Experiment 4: Magnetism

Naim Ayat: 104733125

26 October 2017

Thursday 11:00AM PST

David Bauer

Lab Partner: Adam Jankowski

4.5.1 Worksheet

1. To determine the polarity of the Hall probe, we can use a magnet that has its north and south poles marked. We can then check whether a given face of the Hall probe reads positive or negative on the magnet's north end. If the reading is positive, then that face of the probe is south. If it is negative, then the face is north. This conclusion stems from the fact that northern magnetic fields push out, while southern magnetic fields pull in. Hence, if a face of the probe reads positive on the north end of the magnet, then the polarity of that face must be south because it agrees with the magnetic field. Conversely, if a face of the probe reads negative on the north end of the magnet, then that face must be north because it opposes the magnetic field.
2. Consider that V_{ind} is given by the formula:

$$V_{ind} = I_0 \mu_0 wAnNcos(\omega t) \tag{1}$$

where ω represents the angular frequency on the Faraday coil, μ_0 is the permeability constant, n is the number of turns in the inner coil, N is the number of turns in the outer coil, and A is the cross-sectional area of the inner coil (given by $2\pi r^2$). Now, note that doubling the radius of the inner coil is equivalent to multiplying its cross-sectional area by four, resulting in $A' = 8\pi r^2$. Therefore, we expect the ratio $\frac{V'_{ind}}{V_{ind}} = 4$. In reality, however, increasing the radius of the inner coil while keeping its wire's length constant would reduce the number of turns n. Also, we may have to stretch the radius of the outer coil to accommodate for the larger inner coil, thus reducing the number of outer coil turns N.
3. The amplification factor produced by inserting the ferromagnetic core into the primary coil is given by the ratio of max induced voltages with and without the core:

$$amplification = \frac{V_{ind\,(core)}}{V_{ind\,(no\ core)}} \tag{2}$$

where $V_{ind\,(core)}$ is the maximum induced voltage with the core. We can determine the error of equation (2) with:

$$\delta\, amplification = \sqrt{(\frac{\delta\, V_{ind\,(core)}}{V_{ind\,(no\ core)}})^2 + (\frac{\delta\, V_{ind\,(no\ core)} V_{ind\,(core)}}{V_{ind\,(no\ core)}^2})^2}\,. \tag{3}$$

4. Fig. (1) shows the results of our torsion pendulum experiment.

Material	Magnetism
Cu	Diamagnetic
Al	Paramagnetic
Ta	Paramagnetic
Bi	Diamagnetic
C	Diamagnetic
Fe	Paramagnetic
Ni	Paramagnetic
Glass	Diamagnetic
Rocks	Paramagnetic

Figure 1: Results of a torsional pendulum setup for determining whether a material is paramagnetic, diamagnetic, or inconclusive. This data was gathered by hanging an annular section-shaped magnet from a string and configuring a co-mounted mirror to rotate with the magnet. The mirror deflected a laser beam which reflected from the magnet. By placing a given material about 0.5cm from one of the magnet's poles, we determined each material's magnetism from the position of the laser.

5. If $\boldsymbol{B_2}$ exerts an upward force onto the magnetic dipole $\boldsymbol{m_1}$, then we know that the dipoles must be anti-parallel as per the right-hand-rule. To determine z_0, note that the downward force of $\boldsymbol{m_1}$ is equal and opposite to $\boldsymbol{B_2}$. Thus, we have:

$$F_{\boldsymbol{m_1},z} = Mg = F_{\boldsymbol{B_2}}. \tag{4}$$

In equation (4), F_{B_2} can be given by:

$$F_{\boldsymbol{B_2}} = -\nabla U = -\nabla(\boldsymbol{m_1} \cdot \boldsymbol{B_2}). \tag{5}$$

Now, we combine equations (4) and (5):

$$-\nabla(\boldsymbol{m_1} \cdot \boldsymbol{B_2}) = Mg = \frac{3\boldsymbol{m_1^2}\mu_0}{4\pi z_0^4}. \tag{6}$$

Finally, we solve the right-hand side of equation (6) for z_0:

$$z_0 = \sqrt[4]{\frac{3\mu_0 \boldsymbol{m_1^2}}{4\pi Mg}}. \tag{7}$$

4.5.2 Presentation Report

2. Introduction

This experiment serves to demonstrate and examine the properties of magnetic fields generated by steady-state sources. From measurements of such magnetic fields, we seek to derive the accepted value for the permeability constant, $\mu_0 = 4\pi \times 10^{-7}\frac{H}{m}$.

First, we employ a finite toroidal current distribution with a magnetic field identical to that of an infinite wire (over its volume) to model the magnetic field of an infinite current carrying wire. We configure a Hall probe to move across a linear track to calculate magnetic field at regular intervals in the radial direction. We then perform a linear regression of the linearized data to determine the correlation coefficient and slope, which will represent the permeability of free space μ_0.

Second, we measure the magnetic field in the z-direction generated by a single permanent magnet. We accomplish this by using the Hall probe and measuring the magnetic field values at angles $\theta = 0°$ and $\theta = 90°$. We then verify the proportionality of $\frac{1}{r^3}$ and $\boldsymbol{B_z}(z)$ by graphing data points at regular intervals along the linear track.

Third, we measure the force between two magnets by configuring a magnet to repel and vary vertically above a static permanent magnet placed on a scale. As the upper magnet approaches the lower one, we record the reading on the scale to determine the dipole interactions as a function of varying force. We verify that such an interaction demonstrates proportionality between $\boldsymbol{F}$ and $\frac{1}{r^3}$ using the following formulae:

$$\boldsymbol{F} = -\nabla U \tag{8}$$

$$\text{and } U = -m_1 \cdot \boldsymbol{B_2}. \tag{9}$$

In equation (8), $\boldsymbol{F}$ represents the gradient of potential energy, which is proportional to the magnetic field.

Finally, we run a fourth experiment to demonstrate Faraday's Law by generating a current through two concentric coils; a setup wherein the outer coil is referred to as a "Faraday coil". We note that Faraday's Law is given by:

$$V_{ind} = \frac{-d\Phi}{dt}. \tag{10}$$

This relationship can also by modelled by the waveform

$$I(t) = I_0 \sin(\omega t) \tag{11}$$

when the current is generated sinusoidally. Hence, equation (1) follows. We will also observe the effect a ferromagnetic core introduces to this configuration.

3. Experimental Results

We apply $(17.0 \pm 0.2\,V)$ to the toroidal coil from a DC power supply with a multimeter in series. From the multimeter, we find that current is $(1.580 \pm 0.005)\,A$. Then, we rotate a Hall probe between two of the toroid's wires and find that the probe reaches a maximum value when perpendicular to the toroid, meaning that there is an azimuthal magnetic field of magnitude. We fix the probe to a track such that it is in the center of the inner inductor, where we find that magnetic field is effectively zero $(0.00 \pm 0.02)\,mT$. Hence, we treat the area immediately outside of the inner inductor to be our starting point. This means that we begin at a radius $r = (0.0300 \pm 0.0005)\,m$ away from the center of the inner inductor. We then measure the field strength at intervals of $(0.0500 \pm 0.0005)\,m$ away from the $r = (0.0300 \pm 0.0005)\,m$ position until the probe is completely outside of the magnetic field generated by the toroid. We find this terminal point to occur at $r = (0.1550 \pm 0.0005)\,m$. The relevant measurements are displayed below in Fig. (2).

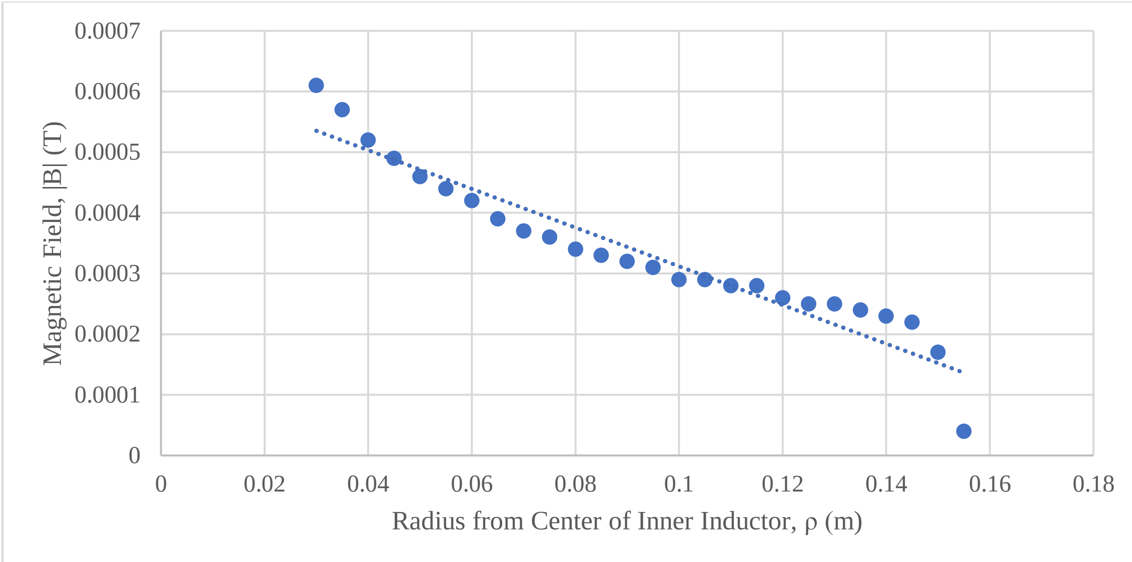

Figure 2: Demonstrating the deterioration of magnetic field strength along the radius of a toroidal magnet. Here, we plot the linearized magnetic field |B| against the radius from the center of the inductor ρ. Note that our measurements ρ begin at $r = (0.0300 \pm 0.0005)\ m$, immediately outside of the inner inductor, since magnetic field inside the inner inductor is effectively zero ($(0.00 \pm 0.02) mT$). The linear equation is $y = (-0.00320 \pm 0.00001)\frac{T}{m}x + (0.000630 \pm 0.00001)\ T$ with a correlation coefficient $r^2 = 0.967$, which models the steady decrease of magnetic field strength as the Hall probe gets further from the outside of the inner coil.

Next, we position a permanent magnet on a block and configure the Hall probe approximately $(0.0500 \pm 0.0005)\ m$ above it. We move the probe along the track, recording changes in the magnetic field at $(0.0050 \pm 0.0005)\ m$ intervals for a total of 22 data collections. This represents the $\theta = 0°$ portion of the experiment. Then, we configure the permanent magnet on a block with the Hall probe approximately $(0.0500 \pm 0.0005)\ m$ away from it at the $\theta = 90°$ horizontal angle. Again, we take 22 recordings at intervals $(0.0050 \pm 0.0005)\ m$ away from the magnet.

To determine the force between two permanent magnets, we configure a "lower" and "upper" magnet such that they repel. The lower is placed on a scale, and we prepare to attach the upper to the track $(0.0300 \pm 0.0005)\ m$ above such that it can move vertically. First, however, we note that the mass of the lower magnet on the scale is $(0.04600 \pm 0.00005)\ kg$, independent of any repulsion from the upper magnet. Using the law $F = ma$ where $a = 9.81\frac{m}{s^2}$ due to gravity, we get that the initial downward force on the lower magnet is $(0.4512 \pm 0.0005)\ N$. We then attach the upper magnet to the track and record a total of 30 scale readings at intervals of $(0.0050 \pm 0.0005)\ m$.

To measure induced voltages across the Faraday Coil, we generated a $1.0000\ kHz$ sine wave to the inner coil with a $(10.70 \pm 0.01)\ \Omega$ resistor in series. As per the oscilloscope, the voltage across the resistor is $(0.38 \pm 0.01)\ V$. Hence, we use equations

$$I = \frac{V}{R} \tag{12}$$

$$\text{and } \delta I = \sqrt{(\frac{V}{R^2}\delta R)^2 + (\frac{\delta V}{R})^2} \tag{13}$$

to determine that the max current through the circuit is $(0.0355 \pm 0.01)\ A$. Note that this current flows through the inner coil, which has turns $n = (175.0 \pm 0.1)$. Hence, there is a voltage on the outer coil which we measure with our oscilloscope for the following turn intervals: $[0 - 500, 0 - 1000, 0 - 1500]$. We re-measure these values after introducing a ferromagnetic core to the center of the inner coil and record the data in Fig. (3).

Num. of Turns, N	**Max induced Voltage,** V_{ind} **(V)**	**Amplification Factor,** $(V_{ind\ (coil)}\ /\ V_{ind})$ **(V)**	**Current in Inner Coil,** I_0 **(A)**
500.0±0.01	0.160±0.004	2.70±0.04	0.0365±0.0009
1000.0±0.01	0.318±0.004	2.71±0.05	0.0365±0.0009
1500.0±0.01	0.461±0.004	2.70±0.04	0.0365±0.0009

Figure 3: Max induced voltage, amplification factor $(V_{ind\ (coil)}\ /\ V_{ind})$, and inner current calculations from oscilloscope readings on the Faraday coil. Error for the number of turns is given by the manufacturer of the coil. Voltage uncertainty comes from oscilloscope noise. Current error is derived from equation (13). Uncertainty in the amplification factor is given by $\sqrt{(\frac{\delta V_{ind\ (coil)}}{V_{ind}})^2 + (\frac{\delta V_{ind} V_{ind\ (coil)}}{V_{ind}^2})^2}$.

4. Analysis

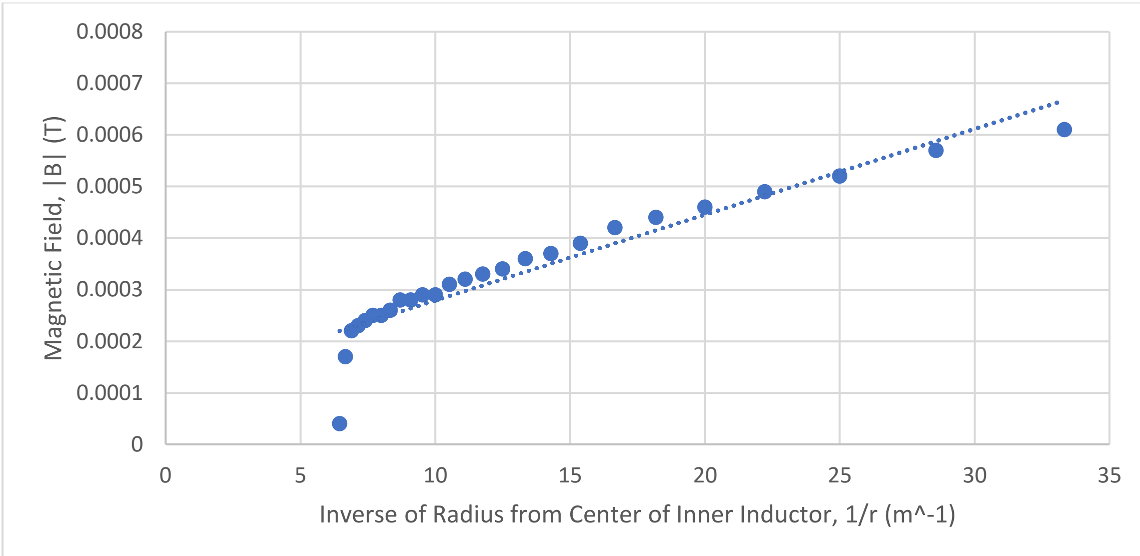

Figure 4: The radial dependence of a toroidal magnetic field over the region between two coils. The linear regression model's slope represents the magnetic field equation $B(x) = \frac{Ix\mu_0}{2\pi}$ where x is the inverse of the radius from the center of the inner inductor $(\frac{1}{r})$. As per our data, the slope is given by $m = (0.000026 \pm 0.000001)\ \frac{N}{A}$ and the y-intercept is $b = (0.00009 \pm 0.0001)\ T$ with $r^2 = 0.951$. The equation in full is $y = (0.000026 \pm 0.000001)\ \frac{N}{A}\ x + (0.00009 \pm 0.0001)\ T$. As anticipated, b contains the value 0 within its error boundary because the magnetic field inside the inner coil (where r is on the range $[0, 0.0300]\ m$) should equal 0.

From Fig. (4), we can determine the experimental value for μ_0. Consider that the slope m represents the following equation:

$$m = (0.000026 \pm 0.000001)\ \frac{N}{A} = \frac{I\mu_0}{2\pi}. \tag{14}$$

Solving for μ_0 in equation (14) yields:

$$\mu_0 = \frac{2\pi m}{I} \frac{N}{A^2}. \tag{15}$$

In equation (15), I is given by the sum of currents on the 100 coils on the toroid. Hence, the best value is:

$$I = 1.52A * 100 = 152A. \tag{16}$$

The error is:

$$\delta I = 100\delta I = 100 * 0.005A = \pm 0.5A. \tag{17}$$

Hence, using equation (15), we determine that our experimental value for the permeability constant is $\mu_{0\ experimental} = (1.1 \pm 0.4) * 10^{-6}\ \frac{N}{A^2}$, where the uncertainty is given by:

$$\delta\mu_{0\ experimental} = \sqrt{(\frac{2\pi\delta m}{I})^2 + (\frac{2\pi\delta m}{I^2})^2}. \tag{18}$$

Knowing that the true value for the permeability of free space is $\mu_0 = 4\pi * 10^{-7}\ \frac{N}{A^2}$, we can calculate the percent error associated with our experimental value with the formula:

$$\%\ error = \left|\frac{experimental - theoretical}{theoretical}\right| * 100. \tag{19}$$

This gives $error = 8.3\%$.

We now compare the measured data in Fig. (3) to theoretical values induced voltage on the Faraday coil. We use the waveform equation (1) with the following input: $t = 0,\ I_0 = (0.0365 \pm 0.0009)\ A,\ \omega = \frac{1000\ rad}{2\pi\ s},\ A = 2\pi r^2,\ n = (175.0 \pm 0.1)\ turns$, and $N = (500.0, 1000.0, 1500.0 \pm 0.1)\ turns$. Moreover, the uncertainty of V_{ind} is given by:

$$\delta V_{ind} = \sqrt{\begin{matrix} \left(\delta I_0\mu_0\omega AnNcos(wt)\right)^2 + \left(I_0\mu_0\omega\delta AnNcos(wt)\right)^2 + \\ (I_0\mu_0\omega A\delta nNcos(wt))^2 + (I_0\mu_0\omega An\delta Ncos(wt))^2. \end{matrix}} \tag{20}$$

We determine the percent error between our experimental and theoretical values of V_{ind} in Fig. (5).

Num. of Turns, N	**Max Induced Voltage,** V_{ind} **(V)**	**Theoretical Max,** $V_{ind\ (theoretical)}$ **(V)**	**Percent Error, %**
500.0±0.01	0.160±0.004	0.270±0.003	40.7%
1000.0±0.01	0.318±0.004	0.661±0.005	51.9%
1500.0±0.01	0.461±0.004	0.822±0.002	43.8%

Figure 5: Comparisons of experimental and theoretical V_{ind} on the Faraday coil. Percent error is calculated with equation (19), and experimental max induced voltage data comes from Fig. (3).

We now calculate the ratios of $V_{ind\ (1500\ turns)}$ to $V_{ind\ (500\ turns)}$ and $V_{ind\ (1500\ turns)}$ to $V_{ind\ (1000\ turns)}$. We expect the results to agree with the ratios of turns, which are around 3 and 1.5, respectively. The results are:

$$\frac{V_{ind\ (1500\ turns)}}{V_{ind\ (500\ turns)}} = 2.9 \pm 0.3 \tag{21}$$

$$\text{and} \quad \frac{V_{ind\ (1500\ turns)}}{V_{ind\ (1000\ turns)}} = 1.4 \pm 0.1. \tag{22}$$

Since the solutions to equations (21) and (22) include 3 and 1.5 in their respective uncertainty boundaries, the results are satisfactory.

In Fig. (6), we plot field strength on the vertical axis and linearized distance $\frac{1}{r^3}$ on the horizontal axis for the field due to a permanent magnet along the z-axis. We determine that the correlation coefficient is $r^2 = 0.995$. Note that the threshold for a conclusive r^2 value is $r^2 > 0.95$ for this experiment. Thus, we conclude that the axes are correlated. In Fig. (7), we plot force on the vertical axis and linearized distance $\frac{1}{r^4}$ on the horizontal axis for the force between dipoles. We get that $r^2 = 0.955$, so we may conclude that these axes are correlated as well.

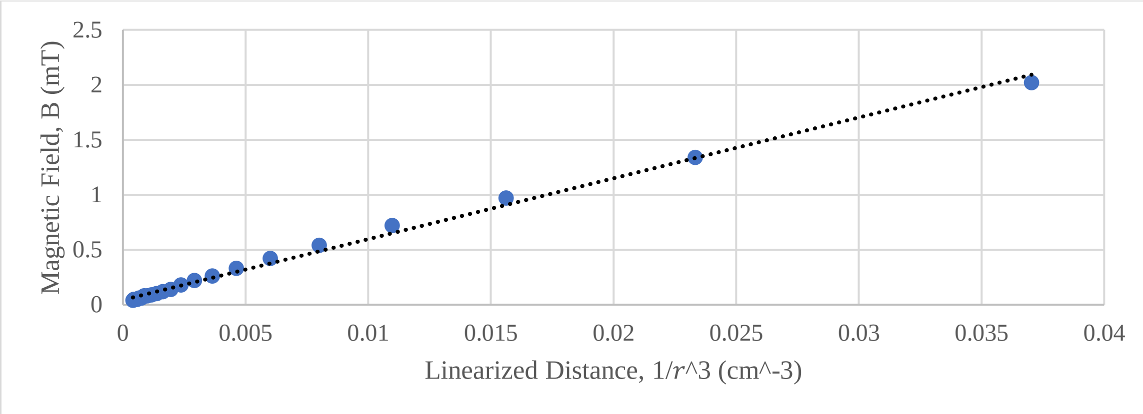

Figure 6: Demonstrating that the field due to a permanent magnet is proportional to $\frac{1}{r^3}$ on the z-component. This consists of 22 data points taken at $(0.0050 \pm 0.0005)\ m$ intervals along the angle $\theta = 0°$. In this case, the linear regression model follows the equation $y = (55.3 \pm 0.8)\ (mT * cm^3)\ x + (0.0436 \pm 0.008)\ mT$ where $x = \frac{1}{r^3}$. Moreover, the correlation coefficient $r^2 = 0.995$ is high enough (defined to be $r^2 > 0.95$) to conclude that the field due to a permanent magnet is, in fact, proportional to $\frac{1}{r^3}$.

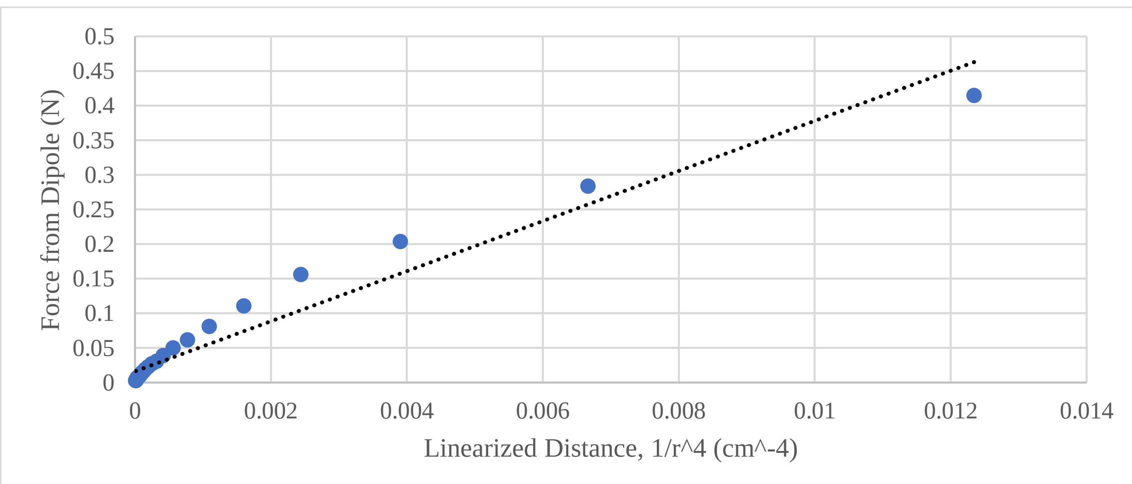

Figure 7: Demonstrating that the force between dipoles is proportional to $\frac{1}{r^4}$ with a linear regression model. This consists of 30 data points taken at $(0.0050 \pm 0.0005)\ m$ intervals while moving the upper dipole vertically. The linear regression model follows the equation $y = (36.1 \pm 0.9)\ (N * cm^4)\ x + (0.016 \pm 0.004)\ N$ where $x = \frac{1}{r^4}$. The correlation coefficient $r^2 = 0.952$ is high enough (defined to be $r^2 > 0.95$) to conclude that the force between dipoles is proportional to $\frac{1}{r^4}$.