Experiment 2: Lorentz Force

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2.3.1 Worksheet

1. Measurement of $\frac{e}{m}$

   (a) Consider the equation for Lorentz force:

   $$\vec{F} = q\vec{v} \times \vec{B} \tag{1}$$

   where $\vec{F}$ is force on the electron beam and $\vec{v}$ is tangent to its circle while pointing to the left (relative to the cathode ray tube operator). As per the right-hand-rule, the magnetic field $\vec{B}$ points in the direction of the operator.

   (b) As accelerating voltage increases, the electron beam thickens and its radius extends. Conversely – as magnetizing current increases, the radius shrinks.

   (c) When the velocity of electrons has only parallel components (not perpendicular) to the magnetic field $\vec{B}$, the beam is a straight horizontal line. In this case, of course, $\overrightarrow{v_\perp} = 0\frac{m}{s}$. Since $\vec{v} \times \vec{B}$ must equal zero, this observation adheres to the Lorentz force equation (1). It is clear that $\vec{F} = 0N$, meaning that no force acts to alter the direction of the beam. Now, when the velocity of electrons has both parallel and perpendicular components to $\vec{B}$, the beam twists in the z-plane, giving the appearance of a helix. In this case, $\vec{v} = \overrightarrow{v_\perp} + \overrightarrow{v_\parallel}$ where $\overrightarrow{v_\perp}$ and $\overrightarrow{v_\parallel}$ are both nonzero. This also follows from equation (1) as the Lorentz force will partially be given by $\overrightarrow{v_\perp} \times \vec{B}$, some perpendicular component of velocity. Thus, perpendicular force twists the beam into a helix.

   (d) Begin with the given statement:

   $$evB = \frac{mv^2}{R}. \tag{2}$$

   Dividing both sides of equation (2) by v yields:

   $$eB = \frac{mv}{R}. \tag{3}$$

   Note that the velocity of electrons stems from potential energy and kinetic energy. Thus, it follows that:

   $$PE = eV = KE = \frac{1}{2}mv^2. \tag{4}$$

   Now, equation (4) may be rewritten as:

   $$2eV = mv^2. \tag{5}$$

Substituting the right-hand-side of equation (5) into equation (2) gives the following:

$$evB = \frac{2eV}{R}. \tag{6}$$

Both sides of equation (6) may be divided by eB, hence:

$$v = \frac{2V}{BR}. \tag{7}$$

Equation (3) in terms of v is:

$$v = \frac{eBR}{m}. \tag{8}$$

The right-hand-side of equation (7) is replaced with the left-hand-side of equation (8) to give:

$$\frac{eBR}{m} = \frac{2V}{BR}. \tag{9}$$

Finally, both sides of equation (9) are divided by BR:

$$\frac{e}{m} = \frac{2V}{B^2R^2}. \tag{10}$$

2. Diodes
   (a) The transformer secondary signal is $(34.0 \pm 0.5)V$ peak-to-peak, and the voltage rating is $12V$. As determined by the oscilloscope employed in this experiment, the RMS voltage of the signal is $(12.2 \pm 0.5)V$. As expected, the voltage rating and RMS voltage are relatively similar.
   (b) The DC level of the full-wave rectified power supply is $(12.1 \pm 0.5)V$. The magnitude of its ripple is $(0.30 \pm 0.5)V$.

2.3.2 Presentation Report

2. Introduction

This experiment serves to confirm the findings of J.J. Thompson's 1897 Nobel Prize-winning study which established the electron charge to mass ratio $\frac{e}{m} \approx 1.76 \times 10^{11} \ \frac{C}{kg}$. A cathode ray tube and an electron gun will be employed to accomplish this. Specifically, the gun will emit a beam of electrons to be altered by the magnetic component of the Lorentz force, which is given by

$$\overrightarrow{F_{e,mag.}} = -e(\vec{v} \times \vec{B}) \tag{11}$$

where $\vec{B}$ represents magnetic field, $\vec{v}$ represents particle velocity, and $-e$ represents charge on an electron.

The purpose of this system is to relate equation (11) with the equation for centripetal force:

$$F_e = \frac{mv^2}{R} \tag{12}$$

as well as the equations for potential and kinetic energy:

$$PE = eV = KE = \frac{mv^2}{2} \tag{13}$$

to ultimately yield Thompson's electron charge to mass ratio:

$$\frac{e}{m} = \frac{2V}{B^2R^2}. \tag{14}$$

In addition, a Hemholtz configuration (Hemholtz coils) will be required to generate a magnetic field $\vec{B}$ perpendicular to velocity $\vec{v}$ the electron beam. The formula

$$B = \frac{8\mu_0 IN}{5\sqrt{5}R_c} \tag{14}$$

represents the magnetic field wherein strength is controlled by current I over two coils of radius $R_c = (0.140 \pm 0.002)\, m$ at a distance R_c apart from one another. $N = (150.00 \pm 0.01)$ denotes the number of windings per coil.

3. Experimental Description and Results

We position the electron gun such that its beam has a velocity perpendicular to the magnetic field $\vec{B}$ generated by the Hemholtz configuration. There are two dials on the cathode ray tube (CRT) apparatus, magnetizing current and accelerating voltage. The current and voltage settings are calculated via two multimeters with accuracy to the nearest thousandth of an ampere and the nearest hundredth of a volt, respectively. Recordings are made at dial intervals of $50V$ and $0.5A$, including all permutations from $100V$ to $200V$ and $1A$ to $2A$. We also observe the dials' qualitative influence on the electron beam. With a millimeter ruler, we measure the radius of the beam – adhering to a best value protocol outlined in section (4). The propagation of uncertainty is also considered.

Accelerating Voltage Vaccel (V)	**Magnetizing Current Icoils (A)**	**Electron Beam Radius R (mm)**	**e/m (C/kg)**
100.31 ± 0.2	1.000 ± 0.005	27.5 ± 0.4	(2.2 ± 0.1) * 10^11
100.41 ± 0.3	1.500 ± 0.005	18.0 ± 0.4	(2.3 ± 0.1) * 10^11
100.08 ± 0.1	2.000 ± 0.005	13.5 ± 0.4	(2.2 ± 0.1) * 10^11
150.51 ± 0.5	1.000 ± 0.005	37.8 ± 0.4	(2.2 ± 0.1) * 10^11
150.02 ± 0.1	1.500 ± 0.005	21.5 ± 0.4	(2.2 ± 0.1) * 10^11
150.14 ± 0.3	2.000 ± 0.005	15.0 ± 0.4	(2.0 ± 0.1) * 10^11
199.91 ± 0.1	1.000 ± 0.005	41.5 ± 0.4	(2.2 ± 0.1) * 10^11
199.72 ± 0.3	1.500 ± 0.005	26.0 ± 0.4	(2.2 ± 0.1) * 10^11
199.33 ± 0.4	2.000 ± 0.005	21.5 ± 0.4	(2.0 ± 0.1) * 10^11

Figure 1: Raw data including uncertainty values gathered from multimeters connected to the CRT apparatus. Changes in the electron beam radius are observed as a result of deviations in accelerating voltage and magnetizing current. Error in columns 1 – 3 (from left) is determined by measuring tool; error in column 4 is determined by propagation of uncertainty formula in section (4).

As displayed by Fig. 1, it is clear that accelerating voltage is directly proportional to the length of the radius, while magnetizing current is inversely proportional to the length of the radius. From the data in Fig. 1, a scatter plot of $V_{accel}\ vs. \frac{B^2R^2}{2}$ is generated (Fig. 2). The slope of the regression line in this chart serves to model the electron charge to mass ratio $\frac{e}{m}$.

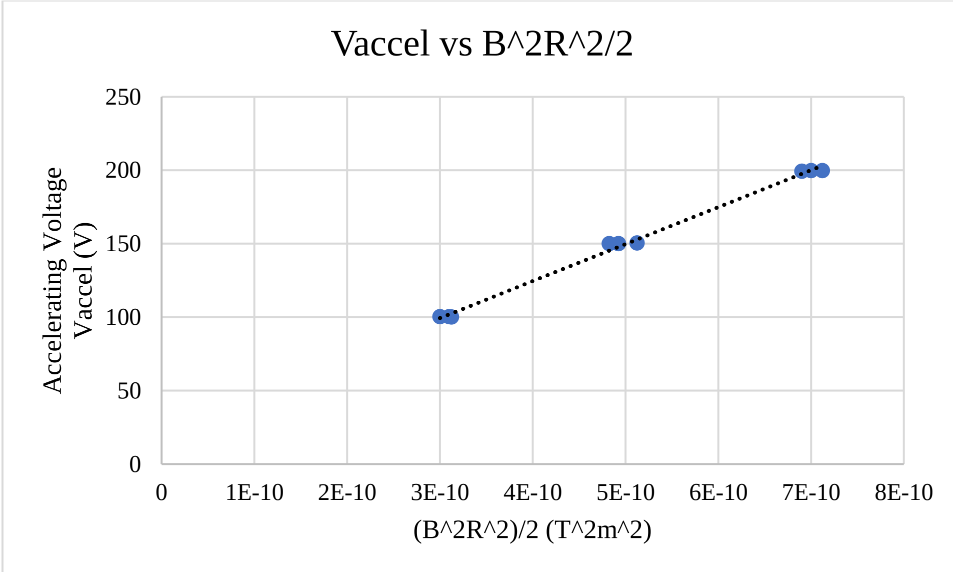

Figure 2: A plot of accelerating voltage vs. $\frac{B^2R^2}{2}$. The slope of the superimposed regression line models the electron charge to mass ratio $\frac{e}{m}$. Therefore, the value of $\frac{e}{m}$ as determined in this experiment is $(2.1 \pm 0.4) \times 10^{11}\ \frac{C}{kg}$, which encompasses the true value of $\frac{e}{m}$ (approximately $1.76 \times 10^{11}\ \frac{C}{kg}$) in its error boundary.

4. Analysis

In Fig. 1, column 3, we determine the uncertainty of the electron beam radius with the following error formula:

$$\delta r = \sqrt{(\frac{\delta r_t}{2})^2 + (\frac{\delta r_b}{2})^2} \tag{15}$$

where r_t represents the measurement taken from the top of the circle and r_b represents the measurement taken from the bottom of the circle. That is – two radius measurements are made per column row, and the best value is then derived from them. Similarly, in Fig. 1, column 4, the uncertainty of $\frac{e}{m}$ is given by:

$$\delta(\frac{e}{m}) = \sqrt{(\frac{2\delta V}{B^2R^2})^2 + (\frac{-4V\delta B}{B^3R^2})^2 + (\frac{-4V\delta R}{B^2R^3})^2}. \tag{16}$$

As per the Hemholtz configuration, we obtain the uncertainty in the magnetic field equation from:

$$\delta B = \sqrt{(\frac{8\delta I \mu_0 N}{5\sqrt{5}R_c})^2 + (\frac{8\delta N \mu_0 I}{5\sqrt{5}R_c})^2 (\frac{-8\delta R_c I \mu_0 N}{5\sqrt{5}R_c^2})^2}. \quad (17)$$

To get the best value for $\frac{e}{m}$, we may use a more general form of the best value formula equations (15-17). Where $N = 9$, the number of rows in Fig. 1, the statistical uncertainty is determined with the following formula:

$$\delta x = \frac{\sigma_x}{\sqrt{N}} = \frac{1}{\sqrt{N}} \sqrt{\frac{1}{\sqrt{N-1}} \sum_{i=1}^{N} (x_i - \bar{x})^2}. \quad (18)$$

This yields $\frac{e}{m} = (2.6 \pm 0.4) \times 10^{11}\ \frac{C}{kg}$. Note that this does not include the true value of $\frac{e}{m}$ $(1.76 \times 10^{11}\ \frac{C}{kg})$ in its error boundary. Hereafter, this method of finding $\frac{e}{m}$ will be referred to as "method 1". The slope of the regression line in Fig. 2 will be referred to as "method 2".

At face value, it seems that method 2 is more accurate; the slope of the regression line is closer to the actual value of $\frac{e}{m}$ than the approximation given by method 1. In fact, the value of $\frac{e}{m}$ as determined by method 2 is $(2.1 \pm 0.4) \times 10^{11}\ \frac{C}{kg}$, which includes the theoretical value in its uncertainty range. Nevertheless, to determine a numerical score for the precision of each method, it is possible to use the percent error formula:

$$\%\ error = \left|\frac{experimental - theoretical}{theoretical}\right| * 100. \quad (19)$$

This calculation delivers the following results:

Method 1: $error = 47.7\%$

Method 2: $error = 19.3\%$.

Therefore, we conclude that method 2 is 28.4% more accurate in determining the true value of $\frac{e}{m}$ as compared to method 1.

5. Conclusion

This experiment was successful in that one of the methods of finding $\frac{e}{m}$ produced a result that includes the theoretical value in its error boundary. Thus, we may claim that we have upheld Thompson's findings. However, we observe that the other method failed to produce a result that encompasses the true value for $\frac{e}{m}$. Our hypothesis is that this inconsistency is the result of multimeter inaccuracies caused by natural mechanical deterioration. It is also likely that poor estimates were used in radius measurements. For optimal results, this experiment should be repeated with new multimeters and a computerized method of calculating the electron beam radius.