**Logo

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**MATH201 - Calculus-I**

**Homework Assignment #1**

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**Due day: 9/24/2024**

**Instruction:**

1. **Push the answer sheet to Github in Word file.**
2. **Overdue homework submission can’t be accepted.**
3. **Takes academic honesty and integrity seriously (Zero Tolerance of Cheating & Plagiarism)**
4. **Researchers measured the blood alcohol concentration (BAC) of eight adult male subjects after rapid consumption of *30* mL of ethanol (corresponding to two standard alcoholic drinks). The table shows the data they obtained by averaging the BAC (in mgymL) of the eight men.**
   1. **Use the readings to sketch the graph of the BAC as a function of *t* in Excel.**

=> Here, I’ve first plotted the data given in the table to create a graph showing Blood Alcohol Concentration (BAC) as a function of time t in hours.

A graph on a white sheet

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* 1. **Use your graph to describe how the effect of alcohol varies with time.**

#### Description of How the Effect of Alcohol Varies with Time:

* **Initial Increase**: The graph shows a sharp increase in BAC from the start (0 hours) reaching a peak of about 0.41 mg/mL by approximately 0.5 hours. This rapid increase indicates the swift absorption of alcohol into the bloodstream.
* **Peak and Plateau**: The peak BAC occurs at around 0.5 hours. It slightly decreases and then somewhat stabilizes for a brief period (up to about 0.75 hours), where it remains close to 0.4 mg/mL.
* **Decrease in BAC**: After reaching the peak, the BAC consistently decreases over time. This decline is gradual and steady from 0.75 hours onward, showcasing the body's metabolism of the alcohol.
* **End of the Observation**: By the end of the 4 hours, the BAC has reduced significantly to 0.01 mg/mL, suggesting most of the alcohol has been metabolized by this point.

This trend provides insights into how alcohol is absorbed and metabolized in the body over a period after consumption. The graph and data can be useful for understanding the effects of alcohol on the body, potentially helping with guidelines for safe drinking practices and understanding the effects of alcohol over time.

|  |  |
| --- | --- |
| ***t* (hours)** | **BAC** |
| 0 | 0 |
| 0.2 | 0.25 |
| 0.5 | 0.41 |
| 0.75 | 0.40 |
| 1 | 0.33 |
| 1.25 | 0.29 |
| 1.5 | 0.24 |
| 1.75 | 0.22 |
| 2.0 | 0.18 |
| 2.25 | 0.15 |
| 2.5 | 0.12 |
| 3.0 | 0.07 |
| 3.5 | 0.03 |
| 4.0 | 0.01 |

1. **Find an expression for the function whose graph is the given curve in the top half of the circle , and then plot it in Excel or any computer language.**

x2+(y−2)2=4

This is the equation of a circle in standard form, where:

* The center of the circle is at (0, 2)
* The radius of the circle is 2 (since 4=22)

We are interested in the top half of this circle, which means we need to express y in terms of x and select the positive square root (since we're only considering the upper half of the circle).

### 2. Solve for y

To find y, solve the given equation for y:

x2 + (y−2)2 = 4

First, isolate (y−2)2:

(y−2)2 = 4− x2

Now, take the square root of both sides:

y−2 = ±√4− x2

Thus, y is:

y = 2± √4− x2

For the top half of the circle, we take the positive square root:

y = 2 + √4− x2

A graph of a function

Description automatically generated

1. **In a certain country, income tax is assessed as follows. There is no tax on income up to *$10,000*. Any income over *$10,000* is taxed at a rate of *10%*, up to an income of *$20,000*. Any income over *$20,000* is taxed at *15%*.**
   1. **Sketch the graph of the tax rate *R* as a function of the income *I* in Excel**

#### Steps to sketch in Excel:

1. Create a column for income values I ranging from 0 to, say, $30,000.
2. Define the tax rate R(I):
   * If I ≤ 10,000, the tax rate R = 0.
   * If 10,000 < I ≤ 20,000, the tax rate R=0.1 (10%).
   * If I > 20,000, the tax rate R=0.15 (15%).  
       
     A graph with a line

     Description automatically generated
   1. **How much tax is assessed on an income of *$14,000*? On *$26,000*?**  
      =>

#### For I = 14,000:

* The first $10,000 is not taxed.
* The remaining $4,000 (from $10,000 to $14,000) is taxed at 10%.

Tax calculation:

Tax on $14,000=0.1×4,000=$400

#### For I = 26,000:

* The first $10,000 is not taxed.
* The next $10,000 (from $10,000 to $20,000) is taxed at 10%.
* The remaining $6,000 (from $20,000 to $26,000) is taxed at 15%.

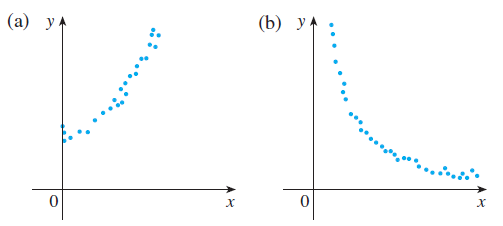
Tax calculation:

Tax on $26,000=0.1×10,000+0.15×6,000=1,000+900=$1,900

* 1. **Sketch the graph of the total assessed tax *T* as a function of the income *I* in Excel.**
     + Steps to sketch in Excel:

1. Use the same income values I from 0 to $30,000.
2. Define the total tax T(I):
   * If I ≤ 10,000, total tax T= 0.
   * If 10,000 < I ≤20,000, total tax T=0.1×(I−10,000).
   * If I > 20,000, total tax T=0.1×10,000+0.15×(I−20,000).  
       
     A graph with a green line

     Description automatically generated
3. Decide what type of function you might choose as a model for the given data as follows by selecting fitting function in Excel. Of course, before fitting, the x-y values should be created based on your observation.



### Graph (a):

* The points in graph (a) show a pattern where the values are **increasing rapidly** as x gets larger.
* This type of growth suggests an **exponential function** might be a good fit because exponential functions show rapid increase as xxx grows.

A typical form of an **exponential function** is:

y=a ⋅ ebx

where a and b are constants.

### Graph (b):

* The points in graph (b) show a **rapid decrease** as x increases. The values seem to decrease quickly at first and then level out.
* This behavior suggests that an **inverse** or **logarithmic function** might be a good fit. Both of these types of functions have decreasing curves that approach an asymptote (a flat line).

A typical form of an **inverse function** is:

y=a/x +b

or a **logarithmic function**:

y = a ⋅ ln(x)+b

A graph of a function

Description automatically generated

1. Anthropologists use a linear model that relates human femur (thighbone) length to height. The model allows an anthropologist to determine the height of an individual when only a partial skeleton (including the femur) is found. Here we find the model by analyzing the data on femur length and height for the eight males given in the following table.
   1. Make a scatter plot of the data in Excel.  
        
      =>   
        
      A graph with a red line

      Description automatically generated
   2. Find and graph the regression line that models the data.
      * **Find the regression line in Excel:**

* After creating the scatter plot, right-click on any of the points and select **Add Trendline.**
* In the **Trendline** options, choose **Linear** to fit a straight line to the data.
* Check the box for **Display Equation on Chart** to show the equation of the regression line.

**Graph the regression line:**

* The regression line will automatically appear on the scatter plot along with its equation. This equation will allow you to predict height based on femur length.
  1. An anthropologist finds a human femur of length *53* cm. How tall was the person?

|  |  |
| --- | --- |
| **Femur length**  **(cm)** | **Height**  **(cm)** |
| 50.1 | 178.5 |
| 48.3 | 173.6 |
| 45.2 | 164.8 |
| 44.7 | 163.7 |
| 44.5 | 168.3 |
| 42.7 | 165.0 |
| 39.5 | 155.4 |
| 38.0 | 155.8 |
|  |  |

Using the equation of the regression line that you find in part (b), substitute x = 53 (femur length in cm) into the equation to calculate the height.

For example, if the regression equation is:

y=mx+b

The green point shows the predicted height for a femur length of 53 cm, which is approximately **182.33 cm**.

1. **The table shows the mean (average) distances *d* of the planets from the sun (taking the unit of measurement to be the distance from the earth to the sun) and their periods *T* time of revolution in years).**
   1. **Fit a power model to the data in Excel**  
      A graph with a line

      Description automatically generated
   2. **Kepler’s Third Law of Planetary Motion states that "The square of the period of revolution of a planet is proportional to the cube of its mean distance from the sun."**

Kepler's Third Law states that T2 ∝ d3, which can be written as T = k \* d(3/2) where

k is a constant.

* 1. **Does your model corroborate Kepler’s Third Law?**

|  |  |  |
| --- | --- | --- |
| **Planet** | **d** | **T** |
| Mercury | 0.387 | 0.241 |
| Venus | 0.723 | 0.615 |
| Earth | 1.000 | 1.000 |
| Mars | 1.523 | 1.881 |
| Jupiter | 5.203 | 11.861 |
| Saturn | 9.541 | 29.457 |
| Uranus | 19.190 | 84.008 |
| Neptune | 30.086 | 164.784 |
|  |  |  |
|  |  |  |

Model corroboration with Kepler's Law:

• After fitting the power model, compare the exponent n from the model to

3/2=1.53/2 = 1.53/2=1.5, which is the exponent predicted by Kepler.

• If the exponent from the power model is close to 1.5, it indicates that the model

supports Kepler’s Third Law.

1. **How is the graph of related to the graph of *f(x)?***
   1. **Sketch the graph of in Excel.**

The graph of y=f(∣x∣) is related to f(x) in the following way:

* The function f(∣x∣) reflects the part of f(x) for x ≥ 0 to the negative side of the x-axis, creating symmetry about the y-axis.
* For example, in the graph above for y = sin(∣x∣), the sine wave for positive values of x is mirrored to the left side (for negative values of x).

This results in a symmetric wave that repeats the pattern on both sides of the y-axis.  
  
A graph of a function

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* 1. **Sketch the graph of in Excel.**

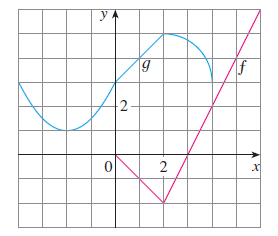
The graph of y=∣x∣ is related to the graph of y=x ​ in the following way:

* The absolute value inside the square root ensures that negative values of x are treated the same as positive values. As a result, the graph is symmetric about the y-axis.
* For negative values of x, the graph reflects the same behavior as for positive x, creating a "V" shape.

In the plot above, you can see how the square root function behaves for both positive and negative x, forming a smooth curve that rises from (0,0) and increases as the absolute value of x increases.  
A graph of a function

Description automatically generated

1. **Use the given graphs of *f* and *g* to evaluate each expression or explain why it is undefined.**
   1. **b. c.**



### Part (a): (g ∘ f)(6)

This means we first find f(6) and then evaluate g at that value.

1. From the graph, f(6) corresponds to y=4, so f(6)=4.
2. Now, evaluate g(4). From the graph of g, g(4) corresponds to y=0.

Thus, (g ∘ f)(6)=g(f(6))=g(4)=0.

### Part (b): (g ∘ g)(−2)

This means we first find g(−2) and then evaluate g again at that value.

1. From the graph, g(−2)corresponds to y=2, so g(−2)=2.
2. Now, evaluate g(2). From the graph of g, g(2) corresponds to y=3.

Thus, (g ∘ g)(−2)=g(g(−2))=g(2)=3.

### Part (c): (f ∘ f)(4)

This means we first find f(4) and then evaluate f again at that value.

1. From the graph, f(4) corresponds to y=2, so f(4)=2.
2. Now, evaluate f(2). From the graph of f, f(2) corresponds to y=0
3. Thus, (f ∘ f)(4)= f(f(4))= f(2)= 0.