Experiment 5: Speed of Sound and Light

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5.5.1 Worksheet

1. (a) The speed of light in terms of the time cursor measurements and the length of the beam path is given by:

$$c = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}. \tag{1}$$

(b) The uncertainty on the measured speed of light is:

$$\delta c = \sqrt{(\frac{-\delta x_1}{\Delta t})^2 + (\frac{\delta x_2}{\Delta t})^2 + (\frac{\delta t_1(\Delta x)}{(\Delta t)^2})^2 + (\frac{-\delta t_2(\Delta x)}{(\Delta t)^2})^2}. \tag{2}$$

From equation (2) and the given values of $\delta t = 1\, ns$, $\delta x = 1\, cm$, $\Delta t = 72\, ns$, and $\Delta x = 21.6m$, we yield $\delta c = (6 * 10^6)\, m/s$.

(c) Assuming we have infinitely precise measurements for beam path lengths, we may disregard the first two terms of equation (2) and set $(\left(\frac{-\delta x_1}{\Delta t}\right)^2 + \left(\frac{\delta x_2}{\Delta t}\right)^2 = 0)$. Nevertheless, we still yield $\delta c = (6 * 10^6)\, m/s$.

(d) Similarly, assuming we have infinitely precise measurements for time, we may disregard the last two terms of equation (2) and set $(\left(\frac{\delta t_1(\Delta x)}{(\Delta t)^2}\right)^2 + \left(\frac{-\delta t_2(\Delta x)}{(\Delta t)^2}\right)^2 = 0)$ to yield $\delta c = (2 * 10^5)\, m/s$.

(e) From parts (c) and (d), we observe that uncertainty in our time measurements (as opposed to uncertainty in length) plays a larger role in determining our total uncertainty δc.

2. For our experiment, the equation for speed of light in terms of time cursor measurements and beam length is given by:

$$c_{best} = \frac{(x_1 + x_2 + x_3 + x_4) - (x_5 + x_6)}{t_2 - t_1} \tag{3}$$

where x_1, x_2, x_3, and x_4 represent the lengths to and from the far mirror, x_5 is the length from the laser to the near mirror, x_6 is the length from the near mirror to the sensor, t_1 is the time when the near mirror curve begins to rise, and t_2 is the time when the far mirror curve begins to rise. The uncertainty is given by:

$$\delta c = \sqrt{(\frac{\delta x_1}{\Delta t})^2 + (\frac{\delta x_2}{\Delta t})^2 + (\frac{\delta x_3}{\Delta t})^2 + (\frac{\delta x_4}{\Delta t})^2 + (\frac{-\delta x_5}{\Delta t})^2 + (\frac{-\delta x_6}{\Delta t})^2 + (\frac{-\delta t(\Delta x)}{(\Delta t)^2})^2}. \tag{4}$$

As per our oscilloscope measurements, the values for x_1 through x_6 are $(3.5290 \pm 0.005)\, m$, $(3.5200 \pm 0.005)\, m$, $(3.6550 \pm 0.005)\, m$, $(3.4420 \pm 0.005)\, m$,

$(0.3620 \pm 0.005)\ m$, and $(0.1250 \pm 0.005)\ m$, respectively. Moreover, we found that $t_2 - t_1 = (47 \pm 2)\ ns$. Using equations (2) and (4), we yield an experimental speed of light $c = (2.9 \pm 0.1) * 10^8\ m/s$. Hence, the defined speed of light $in\ vacuo$ $(v \equiv 299\ 792\ 458\ m/s)$ falls within our error boundary.

3. We use the formula for light in a vacuum as an example of a non-dispersive dispersion relationship (wherein a change in frequency does not affect velocity):

$$v_g = \frac{\partial\omega(k)}{\partial k} = c. \tag{5}$$

Note that v_g represents group velocity and $\omega(k) = ck$. For an instance of a dispersive dispersion relationship (wherein a change in frequency affects velocity), we recall the wave of a non-ideal string. In this case, we have:

$$v_g = \frac{\partial\omega(k)}{\partial k} = \frac{1}{2}\left(\frac{T}{\mu}k^2 + ck^4\right) + 2\left(\frac{Tk}{\mu}\right) + 4ck^3. \tag{6}$$

$$\text{and } \omega(k) = \sqrt{\frac{T}{\mu}k^2 + ck^4}. \tag{7}$$

In equations (6) and (7), μ is string density and T is tension. We observe that frequency changes with k; therefore, velocity changes with frequency.

5.5.2 Presentation Report

2. Introduction

This experiment serves to examine the speed of sound in air and demonstrate an experimental result agreeing with its accepted value of approximately $343\,\frac{m}{s}$. We will employ two separate methods to accomplish this.

First, we recognize that sound is non-dispersive; thus, its phase velocity is given by the formula:

$$v_p = \frac{\omega}{k} \tag{8}$$

where ω is the frequency of the sound waves and k is the wavenumber ($k = \frac{2\pi}{\lambda}$). We then use a RIGOL waveform generator to create several different frequencies and measure their wavelengths λ with a speaker connected to an oscilloscope. Using this data, we create a plot of the dispersion relation (ω vs. k). A linear regression analysis determines the slope, which represents our first experimental v_p.

Second, we generate soundwaves in two directions using a microphone. One direction is towards a moving reflector, and the other is towards a speaker connected to an oscilloscope. From the properties of superposition of standing soundwaves, we note that this configuration will create nodes and antinodes where the signals are in and out of phase, respectively. The distances between antinodes are used to determine wavelengths, and their corresponding frequencies are once again created with the RIGOL waveform generator. Finally, we use equation (8) to determine our second experimental result for v_p.

3. Experimental Results

We begin by using the RIGOL waveform generator to apply a frequency of about $10\ kHz$ to the speaker connected to an oscilloscope. We configure the microphone such that the waveforms appearing on the oscilloscope from the speaker and the microphone are visually identical. We then move the microphone further from the speaker to create phase shifts of π and 2π, knowing that the distance required to create a phase shift of 2π gives the distance of the wavelength. We measure this distance and repeat the procedure with four other frequencies, giving us a total of five spatial frequencies with corresponding wavelength data. We then multiply these frequencies (again, given in kHz by the RIGOL generator) by 2π to convert them to temporal frequencies ω with units $\frac{rad}{s}$. We then determine the wavenumber k for each frequency by using the formula

$$k = \frac{2\pi}{\lambda} \tag{9}$$

where λ represents our experimental wavelengths. We plot the corresponding dispersion relation with spatial frequency k on the x-axis and angular frequency ω on the y-axis in Fig. (1) below.

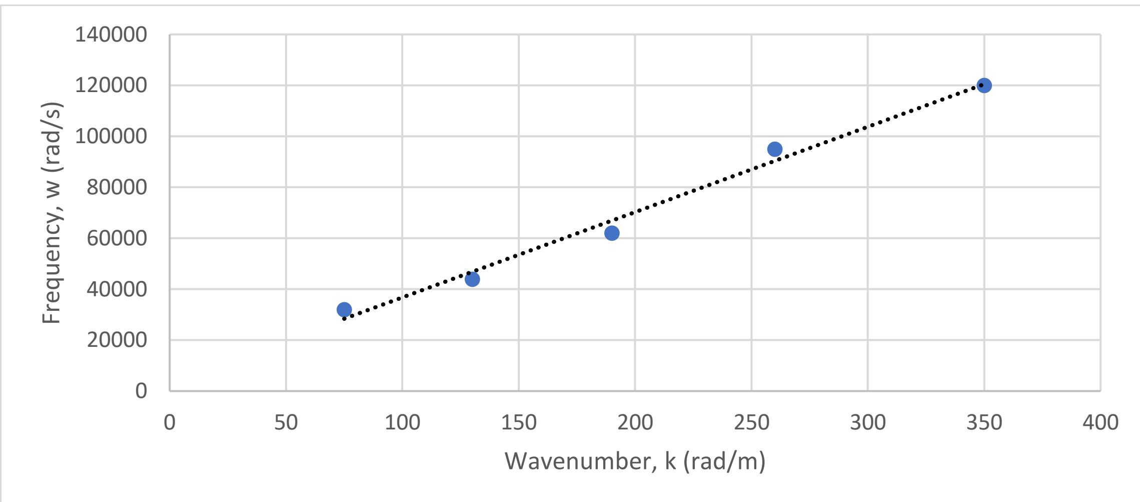

Figure 1: Demonstrating the dispersion relation for travelling sound waves. By equation (8), the phase velocity v_p (or, speed of sound) in this graph is given by the slope of the superimposed regression line. The correlation coefficient is $r^2 = 0.971$. The linear equation follows the form $y = (350 \pm 10)\frac{m}{s}\,x + (900 \pm 1000)\frac{rad}{s}$. Hence, our first experimental value for the speed of sound is $v_p = (350 \pm 10)\frac{m}{s}$. We also note that the y-intercept $(900 \pm 1000)\frac{rad}{s}$ encompasses zero in its uncertainty, as we know a (non-existent) wave with a frequency of zero has a wavenumber of zero.

To begin the second method of calculating v_p, we use a speaker that produces output in two directions and a wheel-mounted reflector to interfere with one of those directions. Then, we connect the reflector to a potentiometer powered by a $2V$ power supply. We redirect its output to a multimeter so that we may determine voltage produced by the potentiometer. At this point, we can use the myDAQ to measure distance traveled by the reflector. With the RIGOL waveform generator, we create sinusoidal soundwaves at frequencies of 5 kHz in both speaker output directions. We connect the microphone to channel 1 of the myDAQ and the potentiometer to channel 0 as the system is set to make continuous measurements. While the myDAQ records data, we move the reflector closer to the speaker in intervals of 5 cm. This generates standing sound waves. After recording these waves, we determine the distance between their corresponding extrema to yield wavelength values. The data gathered via this process is displayed and described in further detail in Fig. (2).

Type of Extrema (Min or Max)	Distance to First Extrema, D_1 (m)	Distance to Next Extrema of Same Type, D_2 (m)	Wavelength, λ (m)
Max	0.260 ± 0.003	0.340 ± 0.003	0.080 ± 0.005
Min	0.305 ± 0.003	0.376 ± 0.003	0.071 ± 0.006
Max	0.340 ± 0.003	0.415 ± 0.004	0.075 ± 0.007
Min	0.376 ± 0.003	0.455 ± 0.005	0.079 ± 0.007
Max	0.415 ± 0.004	0.496 ± 0.005	0.081 ± 0.008
Min	0.455 ± 0.005	0.530 ± 0.005	0.075 ± 0.009
Max	0.496 ± 0.005	0.563 ± 0.005	0.067 ± 0.009
Min	0.530 ± 0.005	0.598 ± 0.006	0.067 ± 0.009

Figure 2: Wavelengths determined by extrema data gathered from a standing sound wave. The first column denotes the type of extrema observed. The second column is its distance as observed by the myDAQ. The third column is the distance for the next extrema of the same type. The final column is the result of column (3) minus column (2). Uncertainties in the two middle columns are given by the propagation of error in the myDAQ system, as well as that of the voltage to distance conversion. The error of wavelength values is calculated by propagating the errors of the two middle columns.

3. Analysis

Our linear regression analysis in Fig. (1) determined the slope of the graph of k vs. ω to be $(350 \pm 10)\frac{m}{s}$. By equation (8), we know that this value represents the speed of sound v_p. Moreover, the correlation coefficient $r^2 = 0.971$ is high enough (defined to be $r^2 > 0.95$) to conclude that the phase velocity is, in fact, proportional to $\frac{\omega}{k}$. We observe that this experimental value of $v_p = (350 \pm 10)\frac{m}{s}$ encompasses the theoretical value for the speed of sound in air, which is about $343\frac{m}{s}$. Knowing this true value for v_p, we can calculate the percent error associated with our experimental value with the formula:

$$\%\, error = \left|\frac{experimental - theoretical}{theoretical}\right| * 100. \quad (10)$$

Thus, our first method for determining the speed of sound gives $error = 2.0\%$.

In our second method, we performed a linear regression on the plot of our potentiometer voltage output vs. the distance travelled by the reflector, which gives the volt-to-meter conversion rate for the myDAQ. We found the slope of this line to be $(1.38 \pm\ 0.1)\frac{v}{m}$. We multiplied each voltage by this conversion value (henceforth denoted by C) to obtain a plot of sound wave amplitude vs. distance. Given that the potentiometer readings have an uncertainty of $0.0005\ V$, the uncertainties of our distance calculations are given by:

$$\delta D = \sqrt{(\delta C V)^2 +}\,(\delta V C)^2 \quad (11)$$

where V represents the voltage across the potentiometer. For the voltage values where the sound wave was at an extremum, we multiplied them by the best value of C to convert to distance. These results populate the middle two columns in Fig. (2). The distances of each extrema to their next corresponding extrema (D_1 to D_2) determine the wavelengths, which is given by $\lambda = D_1 - D_2$. The corresponding uncertainty is:

$$\delta\lambda = \sqrt{(\delta D_2)^2 + (-\delta D_1)^2}. \quad (12)$$

These results comprise the last column in Fig. (2).

Now, we average the wavelengths determined in Fig. (2) to determine a best value λ_{best}, which is given by the formula:

$$\lambda_{best} = \frac{1}{N}\sum_{i=1}^{N}\lambda_i. \quad (13)$$

The uncertainty for this calculation is given by:

$$\delta\lambda_{best} = \sum_{i=1}^{N}\left(\frac{\delta\lambda_i}{N}\right)^2. \quad (14)$$

In our case, the sample size N is 8. Using formulas (13) and (14) on the wavelength values from Fig. (2), we find that $\lambda_{best} = (0.074 \pm 0.003)\ m$. Now, recall that this wave had a frequency of (5.00 ± 0.01) kHz. We may rewrite equation (8) in terms of spatial frequency to obtain:

$$v_p = f\lambda. \tag{15}$$

The error of equation (15) is given by:

$$\delta v_p = \sqrt{(\delta f \lambda)^2 + (f \delta\lambda)^2}. \tag{16}$$

Hence, by equations (15) and (16), we get the experimental speed of sound from our second method to be $v_p = (370 \pm 15)\frac{m}{s}$. As per equation (10), method two gives $error = 7.8\%$.

4. Conclusion

Our objective for this experiment was to calculate the speed of sound with two distinct methods, getting as close as possible to the accepted value of $v_p = 343\frac{m}{s}$. We find that our first method, a linear regression on the dispersion relation for traveling sound waves obtained by measuring phase shifts, was the most accurate. This method yielded a percent error of 2.0%, and our experimental value $v_p = (350 \pm 10)\frac{m}{s}$ encompassed the theoretical value in its uncertainty. Our second method, using equation (15) to determine v_p from wavelength data gathered from a standing sound wave, gave a percent error of 7.8%. Moreover, the experimental value $v_p = (370 \pm 15)$ from this method did not include the theoretical value in its uncertainty.

However, the decrease accuracy from method one to method two was expected. Method two relied heavily on myDAQ readings, which contained noise generated by other lab groups whom we shared the room with. Thus, the noise generated by other experiments and general laboratory noise contributed to a less accurate experimental v_p for method two. If this experiment were to be repeated, it would be beneficial to execute method two in a more controlled environment. Specifically, only one experiment should be conducted at a time. If feasible, a sound-proof room should also be used. This revision to the Experiment

5 protocol would minimize unwanted myDAQ noise and help bring experimental results closer to the theoretical value for v_p.