A_o\omega_c}{s + \beta A_o\omega_c} = \frac{A_o\omega_c}{s + \dfrac{A_o\omega_c}{\alpha_N}} \tag{3.60}$$

where we assumed that $\beta A_o >> 1$, which is quite reasonable in practice.

Applying the same procedure in the case of Fig. 3.20(b), we can obtain, for the gain G_I, the following:

$$G_I \equiv \frac{V_o}{V_i} = -\frac{R_f}{R_1 + R_f}\frac{A}{1 + \beta A} \tag{3.61}$$

where

$$\beta = \frac{R_1}{R_1 + R_f}$$

Then, substituting for A from (3.59), and after some arithmetic manipulations, we obtain

$$G_I = -\alpha_I \frac{\dfrac{A_o\omega_c}{1 + \alpha_I}}{s + \dfrac{A_o\omega_c}{1 + \alpha_I}} \tag{3.62}$$

It can be seen from Eqs. (3.60) and (3.62) that both G_N and G_I have a single-pole behavior. This is to be expected, since $A(s)$ behaves similarly. However, the unexpected is that for equal nominal gains at low frequencies, i.e.,

$$\alpha_N = \alpha_I$$

the useful bandwidth of G_N is larger than that of G_I. In particular, when

$$\alpha_N = \alpha_I = 1$$

the bandwidth of the noninverting amplifier is double the bandwidth of the inverting one.

3.6 The Ideal Operational Transconductance Amplifier (OTA)

The ideal OTA is a differential-input voltage-controlled current source (DVCCS). Its symbol is shown in Fig. 3.21(a), and its operation is defined by the following equation:

$$I_o = g_m(V_1 - V_2) \tag{3.63}$$

The transconductance g_m can be controlled externally by the current I_B. Both voltages V_1 and V_2 are with reference to ground.

The equivalent circuit of the ideal OTA is shown in Fig. 3.21(b). Some simple applications of the OTA are described below [11].

3.6.1 Voltage Amplification

Inverting and noninverting voltage amplification can be achieved using an OTA as shown in Fig. 3.22(a) and 3.22(b), respectively. Any desired gain can be achieved by a proper choice of g_m and R_L. It should be noted that the output voltage V_o is obtained from a source with output impedance equal to R_L. Zero output impedance can be achieved only if such circuits are followed by a buffer or voltage follower.

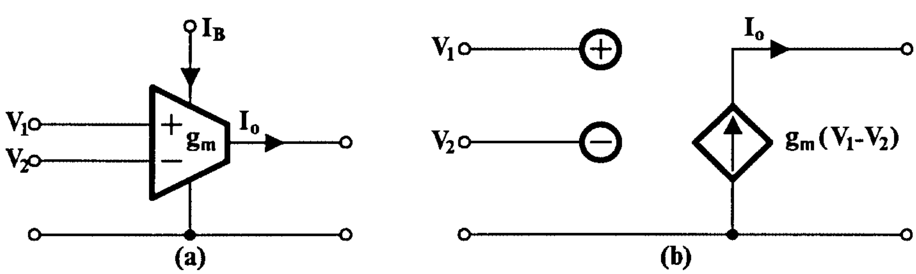


FIGURE 3.21
Ideal operational transconductance amplifier, (a) symbol and (b) equivalent circuit.

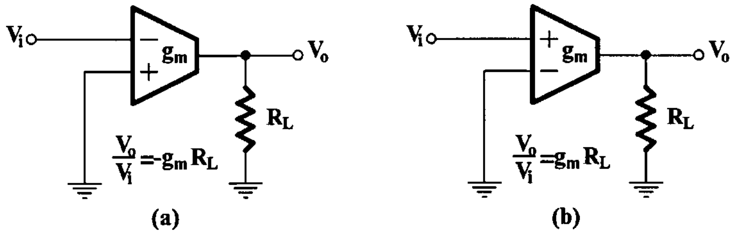


FIGURE 3.22
(a) Inverting and (b) noninverting voltage gain using an ideal OTA.

3.6.2 A Voltage-Variable Resistor (VVR)

A grounded voltage-variable resistor can be easily obtained using the ideal OTA as shown in Fig. 3.23. Since $I_o = -I_i$, we will have the following:

$$Z_i = \frac{V_i}{I_i} = \frac{V_i}{-I_o} = \frac{V_i}{g_m V_i} = \frac{1}{g_m} \tag{3.64}$$

Using two such arrangements cross-connected in parallel, a floating VVR can be obtained. On the other hand, if in Fig. 3.23 the input terminals are interchanged, the input resistance will be $-1/g_m$. Thus, using OTAs, both positive and negative resistors become available without actually having to build them on the chip. These, coupled with capacitors, lead to the creation of the so-called active-C filters discussed later in this book.

3.6.3 Voltage Summation

Voltage summation can be obtained using OTAs, which in effect translate voltages to currents. These are easily summed as shown in Fig. 3.24 for two voltages V_1 and V_2.

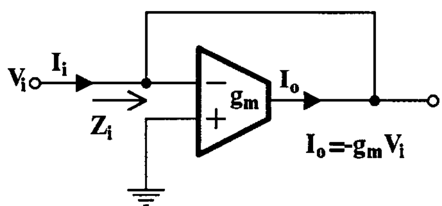


FIGURE 3.23
Grounded voltage-variable resistor.

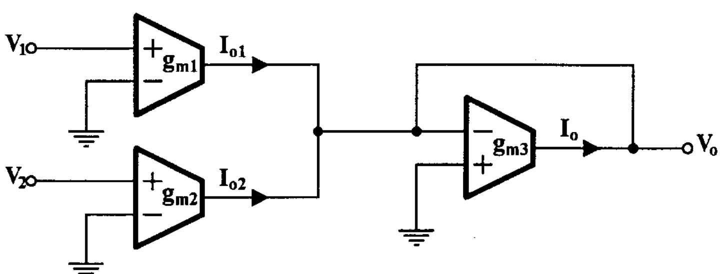


FIGURE 3.24
Voltage summation.

It is clear that

$$I_{01} + I_{02} + I_0 = 0$$

or

$$g_{m1}V_1 + g_{m2}V_2 - g_{m3}V_0 = 0$$

Solving for V_o, we get

$$V_o = \frac{g_{m1}}{g_{m3}}V_1 + \frac{g_{m2}}{g_{m3}}V_2 \tag{3.65}$$

By changing the grounded input of one of the input OTAs, voltage subtraction can be achieved. These operations are useful for the realization of transfer functions.

3.6.4 Integration

The operation of integration can be achieved very conveniently using the OTA as is shown in Fig. 3.25. Clearly,

$$V_o = \frac{I_o}{sC} = \frac{g_m}{sC}(V_1 - V_2) \tag{3.66}$$

It follows that both inverting and noninverting integration is easily achieved. Of course, in all cases, the output impedance of the circuit is nonzero.

If a resistor is connected in parallel with C in Fig. 3.25, the integration will become lossy. On the other hand, connecting the circuit in Fig. 3.23 at the output of that in Fig. 3.25, the integration becomes both lossy and adjustable.

3.6.5 Gyrator Realization

The defining equations of the gyrator can be written in the Y matrix form as follows:

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 & g_1 \\ -g_2 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \tag{3.67}$$

These equations can be interpreted in the form of an equivalent circuit comprising two voltage controlled current sources connected as shown in Fig. 3.26. Thus OTAs, being voltage controlled current sources, are most suitable for the realization of the gyrator—more

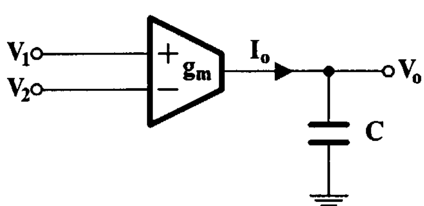


FIGURE 3.25
Integration of the difference of voltages V_1, V_2.

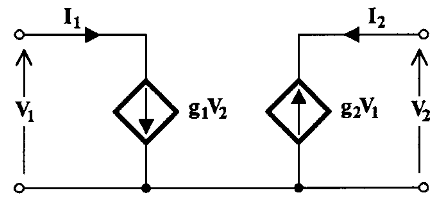


FIGURE 3.26
Gyrator realization using two VCCSs.

suitable than opamps, which are voltage controlled voltage sources. Such a circuit using OTAs is shown in Fig. 3.27. Clearly,

$$I_2 = g_{m1}V_1 \quad \text{and} \quad I_1 = -I_3 = g_{m2}V_2$$

Then the $\boldsymbol{A}$ matrix, not considering Z_L as part of the circuit, will be

$$[\boldsymbol{A}] = \begin{bmatrix} 0 & \frac{1}{g_{m1}} \\ g_{m2} & 0 \end{bmatrix} \tag{3.68}$$

Thus, the circuit consisting of the two OTAs realizes, in general, the ideal active gyrator ($g_{m1} \neq g_{m2}$). In case $g_{m1} = g_{m2}$, the gyrator behaves as a passive circuit.

With Z_L connected as shown in Fig. 3.27, the input impedance Z_i is as follows:

$$Z_i \equiv \frac{V_1}{I_1} = \frac{1}{g_{m1}g_{m2}Z_L} \tag{3.69}$$

If Z_L represents the impedance of a capacitor C_L, the equivalent inductance L_{eq} will be

$$L_{eq} = \frac{C_L}{g_{m1}g_{m2}} \tag{3.70}$$

3.6.6 Practical OTAs

The versatility of the OTA as an active element, as demonstrated above, makes it very useful in VLSI circuits. Also, discrete IC OTAs in bipolar and MOS technology are available.

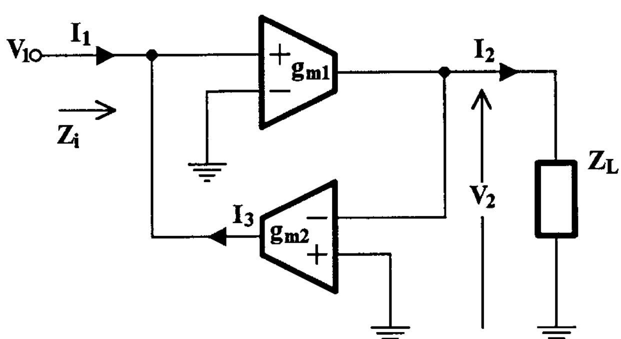


FIGURE 3.27
Gyrator realization using two OTAs.

These practical ICs have certain advantages over opamps as well as disadvantages. Their advantages include higher bandwidths and simpler circuitry. The former make them more useful than the opamps in the design of active filters operating at high frequencies (up to the megahertz region). On the other hand, their simpler circuitry, coupled with the controllability of their g_m, leads to versatility in integration and tuning.

However, they have some drawbacks. Currently available IC OTAs have a performance that is limited by certain imperfections, some of which are similar to those explained in the case of practical IC opamps. Some additional ones, though, need more attention. One important imperfection is the limited range of input voltage (< 20 mV) for linear operation [12, 13]. This problem can be solved by using a potential divider at the input terminals in order to reduce the differential input voltage. This divider, however, reduces the effective input impedance of the OTA.

Other important imperfections include the finite input and output impedances of the OTA, as well as the frequency dependence of transconductance g_m [12, 13].

The input impedance can be modeled by connecting a resistance R_{ic} in parallel with a capacitance C_{ic} from each input terminal of the ideal OTA to ground and a capacitance C_{id} in parallel with a resistance R_{id} between the input terminals. When one of the input terminals is grounded the input impedance is simplified being the parallel combination of the resistances R_{ic}, R_{id} and the capacitance $C_{ic} + C_{id}$.

The OTA output impedance is modeled by the parallel combination of a resistance R_o and a capacitance C_o connected between the OTA output terminal and the ground.

Finally, the frequency dependence of the OTA transconductance can be approximately described by a single pole model given by

$$g_m(s) = \frac{g_{mo}}{1 + s\tau} \tag{3.71}$$

where g_{mo} is the value of g_m at dc, and $\tau = 1/\omega_b$, ω_b being the OTA finite bandwidth.

Also, the phase model is often used, which is described as follows:

$$g_m(j\omega) = g_{mo}e^{-j\varphi} \tag{3.72}$$

In this equation, $\varphi = \omega\tau$ is the phase delay with $\tau = 1/\omega_b$ giving the time delay.

Both Eqs. (3.71) and (3.72) can be further approximately written as

$$g_m(s) \approx g_{mo}(1 - s\tau) \tag{3.73}$$

These OTA g_m models will be used alternatively in later chapters where the OTA is used in filter design as the active element.

In spite of all these imperfections, though, careful design can minimize their effect on the available bandwidth, which remains much higher than that of an opamp. This makes OTAs very useful for the design of active filters at high frequencies, as shown later in this book.

3.6.7 Current Conveyor [14]

The current conveyor (CC) is a three-port active element classified as a current mode device. We introduce the ideal element here, briefly. The definition of the ideal CC type 1 (CCI) is given by means of the following mathematical description with reference to its symbol, shown in Fig. 3.28:

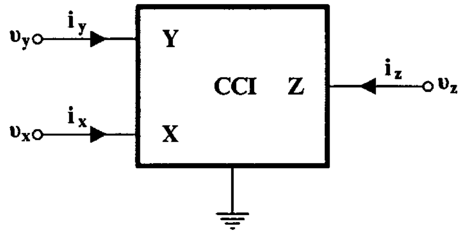


FIGURE 3.28
Symbol of current conveyor, CCI.

$$\begin{bmatrix} i_y \\ \upsilon_x \\ i_z \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & \pm 1 & 0 \end{bmatrix} \begin{bmatrix} \upsilon_y \\ i_x \\ \upsilon_z \end{bmatrix} \tag{3.74}$$

This is the earlier version of the current conveyor, which was followed later by the type-II current conveyor, CCII, mathematically defined by the following equation:

$$\begin{bmatrix} i_y \\ \upsilon_x \\ i_z \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & \pm 1 & 0 \end{bmatrix} \begin{bmatrix} \upsilon_y \\ i_x \\ \upsilon_z \end{bmatrix} \tag{3.75}$$

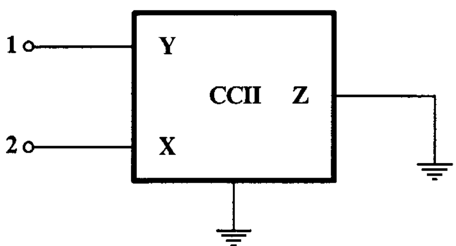


FIGURE 3.29
CCII realizing an ideal VCVS.

The CCII is a more versatile device than the CCI. By means of this, all previously introduced active one- and two-port active elements can be realized. As a first example, consider the situation shown in Fig. 3.29.

If terminal 1 and earth constitute the input port, and terminals 2 and earth the output port, from Eq. (3.75) we can easily obtain the following transmission matrix:

$$[\boldsymbol{T}] = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

which clearly is that of an ideal voltage-controlled voltage source.

As a second example, consider the situation in Fig. 3.30. With $Y(Z)$ and earth representing the input terminals, and X and earth the output terminals, we can easily obtain from Eq. (3.75) the following:

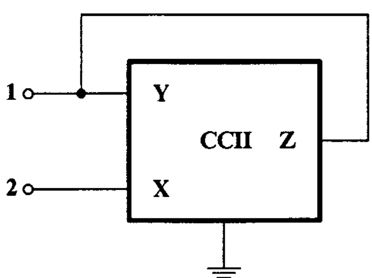


FIGURE 3.30
CCII+ realizing an NIC.

$$\begin{bmatrix} \upsilon_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} \upsilon_2 \\ -i_z \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \upsilon_x \\ -i_x \end{bmatrix} \tag{3.76}$$

Clearly, this equation describes the ideal unity gain INIC, as was shown in Section 3.3.1.

We will consider this active device again in Chapter 12. The reader interested in its applications to filter design may also be referred for example to [15].

3.7 Summary

In continuous-time active filter design, we employ active elements and passive components. The active elements are mostly two-port active building blocks like controlled sources, NICs, GICs, PICs, gyrators, etc. They all can be realized using opamps and/or OTAs. However, opamps and OTAs, themselves being special types of controlled sources, are also used as active elements on their own right.

Because of the importance of these two amplifiers in active filter design, understanding of their imperfections is absolutely necessary. This can help the designer to avoid problems that will surely arise in practical circuits if they are not taken into consideration.

The active elements, which were introduced in this chapter, will be employed in the design of active filters in all subsequent chapters.

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