

Coin toss
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What is the Binomial Distribution?

The binomial distribution models the number of successes in a fixed number of independent yes/no experiments, where each has the same probability of success.

Think of this:

- ✓ Toss a coin 10 times
- What's the probability of getting exactly 6 heads?

That's a binomial problem.

Key Ingredients:

- n = number of trials (e.g., 10 tosses)
- p = probability of success (e.g., 0.5 for heads)
- x = number of successes (e.g., 6 heads)

$n \rightarrow$ no of times coin is tossed 10
 $p \rightarrow$ probability of success 0.5
 $x \rightarrow$ no of successes 6

Plain Text Formula:

$$P(X = x) = C(n, x) \times p^x \times (1 - p)^{(n - x)}$$

Where:

- ✓ $C(n, x)$ is "n choose x" = combinations = number of ways to pick x successes out of n
- p^x is the probability of x successes
- $(1 - p)^{(n - x)}$ is the probability of the remaining being failures

$$\begin{aligned} P(X=x) &= {}^nC_x p^x (1-p)^{n-x} \\ &= \frac{{}^n P_x}{x!} p^x (1-p)^{n-x} \\ &= \frac{{}^n P_x}{x!} \times \left(\frac{1}{2}\right)^{10} = 0.205 \end{aligned}$$

Example:

10 coin tosses, what's the probability of exactly 6 heads?

- $n = 10$
- $x = 6$
- $p = 0.5$

$$P(6\text{heads}) = C(10, 6) * 0.5^6 * 0.5^4 = 210 * (0.015625) * (0.0625) \approx 0.205$$

So there's a ~20.5% chance you'll get exactly 6 heads in 10 tosses.

In Python:

```
from scipy.stats import binom

# Probability of exactly 6 heads in 10 tosses (p = 0.5)
prob = binom.pmf(k=6, n=10, p=0.5)
print(prob) # Output: ~0.205
```

When to Use:

- Email campaign: Will 40 out of 100 people click the link?
- Quality check: How many out of 10 products will be defective?
- A/B testing: Will 60 out of 200 visitors convert?

Summary:

Concept	Value
Type	Discrete

Concept	Value
Formula	$P(X = x) = C(n, x) * p^x * (1 - p)^{(n - x)}$ ✓
Python	<code>scipy.stats.binom.pmf(x, n, p)</code>
Used for	Count of successes in repeated trials