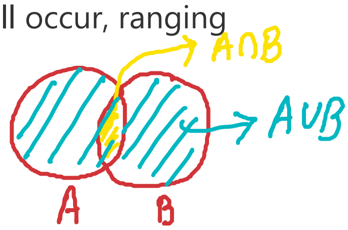
Basic Rules of Probability

1. Important Definitions

In probability, understanding the basic definitions is crucial for grasping more complex concepts.

- **Experiment**: An action or process that leads to one or more outcomes (e.g., rolling a die). Deterministic experiments have predictable outcomes, while probabilistic or random experiments have uncertain outcomes.
- **Outcome:** A possible result of an experiment (e.g., rolling a 3 on a die).
- **Probability (P):** A measure of the likelihood that an event will occur, ranging from 0 (impossible) to 1 (certain).



- **P(AUB):** The probability of event A or event B occurring.
- **P(A∩B):** The probability of both events A and B occurring.
- **Sample Space (S):** The set of all possible outcomes of an experiment.
- **Event (E):** A subset of the sample space.

{HT, TH, TT, HH}

- **Mutually Exclusive Events:** Two events that cannot occur at the same time (e.g., rolling a 2 and rolling a 5 on a die). $P(A \cap B) = 0$
- **Independent Events:** Two events where the occurrence of one does not affect the other. $P(A \cap B) = P(A) \cdot P(B)$
- **Certain Event:** An event that is guaranteed to happen, with a probability of 1.
- **Impossible Event:** An event that cannot happen, with a probability of 0.
- **Exhaustive Events:** A set of events that cover the entire sample space, meaning at least one of them must occur.

{H T}

1, 2, 3, 4, 5, 6,

Example:

Tossing a die:

- Sample space: S = {1, 2, 3, 4, 5, 6}
- Event (E): Rolling an even number = {2, 4, 6}

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2. The Complement Rule

The complement of an event A is the event that A does not occur.

- Notation: A^c
- Rule: $P(A^c) = 1 - P(A)$

$P(A)$

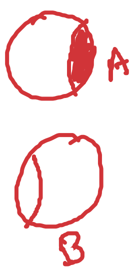
$P(A') = 1 - P(A)$

$P(A^c) = 1 - P(A)$

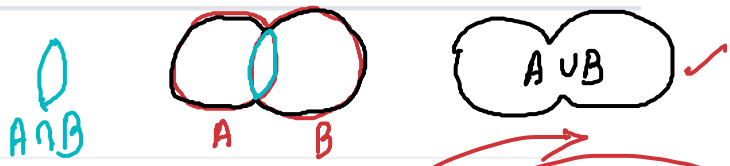
Example:

If the probability of rain today is 0.3, the probability it won't rain is: - P(No Rain) = 1 − 0.3 = 0.7

$1 - 0.3 = 0.7$



3. The Addition Rule



Used to calculate the probability of the union of two events.

$① \ P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$P(A \cup B) = P(A) + P(B)$

For general events:

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

For mutually exclusive events:

$P(A \text{ or } B) = P(A) + P(B)$

$P\ A \cup B = 0.4 + 0.5 - 0.2$
$= 0.7$

Example:

- $P(A) = 0.4$, $P(B) = 0.5$, $P(A \text{ and } B) = 0.2$
- $P(A \text{ or } B) = 0.4 + 0.5 - 0.2 = 0.7$

4. The Multiplication Rule

Used to find the probability that two events occur together.

For Independent events:

P(A and B) = P(A) × P(B) ✓

$P\left(\frac{B}{A}\right) = P(B)$

→ Probability of B given A has occurred

Example: A event A is rolling a 3 on a first throw of a die, and event B is rolling an even number on second throw.

$P(A) = \frac{1}{6}$ $\quad P(B) = \frac{1}{2}$

$P(A \cap B) = P(A) \times P(B)$

$= \frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$

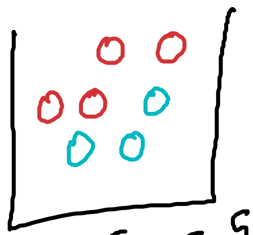
- P(A) = 1/6 (rolling a 3)
- P(B) = 3/6 (rolling a 2, 4, or 6)
- P(A and B) = P(A) × P(B) = (1/6) × (3/6) = 1/12

$P(A \cap B) = P(A) \times P\left(\frac{B}{A}\right)$

For Dependent events:

P(A and B) = P(A) × P(B | A)

Example: A bag contains 5 red and 3 blue balls. Two balls are drawn **without replacement**. Let:



- Event A be drawing a red ball first.
- Event B be drawing a red ball second.
- P(A) = 5/8 (5 red balls out of 8)
- After removing one red ball, there are 4 red left out of 7 total: P(B/A) = 4/7
- P(A and B) = (5/8) X (4/7)= 20/56 = 5/14

$P(A) = \frac{5}{5+3} = \frac{5}{8}$

$P\left(\frac{B}{A}\right) = \frac{4}{7}$

So the probability of drawing two red balls **without replacement** is 5/14

$P(A \cap B) = \frac{5}{\cancel{8}_2} \times \frac{\cancel{4}}{7}$

$= \frac{5}{14}$

5. Independent vs Dependent Events - More examples

- **Independent:** The outcome of one event does not affect the other.
  - Example: Tossing two coins.
- **Dependent:** One event affects the probability of the other.
  - Example: Drawing two cards without replacement.

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6. Mutually Exclusive Events

Two events are **mutually exclusive** if they cannot happen at the same time.

Example:

- Event A: Rolling a 2
- Event B: Rolling a 5
- These are mutually exclusive because a die can't show both at once.

Summary

This lesson covered key probability rules and concepts that are foundational for more advanced topics like distributions and statistical inference.

- Complement Rule: $P(A^c) = 1 - P(A)$
- Addition Rule for unions
- Multiplication Rule for intersections
- Understanding independence and exclusivity.

Homework / Practice

1. A card is drawn from a standard deck. What is the probability of drawing a red card or a queen?
2. If two dice are rolled, what is the probability that both show even numbers?
3. Think of an example from your daily life where two events are dependent.