HW1

Preparation

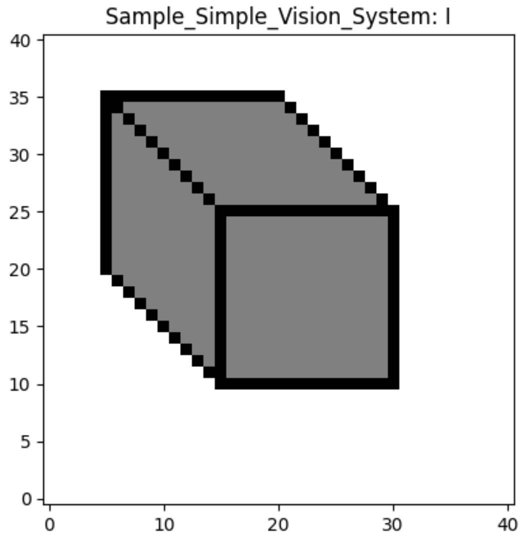
Please make sure you have read Lectures 2 & 3 slides

Please make sure you have read Textbook chapter 2

The notations in this slide deck is a bit different from lecture slides! The goal is to make implementation easier.

Problem overview

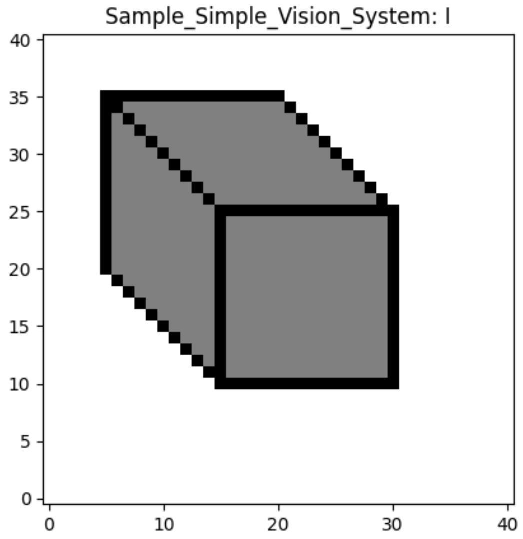
Given

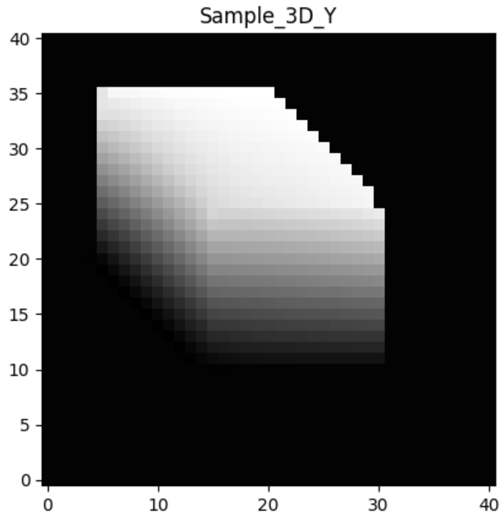


Problem overview

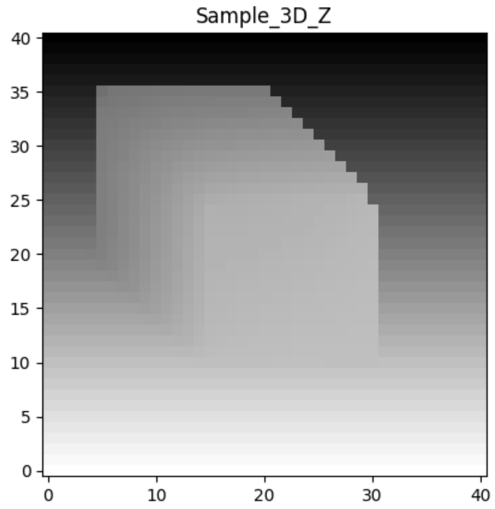
Given

Goal: Generate





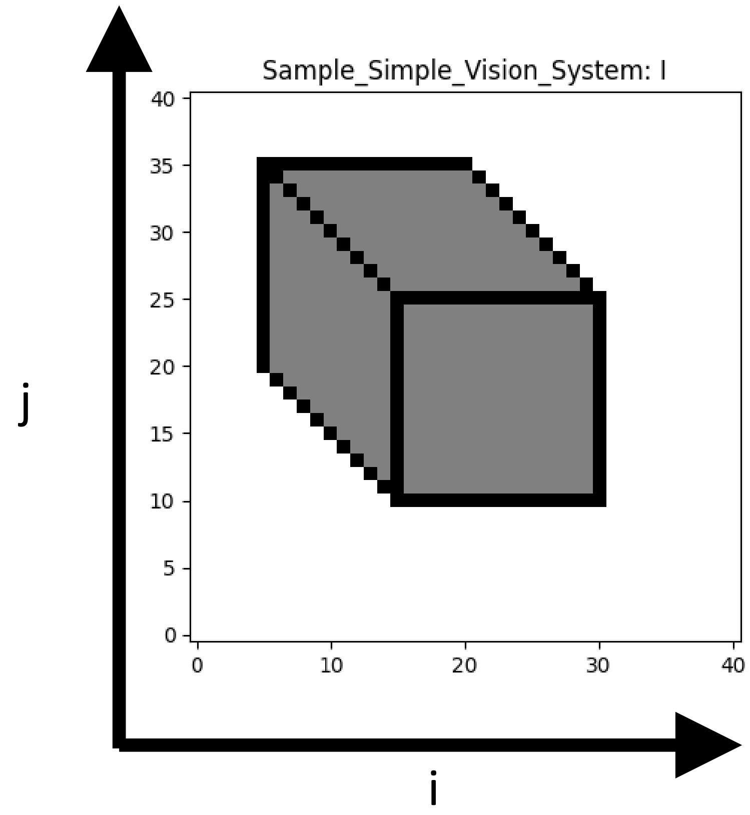
Y: 3D height



Z: 3D depth

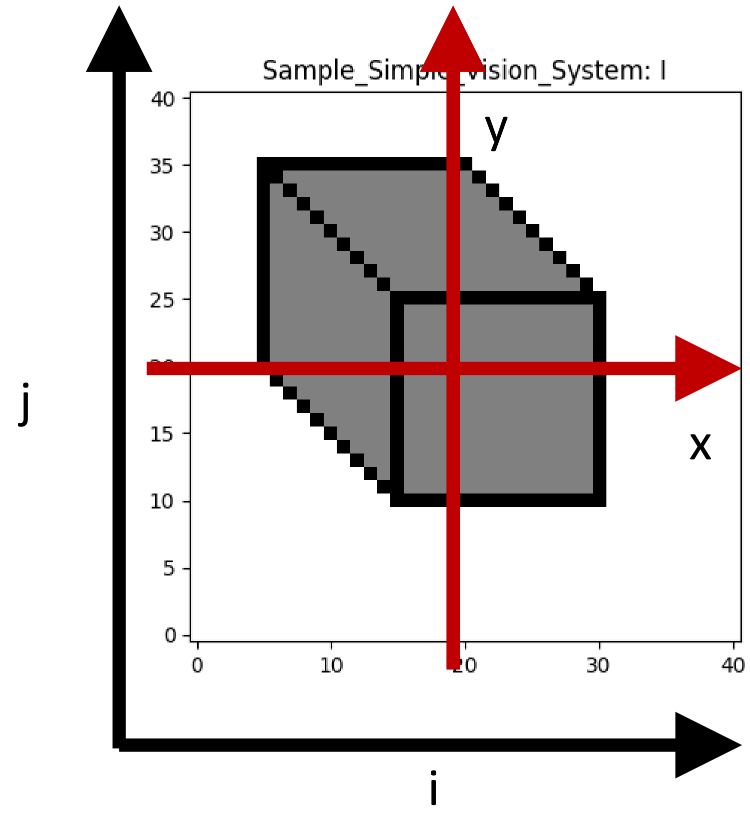
Convention

- In this homework, given a map (or a matrix), say I
  - I[i, j] means the i-th horizontal index (left-right) and j-th vertical index (bottom-up)
  - i >= 0, j >= 0

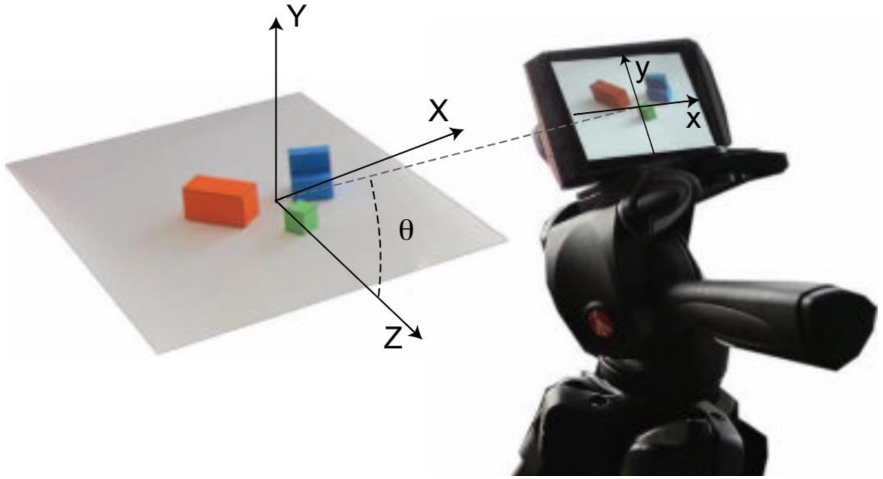


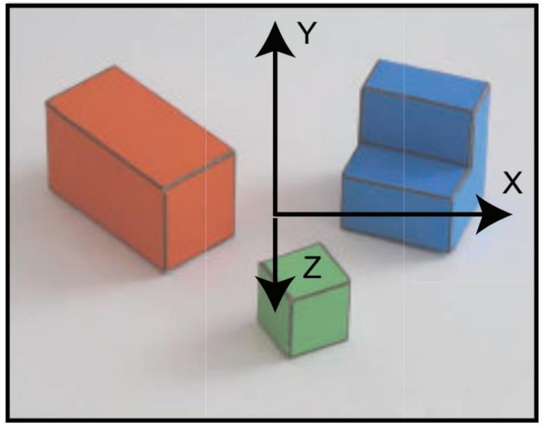
Convention

- There are differences in coordinate systems between (i, j) and (x, y)
- We thus provide a function x, y = image_plane(args, I)
  - x[i, j] is the 2D x location of pixel (i, j)
  - y[i, j] is the 2D y location of pixel (i, j)



Recall (from lectures)





We want to know X(x, y), Y(x, y), and Z(x, y) from the given image!

What we know:

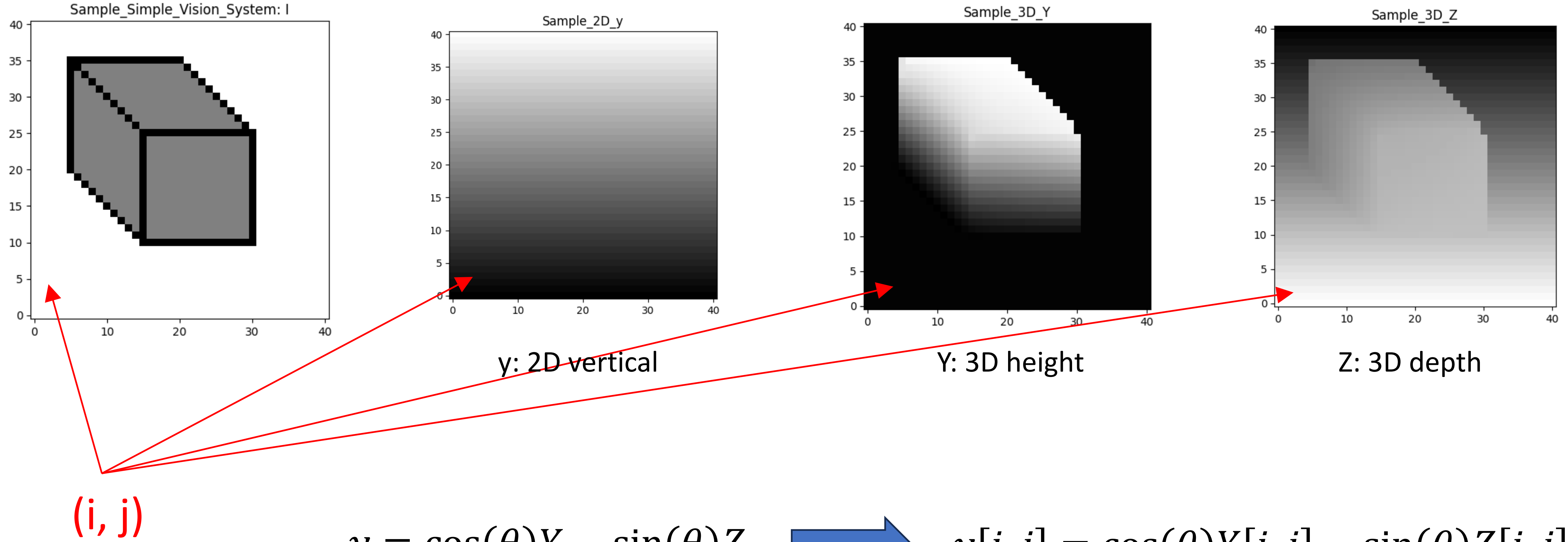
$$x = X$$

$$y = \cos(\theta)Y - \sin(\theta)Z$$

We need some cues from images and the 3D world!

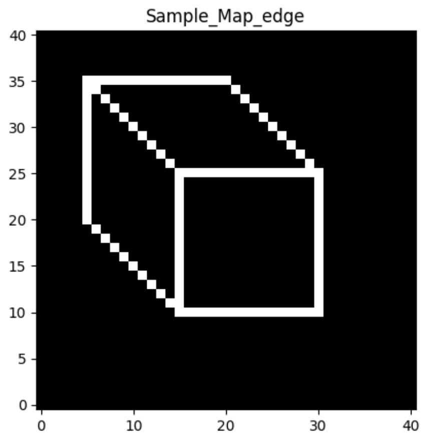
[Figure credit: A. Torralba, P. Isola, and W. T. Freeman, Foundations of Computer Vision.]

For implementation, locations are indexed by [I, j]

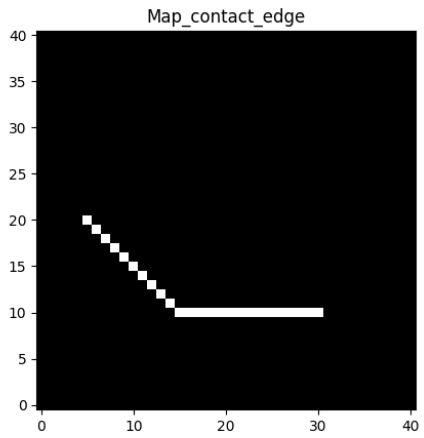


$$y = \cos(\theta)Y - \sin(\theta)Z \quad \Rightarrow \quad y[i,j] = \cos(\theta)Y[i,j] - \sin(\theta)Z[i,j]$$

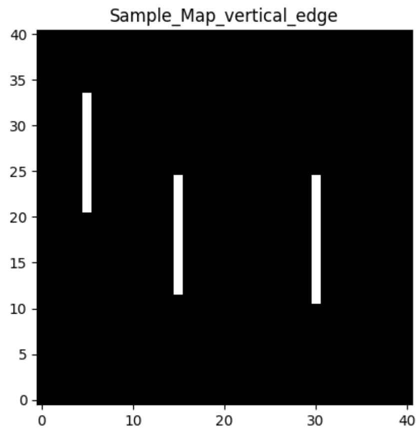
Cue 1: edges (white pixels mean edges)



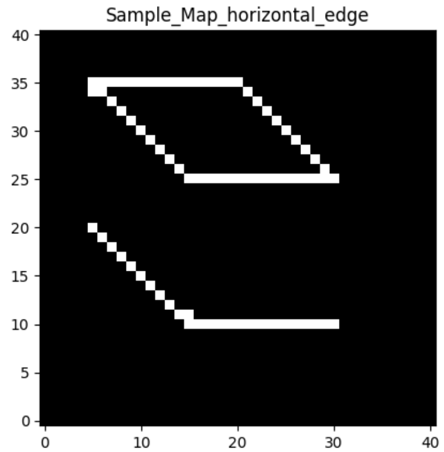
All edges



Contact edges



Vertical edges

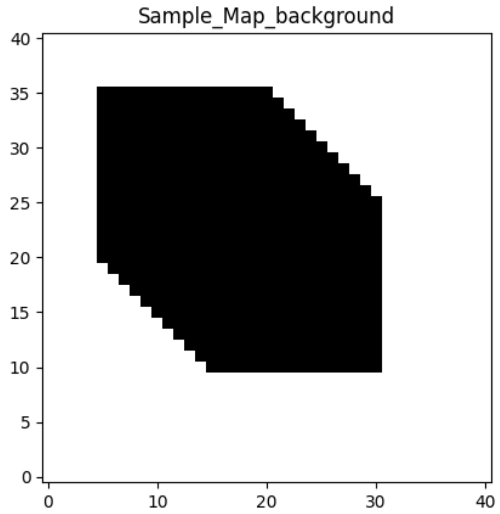


Horizontal

You need to find edge locations!

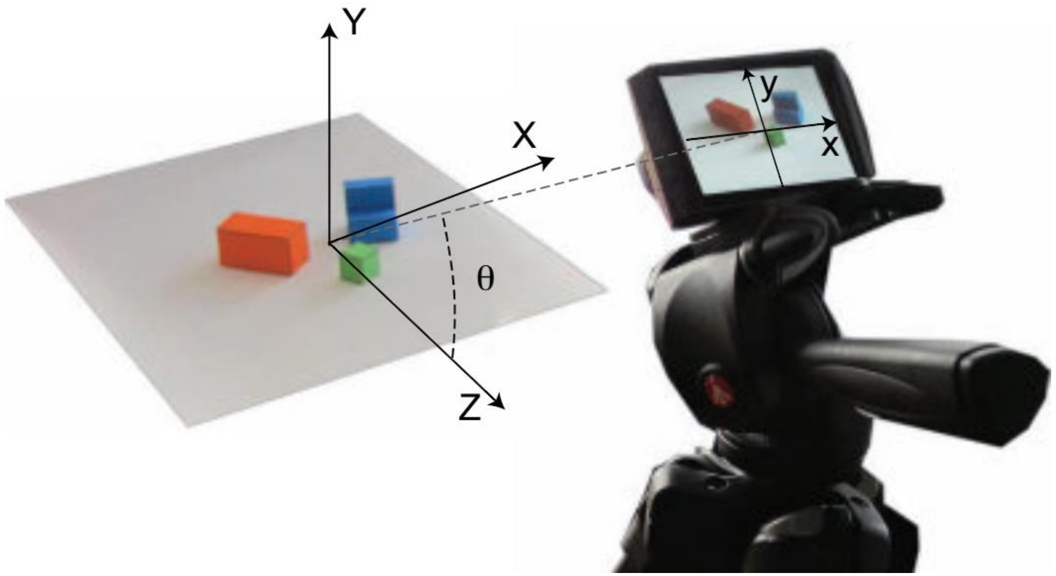
Cue 2: Surfaces & Cue 3: properties from 3D to 2D

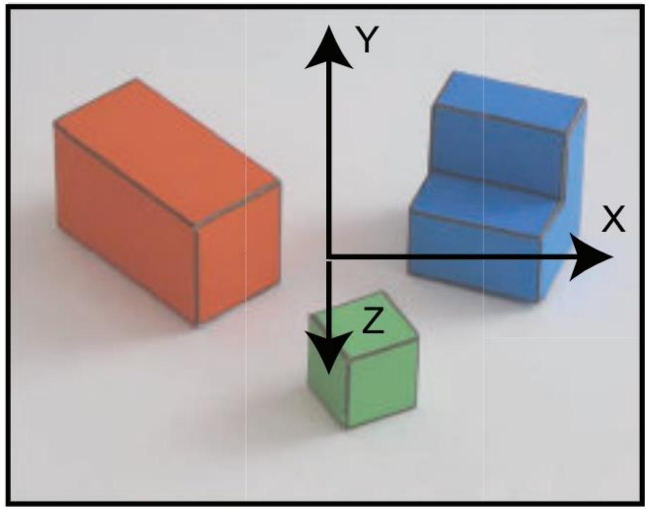
- Separate into **foregrounds (figures)/backgrounds**



- **Not always true, but let's assume it is true**
  - Vertical edges in 2D mean vertical in 3D
  - Non-vertical edges in 2D means horizontal in 3D

Recall (from lectures)





We want to know X(x, y), Y(x, y), and Z(x, y) from the given image!

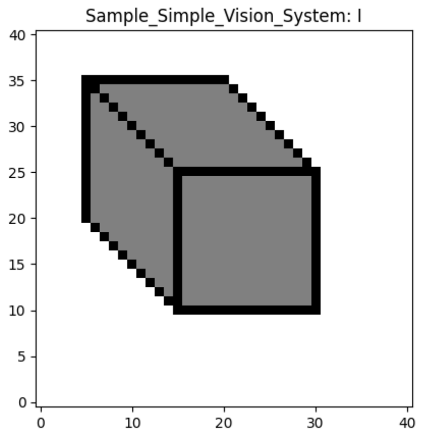
$$x = X$$

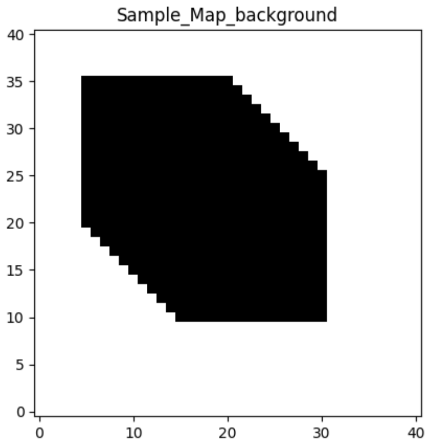
$$y = \cos(\theta)Y - \sin(\theta)Z$$

If we know Y(x, y), we know Z(x, y)

[Figure credit: A. Torralba, P. Isola, and W. T. Freeman, Foundations of Computer Vision.]

Estimating Y[I, j]: cues from the background



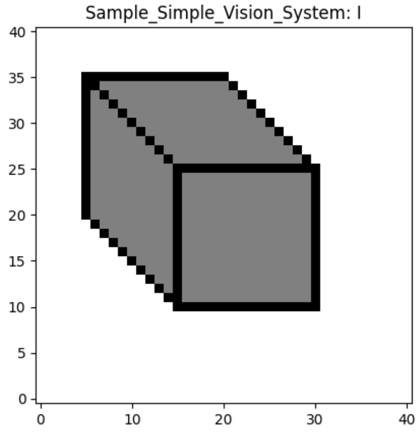


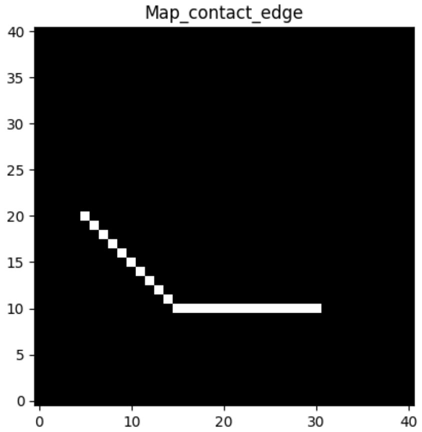
$$\tilde{Y} = \begin{bmatrix} Y(1,1) \\ Y(1,2) \\ \dots \\ Y(1,N) \\ Y(2,1) \\ Y(2,2) \\ \dots \\ Y(2,N) \\ \dots \\ Y(M,N) \end{bmatrix} \Rightarrow [0, \dots, 0, 1, \dots, 0] \begin{bmatrix} Y(1,1) \\ Y(1,2) \\ \dots \\ Y(1,N) \\ Y(2,1) \\ Y(2,2) \\ \dots \\ Y(2,N) \\ \dots \\ Y(M,N) \end{bmatrix} = 0$$

One row for each background pixel

Meaning: these locations have height 0

Estimating Y[I, j]: cues from contact edges



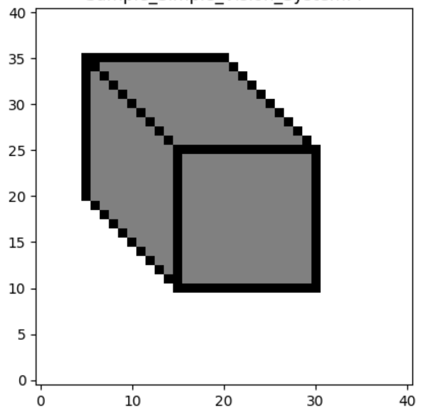


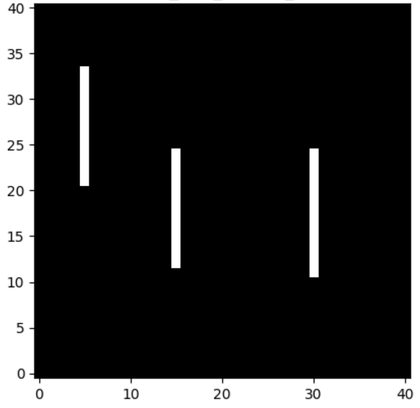
$$\tilde{Y} = \begin{bmatrix} Y(1,1) \\ Y(1,2) \\ \dots \\ Y(1,N) \\ Y(2,1) \\ Y(2,2) \\ \dots \\ Y(2,N) \\ \dots \\ Y(M,N) \end{bmatrix} \Rightarrow [0, \dots, 0, {\color{red}1}, \dots, 0] \begin{bmatrix} Y(1,1) \\ Y(1,2) \\ \dots \\ Y(1,N) \\ Y(2,1) \\ Y(2,2) \\ \dots \\ Y(2,N) \\ \dots \\ Y(M,N) \end{bmatrix} = 0$$

One row for each contact edge pixel

Meaning: these locations have height 0

Estimating Y[I, j]: cues from vertical edges



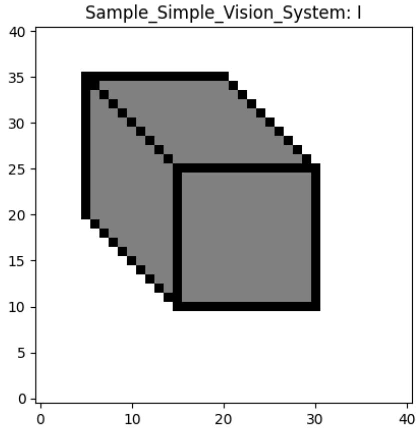


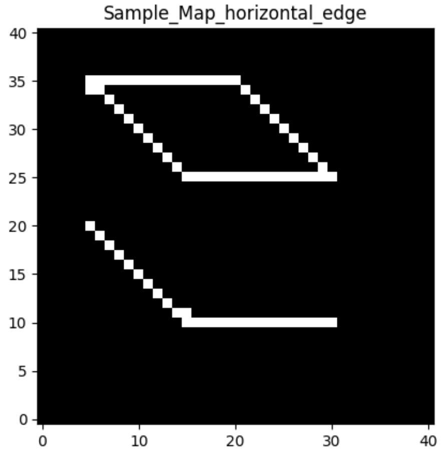
$$Y[i,j] - Y[i,j-1] = 1/\cos(\theta) \Rightarrow [0, \ldots, 0, {\color{red}-1, 1}, \ldots, 0] \begin{bmatrix} Y(1,1) \\ Y(1,2) \\ \ldots \\ Y(1,N) \\ Y(2,1) \\ Y(2,2) \\ \ldots \\ Y(2,N) \\ \ldots \\ Y(M,N) \end{bmatrix} = 1/\cos(\theta)$$

One row for each vertical edge pixel

Meaning: two vertically consecutive pixels have 3D height difference $1/\cos(\theta)$

Estimating Y[I, j]: cues from horizontal edges

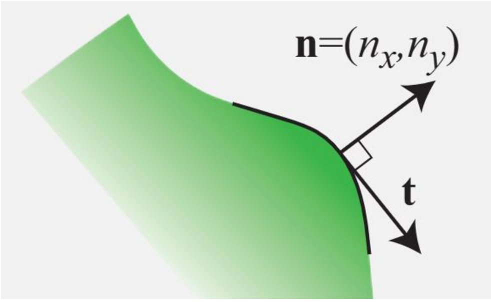




Horizontal edges: Y won't change along the edge

$$-n_y[i,j](Y[i,j]-Y[i-1,j])+n_x[i,j](Y[i,j]-Y[i,j-1])=0$$

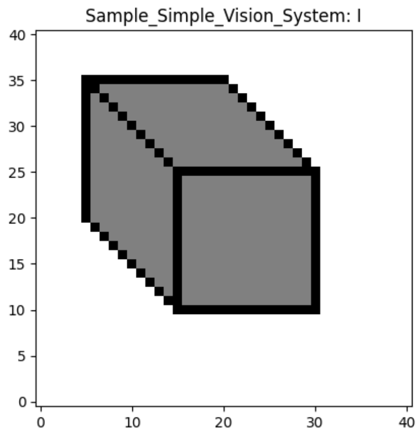
In the code, we have computed n_y and n_x for you!

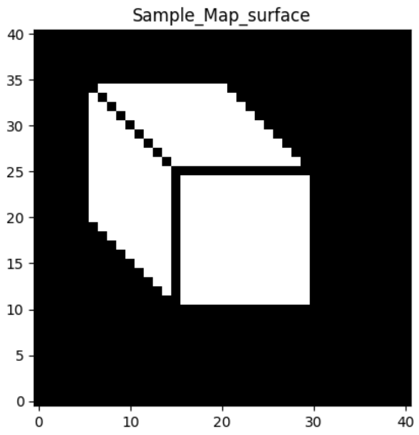


$$[0,\ldots-n_x,(-n_y+n_x),0\ldots0,n_y,\ldots,0]\begin{bmatrix}Y(1,1)\\Y(1,2)\\\ldots\\Y(1,N)\\Y(2,1)\\Y(2,2)\\\ldots\\Y(2,N)\\\ldots\\Y(M,N)\end{bmatrix}=0$$

One row for each horizontal edge pixel

Estimating Y[I, j]: cues from surfaces





Three rows for each surface pixel

Surfaces: flat, not curved

$$2Y[i,j] - Y[i+1,j] - Y[i-1,j] = 0$$

$$2Y[i,j] - Y[i,j+1] - Y[i,j-1] = 0$$

$$Y[i,j] - Y[i,j-1] - Y[i-1,j] + Y[i-1,j-1] = 0$$

$$[0, \dots, -1, 0 \dots, 2, 0, \dots - 1, 0, \dots]\tilde{Y} = 0$$

$$[0, \dots, -1, 0 \dots, 2, 0, \dots - 1, 0, \dots]\tilde{Y} = 0$$

$$[0, \dots, -1, 0 \dots, 1, \dots 0, \dots 1, 0, \dots - 1, 0, \dots]\tilde{Y} = 0$$

Put all the constraints together

$$\begin{bmatrix} - & \boldsymbol{a}_1 & - \\ - & \boldsymbol{a}_2 & - \\ & \dots & \end{bmatrix} \tilde{Y} = \boldsymbol{b}$$

For example,

$$\boldsymbol{A} = \begin{bmatrix} \vdots \\ 0, \dots - n_x, (-n_y + n_x), 0 \dots 0, n_y, \dots, 0 \\ \vdots \\ 0, \dots, 0, -1, 1, \dots, 0 \\ \vdots \\ 0, \dots, -1, 0 \dots, 2, 0, \dots - 1, 0, \dots \\ \vdots \end{bmatrix}$$

$$\boldsymbol{A}\tilde{Y} = \boldsymbol{b}$$

$$\boldsymbol{b} = \begin{bmatrix} \vdots \\ 0 \\ \vdots \\ 1/\cos(\theta) \\ \vdots \\ 0 \\ \vdots \end{bmatrix}$$

Least square solution!

$$\tilde{Y} = \left(\boldsymbol{A}^{\mathrm{T}}\boldsymbol{A}\right)^{-1} \boldsymbol{A}^{\mathrm{T}}\, \boldsymbol{b}$$

A is like 2300-by-1681

Estimating Z from Y and y

$$x = X$$

$$y = \cos(\theta)Y - \sin(\theta)Z$$

$$\Rightarrow$$

$$y[i,j] = \cos(\theta)Y[i,j] - \sin(\theta)Z[i,j]$$

Why linear system? Try this toy example

1	2	3
4	?	6
7	8	9

$$\tilde{Y} = \begin{bmatrix} Y(1,1) \\ Y(1,2) \\ Y(1,3) \\ Y(2,1) \\ Y(2,2) \\ Y(2,3) \\ Y(3,1) \\ Y(3,2) \\ Y(3,3) \end{bmatrix}$$

$$\begin{bmatrix} 1,0,0,0,0,0,0,0,0 \\ 0,1,0,0,0,0,0,0,0 \\ 0,0,1,0,0,0,0,0,0 \\ 0,0,0,1,0,0,0,0,0 \\ 0,0,0,0,0,1,0,0,0 \\ 0,0,0,0,0,0,1,0,0 \\ 0,0,0,0,0,0,0,1,0 \\ 0,0,0,0,0,0,0,0,1 \\ 0,0,0,-1,2,-1,0,0,0 \\ 0,-1,0,0,2,0,0,-1,0 \\ 1,-1,0,-1,1,0,0,0,0 \end{bmatrix} \begin{bmatrix} Y(1,1) \\ Y(1,2) \\ Y(1,3) \\ Y(2,1) \\ Y(2,2) \\ Y(2,3) \\ Y(3,1) \\ Y(3,2) \\ Y(3,3) \end{bmatrix} = \begin{bmatrix} 7 \\ 8 \\ 9 \\ 4 \\ 6 \\ 1 \\ 2 \\ 3 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

By solving it through $\tilde{Y} = \left(\boldsymbol{A}^{\mathrm{T}}\boldsymbol{A}\right)^{-1} \boldsymbol{A}^{\mathrm{T}}\, \boldsymbol{b}$,

$Y(2,2) \approx 5$