

# Bayes' Theorem

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Bayes' Theorem is written as:

$$P(A / B) = [P(B / A) * P(A)] / P(B)$$

where:

- $P(A / B)$  is the probability of event A occurring given that event B has occurred.
- $P(B / A)$  is the probability of event B occurring given that event A has occurred.
- $P(A)$  is the prior probability of event A.

This can also be interpreted as:  $P(A / B) = [P(B / A) * P(A)] / [P(B / A) * P(A) + P(B / \text{No A}) * P(\text{No A})]$

It calculates the probability of event A given that event B has occurred, using the known conditional probability of B given A.

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## Proof of Bayes' Theorem

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We start with the definition of conditional probability:

1.  $P(A / B) = P(A \text{ and } B) / P(B)$
2.  $P(B / A) = P(A \text{ and } B) / P(A)$

From equation (2), we can write:

$$P(A \text{ and } B) = P(B / A) * P(A)$$

Now substitute this into equation (1):

$$P(A / B) = [P(B / A) * P(A)] / P(B)$$

This is Bayes' Theorem.

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## Example: Medical Test Problem

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Suppose a disease affects 1% of the population. A test for the disease has the following characteristics:

- If a person **has** the disease, the test is **positive** 99% of the time  $\rightarrow P(\text{Positive} / \text{Disease}) = 0.99$
- If a person **does not have** the disease, the test is **positive** 5% of the time  $\rightarrow P(\text{Positive} / \text{No Disease}) = 0.05$
- The chance of having the disease  $\rightarrow P(\text{Disease}) = 0.01$
- The chance of not having the disease  $\rightarrow P(\text{No Disease}) = 0.99$

A person takes the test and gets a **positive** result. What is the probability they actually have the disease?

Let:

- A = Disease
- B = Positive test

We apply Bayes' Theorem:

$$P(\text{Disease} / \text{Positive}) = [P(\text{Positive} / \text{Disease}) * P(\text{Disease})] / P(\text{Positive})$$

First, calculate  $P(\text{Positive})$ :

$$P(\text{Positive}) = P(\text{Positive} / \text{Disease}) * P(\text{Disease}) + P(\text{Positive} / \text{No Disease}) * P(\text{No Disease})$$

$$P(\text{Positive}) = 0.99 * 0.01 + 0.05 * 0.99$$

$$P(\text{Positive}) = 0.0099 + 0.0495 = 0.0594$$

Now calculate  $P(\text{Disease} / \text{Positive})$ :

$$P(\text{Disease} / \text{Positive}) = 0.0099 / 0.0594 \approx 0.1667$$

**Conclusion:** Even though the test is 99% accurate, the probability that a person actually has the disease given a positive result is only **16.67%**. This is because the disease is very rare, and false positives affect the result significantly.

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