Economic Modelling 64 (2017) 312–321

Contents lists available at ScienceDirect

Economic Modelling

journal homepage: www.elsevier.com/locate/econmod

Catching the curl: Wavelet thresholding improves forward curve modelling

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ARTICLE INFO

Keywords:
Forward curve
Futures prices
State-space
Kalman filter
Wavelets
Commodities

ABSTRACT

Modelling futures term structures (price forward curves) is essential for commodity-related investments, portfolios, risk management, and capital budgeting decisions. This paper uses a novel strategy, wavelet thresholding, to de-noise futures price data prior to estimation in a state-space framework in order to improve model fit and prediction. Rather than de-noise the raw data, this method de-noises only wavelet coefficients linked to specific timescales, minimizing the amount of information that is accidentally removed. Our findings are that, for the first five futures maturities in our sample data, in-sample (tracking) and 5-day-ahead out-of-sample (forecasting) Root Mean Squared Errors (RMSEs) are smaller both (i) when we increase the number of factors from one to four, and (ii) when we de-noise the data using wavelet thresholding. The improvement due to wavelet thresholding is often greater than the improvement from adding one more factor to the model, which is important because going beyond four factors does not improve model fit. Wavelet-based de-noising thus has the potential to improve considerably the estimation of various economic time series models, helping practitioners and policymakers with better forecasting and risk management.

1. Introduction

Exchange-traded futures contracts and over-the-counter forward contracts have long been essential instruments for price discovery and risk management (Tomek and Peterson, 2001; Williams, 2001), and their importance has only increased since 2000. Indeed, the trade volume and notional value of commodity forward and futures contracts has increased substantially over the period 2003–2008, sometimes referred to as the "financialization" of commodities, with investment inflows rising from very small amounts to about \$250 billion (Irwin and Sanders, 2011; Cheng and Xiong, 2013).

Futures contracts with liquid volume are traded for a large number of maturities, in many cases every month for the first year ahead and at a lower frequency for up to five years into the future. For a given commodity, such as crude oil or corn, this constellation of futures price quotes is called forward curve or futures term structure. It represents aggregate trader information about price expectations and market participant risk aversion (e.g., Schwartz, 1997).[1]

Practitioners and policymakers have great interest in better understanding the entire forward curve or futures term structure (see e.g. Benth et al., 2007; Cortazar et al., 2016; Lautier, 2005), but most academic research focuses on studying the nearby or front-month contract price. Improving the modelling of commodity forward curves, in particular, is essential to traders and portfolio managers, and also matters for the capital budgeting and risk management decisions of corporate firms–especially those involved in oil and gas or in other commodities (e.g. Geman, 2009). Indeed, such firms are known to use strategies such as "pricing against the forward curve", which is essentially valuation using certainty-equivalent cash flows (e.g. Titman and Martin, 2014). This is because, for purposes of valuation, futures or forward prices can be used instead of forecasted spot prices to obtain certainty-equivalent cash flows, which are then discounted at the risk-free rate instead of a risk-adjusted rate such as the cost of equity from the CAPM. Even if a firm does not hedge using futures, this approach provides the correct valuation for a commodity investment project.

Thus, the in-sample tracking and out-of-sample forecasting of commodity forward curves is an important and challenging problem in economic modelling, and it has significant practical ramifications (e.g., Cortazar et al., 2016; Cortazar et al., 2015; Schwartz and Smith,

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[1] Indeed it is well understood that futures prices represent expected future spot prices, but under the risk-neutral probability measure (e.g. Cox and Ross, 1976; Harrison and Kreps, 1979).

http://dx.doi.org/10.1016/j.econmod.2017.03.032
Received 14 February 2017; Received in revised form 27 March 2017; Accepted 30 March 2017
Available online 20 April 2017
0264-9993/

2000). This paper's contribution is to show how to improve this modelling using a new approach based on wavelet thresholding, which is then applied to data on commodity futures contract prices traded at the Chicago Mercantile Exchange (formerly Chicago Board of Trade contracts).

Indeed, the better we can separate the signal from the noise in time series data, the more useful the data becomes for making predictions. This explains why there is a large literature concerned with de-noising data. Wavelet thresholding (Donoho and Johnstone, 1994a, 1994b; Donoho,1995) is a de-noising method that has been mostly overlooked by economists. But what is innovative about it is that instead of de-noising the raw data, it de-noises the data's wavelet coefficients at the finest time scale, which reduces the amount of information that is accidentally removed. The intermediate step (see the Appendix) involves applying to the raw data a discrete wavelet transform prior to de-noising, and then applying an inverse wavelet transform to the de-noised wavelet coefficients, resulting in efficiently filtered data that can be used for estimation.

This paper demonstrates the potential of wavelet thresholding by improving the modelling of forward curves, using a multi-factor price model with several correlated sources of risk. For our sample, de-noising the data this way improves the tracking and forecasting results in most cases, suggesting that the approach should be seriously considered by commodity investment decision-makers, whether for investments, risk management, or capital budgeting.

The economic intuition behind wavelet thresholding in our setting of futures contracts is that price variations occurring below some threshold is only noise. Indeed if variation contributed to the price signal, it should be linked to longer time scales. This reasoning is similar to the argument made by Hasbrouck (2013) who shows how to use wavelet variance decomposition to identify and measure micro-structure volatility and noise. Since price changes contain both information and noise, filtering out the noisy portion prior to fitting the model will improve the efficiency of the estimation. Our empirical results confirm this.[2]

It is fair to ask whether what we call “noise” may in fact be economic news. However, there are at least two strong reasons why it is worth finding better ways to “de-noise” price data prior to fitting a forward curve model. First, there is also an entire literature based on Shiller (1981) that argues (theoretically and empirically) that traders over and under-react to information. So there is strong reason to believe that price changes contain noise and, consequently, increasing the signal/noise ratio is helpful. Thus, if it improves forecasting it must be increasing the signal/noise ratio. Second, the wavelet thresholding approach we propose removes only the component that has a one-day horizon (i.e., the one-day horizon wavelet function). The previous literature (e.g. Hasbrouck, 1991) shows that price changes are least partially noise, and that price changes caused by the arrival of new trades are partly noise and partly information. Our hypothesis is that the portion of price changes that is noise can be identified using the wavelet one-day horizon (timescale) and it is then is removed. Meanwhile the part that is information is identified by the wavelet two-day horizon or longer timescales, and these components of the data are not removed. Thus, we plausibly filter out noise but not useful information.

2. Literature review

2.1. The term structure of commodity futures prices

The futures price F_t for a given date t and maturity T equals the time t expectation of the spot price S_T at maturity T under the risk-neutral probability measure Q (Black, 1976; Cox et al., 1981, 1979; Harrison and Kreps, 1979). It is well understood, therefore, that the futures price is a risk-adjusted forecast of spot price and thus it reflects both the market's expectations as well as a risk adjustment.

$$F(x_t, t, T) = \mathrm{E}_t^Q(S_T) \tag{1}$$

In a simple model of the forward price curve for storable commodities, the following relationship holds at all times:

$$F(t, T) = F(t, t)\exp^{(r+c-\delta)(T-t)} \tag{2}$$

where *F(t,t)* is the futures price for a contract expiring today (i.e., equal to the spot price notwithstanding basis risk), r is the risk-free rate of interest (e.g. 3-month U.S. Treasury bill), c is the cost of carry and δ is the convenience yield. The shape of the forward curve depends only on the net convenience yield: r+c-δ. If $r+c>\delta$, contango results, and if $r+c<\delta$, backwardation results. The convenience yield represents the economic value of holding physical stocks of a commodity, e.g. the benefits of holding inventories to maintain a smooth running commercial operation and avoiding the risk of stock-outs. This definition has, however, been debated in the literature (e.g. Brennan et al., 1997; Williams, 2001). This concept provides a useful way to link commodity inventory levels with the shape of the forward curve. An example of a commodity futures price term structure is presented in Fig. 1 for Chicago Board of Trade corn futures on 6/17/2004, a period when the market was in contango. For some energy and agricultural commodities, the forward curve is also affected by seasonal cycles (Tomek, 1997, 2000; Fackler and Roberts, 1999).

Under the risk-neutral measure, asset price dynamics imply the following relationship (e.g. Fackler and Roberts, 1999):

$$\mu + \delta = r + \sigma\lambda \tag{3}$$

where μ is the actual drift term, δ is the convenience yield, r is the risk-free rate of interest, σ is the diffusion term, and λ is the market price of risk for the state variable in question. The equation may be rearranged to give:

$$\mu - \sigma\lambda = r - \delta \tag{4}$$

which implies that the risk-adjusted drift μ - σλ equals the risk-free rate minus the convenience yield, r - δ. The convenience yield can be estimated, because the left-hand side parameters are estimated from the data using the above model (2), while the 3-month US Treasury bill provides a good proxy for the risk-free interest rate. For multi-factor models as the one described below, additional parameters must be incorporated in the equation, but the approach is similar. .

The most popular approaches used to model the term structure of futures prices are called “reduced-form” and describe the stochastic process generating the futures price term structure using a small number of “factors” each of which is described by a stochastic process.[3] The first approach aims to estimate the unobservable convenience yield of a real or financial asset, which helps explain the shape of the forward curve (e.g., Brennan and Schwartz, 1985; Gibson and Schwartz, 1990). The second approach, which is more general, models the asset price as an affine function of state variables, which are usually unobservable (e.g., Schwartz and Smith, 2000). We therefore use this more general approach to model the forward curve.

2.2. Wavelet-based methods for economic time series

Our main contribution to the literature is to show how wavelet thresholding, applied to the futures price data prior to estimation of the forward curve model, improves the tracking and forecasting perfor-

[2] Although this study is the first to our knowledge to use this empirical strategy, it is worth noting that Haven et al. (2012) use other wavelet methods to de-noise option prices.

[3] An entirely different approach, called “structural”, specifies stochastic processes separately for supply and demand side shocks (see e.g. Pirrong, 2011). This approach is computationally more challenging.

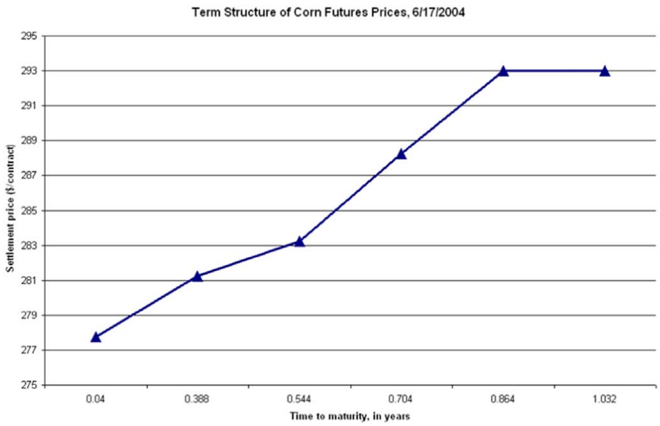

Fig. 1. An example of the term structure of futures prices: daily settlement prices for six nearest maturities, Chicago Board of Trade (CME) corn futures on 6/17/2004.

mance of the model. This method is used to filter out variation below a specific threshold, under the assumption that it contains noise of no economic significance (see e.g. Hasbrouck, 2013). Since wavelets provide an orthogonal decomposition of variance, filtering out this zero-mean variation should result in better model fitting.

It is important to discuss in what ways filtering using wavelets differs from other approaches. Indeed, filters have been widely used in some areas of economics. For example, two popular macroeconomic filters are the Hodrick-Prescott (1980, 1997) filter and the Baxter and King (1999) bandpass filter.

Wavelet thresholding is based on the concept of wavelet variance, which is a statistical method to decompose variance into several additive, orthogonal components, each linked to a specific time scale or time (e.g. Percival and Mondal, 2012; Percival and Walden, 2000). Wavelet methods have been used in numerous economic and financial applications, most commonly to better understand variances and covariances in economic time series. Applications include the hedging effectiveness of futures contracts, financial market integration, volatility spillovers, price or variance comovement between asset classes, and measuring systematic and unsystematic risk.[4] However, to our knowledge no economics study has used wavelet thresholding, and neither has any study tried to improve modelling forward curves using wavelet methods.

Wavelet thresholding or shrinkage (Donoho and Johnstone, 1994a, 1994b; Donoho, 1995) has proven to be a remarkably efficient and accurate method to remove noise and recover the true signal from data in engineering and physical sciences applications. It involves applying a filtering rule not to the raw data, but rather to the wavelet coefficients computed from the data. After applying the thresholding rule, the filtered time series data are recovered from the "thresholded" wavelet coefficients. While the algorithm is most powerful against IID white noise, properly adjusted it provides excellent results when the noise is a dependent and non-IID stochastic process.

There exist a wide variety of filtering methods other than wavelet-based. Two important classes of filters are sinusoidal (Fourier) and polynomial knot (spline) smoothers. These methods, however, have been found to systematically either remove too little or too much noise. The result is a recovered signal that is either over-smoothed or still too noisy to be informative about the true data generating process. In contrast, wavelet thresholding has been found to provide a powerful signal recovery without over-smoothing. In particular, features of the data that are sharp remain so after wavelet thresholding, while previously existing methods tend to dull such sharp features. This is because wavelets have been designed to provide optimal information compression and efficient transformation (e.g., Donoho and Johnstone, 1994a, 1994b; Donoho, 1995; Donoho et al., 1995, 1996).

Unlike the Fourier transform, which is unique, there are numerous possible wavelet transforms. It is reasonable to ask: why favor a wavelet transform over a Fourier transform? There are several reasons that are relevant for this study. First, wavelet transforms are localized in both time and frequency, while the Fourier transform is only localized in frequency (see e.g. Percival and Walden, 2000; Percival and Mondal, 2012). Indeed, if the signal in prices changes with time (as it surely does), the Fourier transform wouldn't inform us as to when the change occurred, while the wavelet transform would, thus improving the resulting signal/noise ratio. Therefore, wavelet transforms are more suitable when prices are potentially non-stationary or irregular (spiky, non-linear), which is a better characterization of commodity prices in particular and asset prices more generally. We believe that it is important to allow for the possibility that the importance of different signals (short-run, medium-run, and long-run) changes over time, especially considering the nearly 30 year sample period we are using in this paper.

Second, the Fourier transform is only applicable to stationary time series, while wavelet transforms are applicable to both stationary and nonstationary time series. Third and last, although it is true that there are many possible wavelet transforms, we carefully justify why we choose the Daubechies Least Asymmetrical wavelet function based on the properties of different wavelets and the properties of the economic time series data we are analyzing (see the Appendix). Essentially, there are four desirable properties we are looking for, especially for economic applications, namely a nonzero number of vanishing moments; compact support; orthogonality and orthonormality; and linear phase. These properties are best respected using the Daubechies Least Asymmetrial wavelet, which is the one used in the paper.

3. Theoretical model

3.1. Forward curve models

Following the literature on commodity futures, we assume that the log of the spot price is an affine function of N different state variables as well as a deterministic cyclical function. The latter captures seasonal variation in commodity supply or demand, or in both. Thus, the logarithm of the spot price P_t is the sum of three components, namely a cyclical function s(t), and two state variables x_t and z_t:

$$\ln(P_t) = s(t) + x_t + z_t \tag{5}$$

The dynamics of each state variable is described by a stochastic differential equation that is solved using the Feynman-Kac partial differential equation approach (Black and Scholes, 1973; Black, 1976; Cortazar and Schwartz, 2003; Routledge et al., 2000). State variables are often unobservable, but in some models they include the spot price or interest rate.[5] The general multivariate stochastic differential equation may be written as follows (see e.g. Cortazar and Naranjo, 2006), where x_t is the state variable, K is a matrix of drift terms (such as mean-reverting parameters), Σ is a matrix of diffusion terms and w_t is a

[4] Since the early contributions by Ramsey et al. (e.g. Ramsey and Zhang, 1997), the literature on wavelets in economics and finance has grown substantially (e.g., Aloui and Hkiri, 2014; Benhmad, 2012; Berger and Uddin, 2016; Conlon and Cotter, 2012; Crowley and Hallett, 2015; Crowley and Hudgins, 2015; Fernandez, 2005; Gallegati and Ramsey, 2013; Gallegati et al., 2014; Gallegati and Semmler, 2014; Gençay et al., 2001, 2003, 2005; Graham and Nikkinen, 2011; Kim and In, 2005, In and Kim, 2006; Kiviaho et al., 2014; Lehkonen and Heimonen, 2014; Marczak and Gómez, 2015; Martín-Barragán et al., 2015; Nachane and Dubey, 2011; Nikkinen et al., 2011; Ramsey and Zhang, 1997; Ranta, 2013; Reboredo and Rivera-Castro, 2013, 2014; Rua and Nunes, 2009, 2012; Sun et al., 2011; Uddin et al., 2013; Vacha and Barunik, 2012). For an excellent overview, see Gençay et al. (2001).

[5] Casassus and Collin-Dufresne (2005) enrich this model by incorporating stochastic interest rates and time-varying risk premia. This paper, however, does not adopt their model because previous research has found that (at least for agricultural commodity futures) interest rate risk has little impact on the model.

Brownian motion (Wiener process).

$$dx_t = -Kx_t dt + \Sigma dw_t \tag{6}$$

$$s_t = \sum_{k=1}^{K} \gamma_k \cos(2\pi kt) + \tilde{\gamma}_k \sin(2\pi kt) \tag{7}$$

In the literature, the canonical approach to capturing the cyclical variation due to seasonality is to estimate the time-varying deterministic function (7), which is assumed identical for any given day in a calendar year (Hannan et al., 1970). This is a function of sines and cosines and thus resembles a truncated Fourier series, but the purpose is not to translate the data into the frequency domain. Rather, this serves a purpose analogous to removing a trend, except in this case we are capturing an oscillating signal, so we can better estimate the state variables (factors). Therefore the best approach is to use a periodic function. In Eq. (7), each coefficient $\gamma_k$and $\tilde{\gamma}_k$ (for k=1, 2, ..., K) takes a constant value which must first be estimated from the data. Each pair of coefficients is linked to a specific cycle in seasonality. The first coefficients, $\gamma_1$and $\tilde{\gamma}_1$, are linked to a cycle that repeats every year. The coefficients for k=2 are linked to a cycle that repeats twice a year, and so forth. Although we could select a large K, according to Sorensen (2002) it is not necessary, as he shows that K=2 is sufficient to capture seasonal cycles in agricultural commodity prices. Therefore we also choose K=2.

We use standard no-arbitrage arguments to solve the spot-futures price relationship and obtain an analytical solution to the futures price as an affine function of the state variables and cyclical function (see e.g. Cortazar and Naranjo, 2006).[6] In this model, the first state variable is a geometric Brownian motion process, while the remaining (N−1) state variables are mean-reverting (Ornstein-Uhlenbeck) processes.[7] Previous research has found that one-factor models (i.e., N=1) typically perform poorly for commodity futures prices, whether the state variable is geometric Brownian motion or Ornstein-Uhlenbeck.

Although commodity price data are mean-reverting over long periods of time, there is also evidence of slow, gradual permanent changes caused by shifts in commodity demand or in technological improvement (e.g., tar sands and shale gas for crude oil). Therefore, the first state variable is defined as geometric Brownian motion, which is non-stationary and represents permanent changes caused for example by economic shocks in technology and preferences:

$$dx_1(t) = \mu x_1(t)dt + \sigma_1 x_1(t)dw_1(t) \tag{8}$$

This state variable is associated with a long run drift term μ, a risk premium λ_1 and a diffusion σ_1. The effect of time-to-maturity is captured by a risk-adjusted drift defined as:

$$\alpha = \mu - \lambda_1 + \frac{1}{2}\sigma^2 \tag{9}$$

State variables x_2 through x_N are defined as Ornstein-Uhlenbeck processes, each with a different speed of mean-reversion captured by κ_n and the long-run mean to which the process is drawn is C:

$$dx_n(t) = -\kappa_n(x_n(t) - C)dt + \sigma_n dw_n(t) \tag{10}$$

The Brownian motions are assumed to be pairwise correlated through a coefficient ρ_{ij}. The term structure of futures price volatility is obtained using the estimated diffusion and correlation parameters:

$$\sigma_F^2(T-t) = \sum_{i=1}^{N} \sum_{j=1}^{N} \sigma_i \sigma_j \exp^{-(\kappa_i+\kappa_j)(T-t)} \tag{11}$$

Note that for the one-factor model, the term structure of volatility reduces to σ^2 and is constant for any time-to-maturity (i.e., it is the Black-Scholes solution). This feature is generally considered to be a poor description of observed data. In contrast, for two or more state variables, volatility has a term structure that is dependent on time to maturity and thus better reflects the data, e.g. Samuelson's hypothesis whereby the volatility term structure is decreasing in time-to-maturity. Overall, the empirical literature has found that it is difficult to improve on a three-factor model (e.g. Cortazar and Schwartz, 2003).

3.2. De-noising futures data using wavelet thresholding

The objective of wavelet thresholding is to determine an optimal value (threshold) using a clear criterion, such as a loss function or minimum risk value (Stein, 1981). Both a threshold choice and a thresholding rule must be carefully selected. First, before using the threshold, a Discrete Wavelet Transform is applied to the data to produce a vector or matrix of wavelet coefficients (see the Appendix for more details). The threshold is used together with the wavelet coefficients. Then, we apply an Inverse Discrete Wavelet Transform (inverse operation of the previous transform) to the filtered or de-noised wavelet coefficients, yielding a filtered version of the original time series with no loss of information (other than the filtering itself). Donoho and Johnstone (1994a, 1994b, 1995) show that a "universal threshold", together with a soft thresholding rule, are both asymptotically optimal and also remarkably robust when used in empirical applications. The universal threshold, assuming a variance of innovations (errors) σ_ϵ^2 and a number of observations T is given by:

$$\delta = \sqrt{s\sigma_e^2 \ln(T)} \tag{12}$$

and the soft thresholding rule applied to wavelet coefficients w is:

$$w^{soft} = \text{sgn}(w)\left(\frac{1}{2}(|w| - \delta + |(|w| - \delta)|)\right) \tag{13}$$

Since the true variance of the innovations is unknown, a mean absolute deviation estimate can be computed as the ratio of the median of wavelet coefficients at the finest timescale over a normalization factor that has been found to be optimal:

$$\hat{\sigma}_{MAD} = \frac{median(w^{j=1})}{0.6745} \tag{14}$$

4. Estimation of the futures term structure model

The family of forward curve models described above can be estimated through specification as hidden component models. These models are commonly used and particularly well suited to estimation by the state-space approach (Durbin and Koopman, 2001).[8] In this class of models, potentially unobservable or latent state variables are estimated together with the model parameters using available data. The standard method is to first derive a reduced form of the theoretical relationship, which will then be estimated in a state-space framework. This reduced form is estimated using the Kalman filter relating the measurement equation, for which the dependent variable is observable, to the transition equation, for which the dependent variable is usually unobservable. To improve convergence, we use Durbin and Koopmans's (2002) exact diffuse prior (initial condition) for the Kalman transition variance, and we exclude the first few observations from the variance calculation.

Our estimation approach is close to Sorensen's (2002), but with two significant differences. First, we de-noise the price data using wavelet thresholding to remove very short-term noise that may obscure meaningful economic variation. Second, we consider not just a two-factor

[6] Cortazar and Naranjo's (2006) framework is based on the affine transformation results of Dai and Singleton (2000), which enable any model satisfying some basic assumptions to be written in a canonical Gaussian form.

[7] Korn (2005) argues in favor of including only mean-reverting state variables in the model, due to weak evidence supporting a nonstationary factor. However, our preference is to allow a gBm factor to capture possible permanent shocks to the commodity market.

[8] Our use of the Kalman filter follows previous work in this area by Schwartz (1997), Schwartz and Smith (2000), Fackler and Roberts (1999), Sorensen (2002), and Korn (2005).

Table 1
Descriptive statistics. This table presents descriptive statistics for the daily time series of front-month corn futures prices and log returns, 1/1988-1/2017. Source: Thomson Datastream. Notes: Log returns are computed using the first nearby futures contract. The series is spliced together by switching to the second nearby futures two weeks prior to front-month contract expiry. The ADF test has as a null hypothesis the absence of a unit root.

	Prices (c/bu)	Log returns
Mean	327.87	.0001
Median	269	0
Minimum	174.75	−.276
Maximum	831.25	.128
Std deviation	132.90	.0173
Skewness	1.53	−1.04
Kurtosis	4.57	22.69
Autocorrelation	.99	.045
ADF test p-value (10 lags)	.38	.00
Nb obs	7305	7305

model but rather several models with a number of factors ranging from one to four. While Sorensen uses all available futures contracts, we use only the first five futures at any given date. The reason is that trading volume for more distant maturities is often low, and these observations may be less reliable as the settlement prices are likely to reflect pricing algorithms used by the clearing committee rather than actual market transactions.

To ensure identification, standard cross-term covariance restrictions are applied. We do not impose restrictions on the market prices of risk, even though the literature suggests that their values are small and sometimes not significantly different from zero. We also allow for correlation between state variables, rather than impose a zero correlation restriction.

The Kalman filter is used to estimate the quasi-maximum likelihood parameters of the state-space model of futures prices. The two most important issues in this estimation problem are solving the reduced form identification problem and providing the Kalman filter with sensible starting values. The identification problem in this case is how to recover structural model parameters from the estimated reduced form model.[9] As for starting values, we initialize the procedure using the estimates found in Sorensen (2002). The state-space model is based on a transition (state) equation and a measurement equation. For each time series date $t = \{1,2,3,...,T\}$, the transition equation is: $X_{t+1} = a + AX_t + \eta_{t+1}$

$$X_{t+1} = a + AX_t + \eta_{t+1} \tag{15}$$

where, for the case of three state variables we have:

$$a = \left(\mu - \frac{1}{2}\sigma^2, 0, 0, 0\right)^T$$
$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-\kappa_2 \Delta} & 0 \\ 0 & 0 & e^{-\kappa_3 \Delta} \end{pmatrix} \tag{16}$$

and the covariance matrix of the state variable innovations, from which are derived parameter identifying restrictions, is:

$$\Omega = \begin{pmatrix} \sigma_1^2 \Delta & \frac{\rho_{12}\sigma_1\sigma_2}{\kappa_2}(1 - e^{-\kappa_2 \Delta}) & \frac{\rho_{13}\sigma_1\sigma_3}{\kappa_3}(1 - e^{-\kappa_3 \Delta}) \\ \frac{\rho_{12}\sigma_1\sigma_2}{\kappa_2}(1 - e^{-\kappa_2 \Delta}) & \frac{\sigma_2^2}{2\kappa_2}(1 - e^{-2\kappa_2 \Delta}) & \frac{\rho_{23}\sigma_2\sigma_3}{\kappa_2+\kappa_3}(1 - e^{-(\kappa_2+\kappa_3)\Delta}) \\ \frac{\rho_{13}\sigma_1\sigma_3}{\kappa_3}(1 - e^{-\kappa_3 \Delta}) & \frac{\rho_{23}\sigma_2\sigma_3}{\kappa_2+\kappa_3}(1 - e^{-(\kappa_2+\kappa_3)\Delta}) & \frac{\sigma_3^2}{2\kappa_3}(1 - e^{-2\kappa_3 \Delta}) \end{pmatrix} \tag{17}$$

where Δ is an increment in the unit of time. In this case, it is 0.04, which is the ratio of one business day over one year (250 business days). The covariance matrix for the case of four state variables follows naturally from the above three-variable matrix. The measurement equation for five maturities, where Y_t is a vector of length five at each point in time, is: $Y_t = c_t + C_t X_t + \varepsilon_t$

$$Y_t = c_t + C_t X_t + \varepsilon_t \tag{18}$$

such that:

$$c_t = s(t) + (\mu + \lambda_1 - 0.5\sigma^2)(T^{(1)} - t), \quad \cdots, s(t) + (\mu + \lambda_1 - 0.5\sigma^2)(T^{(5)} - t), \tag{19}$$

$$C_t = \begin{pmatrix} 1 & e^{-\kappa_2(T^{(1)}-t)} & e^{-\kappa_3(T^{(1)}-t)} \\ \vdots & \vdots & \vdots \\ 1 & e^{-\kappa_2(T^{(5)}-t)} & e^{-\kappa_3(T^{(5)}-t)} \end{pmatrix} \tag{20}$$

and $\sum_t$ is distributed IID Normal with mean zero and covariance $\sigma_\varepsilon^2 \mathbf{I}_t$. The Kalman filter is initialized with starting values for the state variables and covariance, and then computes one-step ahead forecast errors between forecast and actual observations. The exact diffuse prior of Durbin and Koopman (2002) is used to improve the behavior of the transition covariance matrix.[10]

5. Data

We provide an empirical application to corn futures, which are the most actively traded agricultural contract and one of the most traded among all futures. This futures contract is ideally suited as an empirical application for the proposed methodology, given well-documented seasonality and inventory effects which complicate the forward curve over time. This futures contract is traded at the Chicago Mercantile Exchange (formerly Chicago Board of Trade). Daily futures prices for the first five maturities are collected from Datastream over the period of 2/1988 to 2/2017. Over this time period, the forward curve has been both in contango and backwardation.

We present descriptive statistics in Table 1 and a time series plot of the front-month futures price in Fig. 2. Table 1 shows that corn futures prices are nonstationary, right-skewed, and slightly leptokurtic, while futures log returns are stationary and only weakly autocorrelated (first lag statistically but not economically significant), left-skewed, and strongly leptokurtic. The normality of log returns is rejected, but this is neither a problem for wavelet thresholding nor for the state-space estimation, which uses quasi-MLE.

There are 7305 daily observations for each maturity. Observations for 19 October 1987 ("Black Monday") and 11 September 2001 are treated as outliers and are removed from the sample.[11]

6. Results for the one-factor to four-factor models

We estimate the state-space model of the term structure of futures prices (15)–(20) using daily log prices, in the previously described

[9] As explained by Roberts and Fackler (1999), the complete model of the term structure of futures prices for agricultural commodities is over-parameterized, i.e., under-identified. Thus, there is not a unique solution to the estimation problem.

[10] Further estimation details are provided here. Linear ARMA full-information estimation by state space is done in R. Constrained optimization procedures are mainly done in Matlab. Hidden component state space model estimation using the Kalman filter is done mainly in RATS using the DLM procedure, with NONLIN parameter description and constraints and optimization criteria set by NLPAR. Optimization routines are SIMPLEX for the first approximation and BFGS for the actual solution in order to obtain standard errors for the parameters. 200 iterations and 100 sub-iterations are allowed for the BFGS, and up to 5000 trials for the SIMPLEX method. The EXACT diffuse initial conditions of Durbin and Koopman (2002) are used to control the behavior of the non-stationary component of variance in the Kalman filter procedure. The Kalman gain matrix variance is assumed to be scaled proportional to the system variances. The wavelet threshold filtered data contain 16 unfiltered observations at the beginning and end of the sample because the initial and final filtered observations may be affected by boundary effects caused by the wavelet transform.

[11] The measurement unit of corn futures contract positions at the Chicago Board of Trade changed on January 1st 1998 from thousands of bushels to number of contracts, each of which equals five thousand bushels. To ensure consistency in the time series, observations before January 1st 1998 are divided by five, so the unit of measurement is the number of contracts.

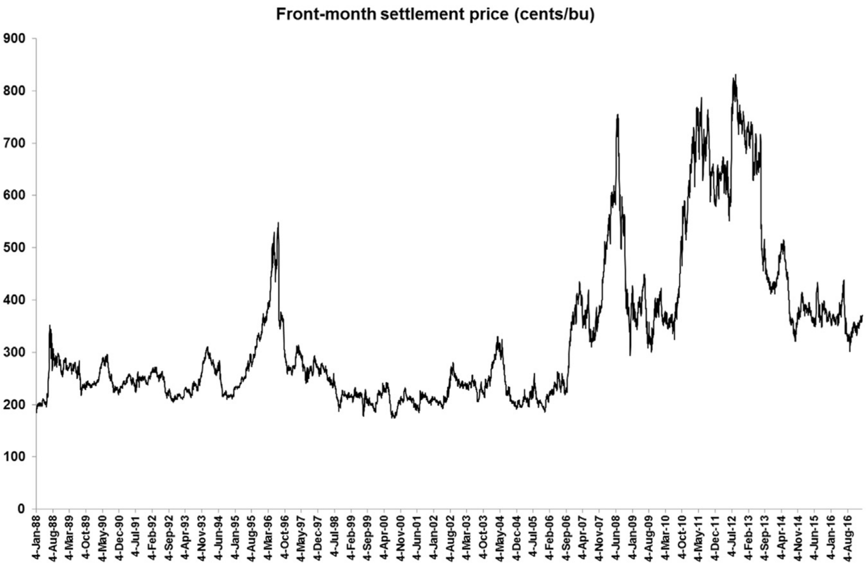

Fig. 2. Daily time series plot of front-month corn futures settlement price, 1/1988-1/2017. Data source: Thomson Datastream.

Table 2
Estimated state-space models for the forward curve, This table presents the estimation results from one- to four-factor models of the term structure of futures prices, using Chicago Board of Trade (CME) corn futures five nearby maturities, from 2/1988 to 1/2015. Results are provided for both full sample and wavelet-filtered sample data. Notes: All parameter estimates are individually significant at least at the 5% level. Source: Authors' calculations using data collected from Thomson Datastream. See footnote 8 for additional details of the state-space estimation procedures.

	One-factor model		Two-factor model		Three-factor model		Four-factor model	
	original	de-noised	original	de-noised	original	de-noised	original	de-noised
μ	.0053	.0048	.0047	.0041	.0091	.0039	.0048	.0013
κ2	.	.	.144	.096	1.06	.074	1.27	2.19
κ3	.	.	.	.	1.18	2.65	.634	2.24
κ4	.	.	.	.	.	.	.49	2.21
σ1	.13	.11	.23	.11	.17	.15	.12	.18
σ 2	.	.	.045	.042	.038	.041	.14	.23
σ 3	.	.	.	.	.087	.036	.092	.19
σ 4	.	.	.	.	.	.	.031	.18
λ1	.014	−.0085	−.102	−.131	−.207	−.196	−.068	.026
λ2	.	.	.064	.163	.129	.168	.071	.017
λ3	.	.	.	.	.136	.145	.048	.029
λ4	.	.	.	.	.	.	.092	−.056
ρ12	.	.	−.27	.21	−.0208	−.0124	.72	.93
ρ13	.	.	.	.	−.763	−.586	.38	.91
ρ14	.	.	.	.	.132	.81	.26	−.90
ρ23	.	.	.	.	.	.	−.92	.93
ρ24	.	.	.	.	.	.	−.71	.89
ρ34	.	.	.	.	.	.	.42	.94

Kalman filter framework and using the RATS software. The previous literature has found state-space estimation is improved by using log prices instead of prices (e.g., Schwartz, 1997). We begin by reporting in Table 2 the baseline values of the parameters estimated for each of the four multi-factor models, both using the original futures price data (panel 1) as well as using the de-noised data (panel 2).[12] For one to four factors, the number of estimated parameters is, respectively, 3, 7, 12 and 18. This implies that the computational burden grows substantially as the number of factors increases. The simplest model nested in the Gaussian *N*-factor framework considers the log of futures prices to be an affine function of one non-stationary state variable in addition to parametric terms:

$$\ln F(t, T) = \mu t + \left(\mu - \lambda + \frac{1}{2}\sigma^2\right)(T-t) + s(t) + x_t + \varepsilon_t \tag{21}$$

$$x_t = \left(\mu - \frac{1}{2}\sigma^2\right) + x_{t-1} + \eta_t \tag{22}$$

where $s(t)$ is the cyclical, deterministic function described earlier and (T-t) is the time to maturity expressed as a fraction of one year.

The parameter estimates suggest that both the non-stationary long-run drift and the risk premium are small, as expected from theory, although all are significant at the 1% level assuming sensible convergence of the numerical derivatives. The diffusion term is consistent with previous estimates found in the literature. Looking at wavelet-filtered one-factor model estimates, the main difference is that the risk premium parameter is now nearly zero. This may be interpreted as evidence that the risk premium is captured by very short run variation. As expected, estimation convergence improves because the variance of the wavelet-

[12] These baseline results are obtained using all data except the most recent 250 observations. The latter are reserved for out-of-sample forecasts.

Table 3
Forward curve in-sample tracking results, This table presents calculations of in-sample tracking Root Mean Squared Errors (RMSE, in percent) for the one-to four-factor models applied to corn futures prices using daily data for the first five maturities. The results are presented for models estimated using the original data, and using the de-noised (wavelet-thresholded) data.

	Original data				De-noised data (wavelet)			
Maturity	**1-factor**	**2-factor**	**3-factor**	**4-factor**	**1-factor**	**2-factor**	**3-factor**	**4-factor**
First	1.25	1.12	0.931	0.824	1.07	0.817	0.732	0.711
Second	1.36	1.44	1.04	0.943	1.13	0.921	0.776	0.728
Third	1.47	1.52	1.26	1.08	1.43	1.13	0.964	0.943
Fourth	1.81	1.76	1.32	1.14	1.48	1.38	1.26	1.22
Fifth	1.90	1.81	1.38	1.18	1.86	1.81	1.30	1.27

filtered data variance is smaller than that of the original data.

The second and additional factors are mean-reverting state variables. These factors help explain the shape of the forward curve (e.g. contango or backwardation) through their interaction with the remaining time to maturity, and can be used to recover estimates of convenience yield and cost of carry. We need at least two mean-reverting state variables to capture mixed shapes of the forward curve (e.g., a hump shape). The two-factor model is:

$$\ln F(t, T) = s(t) + \mu t + (\mu - \lambda_1 + 0.5\sigma^2)(T-t) + x_{1,t} + e^{-\kappa_2 (T-t)} x_{2,t} - \frac{\lambda_2}{\kappa_2}(1 - e^{-\kappa_2 (T-t)}) + 0.5\sigma_1\sigma_2\rho_{12}\left(\frac{1 - e^{-\kappa_2 (T-t)}}{\kappa_2}\right) + \varepsilon_t \tag{23}$$

$$x_t = (\mu - 0.5\sigma^2, 0)' + A x_{t-1} + \eta_t \tag{24}$$

where the matrix A is:

$$A = \begin{pmatrix} 1 & 0 \\ 0 & e^{-\kappa_2 \Delta} \end{pmatrix} \tag{25}$$

Recall that the first state variable is non-stationary geometric Brownian motion, so implicitly we have imposed the restriction κ_1=0. For both the full sample data and the wavelet-filtered data, the parameters are statistically significant at the 5% level or better.

The three-factor model provides a superior fit to the data on days when the forward curve is not smooth but rather kinked. The results suggest that the non-stationary variable has a small but nonzero drift as well as a significant diffusion, while the mean-reverting speed for the other two state variables is fast and consistent with previous findings—larger than Sorensen's (2002) but smaller than Fackler and Roberts's (1999).

The three risk premium parameters confirm the literature's findings that the overall impact of risk premia for agricultural commodity futures is small. The non-stationary state variable is weakly correlated with the first mean-reverting state variable, but strongly negatively correlated with the second. The two stationary state variables are only weakly correlated.

Lastly, we consider the results from estimating a four-factor model, which is characterized by one non-stationary state variable, three stationary state variables, and 17 constant parameters to be estimated. The wavelet-filtered estimates are better overall. In particular, the parameters (mean-reverting, diffusions, and market prices of risk) take on more sensible values. However, the correlation coefficients are large in absolute value.

7. Interpretation of the results and in-sample tracking

To evaluate the tracking and forecasting ability of each model, we consider both in-sample and out-of-sample evidence (see Tables 3 and 4). The in-sample tracking results provide a comparison of how well the different models perform based on the Root Mean Squared Errors (RMSE) criterion, which is computed using deviations between the predicted model price and the actual price. The comparison is made based on the number of factors and whether wavelet thresholding is used or not. These results are based on using all data except for the most recent 250 daily observations. The out-of-sample evidence presents 5-day-ahead forecasts using estimations based on all previous observations. The out-of-sample forecasts use the last 250 daily observations, which have been reserved for this purpose and not included in the sample for estimation.

7.1. In-sample tracking results

Using the model's parameter estimates and the Kalman filter estimated time series of state variables we compute in-sample predictions (tracking) of futures prices for the first five maturities. Futures prices, as predicted from all four models with and without wavelet thresholding, are compared with the actual prices on those days. Using a one-factor model invariably leads to a poor performance, as is known from the literature. Indeed, more factors are needed to track the forward curve (e.g., Cortazar and Schwartz, 2003). Moreover, estimating the models using data that has been previously de-noised using wavelet thresholding leads to improved results, as seen in Table 3.[13] The improved performance is most noticeable for the first couple maturities, and less impressive for more distant maturities.

7.2. Out-of-sample forecasting results

To complement the in-sample evidence, we also present in Table 4 some results of out-of-sample forecasting where we use all available observations up to date t in order to forecast date $t+k$, where k=5 days. Having set aside the last 250 observations, we produce 5-day ahead forecasts and compute RMSEs using these forecasts. As expected, and consistent with the findings of Cortazar and Schwartz (2003) for crude oil futures, we find that RMSEs are larger for out-of-sample results than for in-sample tracking, and moreover that the performance is improved as we go from one- to multi-factor models. However, we also show that the results are noticeably improved when we data are previously de-noised using wavelet thresholding.[14]

In summary, de-noising the raw data leads to tracking and forecasting results that are at least as good as results using the noisy data, at least for the first five maturities. One would expect the benefits of de-noising the data to vary over time as both the theoretical (e.g., DeLong et al., 1990, Easley and O'Hara, 1987) and empirical literature (e.g., Hasbrouck 1991) suggest that the level of noise in prices will vary over time.

[13] The orders of magnitude are consistent with the results of Cortazar and Schwartz (2003), who found a RMSE of 0.42 percent on average in-sample. Although their performance for crude oil futures is better than ours, they used a different commodity futures contract (crude oil) which is more heavily traded.

[14] In terms of magnitude, our results are consistent with those of Cortazar and Schwartz (2003) who obtained, for crude oil futures prices, a RMSE of 0.76 percent on average for out-of-sample forecasting results, roughly twice the value of RMSE for in-sample tracking. The reason their results are better is likely because crude oil futures are more heavily traded and thus contain more information.

Table 4
Forward curve out-of-sample forecasting results, This table presents calculations of out-of-sample five-day-ahead forecasting Root Mean Squared Errors (RMSE, in percent) for the one- to four-factor models applied to corn futures prices using daily data for the first five maturities. The results are presented for models estimated using the original data, and using the de-noised (wavelet-thresholded) data.

	Original data				De-noised data (wavelet)			
Maturity	1-factor	2-factor	3-factor	4-factor	1-factor	2-factor	3-factor	4-factor
First	2.79	1.62	1.38	1.34	2.05	1.31	1.10	1.06
Second	3.02	1.64	1.35	1.32	2.34	1.36	1.08	1.05
Third	3.40	1.82	1.39	1.35	3.06	1.65	1.15	1.12
Fourth	3.64	1.96	1.51	1.47	3.32	1.83	1.23	1.20
Fifth	3.98	2.0	1.83	1.76	3.78	1.89	1.47	1.41

8. Conclusion

Financial risk management strategies are improved by optimally using the information content of the forward curve—the term structure of futures prices for a given asset. However, price changes contain both information and noise, and that noise reduces the efficiency of estimations and thus the accuracy of term structure forecasts. This paper is the first to use wavelet thresholding to de-noise futures-price data prior to estimation of a state-space model allowing for one or several factors, i.e. latent state variables (Schwartz, 1997). The economic intuition is analogous to Hasbrouck's (2013), who shows how the wavelet variance can help distinguish types of microstructure noise. An innovation of the methodology, relative to existing filtering methods, is that de-noising is done at the level of wavelet coefficients, and not the raw data. This has been shown to avoid either over- or under-smoothing. The results show that wavelet thresholding produces tracking and forecasting results that are generally superior, especially for the first few maturities, to those that are based on data that has not been de-noised.

Acknowledgements

We thank the Editor, Sushanta Mallick, as well as an anonymous referee for very helpful comments. We also thank Tim Mount, Hazem Daouk, and participants at the WAEA and NCCC-134 conferences. Any errors are ours alone.

Appendix. A brief overview of wavelets for economic and financial time series

The purpose of this appendix is to review important elements of wavelet selection for the purposes of time series analysis of economic and financial data. We describe the properties that make particular wavelets optimal for a given application as well as trade-offs involved in the selection of an ideal wavelet. In time series analysis, desirable wavelet properties include symmetry, moment preservation, orthogonality between levels of decomposition, perfect reconstruction, correct time alignment (linear/zero phase), minimization of spurious artifacts and boundary effects, and compact support.

To illustrate the usefulness of these properties, we focus on the Daubechies (1988) wavelet class, which the literature has found to be the best for empirical time series work using economic and financial data. We also discuss properties of the original wavelet, discovered by Haar (1910), which is the simplest to construct and also a nested special case of the Daubechies wavelet. A large number of wavelets have been defined but only those of Daubechies and Haar appear to be consistently useful to economists. A thorough treatment of wavelet properties is found in Daubechies (1992, 1993), Ogden (1997) and Vidakovic (1999).

The four key properties for wavelets in time series analysis are:

1. A nonzero number of vanishing moments
2. Compact support
3. Orthogonality and orthonormality
4. Linear phase

To explain the importance of a nonzero number of vanishing moments, we introduce the two principal conditions of a wavelet. First, a wavelet is a function $\psi(\cdot)$ defined on the extended Real line such that the admissibility condition is satisfied:

$$\int \psi(t) = dt$$

Second, a wavelet is generally required to satisfy the unit energy (variance) condition:

$$\int \psi^2(t)dt = 1$$

Then, a greater requirement is for the wavelet to have a number N of vanishing moments such that, for $k=\{0, ..., N-1\}$ the wavelet satisfies:

$$\int t^k \psi_0(t)dt \equiv 0$$

A greater number of vanishing moments is particularly important for the wavelet-based analysis of long-range dependence (see Teyssiere and Abry (2006)). The literature also refers to filters associated with wavelet transforms and the length of a filter is precisely twice its number of vanishing moments. A large number of vanishing moments increases however the size of the wavelet and may generate spurious artifacts in the transformed data. The Daubechies regular and least asymmetrical wavelets among others have an arbitrary number of vanishing moments such that the researcher can select the most appropriate number. In contrast, the simple Haar wavelet has zero vanishing moments as it is piecewise linear.

Compact or finite support captures local variation more accurately. The wavelet oscillates locally and quickly fades away on the left and on the right. In contrast, sines and cosines oscillate indefinitely. The Haar and Daubechies (regular and least asymmetrical) are three of the only four wavelets that are both compactly supported and orthogonal wavelets.

Orthogonality means that for a wavelet timescale representation of the data, the different levels are uncorrelated which implies the perfect reconstruction property holds. Suppose that we want to know how much of a time series variance is explained by variation at the short-run, medium-run, and long-run horizons. Orthogonality implies that the perfect reconstruction property holds and therefore enables an accurate deconstruction of a time series into different levels or time horizons. Orthonormality further ensures unit energy (variance), which means the decomposed data remains accurate to scale. Both the Daubechies and Haar wavelets are orthonormal.

Linear phase ensures correct time localization. For example, we may wish to determine the precise date of a mean or variance change-point in a time series. Linear phase is also a necessary and sufficient condition for perfect symmetry, a property that only the Haar wavelet possesses. Since excessive asymmetry is undesirable, Daubechies developed a Least Asymmetrical wavelet that has essentially correct time localization and is therefore often used in economic applications.

As with nonparametric regression and frequency domain analysis, wavelet analysis involves dealing with the problem of boundary effects. The theory behind wavelets has been developed under the assumption of an infinite number of observations, but sampled data in economics and other non-experimental sciences are necessarily finite. If no correction is made, the computed wavelet coefficients will be overstated at the beginning and end of the sample.

Two general solution methods are, first, to discard those biased observations by truncating the sample a few observations after the beginning and before the end and, second, to artificially extend the time series for purposes of wavelet analysis but only include the true observations in the economic analysis and interpretation of results. The time series can be extend by padding with zeros, reflecting (symmetrically) the observations at the sample's endpoints, or assuming the sample repeats periodically. Cohen et al. (1993) have found that zero-padding creates large artifacts in the data and reflecting the data causes the orthonormality property to be lost. Periodization is therefore the least harmful method unless the researcher can afford to discard some observations at both endpoints.

To obtain a frequency domain representation of time series data suitable for spectral analysis, the Fourier transform is applied to the data (see e.g. Hamilton, 1994). The workhorse of wavelet-based time series analysis is the Discrete Wavelet Transform (DWT). Unlike the Fourier transform, which is unique, wavelet transforms are numerous because each one is constructed from a specific wavelet function and filter length. For all wavelets, the resulting Discrete Wavelet Transform is the inner product (convolution) of the data with translations and dilations of the wavelet function. The outcome is a wavelet coefficient vector of the same length as the original data. The wavelet coefficients contain information in both the time and scale domain, where the scale corresponds to different length time periods. For example, if the original data are daily observations, then the scales would be daily, weekly, monthly and so forth.

Data in this paper are sampled daily over a period of two decades. This means the wavelet transform requirement of a sample of dyadic length (base two) is not overly restrictive. Many economic datasets, e.g. macroeconomic time series, however, consist of much shorter time series where each observation is needed. A different transform, called the maximum overlap discrete wavelet transform, may be applied to data of any length. The downside is that it loses the orthonormality property, which implies a loss of efficiency and a more conservative interpretation of the results.

The second reason to use the translation-invariant wavelet transform is that, as implied by its name, its localization in time remains accurate, whereas the basic discrete wavelet transform has a small bias. For instance, after it is found that there exist in the data one or more change-points or structural breaks, the translation-invariant transform should be used to actually date the change-point or break.

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