Experiment 3: AC Circuits

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3.4.1 Worksheet

1. The BODE analyzer software used in this experiment can sample frequencies over the range $[20\ Hz, 20\ kHz]$ and generate curves for frequency vs. gain ($\left|\frac{v_{out}}{v_{in}}\right|$) on a circuit. Consider the formula

$$f_{res} = \frac{1}{2\pi\sqrt{LC}} \tag{1}$$

where f is frequency, L is inductance, and C is capacitance. Increasing L and C would increase the value of the right-hand-side denominator, thus decreasing the resonant frequency f_{res}. However, in the case that L and C are at their maximum available values, we must physically alter the circuit such that its inductors are in series. This is because inductors in series are additive, meaning total inductance in the circuit would increase. Similarly, we could increase capacitance by utilizing the additive property of capacitors and configuring them to be in parallel.

2. Consider that the Q-factor of a circuit is given by

$$Q = \frac{f_{res}}{f_2 - f_1} \tag{2}$$

where f_1 and f_2 are frequencies on the plot of frequency vs. gain that intersect with the line $gain = (\frac{v_{out}}{\sqrt{2}})$. This relationship indicates that Q-factor increases as gain's slope and peak increases. Hence, moving driving frequency away from resonance in circuit 1 (high Q-factor, $Q_1 \gg 1$) would decrease the amplitude response quicker than moving driving frequency away from resonance in circuit 2 (low Q-factor, $Q_2 < 1$).

3. Assume that a car antenna can pick up any signal in the radio frequency spectrum ($f_{radio}\ \epsilon\ [3\ kHz, 300\ GHz]$) without loss and that we can see a signal that is a superposition of all these frequencies on a scope of $V_{antenna}$. We are given the inductance of an RLC circuit with a variable capacitor: $L = 1\ mH$. To access the entire spectrum of radio frequencies, capacitance C must be in the range $C\ \epsilon\ [3\ \mu\text{F}, (3\times10^{-16})\ \mu\text{F}]$ because it must adhere to equation (1). Moreover, given a single value capacitor (1 μF) and a variable inductor, the inductance range necessary to tune into the FM band ($f_{FM}\ \epsilon\ [88\ MHz, 108\ MHz]$) alone is $L\ \epsilon\ [(3.27\times10^{-6})\ \mu\text{H}, (2.17\times10^{-6})\ \mu\text{H}]$. This is also a result of equation (1).

4. Suppose we want to find values for resistance R and capacitance C which will allow the myDAQ to measure a signal with a frequency of about 3 Hz. It would be ideal to use a low-pass filter, since low frequencies would be able to pass while higher frequencies would be weakened after a certain cutoff signal, which is given by

$$f_c = \frac{1}{2\pi RC}\,. \tag{3}$$

Therefore, we select values $R = 2\Omega$ and and $C = .025F$ to achieve a maximum signal of about $3\,Hz$ by equation (3). Additionally, suppose we have a desired frequency $f_{desired} = 1\ GHz$ inside noisy frequency with noise $\geq \mp 200\ MHz$ from $f_{desired}$. In this case, we should use a combination of high- and low-pass filters to create a narrow band for the desired frequency to pass through. We can attenuate unwanted low frequencies by setting the high-pass filter to $R = (2.0\times10^{-4})\Omega$ and $C = 1\mu F$. Similarly, we can attenuate unwanted high frequencies by setting the low-pass filter to $R = (1.4\times10^{-4})\Omega$ and $C = 1\mu F$. Again, as per equation (3), this configuration would create a band with minimum frequency $0.79\,GHz$ and maximum frequency $1.2\,GHz$, thus allowing our desired frequency of 1 GHz to pass through.

3.4.2 Presentation Report

2. Introduction

This experiment serves to demonstrate and examine the basic properties of AC circuits. We explore the transient state behavior of RC, RL, and resonant RLC circuits. To accomplish this, we employ a RIGOL Waveform Generator to measure transient time τ in the RC and RL circuits, as well as resonant frequency f_{res} and quality factor Q for the RLC circuit.

For τ in the RC circuit, we create a square wave function in the RIGOL Generator. We find that the amplitude of AC voltage is $(4.0 \pm 0.1)\ V$ peak-to-peak, as indicated by the machine's display. Upon linearizing one period of the waveform, we yield the following relationship between voltage, time, current, and resistance:

$$\frac{-t}{RC} = \ln\left(\frac{v_b - v(t)}{v_b}\right). \tag{4}$$

Here, v_b represents input voltage and $v(t)$ represents the voltage of the capacitor over time. We create a linear regression model to attain the slope, from which we may use the relationship

$$\tau = RC \tag{5}$$

to solve for transient time. These calculations are made explicit in section (3) of the report.

After completing our observation of the RC circuit, we employ an oscilloscope to measure values from the RL circuit. Again, we create a waveform with the RIGOL machine. Then, we send the current through a circuit with an inductor and a resistor in series. In this configuration, the relationship

$$\tau = \frac{L}{R} \tag{6}$$

holds. Using the oscilloscope's cursor, we find τ.

We now create the RLC circuit, which uses the same inductor from the RL circuit. Thus, inductance L is constant across these two configurations. From this information, we calculate f_{res} via equation (1). At this point, we determine the frequency that maximizes gain with two methods. In the first method, we have the waveform generator create a sine curve around f_{res} such that amplitude is maximized and the circuit is connected to our oscilloscope. We observe the frequency which corresponds to the maximum voltage, which in turn yields maximum gain ($\left|\frac{v_{out}}{v_{in}}\right|$). In the second method, we connect our circuit to the myDAQ configuration and use the BODE analyzer software to plot frequency vs. gain over the range $[20\ Hz, 20\ kHz]$. Once more, we observe the frequency which corresponds to the maximum gain.

3. Analysis

To find τ in the RC circuit, we linearized voltage across the capacitor and plotted a regression model of the linearization vs. time. We began with the formula:

$$v_c(t) = v_b(1 - e^{\frac{-t}{RC}}) \tag{7}$$

where v_b is input voltage and v_c represents voltage across the capacitor. Equation (7) can be rearranged to form equation (4), which follows the log-linear form:

$$\ln(y) = mx + b. \tag{8}$$

Here, m is given by $\frac{-1}{RC}$, y is given by $\frac{v_b - v(t)}{v_b}$, x represents t, and $b = 0$.

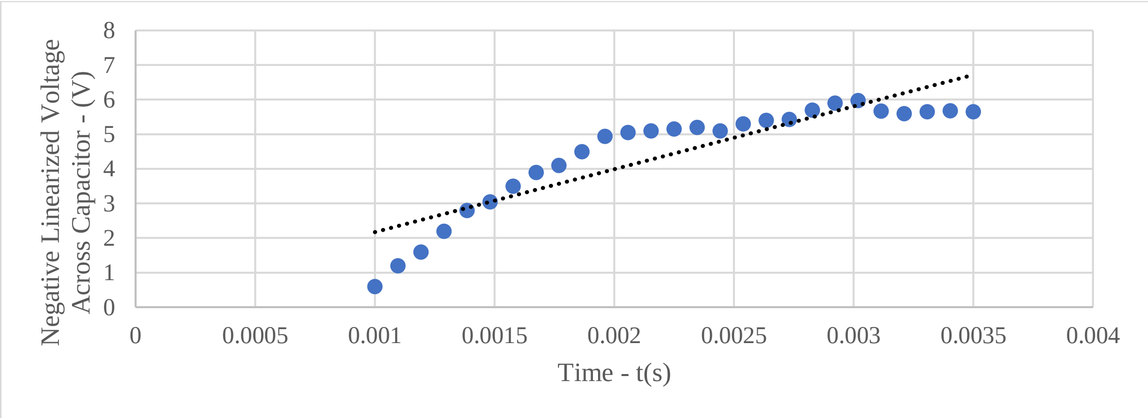

Figure 1: Negation of linearized voltage representing transient time τ for an RC circuit. As per a Microsoft Excel regression analysis, the slope is given by $(1800 \pm 200)Hz$ and the y-intercept is $(0.2 \pm 0.4)V$. Thus, $\ln(y) = \left((1800 \pm 200)Hz\right)x + (0.2 \pm 0.4)V$. This y-intercept encompasses the expected y-intercept of $0V$, so the calculation is reasonable.

From Fig. (1), we calculated the inverse of the slope m to obtain a best value for transient time: $\tau = (0.00055 \pm 0.00006)\ s$. The margin of error is given by the formula for relative error, which in this case is:

$$\frac{\delta\frac{1}{RC}}{\frac{1}{RC}} = \frac{\delta\tau}{\tau}. \tag{9}$$

We also used equation (5) to obtain a second calculation for transient time. This yielded $\tau = (0.00049 \pm 0.00002)\ s$, given a resistor of $(0.500 \pm 0.005)\ k\Omega$ and a capacitor of $(1.00 \pm 0.05)\ \mu F$. The uncertainty for τ in this calculation is given by:

$$\delta_\tau = \sqrt{(R\delta C)^2 + (C\delta R)^2}. \tag{10}$$

For the RL circuit, a result for τ was given by the oscilloscope. After creating a square wave function of peak-to-peak amplitude $(4.0 \pm 0.1)\ V$, we sought to determine where voltage across our resistor of $(0.09930 \pm 0.0005)\ k\Omega$ was 63.2% of the amplitude. This result was determined to be $(2.51 \pm 0.04)\ V$ as per the oscilloscope's cursor function. Transient time at this location was given by $\tau = (52.001 \pm 0.005)\ \mu s$. From equation (6), we calculated inductance to be $L = (0.05161 \pm 0.00003)\ H$. The uncertainty for this result is given by the formula

$$\delta L = \sqrt{(T\delta R)^2 + (R\delta T)^2}. \tag{11}$$

We calculated f_{res} for our RLC circuit with equation (1) using the same values for resistance and inductance from the RL circuit. Capacitance was given by $C = (1.00 \pm 0.05)\ \mu F$. This resulted in a best value of $f_{res} = 700.4\ Hz$, which we will use as the accepted value in error calculations for two alternate methods of determining f_{res}.

For the first method, which will henceforth be referred to as method 1, we created a sine wave on the RIGOL generator with maximum amplitude and a frequency $700\ Hz$. We then adjusted the frequency such that we could determine the maximum possible value of V by eye, which equates to maximum gain. From method 1, we found $f_{res} = (570 \pm 30)\ Hz$.

For method 2, we employed the myDAQ system and the BODE software to generate a graph of frequency vs. gain for the RLC circuit over the range $[20\ Hz, 20\ kHz]$.

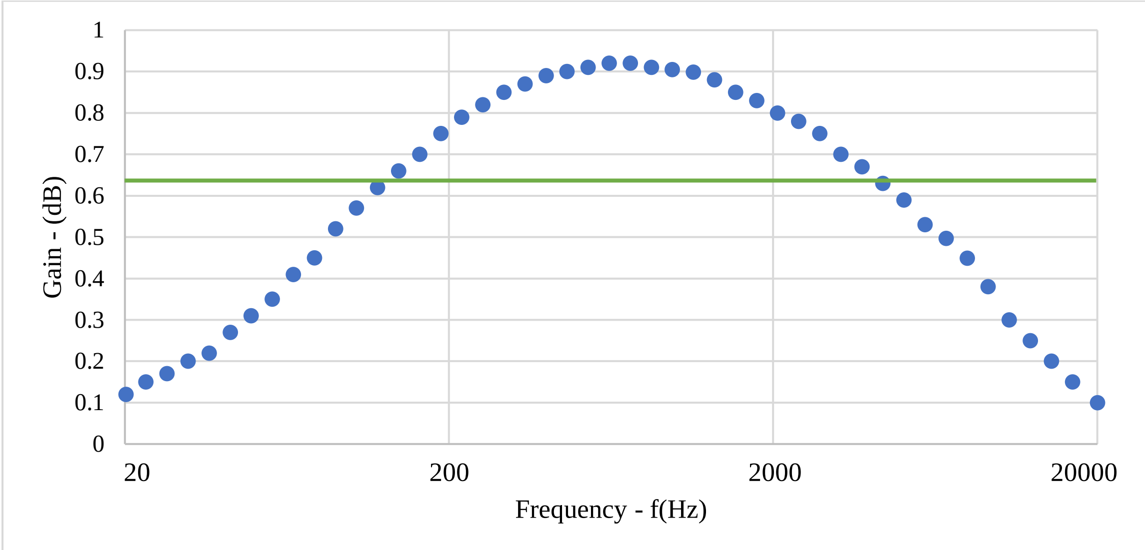

Figure 2: Sine wave on RIGOL generator representing gain on a resonant RLC circuit. Visually, f_{res} is demonstrated by the peak of the wave on the range $[20\ Hz, 20\ kHz]$. This maximum value occurs at $(510 \pm 50)\ Hz$. The gain reaches $0.910\ dB$ at this point. The green horizontal line intersects the wave where $v_{out} = \frac{v_{max}}{\sqrt{2}}$, at the points $f_1 = (140 \pm 5)\ Hz$ and $f_2 = (3400 \pm 100) Hz$.

The sine wave generated by this method is shown in Fig. (2) – it resulted in a resonant frequency value of $f_{res} = (510 \pm 50)\ Hz$, which is further from the accepted value of $700.4\ Hz$ than the result from method 1. To determine a numerical score for the precision of each method, we use the percent error formula:

$$\%\ error = \left|\frac{experimental - theoretical}{theoretical}\right| * 100. \qquad (12)$$

This calculation delivers the following results:

Method 1: $error = 18.6\%$

Method 2: $error = 27.2\%$.

Therefore, we conclude that method 1 was 8.6% more accurate in determining the true value of f_{res} as compared to method 1.

Before the experiment, we hypothesized the opposite would be true – method 2 seemed to be more accurate than method 1 because it relied more on technology than the human eye. However, method 2 still required a final frequency estimate to be made by eye, so a larger inaccuracy in this reading compared to those in method 1 may have contributed to a larger error result. Regarding this possibility, note the Q-factor of the RLC circuit as given by equation (2), which is approximately $Q = 0.22$. This result indicates that the system had a low quality factor (defined as $Q < 0.5$) and was thus overdamped; meaning the peak of the sine wave was not sharp enough to produce a clear (by eye) value for f_{res}.