

# Discussion #1 Problems and Solutions

CSM51A / EEM16 Spring 2017

April 7, 2017

Problems labeled with Textbook tags are taken from the class textbook (Dally & Harting).

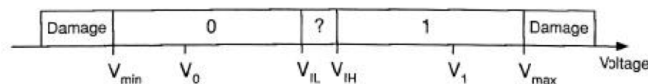
## [Textbook 1.2] Noise Margins

Two wires have been placed close together on a chip. They are so close, in fact, that the larger wire (aggressor) couples to the smaller wire (the victim) and causes the voltage on the victim wire to change. Using the data from Table 1.1, determine the following.

**Table 1.1. Encoding of binary signals for 2.5 V LVCMOS logic**

Signals with voltage in  $[-0.3, 0.7]$  are considered to be a 0. Signals with voltage in  $[1.7, 2.8]$  are considered to be a 1. Voltages in  $[0.7, 1.7]$  are undefined. Voltages outside  $[-0.3, 2.8]$  may cause permanent damage.

Parameter	Value	Description
$V_{\min}$	$-0.3\text{ V}$	absolute minimum voltage below which damage occurs
$V_0$	$0.0\text{ V}$	nominal voltage representing logic “0”
$V_{OL}$	$0.2\text{ V}$	maximum output voltage representing logic “0”
$V_{IL}$	$0.7\text{ V}$	maximum voltage considered to be a logic “0” by a module input
$V_{IH}$	$1.7\text{ V}$	minimum voltage considered to be a logic “1” by a module input
$V_{OH}$	$2.1\text{ V}$	minimum output voltage representing logic “1”
$V_1$	$2.5\text{ V}$	nominal voltage representing logic “1”
$V_{\max}$	$2.8\text{ V}$	absolute maximum voltage above which damage occurs



**Figure 1.1.** Encoding of two symbols, 0 and 1, into voltage ranges. Any voltage in the range labeled 0 is considered to be a 0 symbol. Any voltage in the range labeled 1 is considered to be a 1 symbol. Voltages between the 0 and 1 ranges (the ? range) are undefined and represent neither symbol. Voltages outside the 0 and 1 ranges may cause permanent damage to the equipment receiving the signals.

---

(a) If the victim wire is at  $V_{OL}$ , what is the most the aggressor can push it down without causing a problem?

---

**Solution:**

---

(b) If the victim wire is at 0 V, what is the most the aggressor can push it down without causing a problem?

---

**Solution:**

---

(c) If the victim wire is at  $V_{OH}$ , what is the most the aggressor can push it down without causing a problem?

---

**Solution:**

---

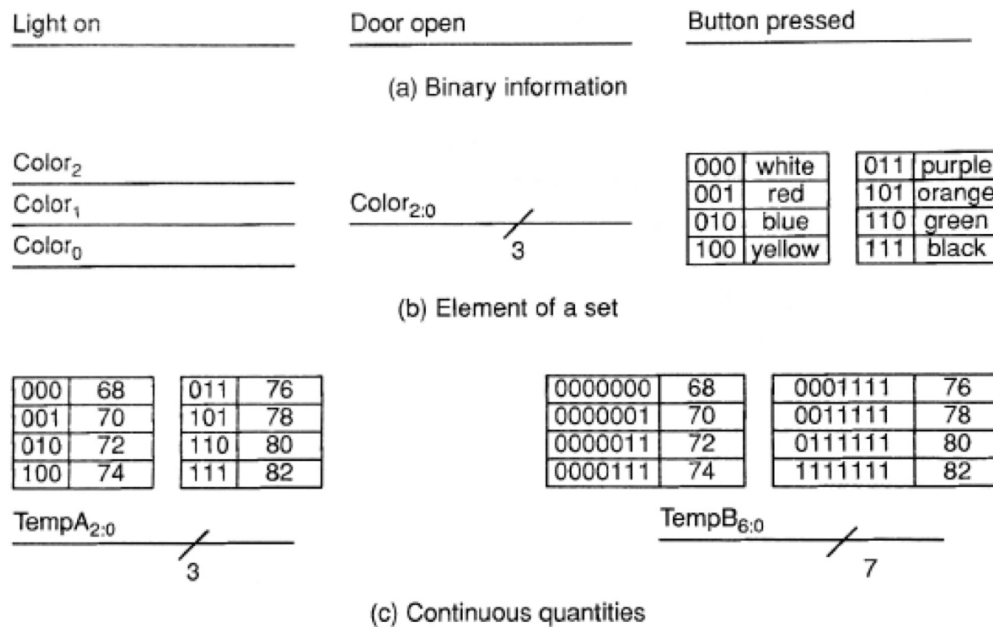
(d) If the victim wire is at  $V_{DD}$ , what is the most the aggressor can push it up without causing a problem?

---

**Solution:**

## [Textbook 1.9] Gray Codes

A continuous value that has been quantized into  $N$  states can be encoded into an  $n = \lceil \log_2 N \rceil$  bit signal in which adjacent states differ in at most one bit position. Show how the eight temperatures of Figure 1.7(c) can be encoded into three bits in this manner. Make your encoding such that the encodings of 82°F and 68°F also differ in just one bit position.



**Figure 1.7.** Representing information with digital signals. (a) Binary-valued predicates are represented by a single-bit signal. (b) Elements of sets with more than two elements are represented by a group of signals. In this case one of eight colors is denoted by a three-bit signal  $Color_{2:0}$ . (c) A continuous quantity, like temperature, is *quantized*, and the resulting set of values is encoded by a group of signals. Here one of eight temperatures can be encoded as a three-bit signal  $TempA_{2:0}$  or as a seven-bit *thermometer-coded* signal  $TempB_{6:0}$  with at most one transition from 0 to 1.

**Solution: :**

## [Textbook 1.11] Encoding Playing Cards

Suggest a binary representation for playing cards — a set of binary signals that uniquely identifies one of the 52 cards in a standard deck. What different representations might be used to (i) optimize density (minimum number of bits per card) or (ii) simplify operations such as determining if two cards are of the same suit or rank? Explain how you can check to see if two cards are adjacent (rank differing by one) using a specific representation.

---

**Solution:** :

## [Textbook 3.6] De Morgan's Theorem

Using perfect induction, prove De Morgan's theorem with four variables. Specifically

$$\overline{w \wedge x \wedge y \wedge z} = \bar{w} \vee \bar{x} \vee \bar{y} \vee \bar{z} \quad (1)$$

and

$$\overline{w \vee x \vee y \vee z} = \bar{w} \wedge \bar{x} \wedge \bar{y} \wedge \bar{z} \quad (2)$$

---

**Solution:**

## [Textbook 3.9] Simplifying Boolean Equations, II

Reduce the following Boolean expression to a minimum number of literals:

$$(x \wedge y \wedge z) \vee (\bar{x} \wedge y) \vee (x \wedge y \wedge \bar{z}).$$

---

**Solution:**

## [Textbook 3.10] Simplifying Boolean Equations, III

Reduce the following Boolean expression to a minimum number of literals:

$$((y \wedge \bar{z}) \vee (\bar{x} \wedge w)) \wedge ((x \wedge \bar{y}) \vee (z \wedge \bar{w})).$$

---

**Solution:**