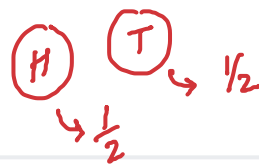


Uniform Distribution



What It Is:

✓ A uniform distribution is when every outcome is equally likely.

Simple Example: Rolling a Fair Die

✓ Possible outcomes: {1, 2, 3, 4, 5, 6}

✓ Each number has a $\frac{1}{6}$ chance → That's a discrete uniform distribution

$$P(1) = \frac{1}{6} \quad P(2) = \frac{1}{6} \quad P(3) = \frac{1}{6}$$

$$P(6) = \frac{1}{6}$$

Continuous Version:

$$P(3) = \frac{1}{6}$$

6

$$\underbrace{x + x + x + x + x + x}_{=1} = 1$$

$$x = \frac{1}{6}$$

Let's say we randomly pick a number between 0 and 1.

- Every value in that range is **equally likely**.
- That's a **continuous uniform distribution**.

$$\begin{matrix} a= & & b= \\ 0 & \text{---} & 1 \end{matrix}$$

$$P(x) = \frac{1}{b-a} = \frac{1}{1} \quad 0 \leq x \leq 1$$

✓ PMF \Rightarrow Probability mass function

Graphs to Visualize

✓ PDF \rightarrow Probability Density function

PDF



✓ 1. Discrete Uniform (like a die roll)

Outcome: 1 2 3 4 5 6

Probability: |---|---|---|---|---|---|

$\frac{1}{6}$ for each \rightarrow flat bars

$$P(0.5 \leq x \leq 0.6) = \int_{0.5}^{0.6} \frac{1}{b-a} dx$$

$$0.5 = \frac{0.6 - 0.5}{1} = 0.1$$

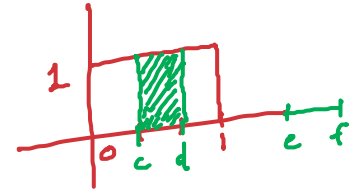
Discrete Uniform Distribution Formula:

✓ $P(X = x) = 1 / n$

- Where n is the number of possible outcomes (e.g., for a 6-sided die, $P(\text{rolling a 4}) = 1/6$)

2. Continuous Uniform (0 to 1)

- It's just a **flat horizontal line** from $x = 0$ to $x = 1$ ✓
- The probability density is constant (say 1.0) across that interval ✓



Continuous Uniform Distribution Formula:

✓ $f(x) = 1 / (b - a)$ for values between a and b

- Outside the range a to b , the probability is 0

PDF → Probability density.
PMF ✓ → Exact Probabilities

Why It's Useful in Data Science

- ✓ It models **pure randomness**
- ✓ It's used to **simulate random choices**
- Used in:
 - Generating random numbers (`np.random.uniform`)
 - Monte Carlo simulations
 - Bootstrapping

In Code (Python):

```
import numpy as np
import matplotlib.pyplot as plt
```

```
# Continuous uniform from 0 to 1
samples = np.random.uniform(0, 1, 10000)

plt.hist(samples, bins=50, density=True, alpha=0.6, color='skyblue')
plt.title("Continuous Uniform Distribution (0 to 1)")
plt.xlabel("Value")
plt.ylabel("Probability Density")
plt.grid(True)
plt.show()
```

You'll see a **flat histogram** showing uniform probability.

Summary:

Property	Uniform Distribution
Type	Discrete or Continuous
Shape	Flat
Real-world example	Die roll, random number gen
Python function	<code>np.random.uniform(a, b)</code>