Fraudulent Claim Detection By Paras Mehta and Shaik Muhammad Naim

1. Data Loading

```
df = pd.read_csv("insurance_claims.csv")
df.head() # check the records
df.shape #Check the shape
# Inspect the features in the dataset
df.info()
```

2. Data Cleaning

- 2.1. Handle null values
 - 2.1.1. Examine the columns to determine if any value or column needs to be treated

```
# Check the number of missing values in each column
df = df.replace('?', pd.NA)
df.isnull().sum()
```

2.1.2. Handle rows containing null values

```
# Handle the rows containing null values

df["collision_type"] = df["collision_type"].fillna('unknown')

df["property_damage"] = df["property_damage"].fillna('unknown')

df["authorities_contacted"] = df["authorities_contacted"].fillna('unknown')

df["police_report_available"] = df["police_report_available"].fillna('unknown')

df.isnull().sum()
```

- 2.2. Identify and handle redundant values and columns
 - 2.2.1. Examine the columns to determine if any value or column needs to be treated

```
# Write code to display all the columns with their unique values and counts and check for redundant values redundant cols = []
```

```
for col in df.columns:
    nunique = df[col].nunique(dropna =False)
    total = len(df[col])
    if nunique <= 1:
        redundant_cols.append(col)

elif nunique/total > 0.95:
    redundant_cols.append(col)

print ("Redundant column")
print (redundant_cols)
for col in df.columns:
    print(f"\nColumn :{col}")
    print("Unique count: ", df[col].nunique(dropna=False))
    print("Sample value: ", df[col].dropna().unique()[:10])
```

2.2.2. Identify and drop any columns that are completely empty

```
# Identify and drop any columns that are completely empty df = df.drop(columns=['_c39'],axis=1 )
```

2.2.3. Identify and drop rows where features have illogical or invalid values, such as negative values for features that should only have positive values

```
# Identify and drop rows where features have illogical or invalid values, such as negative values for features that should only have positive values #Removing the umbrella_limit that is less than 0 df = df[df["umbrella_limit"]>=0]
```

2.2.4. Identify and remove columns where a large proportion of the values are unique or near-unique, as these columns are likely to be identifiers or have very limited predictive power

```
# Identify and remove columns that are likely to be identifiers or have very limited predictive power

#policy_number → unique per record, no predictive power.

#policy_bind_date → almost unique, timestamp-like, not directly predictive.

#insured_zip → 999 unique values (almost every row unique).

#incident_date → 999 unique values, not predictive unless transformed to "day of week / month".

#incident_location → 999 unique values (acts like ID).

#total_claim_amount → redundant (sum of injury_claim + property_claim +
```

```
vehicle_claim).
drop_cols = [
   "policy_number",
   "policy_bind_date",
   "insured_zip",
   "incident_date",
   "incident_location",
   "total_claim_amount" # redundant with injury/property/vehicle claims
]
df_clean = df.drop(columns=drop_cols)
# Check the dataset
df_clean.head()
```

2.3. Fix Data Types

```
# Fix the data types of the columns with incorrect data types
#Make fraud_reported to 1 or 0 1 means fraud claim and 0 means not
fraud
df_clean["fraud_reported"] = df_clean["fraud_reported"].map({"Y": 1, "N":
0}).astype(int)

# change categorial column to type as category
categorical_cols = [
    "policy_state", "policy_csl", "insured_sex", "insured_education_level",
    "insured_occupation", "insured_hobbies", "insured_relationship",
    "incident_type", "collision_type", "incident_severity",
    "authorities_contacted", "incident_state", "incident_city",
    "property_damage", "police_report_available", "auto_make",
    "auto_model"
]
df_clean[categorical_cols] = df_clean[categorical_cols].astype("category")

# Check the features of the data again
df_clean.info()
```

3. Train-Validation Split

3.1. Import required libraries

```
# Import train-test-split from sklearn.model_selection import train_test_split
```

3.2. Define feature and target variables

```
# Put all the feature variables in X
X = df_clean.drop(columns=["fraud_reported"], axis =1)
# Put the target variable in y
y = df_clean["fraud_reported"]
```

3.3. Split the data

```
# Split the dataset into 70% train and 30% validation and use stratification on the target variable
X_train,X_test,y_train,y_test = train_test_split(X,y, test_size=0.3, stratify=y, random_state=42)
# Reset index for all train and test sets
X_train = X_train.reset_index(drop=True)
X_test = X_test.reset_index(drop=True)
y_train = y_train.reset_index(drop=True)
y_test = y_test.reset_index(drop=True)
```

4. EDA on training data

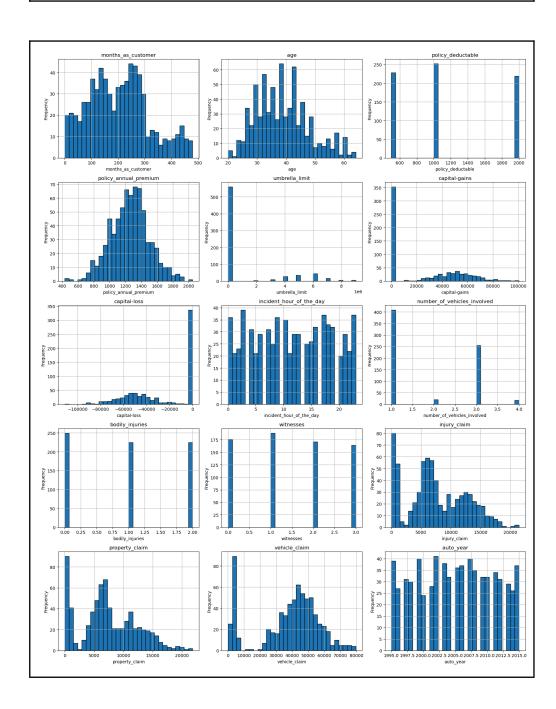
- 4.1. Perform univariate analysis
 - 4.1.1. Identify and select numerical columns from training data for univariate analysis

```
# Select numerical columns
numericals_col =
X_train.select_dtypes(include=["int64","float64"]).columns.tolist()
print("numerical column", numericals_col)
```

4.1.2. Visualise the distribution of selected numerical features using appropriate plots to understand their characteristics

```
plt.figure(figsize=(16, 20)) # adjust size for readability for i, col in enumerate(numericals_col, 1):
    plt.subplot(5, 3, i) # 5 rows, 3 columns of subplots
    X_train[col].hist(bins=30, edgecolor='black')
    plt.title(col)
    plt.xlabel(col)
    plt.ylabel("Frequency")
```

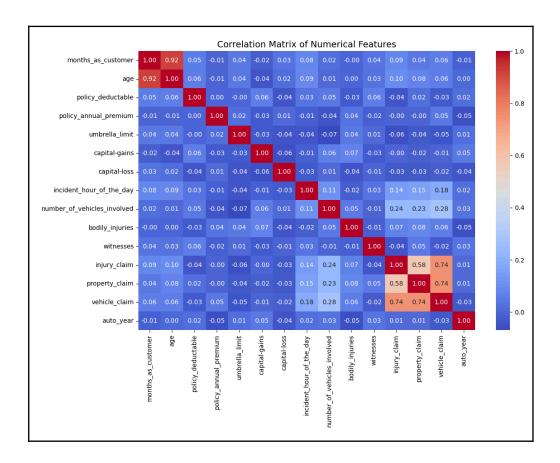
plt.tight_layout() plt.show()



4.2. Perform correlation analysis

Create correlation matrix for numerical columns corr_matrix = X_train[numericals_col].corr() # Plot Heatmap of the correlation matrix plt.figure(figsize=(12, 8))

```
sns.heatmap(corr_matrix, annot=True, cmap="coolwarm", fmt=".2f", cbar=True)
plt.title("Correlation Matrix of Numerical Features", fontsize=14)
plt.show()
```

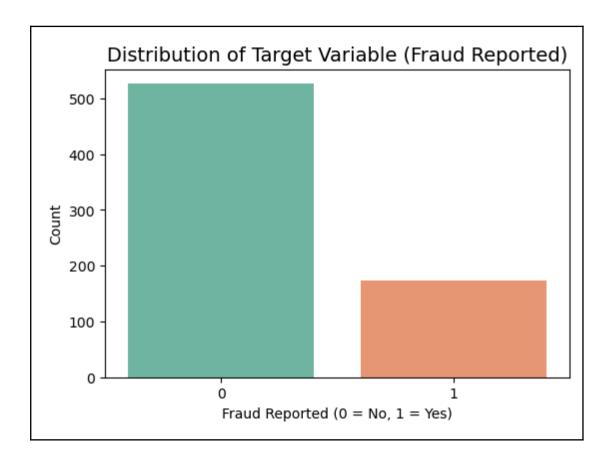


4.3. Check class balance

```
# Plot a bar chart to check class balance
print(y_train["fraud_reported"].value_counts())
print("\nClass distribution (%):")
print(y_train["fraud_reported"].value_counts(normalize=True) * 100)
plt.figure(figsize=(6,4))
sns.countplot(x="fraud_reported", data=y_train, palette="Set2")
plt.title("Distribution of Target Variable (Fraud Reported)", fontsize=14)
plt.xlabel("Fraud Reported (0 = No, 1 = Yes)")
plt.ylabel("Count")
plt.show()

--output-
fraud_reported
0 526
1 173
Name: count, dtype: int64
```

```
Class distribution (%):
fraud_reported
0 75.250358
1 24.749642
Name: proportion, dtype: float64
```



4.4. Perform bivariate analysis

4.4.1. Target likelihood analysis for categorical variables

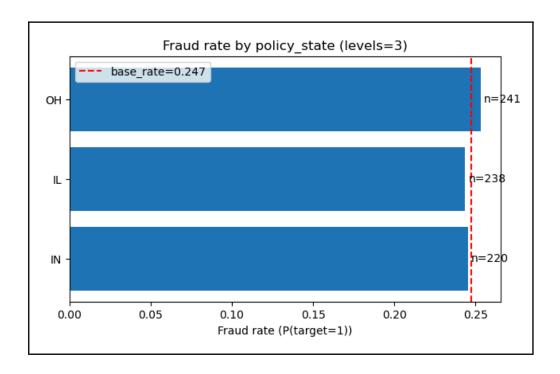
```
# Write a function to calculate and analyse the target variable likelihood for categorical features

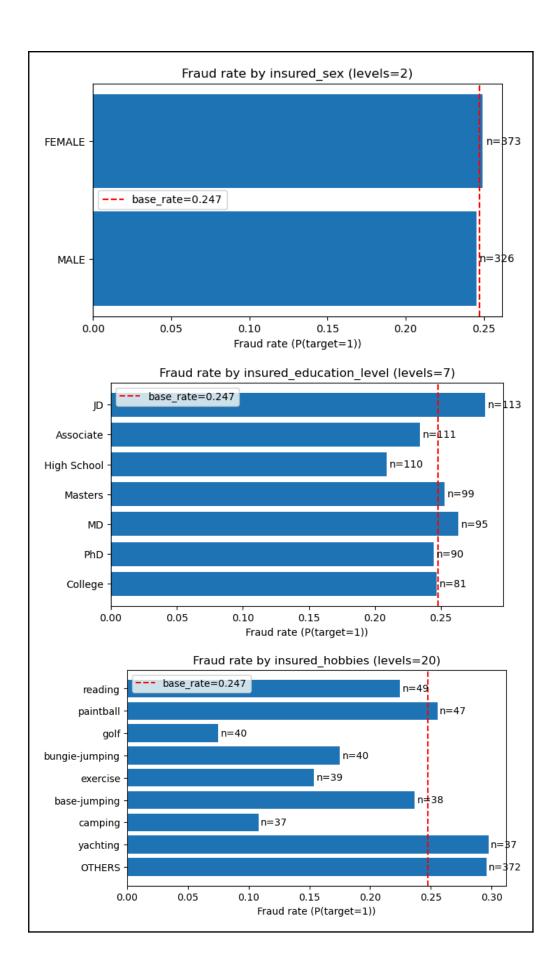
def analyze_cat_target_likelihood(X_train, y_train, categorical_cols=None, min_count=20, rate_diff_threshold=0.05, plot=False, top_n_levels_plot=10, figsize=(7,4)):

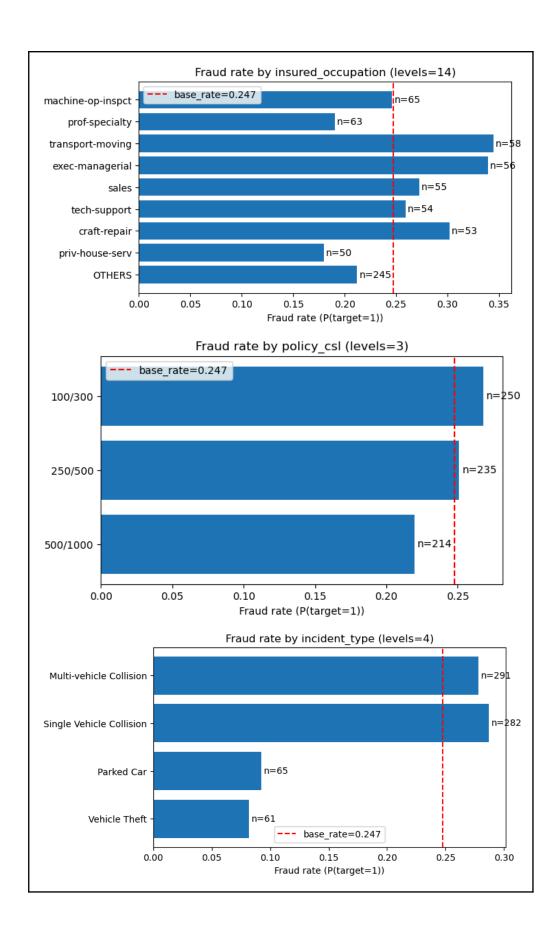
Analyze P(target=1) for each level of categorical features using X_train and y_train.
```

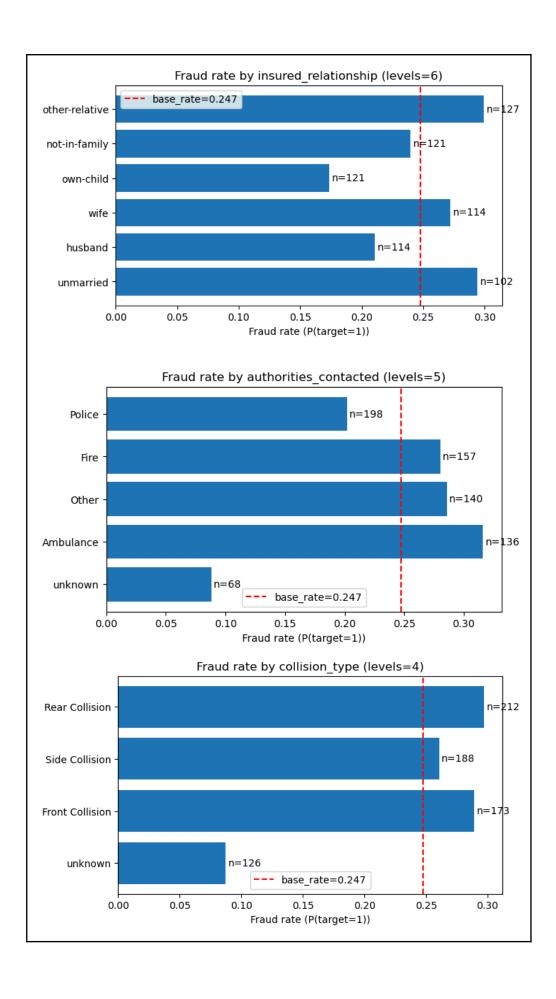
```
Robust to y_train being a Series or a single-column DataFrame.
  # Ensure y is a 1-D numeric Series
  if isinstance(y_train, pd.DataFrame):
     if y_train.shape[1] == 1:
       y = y train.iloc[:, 0].reset index(drop=True)
     else:
       # more than one column - take first column (user should supply
single-target)
       y = y_train.iloc[:, 0].reset_index(drop=True)
  else:
     y = pd.Series(y_train).reset_index(drop=True)
  # Coerce to numeric (0/1)
  y = y.astype(int)
  # Reset X index to align
  X = X train.reset index(drop=True).copy()
  # Auto-detect categorical columns if not provided
  if categorical_cols is None:
     categorical_cols = X.select_dtypes(include=[
"category"]).columns.tolist()
  categorical cols = [c for c in categorical cols if c in X.columns]
  per_feature = {}
  summary rows = []
  # scalar base rate
  base rate = float(y.mean())
  for col in categorical cols:
     # skip if column has no non-null values
     if X[col].dropna().shape[0] == 0:
       continue
     temp = pd.concat([X[col], y], axis=1)
     temp.columns = [col, "target"]
     grp = temp.groupby(col)["target"].agg(total_count="count",
fraud_cases="sum").reset_index()
     grp["fraud_rate"] = grp["fraud_cases"] / grp["total_count"]
     grp["low support"] = grp["total count"] < min count</pre>
     grp = grp.sort values("fraud rate",
ascending=False).reset index(drop=True)
     # feature-level metrics
     max rate = float(grp["fraud rate"].max())
     min_rate = float(grp["fraud_rate"].min())
     rate range = max rate - min rate
     n levels = grp.shape[0]
```

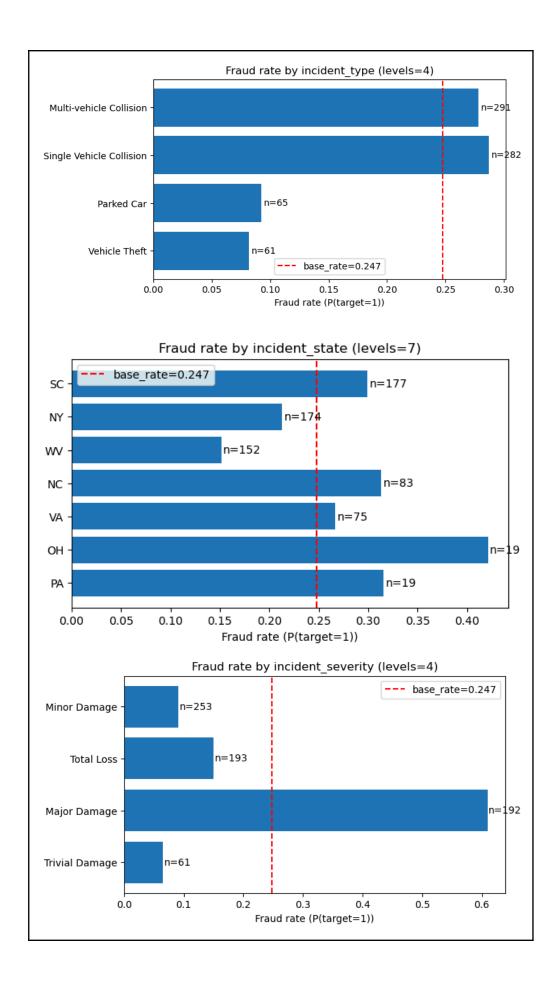
```
weighted_rate = float((grp["fraud_rate"] * grp["total_count"]).sum() /
grp["total count"].sum())
     n_low_support_levels = int(grp["low_support"].sum())
     is weak = rate_range < rate_diff_threshold
     per_feature[col] = grp
     summary rows.append({
        "feature": col,
        "n_levels": n_levels,
       "max rate": max rate,
       "min_rate": min_rate,
       "rate range": rate range,
       "weighted_rate": weighted_rate,
       "base rate": base rate,
       "is weak": is weak,
        "n low_support_levels": n_low_support_levels
     # optional plotting
     if plot:
       plot_df = grp.sort_values("total_count", ascending=False).copy()
       if plot df.shape[0] > top n levels plot:
          top = plot_df.head(top_n_levels_plot).copy()
          others = plot_df.iloc[top_n_levels_plot:].copy()
          others_row = {
             col: "OTHERS",
             "total count": others["total count"].sum(),
             "fraud cases": others["fraud cases"].sum()
          others_row["fraud_rate"] = others_row["fraud_cases"] /
others_row["total_count"] if others_row["total_count"]>0 else np.nan
          others row["low support"] =
others["total count"].lt(min count).all()
          top = pd.concat([top, pd.DataFrame([others row])],
ignore_index=True)
          plot_df = top
       plt.figure(figsize=figsize)
       plt.barh(plot_df[col].astype(str), plot_df["fraud_rate"])
       plt.xlabel("Fraud rate (P(target=1))")
       plt.title(f"Fraud rate by {col} (levels={n_levels})")
       plt.gca().invert yaxis()
       for i, row in enumerate(plot df.itertuples()):
          # protect against NaN fraud_rate
          if pd.notna(getattr(row, "fraud rate", None)):
             plt.text(getattr(row, "fraud rate") + 0.002, i,
f"n={int(row.total_count)}", va="center")
       # draw base rate as line
       plt.axvline(x=base rate, color="red", linestyle="--",
label=f"base rate={base rate:.3f}")
        plt.legend()
       plt.show()
  summary =
```

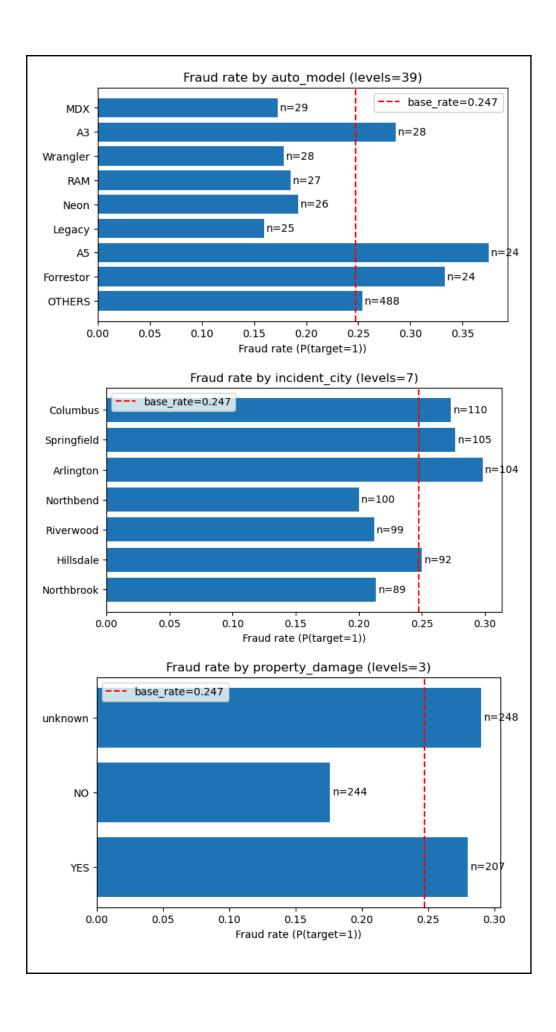


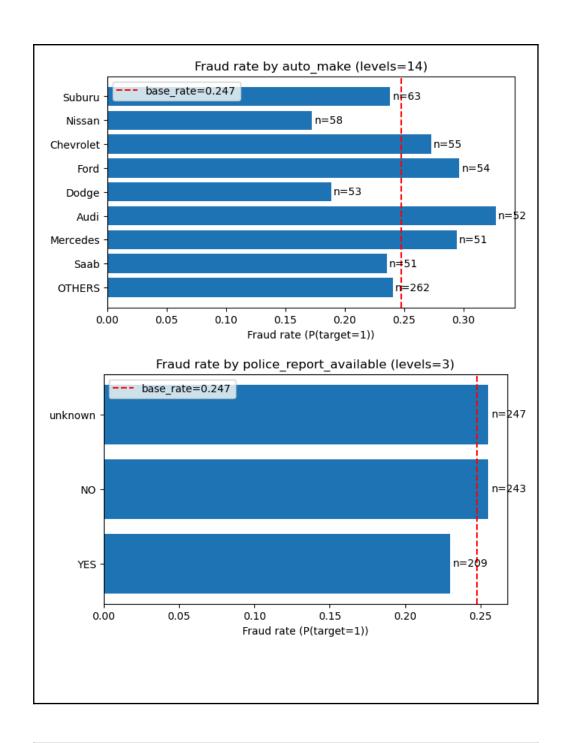












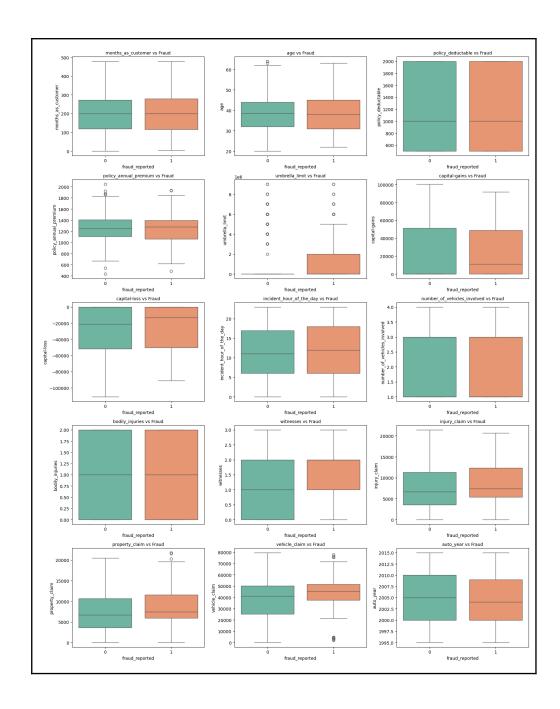
<pre>#output n_levels max_rate min_rate</pre>	rate_	_range \	
feature insured_hobbies 0.831250	20	0.906250	0.075000
incident_severity 0.543801	4	0.609375	0.065574
auto_model 0.500000	39	0.500000	0.000000
incident_state	7	0.421053	0.151316
authorities_contacted	5	0.316176	0.088235
collision_type	4	0.297170	0.087302

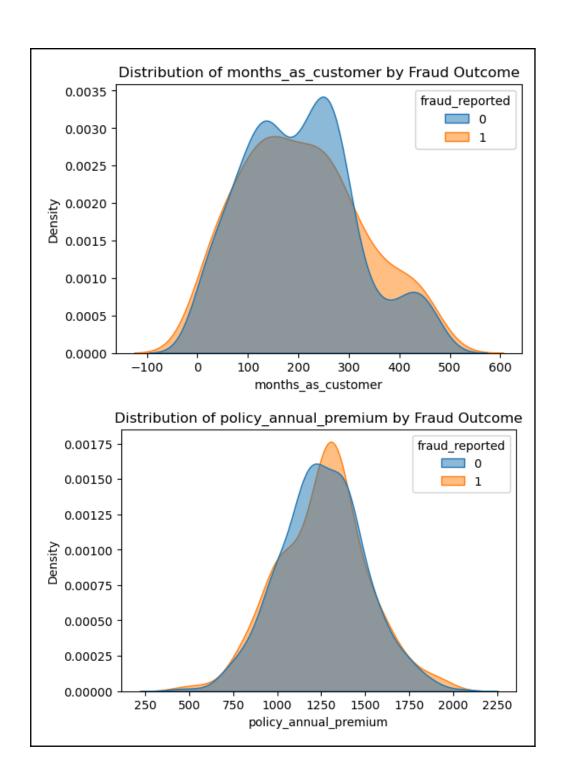
0.209868	
incident_type	4 0.287234 0.081967
0.205267	
	14 0.344828 0.142857
0.201970	
auto_make	14 0.326923 0.140000
0.186923	
insured relationship	6 0.299213 0.173554
0.125659	
property_damage	3 0.290323 0.176230
0.114093	
incident city	7 0.298077 0.200000
0.098077	
insured education level	7 0.283186 0.209091
0.074095	
policy csl	3 0.268000 0.219626
0.048374	
police_report_available	3 0.255144 0.229665
0.025479	
policy_state	3 0.253112 0.243697
0.009415	
insured sex	2 0.249330 0.245399
0.003931	
	<pre>weighted_rate base_rate is_weak \</pre>
feature	
insured_hobbies	0.247496 0.247496 False
incident_severity	0.247496 0.247496 False
auto_model	0.247496 0.247496 False
incident_state	0 247496
authorities_contacted	0.247496 0.247496 False
collision_type	0.24/496
incident_type	0.247496 0.247496 False
insured_occupation	0.247496 0.247496 False
auto_make	0.247496 0.247496 False
insured_relationship	0.247496 0.247496 False
property_damage	0.247496 0.247496 False
incident_city	0.247496 0.247496 False
insured_education_level	0.247496 0.247496 False
policy_csl	0.247496 0.247496 True
police_report_available	0.247496 0.247496 True
policy_state	0.247496 0.247496 True
insured_sex	0.247496 0.247496 True
	n_low_support_levels
feature	
insured_hobbies	1
incident_severity	0
auto_model	25
incident_state	2
authorities_contacted	0
collision_type	0
incident_type	0
insured_occupation	0
auto_make	0
insured_relationship	0
property_damage	0
incident_city	0
<pre>insured_education_level</pre>	0
policy_csl	0

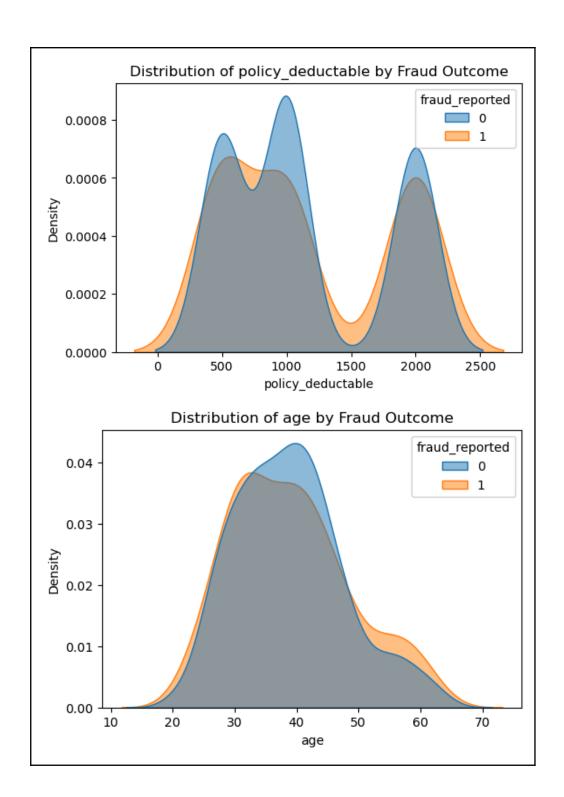
```
0
police report available
policy state
                                   0
insured sex
 property damage total count fraud cases fraud rate
low support
     unknown
                                72
0
                    248
                                     0.290323
False
           NO 244
                              43 0.176230
False
          YES
                     207
                              58 0.280193
False
```

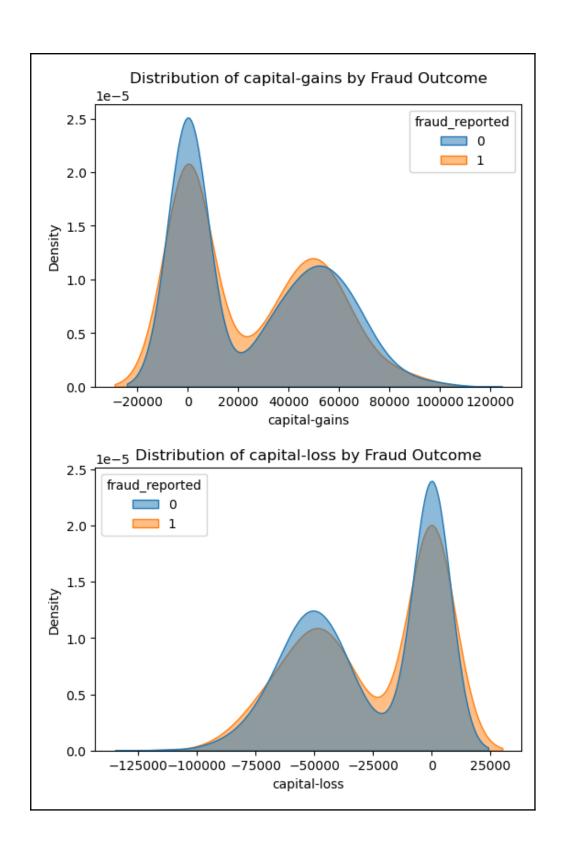
4.4.2. Explore the relationships between numerical features and the target variable to understand their impact on the target outcome using appropriate visualisation techniques to identify trends and potential interactions

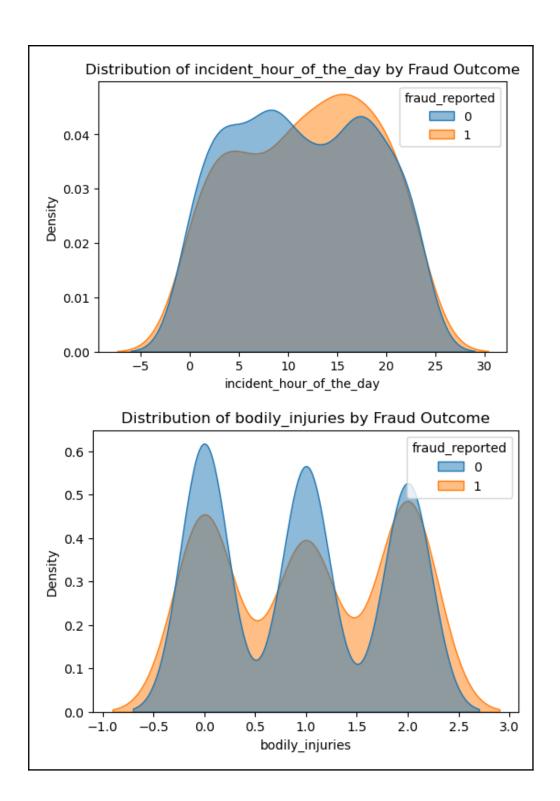
```
# Visualise the relationship between numerical features and the target
variable to understand their impact on the target outcome
# Ensure target is numeric (0/1)
y = y_train if isinstance(y_train, pd.Series) else y_train.squeeze()
Xy = X_{train.copy()}
Xy["fraud_reported"] = y
# Select numerical columns
numerical cols = X train.select dtypes(include=["int64",
"float64"]).columns.tolist()
# Plot boxplots for each numerical column vs target
plt.figure(figsize=(16, 20))
for i, col in enumerate(numerical_cols, 1):
  plt.subplot(5, 3, i)
  sns.boxplot(x="fraud_reported", y=col, data=Xy, palette="Set2")
  plt.title(f"{col} vs Fraud", fontsize=10)
plt.tight layout()
plt.show()
# Plot distribution (histogram + KDE) for each numeric column split by fraud
for col in numerical cols:
  plt.figure(figsize=(6,4))
  sns.kdeplot(data=Xy, x=col, hue="fraud reported".
common norm=False, fill=True, alpha=0.5)
  plt.title(f"Distribution of {col} by Fraud Outcome")
  plt.show()
```

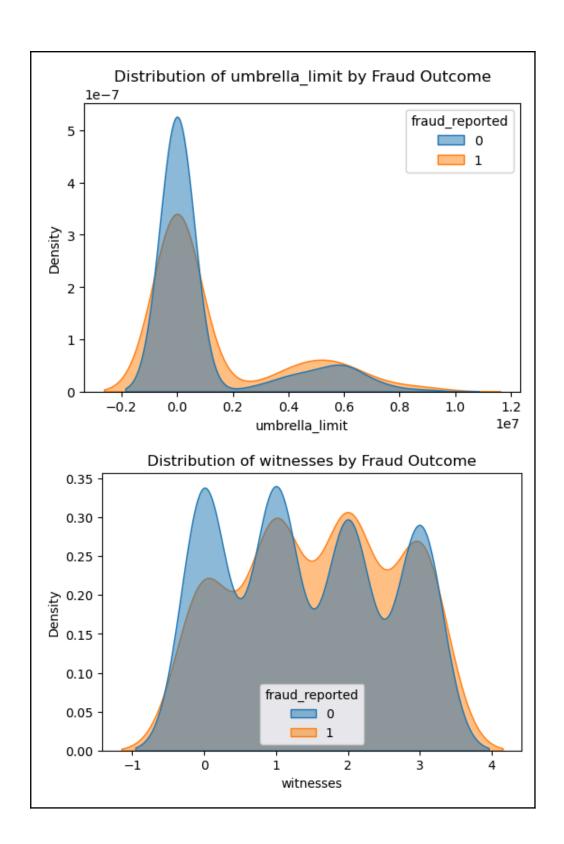


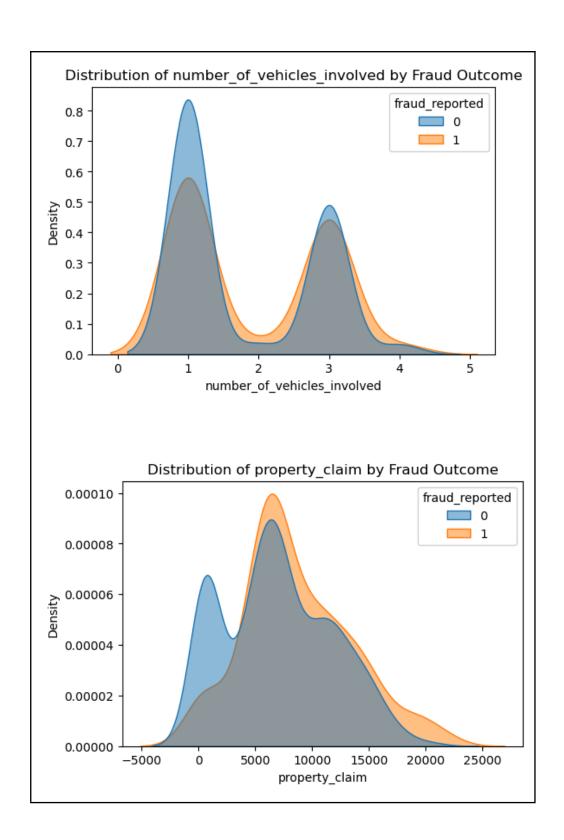


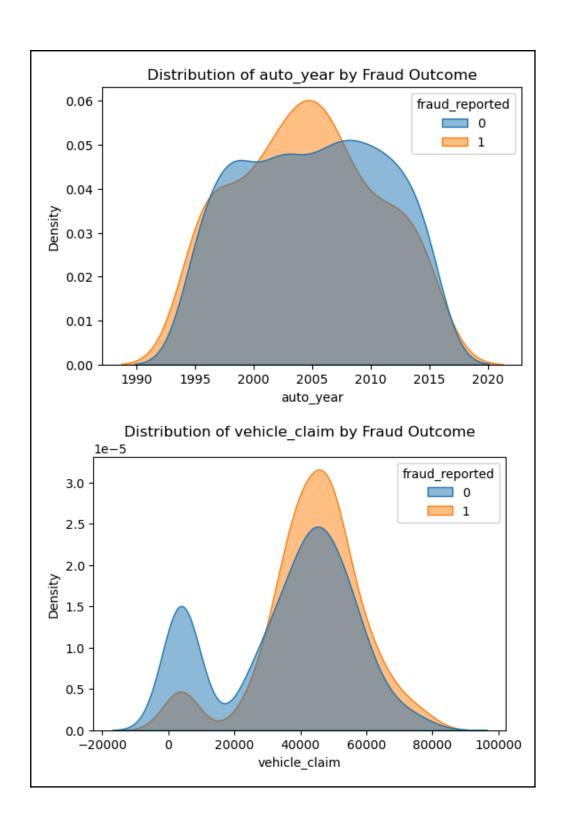


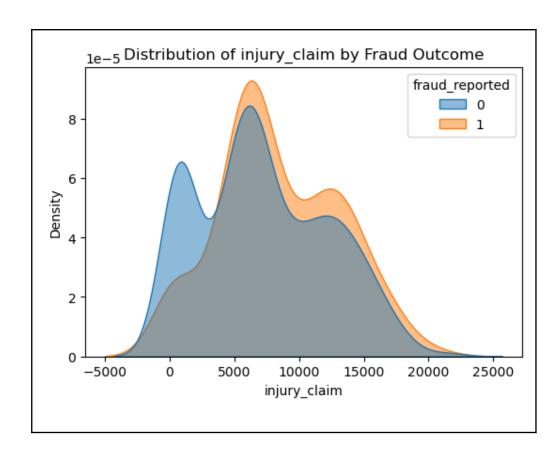












5. Feature Engineering

5.1. Perform resampling

```
# Import RandomOverSampler from imblearn library from imblearn.over_sampling import RandomOverSampler

# Perform resampling on training data from collections import Counter

# Before resampling print("Class distribution before resampling:", Counter(y_train))

# Apply RandomOverSampler ros = RandomOverSampler(random_state=42)
X_train_res, y_train_res = ros.fit_resample(X_train, y_train)

# After resampling print("Class distribution after resampling:", Counter(y_train_res))

# Shapes print("Original training shape:", X_train.shape, y_train.shape) print("Resampled training shape:", X_train_res.shape, y_train_res.shape)
```

```
#output
Class distribution before resampling: Counter({0: 526, 1: 173})
Class distribution after resampling: Counter({0: 526, 1: 526})
Original training shape: (699, 32) (699,)
Resampled training shape: (1052, 32) (1052,)
```

5.2. Feature Creation

```
# Create new features based on your understanding for both training and
validation data
import numpy as np
import pandas as pd
def create new features(df):
  df_new = df.copy()
  # --- Policy / Customer ---
  df_new["policy_premium_per_month"] =
df_new["policy_annual_premium"] / 12
  df_new["tenure_bucket"] = pd.cut(
    df_new["months_as_customer"],
    bins=[0, 12, 36, 60, 120, np.inf],
    labels=["<1yr", "1-3yr", "3-5yr", "5-10yr", "10+yr"]
  # --- Claims ---
  df_new["claim_component_sum"] = (
    df_new["injury_claim"] + df_new["property_claim"] +
df new["vehicle claim"]
  df new["claim component nonzero"] = (
    (df_new[["injury_claim", "property_claim", "vehicle_claim"]] >
0).sum(axis=1)
  df new["avg claim component"] = (
    df new["claim component sum"] /
(df new["claim component nonzero"] + 1e-5)
  df new["is single component claim"] = (
    (df_new["claim_component_nonzero"] == 1).astype(int)
  # --- Ratios / interactions ---
  df_new["claim_to_premium_ratio"] = (
    df new["claim component sum"] / (df new["policy annual premium"]
+ 1e-5)
  df_new["age_x_premium"] = df_new["age"] *
df new["policy annual premium"]
```

```
df_new["claim_per_vehicle"] = (
     df new["claim component sum"] /
(df_new["number_of_vehicles_involved"] + 1e-5)
  # --- Buckets ---
  df_new["age_bucket"] = pd.cut(
     df new["age"],
     bins=[0, 25, 40, 60, np.inf],
     labels=["young", "adult", "mid-age", "senior"]
  df_new["vehicle_age"] = 2025 - df_new["auto_year"]
  return df new
# Apply to both train & validation sets
X_train_fe = create_new_features(X_train_res)
X test fe = create new features(X test)
print("Train shape before:", X train res.shape, "after feature engineering:",
X train fe.shape)
print("Validation shape before:", X_test.shape, "after feature engineering:",
X_test_fe.shape)
```

```
#output

Train shape before: (1052, 32) after feature engineering:
  (1052, 43)

Validation shape before: (300, 32) after feature engineering:
  (300, 43)
```

5.3. Handle redundant columns

```
# Drop redundant columns from training and validation data
X_train_fe["claim_component_sum"] = (
    X_train_fe["injury_claim"] +
    X_train_fe["property_claim"] +
    X_train_fe["vehicle_claim"]
)

X_test_fe["claim_component_sum"] = (
    X_test_fe["injury_claim"] +
    X_test_fe["property_claim"] +
    X_test_fe["vehicle_claim"]
)
drop_cols = ["injury_claim", "property_claim",
"vehicle_claim","months_as_customer","policy_state","insured_sex"]
```

```
X_train_fe = X_train_fe.drop(columns=drop_cols, errors="ignore")
X_test_fe = X_test_fe.drop(columns=drop_cols, errors="ignore")

# Check the data
X_train_fe.head()
```

5.4. Combine values in Categorical Columns

```
# Combine categories that have low frequency or provide limited predictive
information
def collapse_rare_levels(df, col, min_count=50, new_label="Other"):
  Replace rare levels in a categorical column with 'Other'.
  value counts = df[col].value counts()
  rare levels = value counts[value counts < min count].index
  df[col] = df[col].replace(rare_levels, new_label)
  return df
categorical cols = [
  "auto model", "insured hobbies", "incident state", "insured occupation"
for col in categorical cols:
  if col in X_train_fe.columns:
     X_train_fe = collapse_rare_levels(X_train_fe, col, min_count=50)
     X_test_fe = collapse_rare_levels(X_test_fe, col, min_count=50)
# Inspect results
for col in categorical cols:
  print(f"\n{col} levels after collapsing:")
  print(X_train_fe[col].value_counts())
###OUTPUT####
auto model levels after collapsing:
auto_model
Other 1002
A5
         50
Name: count, dtype: int64
insured hobbies levels after collapsing:
insured hobbies
Other
                   431
                  101
chess
paintball reading
                  73
                    65
bungie-jumping 64
exercise
                   56
skydiving
                   55
base-jumping
                    53
```

```
yachting
                     53
                   51
board-games
                    50
hiking
Name: count, dtype: int64
incident state levels after collapsing:
incident state
      257
SC
NY
         194
135
WV
NC
VA 109
Other 74
Name: count, dtype: int64
insured occupation levels after collapsing:
insured_occupation
exec-managerial
                      99
                      96
machine-op-inspct
prof-specialty
                      95
                      93
Other
transport-moving 92
tech-support 86
craft-repair 83
                     82
sales
armed-forces adm-clerical
                     73
                      71
other-service
                     66
priv-house-serv 63 farming-fishing 53
Name: count, dtype: int64
```

5.5. Dummy variable creation

5.5.1. Identify categorical columns for dummy variable creation

```
# Identify the categorical columns for creating dummy variables
# Identify categorical columns
categorical_cols =
X_train_fe.select_dtypes(include=["category"]).columns.tolist()

# Identify numeric columns
numeric_cols = X_train_fe.select_dtypes(include=["int64",
    "float64"]).columns.tolist()

print("Categorical columns for dummies:", categorical_cols)
print("Numeric columns:", numeric_cols)

###Output###
Categorical columns for dummies: ['policy_csl',
    'insured_education_level', 'insured_occupation',
    'insured_hobbies', 'insured_relationship',
    'incident_type', 'collision_type', 'incident_severity',
    'authorities_contacted', 'incident_state',
```

```
'incident_city', 'property_damage',
'police_report_available', 'auto_make', 'auto_model',
'tenure_bucket', 'age_bucket']
Numeric columns: ['age', 'policy_deductable',
'policy_annual_premium', 'umbrella_limit',
'capital-gains', 'capital-loss',
'incident_hour_of_the_day',
'number_of_vehicles_involved', 'bodily_injuries',
'witnesses', 'auto_year', 'policy_premium_per_month',
'claim_component_sum', 'claim_component_nonzero',
'avg_claim_component', 'is_single_component_claim',
'claim_to_premium_ratio', 'age_x_premium',
'claim_per_vehicle', 'vehicle_age']
```

5.5.2. Create dummy variables for categorical columns in training data

```
# Create dummy variables using the 'get dummies' for categorical
columns in training data
# Create dummy variables (one-hot encoding)
X train dummies = pd.get dummies(X train fe,
columns=categorical cols, drop first=True)
print("Original train shape:", X_train_fe.shape)
print("After get dummies shape:", X train dummies.shape)
print("New columns added:", X_train_dummies.shape[1] -
X train fe.shape[1])
bool cols =
X train dummies.select dtypes(include=["bool"]).columns
X_train_dummies[bool_cols] =
X_train_dummies[bool_cols].astype(int)
###OUTPUT###
Original train shape: (1052, 37)
After get dummies shape: (1052, 104)
New columns added: 67
```

5.5.3. Create dummy variables for categorical columns in validation data

```
# Create dummy variables using the 'get_dummies' for categorical columns in validation data
X_test_dummies = pd.get_dummies(X_test_fe, columns=categorical_cols,
```

```
drop_first=True)

print("Original train shape:", X_test_fe.shape)
print("After get_dummies shape:", X_test_dummies.shape)
print("New columns added:", X_test_dummies.shape[1] -
    X_test_fe.shape[1])
bool_cols = X_test_dummies.select_dtypes(include=["bool"]).columns
    X_test_dummies[bool_cols] = X_test_dummies[bool_cols].astype(int)

X_train_dummies, X_test_dummies = X_train_dummies.align(
    X_test_dummies, join="left", axis=1, fill_value=0
)

###Output###
Original train shape: (300, 37)
After get_dummies shape: (300, 79)
New columns added: 42
```

5.6. Feature scaling

```
# Import the necessary scaling tool from scikit-learn from sklearn.preprocessing import StandardScaler

# Scale the numeric features present in the training data scaler = StandardScaler() scaler.fit(X_train_dummies[numeric_cols]) 
X_train_scaled = X_train_dummies.copy() 
X_train_scaled[numeric_cols] = scaler.transform(X_train_dummies[numeric_cols]) 
# Scale the numeric features present in the validation data 
X_test_scaled = X_test_dummies.copy() 
X_test_scaled[numeric_cols] = scaler.transform(X_test_dummies[numeric_cols])
```

6. Model Building

6.1. Feature selection

6.1.1. Import necessary libraries

Import necessary libraries from sklearn.feature_selection import RFECV from sklearn.linear_model import LogisticRegression from sklearn.model_selection import StratifiedKFold import matplotlib.pyplot as plt

6.1.2. Perform feature selection

```
# Apply RFECV to identify the most relevant features
logreg = LogisticRegression(max_iter=1000, solver='liblinear',
class_weight='balanced')
# RFECV with 5-fold Stratified CV
rfecv = RFECV(
   estimator=logreg,
   step=1,
                           # remove one feature at a time
   cv=StratifiedKFold(5), # stratified to preserve fraud/non-fraud
ratio
   scoring='roc auc',
                                # optimize for ROC-AUC (good for
imbalanced datasets)
   n_jobs=-1
# Fit RFECV on training data
rfecv.fit(X_train_scaled, y_train_res)
# Display the features ranking by RFECV in a DataFrame
rfecv results = pd.DataFrame({
   "Feature": X_train_scaled.columns,
   "Rank": rfecv.ranking_,
   "Selected": rfecv.support_
}).sort values("Rank")
print("RFECV Feature Ranking:")
print(rfecv_results)
####OUTPUT###
RFECV Feature Ranking:
                                                Feature Rank Selected
     Feature Rank Selected
age 1 True
incident_city_Hillsdale 1 True
incident_city_Columbus 1 True
incident_state_WV 1 True
incident_state_VA 1 True
incident_state_VA 1 True
age_bucket_senior 1 True
auto_make_Chevrolet 2 False
insured_occupation_craft-repair 3 False
incident_type_Parked Car 4 False
insured_occupation_farming-fishing 5 False
74
73
72
71
103
85
29
55
31
[104 rows x 3 columns]
```

6.1.3. Retain the selected features

```
# Put columns selected by RFECV into variable 'col'
col = X train scaled.columns[rfecv.support ].tolist()
print("Number of features selected:", len(col))
print("Selected features (col):", col)
##OUTPUT##
Number of features selected: 100
Selected features (col): ['age', 'policy deductable',
'policy annual premium', 'umbrella limit',
'capital-gains', 'capital-loss',
'incident hour of the day',
'number of vehicles involved', 'bodily injuries',
'witnesses', 'auto year', 'policy premium per month',
'claim component sum', 'claim component nonzero',
'avg claim component', 'is single component claim',
'claim to premium ratio', 'age x premium',
'claim per vehicle', 'vehicle age',
'policy csl 250/500', 'policy csl 500/1000',
'insured education level College',
'insured education level High School',
'insured education level JD',
'insured education level MD',
'insured education level Masters',
'insured education level PhD',
'insured occupation armed-forces',
'insured occupation exec-managerial',
'insured occupation Other',
'insured occupation machine-op-inspct',
'insured occupation other-service',
'insured occupation priv-house-serv',
'insured occupation prof-specialty',
'insured occupation sales',
'insured occupation tech-support',
'insured occupation transport-moving',
'insured hobbies Other', 'insured hobbies board-games',
'insured hobbies bungie-jumping',
'insured hobbies chess', 'insured hobbies exercise',
'insured hobbies hiking', 'insured hobbies paintball',
'insured hobbies reading', 'insured hobbies skydiving',
'insured hobbies yachting',
'insured relationship not-in-family',
'insured relationship other-relative',
'insured relationship own-child',
```

```
'insured relationship unmarried',
'insured relationship wife', 'incident type Single
Vehicle Collision', 'incident_type_Vehicle Theft',
'collision type Rear Collision', 'collision type Side
Collision', 'collision type unknown',
'incident severity Minor Damage',
'incident severity Total Loss',
'incident severity Trivial Damage',
'authorities contacted Fire',
'authorities contacted Other',
'authorities contacted Police',
'authorities contacted unknown', 'incident state NY',
'incident state Other', 'incident state SC',
'incident state VA', 'incident state WV',
'incident city Columbus', 'incident city Hillsdale',
'incident city Northbend', 'incident city Northbrook',
'incident_city_Riverwood', 'incident_city_Springfield',
'property damage YES', 'property damage unknown',
'police report available YES',
'police_report_available_unknown', 'auto_make_Audi',
'auto make BMW', 'auto make Dodge', 'auto make Ford',
'auto make Honda', 'auto_make_Jeep',
'auto_make_Mercedes', 'auto make Nissan',
'auto make Saab', 'auto make Suburu',
'auto make Toyota', 'auto make Volkswagen',
'auto model A5', 'tenure bucket 1-3yr',
'tenure bucket 3-5yr', 'tenure bucket 5-10yr',
'tenure_bucket_10+yr', 'age_bucket_adult',
'age bucket mid-age', 'age bucket senior']
```

6.2. Build Logistic Regression Model

6.2.1. Select relevant features and add constant in training data

```
# Select only the columns selected by RFECV

X_train_rfecv = X_train_scaled[col].copy()

X_test_rfecv = X_test_scaled[col].copy()

# Import statsmodels and add constant import statsmodels.api as sm

X_train_rfecv_const = sm.add_constant(X_train_rfecv, has_constant="add")

# Check the data print("Shape with constant:", X_train_rfecv_const.shape)
```

print("First few columns:", X_train_rfecv_const.head())

6.2.2. Fit logistic regression model

```
# Fit a logistic Regression model on X_train after adding a constant
and output the summary
X train rfecv const re = X train rfecv const
logit model = sm.Logit(y train res, X train rfecv const)
result = logit model.fit()
print(result.summary())
##OUTPUT##
Warning: Maximum number of iterations has been exceeded.
       Current function value: 0.316231
       Iterations: 35
                    Logit Regression Results
_____
Dep. Variable: fraud_reported No. Observations: 1052
                         Logit Df Residuals:
Model:
953
Method:
                           MLE Df Model:
98
Date:
              Tue, 09 Sep 2025
                               Pseudo R-squ.:
0.5438
Time:
                       05:54:25
                               Log-Likelihood:
-332.68
                         False LL-Null:
converged:
-729.19
Covariance Type:
                     nonrobust LLR p-value:
2.979e-109
_____
_____
                                   coef
                                         std
              P > |z| [0.025
                                 0.975]
const
                                  3.4216
1421.372 0.002 0.998 -2782.416 2789.259
                                 -0.5721
age
age
0.668 -0.857 0.392 -1.881
                                    0.737
policy_deductable
                                 0.1336
0.360
                                 -0.1740
policy_annual_premium
       nan nan
                         nan
                                    nan
                                 0.2921
umbrella limit
0.118 2.478 0.013 0.061
                                    0.523
                                 0.1379
capital-gains
0.115 1.194 0.232 -0.088
                                    0.364
                                 -0.0721
capital-loss
                0.535 -0.300
0.116 -0.620
                                    0.156
incident hour of_the_day
                                 -0.1377
0.117 \quad -1.\overline{180} \quad 0.238
                        -0.366
                                    0.091
```

```
number of vehicles involved
                                   -0.6196
0.509 -1.218 0.223 -1.617
                                      0.378
bodily injuries
                                    0.2030
              0.079
0.115
     1.759
                            -0.023
                                       0.429
witnesses
                                    0.4500
         3.817 0.000
0.118
                           0.219
                                      0.681
auto year
                                   -0.1431
5.85e+06 -2.45e-08 1.000
                            -1.15e+07 1.15e+07
policy_premium per month
                                   -0.1740
nan nan nan
                           nan
                                      nan
                                   -3.0214
claim component sum
2.150 -1.405
                  0.160 -7.235
                                      1.192
                                    0.2103
claim component nonzero
0.536 	 0.39\overline{2} 	 0.695
                           -0.841
                                    1.262
                                    2.6595
avg_claim_component
2.062 1.290
                 0.197
                           -1.382
                                      6.701
is_single_component_claim
                                    1.4653
1.62e+04 9.02e-05 1.000
                            -3.18e+04
                                    3.18e+04
                                    0.7455
claim to premium ratio
0.484 1.541 0.123
                           -0.203
                                      1.694
age_x_premium
                                    1.3831
               0.096
0.8\overline{3}2 1.662
                           -0.248
                                     3.014
claim_per_vehicle
                                    0.1044
                           -1.079
0.604 0.173
                 0.863
                                      1.288
vehicle age
                                    0.1431
5.85e+06 2.45e-08 1.000 -1.15e+07
                                      1.15e+07
policy csl 250/500
                                    0.5719
0.278 2.056
                 0.040
                           0.027
                                      1.117
policy csl 500/1000
                                   -0.2904
0.288 -1.007
                 0.314
                            -0.855
                                      0.275
insured education level College
                                   -0.0756
0.471 -0.160 0.873
                           -0.999
                                      0.848
insured_education level High School
                                   -0.2499
0.426 \quad -0.587 \quad 0.557 \quad -1.084
                                      0.585
insured education level JD
                                    0.8027
0.407 1.972 0.049
                           0.005
                                      1.601
insured education level MD
                                    0.5856
0.424 1.381 0.167
                            -0.245
                                      1.417
                                    0.2699
insured education level Masters
                            -0.563
0.425
      0.635 0.525
                                      1.102
insured education level PhD
                                    0.8223
                            -0.028
0.434
      1.896 0.058
                                      1.672
insured_occupation_armed-forces
                                    0.5292
0.530 0.999 0.318
                           -0.509
                                      1.568
                                    0.3991
insured occupation exec-managerial
0.484 0.824 0.410 -0.550
                                      1.348
insured occupation Other
                                   -1.3333
0.455 -2.929 0.003
                                     -0.441
                           -2.225
insured occupation machine-op-inspct
                                   -0.0128
0.454 -0.028 -0.977 -0.902
                                     0.877
                                   -0.8327
insured_occupation_other-service
0.544 -1.531 0.126 -1.899
                                     0.233
insured occupation priv-house-serv
                                   -1.3099
0.591 -2.217 0.027 -2.468
                                    -0.152
insured occupation prof-specialty
                                    0.1805
0.473 0.382 0.703
                           -0.746
                                      1.107
insured occupation sales
                                   -0.1975
0.563 -0.351 0.726
                            -1.301
                                      0.906
insured occupation tech-support
                                   -0.2887
                            -1.232
                                      0.654
0.481 -0.600 0.548
```

insured_occupation_transport-movi	na	0 8429
1 000 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	0 015	1 700
0.438 1.926 0.054	-0.015	1.700
insured_hobbies_Other		-0.3305
0.497 - 0.666 0.506	-1.304	0.643
insured hobbies board-games		-0.3108
	-1.713	1.091
insured hobbies bungie-jumping	1.710	-1.1174
	0 540	
	-2.542	0.308
insured_hobbies_chess		6.4090
$0.839 7.63\overline{5} 0.000$	4.764	8.054
insured hobbies exercise		-0.4941
0.6680.740 0.459	-1.803	0.815
insured hobbies hiking	1.000	-1.0540
1 F10 0 121	0 400	
0.698 -1.510 0.131	-2.422	0.314
insured_hobbies_paintball		-0.2730
0.633 -0.431 0.666	-1.515	0.969
insured hobbies reading		-0.6998
0.667 - 1.050 0.294	-2.007	0.607
insured hobbies skydiving	2.007	-0.9335
	0 204	
$0.740 - 1.26\overline{2} 0.207$	-2.384	0.517
insured_hobbies_yachting		-0.0363
0.711 -0.051 0.959	-1.429	1.357
insured relationship not-in-famil	_V	0.9425
0.409 2.303 0.021	0.140	1.745
insured_relationship_other-relati		0.5917
0.396 1.493 0.135	-0.185	
insured_relationship_own-child		-0.6324
0.418 -1.513 0.130	-1.452	0.187
insured_relationship_unmarried		0.9204
0.420 - 2.194 0.028	0.098	1.743
insured relationship wife	0.000	-0.0186
Insured_relactionship_wire	0 050	-0.0186
0.429 -0.043 0.965	-0.859	0.822
<pre>incident_type_Single Vehicle Coll</pre>		
1.166 -1.130 0.259	-3.603	0.968
incident_type_Vehicle Theft		-0.3085
	-1.846	1.229
collision_type_Rear Collision		0.4204
0 206 1 421 0 155	-0.160	1.000
	-0.160	
collision_type_Side Collision		-0.5707
0.317 -1.800 0.072	-1.192	0.051
collision type unknown		-0.2411
$1.452 -0.1\overline{6}6 0.868$	-3.087	2.605
incident severity Minor Damage		-4.5796
	-5.321	-3.838
	J.JZI	
incident_severity_Total Loss		-3.6086
0.311 -11.603 0.000	-4.218	-2.999
<pre>incident_severity_Trivial Damage</pre>		-5.4819
0.876 -6.260 0.000	-7.198	-3.765
authorities contacted Fire		-0.2997
0.347 -0.862 0.388	-0.981	0.381
	-0.901	
authorities_contacted_Other	0 0 0 1 1	0.3026
0.344 0.880 0.379	-0.371	0.976
authorities_contacted_Police		0.0204
0.343	-0.651	0.692
authorities contacted unknown		0.0175
0.879 0.020 0.984	-1.705	1.740
	1.700	
incident_state_NY	1 000	-0.4862
0.409 -1.189 0.234	-1.288	0.315
incident_state_Other		-0.1878
0.614 -0.306 0.760	-1.391	1.015

Incident				
0.399	incident state SC			-0 1800
Incident		0 652	-0 962	
0.463 1.558 0.119 -0.186 1.630 incident_state_WV -1.0632 -0.022 0.429 -2.479 0.013 -1.904 -0.222 incident_city_Columbus 0.0793 0.0793 0.0793 0.398 0.199 0.842 -0.701 0.859 incident_city_Northbend -0.666 -0.652 1.020 incident_city_Northbrook -0.497 -0.4012 0.458 -0.875 0.381 -1.300 0.497 incident_city_Northbrook 0.2578 0.4012 0.2578 0.434 -0.594 0.552 -0.592 1.108 incident_city_Springfield 0.256 -0.592 1.108 incident_city_Springfield 0.293 0.295 0.6012 0.295 2.037 0.042 0.023 1.180 0.296 1.557 0.119 -0.111 0.971 police_report_available_unknown 0.042 0.234 0.238 0.295 -1.413 0.158		0.002	0.502	
Incident state WV		0 110	0 106	
0.429 -2.479 0.013 -1.904 -0.222 incident_city_Columbus 0.0793 0.0793 incident_city_Hillsdale 0.1840 0.427 0.431 0.666 -0.652 1.020 incident_city_Northbend -0.0163 -0.414 -0.039 0.969 -0.828 0.795 incident_city_Northbrook 0.458 -0.875 0.381 -1.300 0.497 incident_city_Riverwood 0.2578 0.434 0.594 0.552 -0.592 1.108 incident_city_Springfield 0.560 0.576 -0.585 1.052 property_damage_uknown 0.295 0.6012 0.6012 0.295 2.037 0.042 0.023 1.180 property_damage_uknown 0.4298 0.4298 0.295 -1.413 0.158 -0.995 0.161 0.295 -1.413 0.158 -0.995 0.161 0.295 -1.413 0.158 -0.995 0.161 0.606 3.038 <td< td=""><td></td><td>0.119</td><td>-0.100</td><td></td></td<>		0.119	-0.100	
Incident_city_Columbus 0.0793 0.398 0.199 0.842 -0.701 0.859 incident_city_Hillsdale 0.427 0.431 0.666 -0.652 1.020 incident_city_Northbend -0.0163 0.795 incident_city_Northbend -0.0163 0.4795 incident_city_Northbrook -0.4012 0.458 -0.875 0.381 -1.300 0.497 incident_city_Northbrook -0.497 incident_city_Springfield 0.2578 0.434 0.594 0.552 -0.592 1.108 incident_city_Springfield 0.2338 0.418 0.560 0.576 -0.585 1.052 property_damage_YES 0.6012 0.295 2.037 0.042 0.023 1.180 property_damage_unknown 0.4298 0.276 1.557 0.119 -0.111 0.971 0.971 0.976 0.276 1.557 0.119 -0.111 0.971 0.971 0.277 -1.099 0.272 -0.846 0.238 0.277 -1.099 0.272 -0.846 0.238 0.2247 0.502 0.293 0.2247 0.502 0.293 0.2247 0.502 0.293 0.2247 0.502 0.293 0.2247 0.502 0.293 0.2247 0.502 0.293 0.2247 0.502 0.293 0.2247 0.502 0.293 0.2247 0.502 0.293 0.222 0.167 0.2420 0.526 0.799 0.424 -0.610 0.4200 0.526 0.799 0.424 -0.610 0.4200 0.526 0.799 0.424 -0.610 0.4200 0.526 0.799 0.424 -0.610 0.4200 0.526 0.799 0.424 -0.610 0.4200 0.525 0.541 0.046 0.241 0.4200 0.525 0.551 0.577 0.233 0.515 0.453 0.651 -0.777 0.2333 0.515 0.453 0.651 -0.777 0.2333 0.515 0.453 0.551 -0.777 0.243 0.500 0.1145 0.252 -0.408 0.5523 0.552 0.599 1.295 0.195 -0.398 0.4392 0.4392 0.546 0.778 0.436 -0.667 0.545 0.078 0.599 0.295 0.195 0.398 0.4392 0.4392 0.4400 0.778 0.436 0.667 0.4300 0.778 0.436 0.667 0.4300 0.778 0.436 0.667 0.4300 0.778 0.436 0.667 0.4300 0.778 0.436 0.667 0.4300 0.778 0.436 0.667 0.4300 0.999 0.447 0.086 0.392 -2.668 0.046 0.046 0.046 0.046 0.046 0.046 0.046 0.046 0.046 0.046 0.046 0.046 0.046 0.046 0.046 0.046 0.046 0.0		0 010	1 004	
0.189			-1.904	
Incident_city_Hillsdale				
0.427			-0.701	0.859
Incident_city_Northbend				0.1840
0.414 -0.039 0.969 -0.828 0.795 incident_city_Northbrook -0.4012 0.497 incident_city_Riverwood 0.2578 0.434 0.594 0.552 -0.592 1.108 incident_city_Springfield 0.2338 0.418 0.560 0.576 -0.585 1.052 property_damage_YES 0.6012 0.023 1.180 property_damage_unknown 0.4298 0.4298 0.276 1.557 0.119 -0.111 0.971 police_report_available_YES 0.0469 0.4298 0.2498 0.276 1.557 0.119 -0.111 0.971 police_report_available_YES 0.0469 0.4169 0.4298 0.276 -1.413 0.158 -0.995 0.161 police_report_available_WES 0.095 0.0416 0.236 0.238 0.277 -1.099 0.272 -0.846 0.238 0.228 0.646 3.038 0.002 0.696 3.228	$0.427 0.\overline{431}$	0.666	-0.652	1.020
0.414 -0.039 0.969 -0.828 0.795 incident_city_Northbrook -0.4012 0.497 incident_city_Riverwood 0.2578 0.434 0.594 0.552 -0.592 1.108 incident_city_Springfield 0.2338 0.418 0.560 0.576 -0.585 1.052 property_damage_YES 0.6012 0.023 1.180 property_damage_unknown 0.4298 0.4298 0.276 1.557 0.119 -0.111 0.971 police_report_available_YES 0.0469 0.4298 0.2498 0.276 1.557 0.119 -0.111 0.971 police_report_available_YES 0.0469 0.4169 0.4298 0.276 -1.413 0.158 -0.995 0.161 police_report_available_WES 0.095 0.0416 0.236 0.238 0.277 -1.099 0.272 -0.846 0.238 0.228 0.646 3.038 0.002 0.696 3.228	incident city Northbe	nd		-0.0163
Incident_city_Northbrook			-0.828	0.795
0.458				
incident_city_Riverwood	0 458 -0 875	0 381	-1 300	
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Incident_city_Springfield			0 500	
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0.564 0.778 0.436 -0.667 1.545 auto_model_A5 0.0999 0.746 0.134 0.894 -1.363 1.563 tenure_bucket_1-3yr -0.8109 0.947 -0.856 0.392 -2.668 1.046 tenure_bucket_3-5yr -0.0136 0.959 -0.014 0.989 -1.894 1.866 tenure_bucket_5-10yr -0.2252 0.799 -0.282 0.778 -1.791 1.341 tenure_bucket_10+yr -1.2530		3.133	0.000	
auto_model_A5 0.0999 0.746 0.134 0.894 -1.363 1.563 tenure_bucket_1-3yr -0.8109 0.947 -0.856 0.392 -2.668 1.046 tenure_bucket_3-5yr -0.0136 0.959 -0.014 0.989 -1.894 1.866 tenure_bucket_5-10yr -0.2252 0.799 -0.282 0.778 -1.791 1.341 tenure_bucket_10+yr -1.2530		0 436	-0 667	
0.746 0.134 0.894 -1.363 1.563 tenure_bucket_1-3yr -0.8109 0.947 -0.856 0.392 -2.668 1.046 tenure_bucket_3-5yr -0.0136 -0.0136 1.866 0.959 -0.014 0.989 -1.894 1.866 tenure_bucket_5-10yr -0.2252 0.799 -0.282 0.778 -1.791 1.341 tenure_bucket_10+yr -1.2530		0.430	-0.007	
tenure_bucket_1-3yr		0 004	1 262	
0.947 -0.856 0.392 -2.668 1.046 tenure_bucket_3-5yr -0.0136 0.959 -0.014 0.989 -1.894 1.866 tenure_bucket_5-10yr -0.2252 0.799 -0.282 0.778 -1.791 1.341 tenure_bucket_10+yr -1.2530		U.894	-1.363	
tenure_bucket_3-5yr -0.0136 0.959 -0.014 0.989 -1.894 1.866 tenure_bucket_5-10yr -0.2252 0.799 -0.282 0.778 -1.791 1.341 tenure_bucket_10+yr -1.2530			_	
0.959 -0.014 0.989 -1.894 1.866 tenure_bucket_5-10yr -0.2252 0.799 -0.282 0.778 -1.791 1.341 tenure_bucket_10+yr -1.2530		0.392	-2.668	
tenure_bucket_5-10yr -0.2252 0.799 -0.282 0.778 -1.791 1.341 tenure_bucket_10+yr -1.2530	tenure_bucket_3-5yr			-0.0136
0.799 -0.282 0.778 -1.791 1.341 tenure_bucket_10+yr -1.2530	$0.959 - 0.\overline{0}14$	0.989	-1.894	1.866
0.799 -0.282 0.778 -1.791 1.341 tenure_bucket_10+yr -1.2530	tenure bucket 5-10yr			-0.2252
tenure_bucket_10+yr -1.2530		0.778	-1.791	
		· -		
0.755		0.110	-2 791	
			• , , , _	J.200

```
age bucket adult
                                 -0.5675
0.656 - 0.865
             0.387 -1.853
                                    0.718
age bucket mid-age
                                 -0.6122
              0.469
                          -2.271
0.846 -0.723
                                    1.047
                                 -2.2176
age_bucket_senior
              0.113 -4.959
1.399
     -1.586
                                    0.524
```

```
from sklearn.linear_model import LogisticRegression from sklearn import metrics logsk = LogisticRegression(C=1e9) logsk.fit(X_train_rfecv, y_train_res)
```

6.2.3. Evaluate VIF of features to assess multicollinearity

```
# Import 'variance_inflation_factor'
from statsmodels.stats.outliers influence import
variance inflation factor
# Make a VIF DataFrame for all the variables present
def calculate_vif(X: pd.DataFrame):
  Create a VIF DataFrame for all numeric columns in DataFrame X.
  # Ensure numeric only
  X numeric = X.select dtypes(include=[np.number])
  # Add small constant if any column is constant (to avoid division
by zero)
  X_numeric = X_numeric.copy()
  for col in X numeric.columns:
    if X numeric[col].nunique() == 1:
       X \text{ numeric[col]} = X \text{ numeric[col]} + 1e-6
  vif data = pd.DataFrame()
  vif data["Feature"] = X numeric.columns
  vif_data["VIF"] = [
    variance inflation factor(X numeric.values, i)
    for i in range(X_numeric.shape[1])
  return vif_data.sort_values("VIF", ascending=False)
# Example: run on your one-hot encoded training set
```

```
vif_df = calculate_vif(X_train_rfecv)
print(vif_df[vif_df['VIF'] > 5])
```

6.2.4. Make predictions on training data

```
# Predict the probabilities on the training data
y_train_pred_prob = logsk.predict_proba(X_train_rfecv)[:, 1]

# Ensure it's a flat 1-D array
y_train_pred_prob = np.asarray(y_train_pred_prob).reshape(-1,)

print("Shape:", y_train_pred_prob.shape)
print("First 10 predicted probabilities:", y_train_pred_prob[:10])

##OUTPUT##
Shape: (1052,)
First 10 predicted probabilities: [0.7028878  0.13683076
0.17926176  0.03150495  0.06332432  0.49219636
0.86344362  0.02615324  0.24287583  0.25465523]
```

6.2.5. Create a DataFrame that includes actual fraud reported flags, predicted probabilities, and a column indicating predicted classifications based on a cutoff value of 0.5

```
# Create a new DataFrame containing the actual fraud reported flag
and the probabilities predicted by the model
train pred df = pd.DataFrame({
  "fraud reported": y train res.values, # actual
  "pred_prob": y_train_pred_prob
                                           # predicted probability
(fraud)
})
# Create new column indicating predicted classifications based on a
cutoff value of 0.5
cutoff = 0.5
train_pred_df["pred_class"] = (train_pred_df["pred_prob"] >=
cutoff).astype(int)
print(train pred df.head())
##OUTPUT##
  fraud_reported pred_prob pred_class
                  0^{-}0.\overline{7}02888
                     0.136831
1
                  \cap
2
                  0 0.179262
3
                  0 0.031505
4
                       0.063324
```

6.2.6. Check the accuracy of the model

```
# Import metrics from sklearn for evaluation from sklearn import metrics

# Check the accuracy of the model accuracy = metrics.accuracy_score(train_pred_df["fraud_reported"], train_pred_df["pred_class"])

print(f"Training Accuracy: {accuracy:.4f}")

##OUTPUT##
Training Accuracy: 0.8888
```

6.2.7. Create a confusion matrix based on the predictions made on the training data

```
# Create confusion matrix
cm = metrics.confusion_matrix(train_pred_df["fraud_reported"],
train_pred_df["pred_class"])

# Print numeric confusion matrix
print("Confusion Matrix:\n", cm)
##OUTPUT##
Confusion Matrix:
[[465 61]
[ 56 470]]
```

6.2.8. Create variables for true positive, true negative, false positive and false negative

```
# Create variables for true positive, true negative, false positive and false negative tn, fp, fn, tp = cm.ravel()

print("True Negatives:", tn)
print("False Positives:", fp)
print("False Negatives:", fn)
print("True Positives:", tp)

##OUTPUT##

True Negatives: 465
False Positives: 61
False Negatives: 56
True Positives: 470
```

6.2.9. Calculate sensitivity, specificity, precision, recall and F1-score

```
# Calculate the sensitivity
sensitivity = tp / (tp + fn)
print(f"Sensitivity (Recall): {sensitivity:.4f}")
# Calculate the specificity
specificity = tn / (tn + fp)
print(f"Specificity: {specificity:.4f}")
# Calculate Precision
precision = tp / (tp + fp)
print(f"Precision : {precision:.4f}")
# Calculate Recall
recall = tp / (tp + fn)
print(f"Recall : {recall:.4f}")
# Calculate F1 Score
f1 score = 2 * (precision * recall) / (precision + recall)
print(f"F1 Score : {f1_score:.4f}")
##OUTPUT##
Sensitivity (Recall): 0.8935
Specificity: 0.8840
Precision : 0.8851
Recall : 0.8935
F1 Score : 0.8893
```

6.3. Find the Optimal Cutoff

6.3.1. Plot ROC Curve to visualise the trade-off between true positive rate and false positive rate across different classification thresholds

```
# Import libraries or function to plot the ROC curve from sklearn.metrics import roc_curve, roc_auc_score

# Define ROC function
def plot_roc_curve(y_true, y_prob, title="ROC Curve"):

# Compute ROC values
fpr, tpr, thresholds = roc_curve(y_true, y_prob)
auc_score = roc_auc_score(y_true, y_prob)

# Plot
plt.figure(figsize=(6,5))
plt.plot(fpr, tpr, color="blue", label=f"AUC = {auc_score:.4f}")
plt.plot([0,1], [0,1], linestyle="--", color="gray", label="Random Guess")
```

```
plt.xlabel("False Positive Rate (1 - Specificity)")
  plt.ylabel("True Positive Rate (Sensitivity)")
  plt.title(title)
  plt.legend(loc="lower right")
  plt.grid(True)
  plt.show()
  return auc_score
# Call the ROC function
auc_train = plot_roc_curve(train_pred_df["fraud_reported"],
                   train pred df["pred prob"],
                  title="ROC Curve - Training Data")
print("Training AUC:", auc_train)
                           ROC Curve - Training Data
    1.0
    0.8
 True Positive Rate (Sensitivity)
    0.6
    0.4
    0.2
                                                           AUC = 0.9403
                                                           Random Guess
    0.0
                                                                       1.0
          0.0
                      0.2
                                   0.4
                                               0.6
                                                           0.8
                         False Positive Rate (1 - Specificity)
```

6.3.2. Predict on training data at various probability cutoffs

```
# Create columns with different probability cutoffs to explore the impact of cutoff on model performance
# Define cutoffs to test
cutoffs = [0.3, 0.4, 0.5, 0.6, 0.7]

# Create new columns for each cutoff for c in cutoffs:
    col_name = f"pred_class_{str(c).replace('.', ")}"
```

```
train_pred_df[col_name] = (train_pred_df["pred_prob"] >=
c).astype(int)
print(train pred df.head())
##OUTPUT##
fraud_reported pred_prob pred_class pred_class_03
pred_class_04 \
                    0.702888
                                                      1
1
1
                0
                   0.136831
                                       0
                                                      0
0
2
                0
                   0.179262
                                       0
                                                      0
0
3
                  0.031505
                                       0
                                                      0
                0
0
                                       0
4
                0
                  0.063324
                                                      0
0
  pred class 05 pred class 06 pred class 07
0
1
              0
                             0
2
               0
                             0
3
               0
                             0
4
```

6.3.3. Plot accuracy, sensitivity, specificity at different values of probability cutoffs

```
# Create a DataFrame to see the values of accuracy, sensitivity, and
specificity at different values of probability cutoffs
from sklearn metrics import confusion matrix, accuracy score
# Define a range of cutoffs (finer granularity than before)
cutoffs = np.arange(0.1, 0.91, 0.1) # 0.1 to 0.9 in steps of 0.1
results = []
for c in cutoffs:
  # Predicted class at cutoff
  y_pred_class = (train_pred_df["pred_prob"] >= c).astype(int)
  # Confusion matrix
  tn, fp, fn, tp = confusion_matrix(train_pred_df["fraud_reported"],
y pred class).ravel()
  # Metrics
  accuracy = accuracy_score(train_pred_df["fraud_reported"],
y_pred_class)
  sensitivity = tp / (tp + fn) if (tp+fn) > 0 else 0
  specificity = tn / (tn + fp) if (tn+fp) > 0 else 0
  results.append({
```

```
"Cutoff": round(c, 2),
        "Accuracy": round(accuracy, 4),
        "Sensitivity": round(sensitivity, 4),
        "Specificity": round(specificity, 4)
    })
cutoff df = pd.DataFrame(results)
print(cutoff df)
##OUTPUT##
   Cutoff Accuracy Sensitivity Specificity
           0.1 0.7795 0.9848 0.5741
                        0.8337
           0.2

      0.2
      0.8337
      0.9696

      0.3
      0.8631
      0.9544

      0.4
      0.8745
      0.9183

      0.5
      0.8888
      0.8935

      0.6
      0.8774
      0.8517

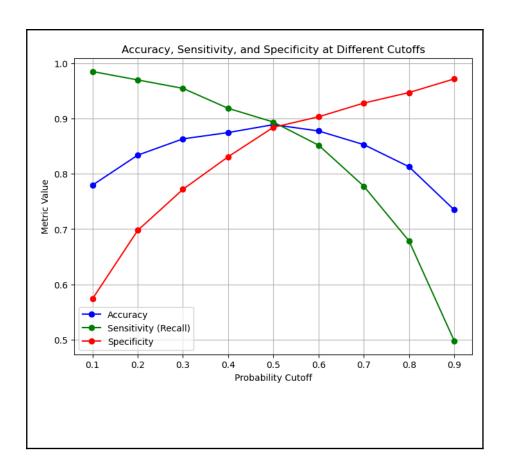
      0.7
      0.8527
      0.7776

      0.8
      0.8127
      0.6787

      0.9
      0.7348
      0.4981

1
                                                0.9696
                                                                           0.6977
                                                                         0.7719
2
                                                                         0.8308
3
                                                                       0.8840
4
5
                                                                         0.9030
6
                                                                          0.9278
7
                                                                           0.9468
8
                                                                            0.9715
```

```
# Plot accuracy, sensitivity, and specificity at different values of
probability cutoffs
# Plot Accuracy, Sensitivity, Specificity vs Cutoff
plt.figure(figsize=(8,6))
plt.plot(cutoff_df["Cutoff"], cutoff_df["Accuracy"], marker="o",
label="Accuracy", color="blue")
plt.plot(cutoff_df["Cutoff"], cutoff_df["Sensitivity"], marker="o",
label="Sensitivity (Recall)", color="green")
plt.plot(cutoff df["Cutoff"], cutoff df["Specificity"], marker="o",
label="Specificity", color="red")
plt.xlabel("Probability Cutoff")
plt.ylabel("Metric Value")
plt.title("Accuracy, Sensitivity, and Specificity at Different Cutoffs")
plt.legend()
plt.grid(True)
plt.show()
```



6.3.4. Create a column for final prediction based on optimal cutoff

```
# Create a column for final prediction based on the optimal cutoff optimal_cutoff = 0.5 train_pred_df["final_pred"] = (train_pred_df["pred_prob"] >= optimal_cutoff).astype(int) print(train_pred_df.head())
```

6.3.5. Calculate the accuracy

```
# Check the accuracy now accuracy_final = accuracy_score(train_pred_df["fraud_reported"], train_pred_df["final_pred"])

print(f"Accuracy at optimal cutoff: {accuracy_final:.4f}")

##OUTPUT##

Accuracy at optimal cutoff: 0.8888
```

6.3.6. Create confusion matrix

```
# Create the confusion matrix once again
# Create confusion matrix
cm = metrics.confusion_matrix(train_pred_df["fraud_reported"],
train_pred_df["final_pred"])

# Print numeric confusion matrix
print("Confusion Matrix:\n", cm)
##OUTPUT##
Confusion Matrix:
[[465 61]
[ 56 470]]
```

6.3.7. Create variables for true positive, true negative, false positive and false negative

```
# Create variables for true positive, true negative, false positive and false negative tn, fp, fn, tp = cm.ravel()

print("True Negatives:", tn)
print("False Positives:", fp)
print("False Negatives:", fn)
print("True Positives:", tp)
##OUTPUT##
True Negatives: 465
False Positives: 61
False Negatives: 56
True Positives: 470
```

6.3.8. Calculate sensitivity, specificity, precision, recall and F1-score of the model

```
# Calculate the sensitivity

sensitivity = tp / (tp + fn)
print(f"Sensitivity (Recall): {sensitivity:.4f}")

# Calculate the specificity
specificity = tn / (tn + fp)
print(f"Specificity: {specificity:.4f}")

# Calculate Precision
precision = tp / (tp + fp)
print(f"Precision : {precision:.4f}")

# Calculate Recall
recall = tp / (tp + fn)
print(f"Recall : {recall:.4f}")

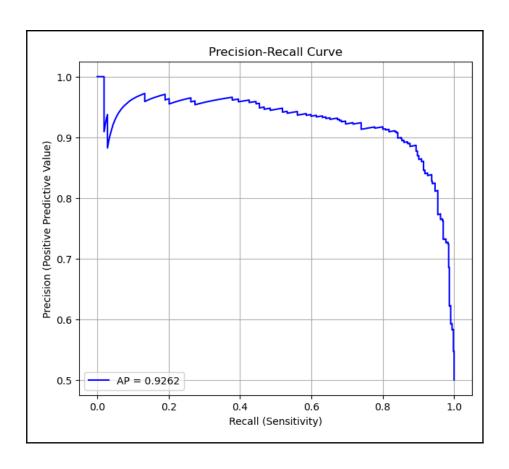
# Calculate F1 Score
```

```
f1_score = 2 * (precision * recall) / (precision + recall)
print(f"F1 Score : {f1_score:.4f}")

##OUTPUT##
Sensitivity (Recall): 0.8935
Specificity: 0.8840
Precision : 0.8851
Recall : 0.8935
F1 Score : 0.8893
```

6.3.9. Plot precision-recall curve

```
# Import precision-recall curve function
from sklearn.metrics import
precision recall curve, average precision score
# Plot precision-recall curve
precision, recall, thresholds = precision_recall_curve(
  train_pred_df["fraud_reported"], train_pred_df["pred_prob"]
ap_score =
average precision score(train pred df["fraud reported"],
train pred df["pred prob"])
plt.figure(figsize=(7,6))
plt.plot(recall, precision, color="blue", label=f"AP = {ap_score:.4f}")
plt.xlabel("Recall (Sensitivity)")
plt.ylabel("Precision (Positive Predictive Value)")
plt.title("Precision-Recall Curve")
plt.legend(loc="lower left")
plt.grid(True)
plt.show()
```



6.4. Build Random Forest Model

6.4.1. Import necessary libraries

Import necessary libraries from sklearn.ensemble import RandomForestClassifier from sklearn.metrics import classification_report from sklearn.model_selection import cross_val_score, GridSearchCV

6.4.2. Build the random forest model

```
# Build a base random forest model

rf_model = RandomForestClassifier(
    n_estimators=5, # number of trees
    random_state=42, # reproducibility
    n_jobs=-1 # use all CPU cores
)

rf_model.fit(X_train_rfecv, y_train_res)
```

6.4.3. Get feature importance scores and select important features

```
# Get feature importance scores from the trained model
importances = rf model.feature importances
# Create a DataFrame to visualise the importance scores
feat importances = pd.DataFrame({
  "Feature": X_train_rfecv.columns,
  "Importance": importances
}).sort values(by="Importance", ascending=False)
print(feat_importances.head(10)) # Top 10 features
# Plot top 15 features
plt.figure(figsize=(10,6))
plt.barh(feat importances["Feature"].head(15)[::-1],
     feat importances["Importance"].head(15)[::-1],
      color="skyblue")
plt.xlabel("Importance Score")
plt.ylabel("Feature")
plt.title("Top 15 Feature Importances - Random Forest")
plt.show()
##OUTPUT##
                                 Feature Importance
58
    incident_severity_Minor Damage     0.097077
16
               claim_to_premium_ratio     0.057755
17
                          age_x_premium 0.056275
12
                   claim_component_sum 0.052069
11
           policy_premium_per_month 0.046125
14
                  avg_claim_component 0.043906
2
                policy annual premium 0.038038
41
                insured hobbies chess 0.036892
6
            incident hour of the day
                                                0.035328
0
                                                0.032662
                                Top 15 Feature Importances - Random Forest
  incident severity Minor Damage
      claim_to_premium_ratio
          age_x_premium
       claim_component_sum
    policy premium per month
       avg claim component
      policy_annual_premium
       insured_hobbies_chess
     incident_hour_of_the_day
         claim per vehicle
            auto year
            vehicle_age
    incident_severity_Total Loss
           capital-gains
                            0.02
                                                            0.08
                                      0.04
                                                                      0.10
                                         Importance Score
```

Select features with high importance scores top_features = feat_importances[feat_importances["Importance"] >= 0.01]["Feature"].tolist()

print("Selected top features:\n", top_features)
Create a new training data with only the selected features
X_train_selected = X_train_rfecv[top_features].copy()
X_test_selected = X_test_rfecv[top_features].copy()

```
##OUTPUT##
Selected top features:
   ['incident_severity_Minor Damage',
   'claim_to_premium_ratio', 'age_x_premium',
   'claim_component_sum', 'policy_premium_per_month',
   'avg_claim_component', 'policy_annual_premium',
   'insured_hobbies_chess', 'incident_hour_of_the_day',
   'age', 'claim_per_vehicle', 'auto_year', 'vehicle_age',
   'incident_severity_Total Loss', 'capital-gains',
   'capital-loss', 'insured_hobbies_Other',
   'collision_type_unknown', 'policy_csl_500/1000',
   'tenure_bucket_10+yr', 'incident_state_WV',
   'policy_csl_250/500']
```

6.4.4. Train the model with selected features

Fit the model on the training data with selected features rf_model.fit(X_train_selected, y_train_res)

6.4.5. Generate predictions on the training data

```
# Generate predictions on training data
y_train_rf_pred = rf_model.predict(X_train_selected)
y_train_rf_prob = rf_model.predict_proba(X_train_selected)[:, 1]
```

6.4.6. Check accuracy of the model

```
# Check accuracy of the model from sklearn.metrics import accuracy_score, classification_report, confusion_matrix print("Training Accuracy:", accuracy_score(y_train_res, y_train_rf_pred)) print("\nClassification Report:\n", classification_report(y_train_res, y_train_rf_pred))
```

##OUTPUT## Training Accuracy: 0.9933460076045627				
Classification	Report: precision	recall	f1-score	support
0 1	1.00 0.99	0.99	0.99	526 526
accuracy macro avg weighted avg	0.99	0.99	0.99 0.99 0.99	1052 1052 1052

6.4.7. Create confusion matrix

```
# Create the confusion matrix to visualise the performance
cm = confusion_matrix(y_train_res, y_train_rf_pred)
print("\nConfusion Matrix:\n", cm)

##OUTPUT##
Confusion Matrix:
```

Confusion Matrix: [[521 5] [2 524]]

6.4.8. Create variables for true positive, true negative, false positive and false negative

```
# Create variables for true positive, true negative, false positive and false negative tn, fp, fn, tp = cm.ravel()

print("True Negatives:", tn)
print("False Positives:", fp)
print("False Negatives:", fn)
print("True Positives:", tp)

##OUTPUT##
```

True Negatives: 521
False Positives: 5
False Negatives: 2
True Positives: 524

6.4.9. Calculate sensitivity, specificity, precision, recall and F1-score of the model

```
# Calculate the sensitivity
sensitivity = tp / (tp + fn)
print(f"Sensitivity (Recall): {sensitivity: 4f}")
# Calculate the specificity
specificity = tn / (tn + fp)
print(f"Specificity: {specificity:.4f}")
# Calculate Precision
precision = tp / (tp + fp)
print(f"Precision : {precision:.4f}")
# Calculate Recall
recall = tp / (tp + fn)
print(f"Recall : {recall:.4f}")
# Calculate F1 Score
f1_score = 2 * (precision * recall) / (precision + recall)
print(f"F1 Score : {f1_score:.4f}")
##OUTPUT##
Sensitivity (Recall): 0.9962
Specificity: 0.9905
Precision : 0.9905
Recall : 0.9962
F1 Score : 0.9934
```

6.4.10. Check if the model is overfitting training data using cross validation

```
# Use cross validation to check if the model is overfitting
from sklearn.model selection import cross validate, StratifiedKFold
from sklearn.metrics import make scorer, accuracy score,
precision score, recall score, f1 score, roc auc score
y arr = np.asarray(y train res).reshape(-1,)
# Define scoring metrics
scoring = {
  "accuracy": make scorer(accuracy score),
  "precision": make_scorer(precision_score, zero_division=0),
  "recall": make scorer(recall score, zero division=0),
  "f1": make scorer(f1_score, zero_division=0),
  "roc_auc": "roc_auc"
}
# Stratified K-Fold CV
cv = StratifiedKFold(n splits=5, shuffle=True, random state=42)
# Run cross-validation
cv_results = cross_validate(
  rf model,
  X_train_selected, y_arr,
```

```
cv=cv.
  scoring=scoring,
  return train score=True,
  n jobs=-1
# Build summary DataFrame
summary = pd.DataFrame({
  "Metric": ["Accuracy", "Precision", "Recall", "F1", "ROC-AUC"],
  "Train Mean": [cv results[f"train {m}"].mean() for m in
["accuracy", "precision", "recall", "f1", "roc_auc"]],
  "CV Mean": [cv results[f"test {m}"].mean() for m in
["accuracy", "precision", "recall", "f1", "roc_auc"]],
  "Train Std": [cv results[f"train {m}"].std() for m in
["accuracy", "precision", "recall", "f1", "roc_auc"]],
  "CV Std": [cv_results[f"test_{m}"].std() for m in
["accuracy", "precision", "recall", "f1", "roc_auc"]],
})
# Add delta column (train - cv)
summary["Delta"] = summary["Train Mean"] - summary["CV Mean"]
print(summary)
##OUTPUT##
Metric Train Mean CV Mean Train Std
                                               CV Std
Delta
0 Accuracy 0.986217 0.878321
                                         0.003324 0.008904
0.107896
1 Precision
                0.982111 0.849797
                                         0.004308 0.018230
0.132315
                0.990497 0.920126
      Recall
                                         0.005415 0.020598
0.070371
3
          F1
                 0.986273 0.883214
                                         0.003320 0.008087
0.103059
    ROC-AUC
                 0.998989 0.947585
                                         0.000462 0.013671
0.051404
```

6.5. Hyperparameter Tuning

6.5.1. Use grid search to find the best hyperparameter values

```
# Use grid search to find the best hyperparamter values param_grid = {
    "n_estimators": [100, 200, 300,400,500], # number of trees
    "max_depth": [5, 10, 15, 20,25], # tree depth
    "min_samples_split": [2, 10, 20], # min samples to split a node
    "min_samples_leaf": [1, 5, 10], # min samples per leaf
    "max_features": ["sqrt", "log2"], # features per split
    "class_weight": [None, "balanced"] # handle imbalance
```

```
rf = RandomForestClassifier(random state=42, n jobs=-1)
grid search = GridSearchCV(
  estimator=rf,
  param_grid=param_grid,
  scoring="roc auc", # optimize for ROC-AUC (better for fraud
detection)
  cv=5,
  n jobs=-1,
  verbose=2
# Fit on training set (using selected features)
grid search.fit(X train selected, y train res)
# Best Hyperparameters
print("Best Parameters:", grid search.best params )
print("Best ROC-AUC Score:", grid search.best score )
##OUTPUT##
Best Parameters: {'class_weight': None, 'max_depth': 20,
'max_features': 'sqrt', 'min_samples_leaf': 1,
'min_samples_split': 2, 'n_estimators': 500}
Best_ROC-AUC_Score: 0.9865543148076841
```

6.5.2. Build a random forest model based on hyperparameter tuning results

```
# Building random forest model based on results of hyperparameter tuning best_params=grid_search.best_params_
rf_tuned = RandomForestClassifier(**best_params)
rf_tuned.fit(X_train_selected, y_train_res)
```

6.5.3. Make predictions on training data

```
# Make predictions on training data
y_train_pred = rf_tuned.predict(X_train_selected)
y_train_prob = rf_tuned.predict_proba(X_train_selected)[:,1]
```

6.5.4. Check accuracy of Random Forest Model

6.5.5. Create confusion matrix

```
# Create the confusion matrix
cm = confusion_matrix(y_train_res, y_train_pred)
print("\nConfusion Matrix:\n", cm)
##OUTPUT##
Confusion Matrix:
[[526 0]
[ 0 526]]
```

6.5.6. Create variables for true positive, true negative, false positive and false negative

```
# Create variables for true positive, true negative, false positive and
false negative
tn, fp, fn, tp = cm.ravel()

print("True Negatives:", tn)
print("False Positives:", fp)
print("False Negatives:", fn)
print("True Positives:", tp)
##OUTPUT##
True Negatives: 526
False Positives: 0
False Negatives: 0
True Positives: 526
```

6.5.7. Calculate sensitivity, specificity, precision, recall and F1-score of the model

```
# Calculate the sensitivity
sensitivity = tp / (tp + fn)
print(f"Sensitivity (Recall): {sensitivity:.4f}")
# Calculate the specificity
specificity = tn / (tn + fp)
print(f"Specificity: {specificity:.4f}")
# Calculate Precision
precision = tp / (tp + fp)
print(f"Precision : {precision:.4f}")
# Calculate Recall
recall = tp / (tp + fn)
print(f"Recall : {recall: 4f}")
# Calculate F1 Score
f1_score = 2 * (precision * recall) / (precision + recall)
print(f"F1 Score : {f1 score:.4f}")
##OUTPUT##
Sensitivity (Recall): 1.0000
Specificity: 1.0000
Precision : 1.0000
Recall : 1.0000
F1 Score : 1.0000
```

- 7. Prediction and Model Evaluation
 - 7.1. Make predictions over validation data using logistic regression model
 - 7.1.1. Make predictions over validation data

```
# Make predictions on the validation data and store it in the variable 'y_validation_pred'
y_validation_pred = logsk.predict_proba(X_test_rfecv)[:, 1]
```

7.1.2. Create DataFrame with actual values and predicted values for validation data

fraud_reported	pred_prob

1	0	0.042975
2	0	0.241139
3	0	0.142840
4	0	0.119074

7.1.3. Make final prediction based on cutoff value

```
# Make final predictions on the validation data using the optimal
print("Optimal cutoff:", optimal_cutoff)
test pred df["final pred"] = (test pred df["pred prob"] >=
optimal cutoff).astype(int)
print(test_pred_df.head())
##OUTPUT##
Optimal cutoff: 0.5
   fraud_reported pred_prob final_pred
                0 0.734447
0 0.042975
1
2
                    0.241139
                 0
                    0.142840
3
                0
4
                 0 0.119074
```

7.1.4. Check the accuracy of logistic regression model on validation data

```
# Check the accuracy
print("Test Accuracy:", accuracy_score(y_test,
test_pred_df["final_pred"]))
##OUTPUT##
Test Accuracy: 0.74
```

7.1.5. Create confusion matrix

```
# Create the confusion matrix
cm = confusion_matrix(y_test, test_pred_df["final_pred"])
print("\nConfusion Matrix:\n", cm)
##OUTPUT##
Confusion Matrix:
```

```
[[178 48]
[ 30 44]]
```

7.1.6. Create variables for true positive, true negative, false positive and false negative

```
# Create variables for true positive, true negative, false positive and false negative tn, fp, fn, tp = cm.ravel()

print("True Negatives:", tn)
print("False Positives:", fp)
print("False Negatives:", fn)
print("True Positives:", tp)

##OUTPUT##

True Negatives: 178
False Positives: 48
False Negatives: 30
True Positives: 44
```

7.1.7. Calculate sensitivity, specificity, precision, recall and f1 score of the model

```
# Calculate the sensitivity
sensitivity = tp / (tp + fn)
print(f"Sensitivity (Recall): {sensitivity:.4f}")
# Calculate the specificity
specificity = tn / (tn + fp)
print(f"Specificity: {specificity:.4f}")
# Calculate Precision
precision = tp / (tp + fp)
print(f"Precision : {precision:.4f}")
# Calculate Recall
recall = tp / (tp + fn)
print(f"Recall : {recall:.4f}")
# Calculate F1 Score
f1 score = 2 * (precision * recall) / (precision + recall)
print(f"F1 Score : {f1 score:.4f}")
##OUTPUT##
Sensitivity (Recall): 0.5946
Specificity: 0.7876
Precision : 0.4783
Recall : 0.5946
F1 Score : 0.5301
```

- 7.2. Make predictions over validation data using random forest model
 - 7.2.1. Select the important features and make predictions over validation data

```
# Select the relevant features for validation data
X_test_selected
# Make predictions on the validation data
y_test_pred = rf_tuned.predict(X_test_selected)
y_test_prob = rf_tuned.predict_proba(X_test_selected)[:,1]
```

7.2.2. Check accuracy of random forest model

```
# Check accuracy
print("Test Accuracy:", accuracy_score(y_test, y_test_pred))
##OUTPUT##
Test Accuracy: 0.77
```

7.2.3. Create confusion matrix

7.2.4. Create variables for true positive, true negative, false positive and false negative

```
# Create variables for true positive, true negative, false positive and false negative tn, fp, fn, tp = cm.ravel()

print("True Negatives:", tn)
print("False Positives:", fp)
print("False Negatives:", fn)
print("True Positives:", tp)
##OUTPUT##
```

```
True Negatives: 198
False Positives: 28
False Negatives: 41
True Positives: 33
```

7.2.5. Calculate sensitivity, specificity, precision, recall and F1-score of the model

```
# Calculate the sensitivity
sensitivity = tp / (tp + fn)
print(f"Sensitivity (Recall): {sensitivity:.4f}")
# Calculate the specificity
specificity = tn / (tn + fp)
print(f"Specificity: {specificity:.4f}")
# Calculate Precision
precision = tp / (tp + fp)
print(f"Precision : {precision:.4f}")
# Calculate Recall
recall = tp / (tp + fn)
print(f"Recall : {recall:.4f}")
# Calculate F1 Score
f1 score = 2 * (precision * recall) / (precision + recall)
print(f"F1 Score : {f1_score:.4f}")
##OUTPUT##
Sensitivity (Recall): 0.4459
Specificity: 0.8761
Precision : 0.5410
Recall : 0.4459
F1 Score : 0.4889
```

8. Evaluation and Conclusion

Logistic Regression Results

Training Performance Accuracy: 0.8888 (very strong) Confusion Matrix: [[465, 61], [56, 470]] TN = 465, FP = 61, FN = 56, TP = 470 Sensitivity (Recall): 0.8935 - model correctly catches ~89% of fraud cases. Specificity: 0.8840 - also good at correctly identifying legitimate claims. Precision: 0.8851 - 89% of predicted frauds are actual fraud. F1 Score: 0.8893 - balanced performance. Training metrics look very strong and balanced across all measures.

Validation (Test) PerformancE Accuracy: 0.7400 (drops from training → sign of overfitting / weaker generalization) Confusion Matrix: [[178, 48], [30, 44]] TN = 178, FP = 48, FN = 30, TP = 44 Sensitivity (Recall): 0.5946 - only 59% of frauds are detected (misses 41%). Specificity: 0.7876 - does better at identifying legitimate claims. Precision: 0.4783 - less than half of

predicted frauds are actually fraud. F1 Score: 0.5301 - moderate balance, but weak compared to training On validation data, the model performance drops significantly, especially in Precision and Recall.

Conclusion for logistic regression Logistic Regression fits the training data very well, showing balanced and strong performance However, on validation data, it generalizes poorly: Accuracy drops to 74%. Recall falls to 59% (model misses many fraud cases). Precision is low (48%), meaning high false positives. This suggests overfitting or that linear decision boundaries are insufficient for capturing fraud patterns.

Random Forest Results

Training Performance (before and after tuning) Baseline RF (n_estimators=10) Accuracy: 0.9933 Confusion Matrix: [[521, 5], [2, 524]] Sensitivity (Recall): 0.9962 Specificity: 0.9905 F1 Score: 0.9934 Already extremely high performance on training set.

After GridSearchCV tuning (best params: depth=20, n_estimators=500, etc.) Training Accuracy: 1.0000 Confusion Matrix: [[526, 0], [0, 526]] Sensitivity, Specificity, Precision, Recall, F1: all = 1.0 Perfect fit on training data \rightarrow a clear sign of overfitting.

Test (Validation) Performance Accuracy: 0.7400 (same as Logistic Regression test accuracy). Confusion Matrix: [[178, 48], [30, 44]] TN = 178, FP = 48, FN = 30, TP = 44 Sensitivity (Recall): $0.5946 \rightarrow \text{catches} \sim 59\%$ of fraud (misses 41%). Specificity: $0.7876 \rightarrow \text{correctly identifies} \sim 79\%$ legitimate claims. Precision: $0.4783 \rightarrow \text{less}$ than half of flagged frauds are true fraud. F1 Score: $0.5301 \rightarrow \text{weak}$ balance of precision & recall. Despite perfect training performance, the test metrics collapse to the same level as Logistic Regression.

Conclusion Random Forest massively overfits: It memorizes the training set (100% accuracy). But generalization to test set is poor (only 74% accuracy). Performance on test set (Accuracy = 0.74, Recall = 0.59, Precision = 0.48, F1 = 0.53) is almost identical to Logistic Regression's test performance. This means Random Forest did not improve generalization, even though it overfits more aggressively than Logistic Regression.

Conlusion between Losistic classification and Random forest

Logistic Regression vs Random Forest — Model Comparison

Model Performance

Metric Logistic Random Forest Regression

Training Accuracy	0.8888	1.0000 (after tuning)
Training Recall	0.8935	1.0000
Training Precision	0.8851	1.0000
Training F1 Score	0.8893	1.0000
Test Accuracy	0.7400	0.7400
Test Recall	0.5946	0.5946
Test Precision	0.4783	0.4783
Test F1 Score	0.5301	0.5301

Conclusion

1. On training data

- Logistic Regression performs very well but not perfect.
- Random Forest (especially after tuning) achieves perfect fit (100%), which is a strong sign of overfitting.

2. On validation/test data

- Both models perform similarly (Accuracy ≈ 0.74, Recall ≈ 0.59, Precision ≈ 0.48, F1 ≈ 0.53).
- Neither model generalizes well; Random Forest's extra complexity did not improve performance compared to Logistic Regression.

3. Interpretability vs Complexity

- Logistic Regression is simpler, interpretable, and stable.
- Random Forest is more complex and overfits easily without improving test results.

4. Overall

- Logistic Regression is a better baseline model here: it generalizes almost as well as Random Forest, but with less overfitting risk and easier interpretability.
- Random Forest needs stronger regularization/tuning or may require additional feature engineering / resampling techniques to outperform Logistic Regression.