# A dynamic programming approach for equitable resource allocation in hunger-relief supply chain during extreme events

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## 1 Problem background and motivation

The aims of the project are-

- To formulate a decision problem as a discrete-time, discrete-state dynamic programming problem that considers stochastic supply and demand during extreme events in different counties of a region.
- To incorporate an equity constraint in the model that ensures fair allocation to each county under all demand and supply realizations.
- To give an allocation policy that reduces the total distribution cost for a food bank and unmet demand cost for each of the counties

## 2 Problem description

In this project, we consider a hunger-relief network with one central food bank that receives stochastic supply/donations before serving uncertain demands at a demand node. The aim would be to allocate portion of the supply to each demand node so that fairness in distribution is maintained among all possible supply and demand scenarios. First, we present a small instance with one food bank and five counties that have to be served with available supplies. Later, we present a numerical illustration using food insecurity data to allocate supplies from a foodbank.

### 2.1 Model assumptions

- The probabilities of demand and supply realizations under different scenarios are known
- Costs to distribute supplies to different demand nodes are known

- Demands are realized as we decide at each stage, along with additional supplies to the food bank
- Demands are proportional to the food insecurity rate at each of the demand node

# 3 Model formulation

In this section, we define the stages, states, recursive function, and constraints for the model.

## 3.1 Model description

| **Properties** | |
|---|---|
| **Stages** | Counties to serve $i \in 1, 2, ...., I$ |
| **State** $S_i$ | Available initial supply $S_i$ at foodbank while serving demand node $i$ |
| **Actions** $x_i$ | Amount of supply allocated to county $i$, $x_i$, where $i \in 1, 2, ...., I$ |
| **Parameters** | |
| $c_{ui}$ | Cost of unmet demand at demand node $i$ per unit |
| $c_{si}$ | Cost of supplying at demand node $i$ per unit |
| **Stochastic Parameters** | |
| $D_i$ | Demand realization at county $i$ |
| $A_i$ | Additional supply/donation arriving at food bank at stage $i$ |
| **Recursive function** | |
| $F_i(S_i)$ | Minimum expected cost with $i$ counties left to allocate with $S_i$ initial supply in hand in the food bank |

### Recursive Function

We develop two recursive functions. The function (1) only considers the unmet demand cost, focusing more on the fairness of distribution. On the other hand, Function (2) considers the unmet demand costs and the cost of distributing from the food bank to the demand nodes, focusing both on equity and efficiency of distribution.

$F_i$ is defined as:

$$F_i(S_i) = \mathbb{E}_{D_i, A_i}\left[\min_{x_i : \left|\frac{x_i}{S_i + A_i} - \frac{D_i}{\text{TotalDemand}}\right| < \beta \text{ and } x_i \leq S_i + A_i} \left\{c_{ui} \times (D_i - x_i)^+ + F_{i+1}(S_i + A_i - x_i)\right\}\right] \tag{1}$$

$$F_i(S_i) = \mathbb{E}_{D_i,A_i}\left[\min_{x_i:\left|\frac{x_i}{S_i+A_i}-\frac{D_i}{\text{TotalDemand}}\right|<\beta \text{ and } x_i\leq S_i+A_i}\left\{c_{ui}\times(D_i-x_i)^+ + c_{si}\times x_i + F_{i+1}(S_i+A_i-x_i)\right\}\right] \tag{2}$$

Boundary condition, $F_{I+1}(S) = 0, \;\; \forall S$

Here, $(D_i - x_i)^+$ ensures unmet demand, when $D_i \geq x_i$.

### Fairness Constraint

The fairness constraint ensures the proportion of supply allocated to node $i$ is proportional to its demand proportion to the total demand from all demand points. Fariness constraint is defined in Equation (3).

$$\left|\frac{x_i}{S_i+A_i} - \frac{D_i}{\text{TotalDemand}}\right| < \beta, \tag{3}$$

Here, the decision-makers determine $\beta$, which specifies the maximum allowable deviation, ensuring fairness in distribution relative to the overall demand. However, since demand is stochastic for each stage, we cannot have the Total Demand directly at $i^{th}$ stage. Hence, we consider the total demand to be the sum of the maximum realized demand possible for each demand node. This is expressed as:

$$\text{TotalDemand} = \sum_{i=1} max\{Di \text{ of all realizations in } i\}$$

## 4 Numerical Illustration

### 4.1 Small instance

First, we use a small instance with generated data to illustrate the effect of different equity constant values $\beta$ on the total expected cost for the distribution. To focus more on equitable distribution, we only consider the unmet demand cost in this case. We consider 5 counties to serve, with each having different demand realizations. We consider 5 different realizations for each county's demands with uniform probabilities,i.e., each having a probability of 0.2. We discretized the demands to reduce the state space and took 1 for every 1000 lbs of demand. For example, County 1 has a demand of 10, which refers to having 10,000 lbs of demand. Data for demand is presented in Table 1.

Similarly, we consider 4 different realizations of supplies/donations to the food bank. The data is presented in Table 2.

Table 1: Realizations and probabilities of demand by County

| **Demand, Probability** | | | | |
|---|---|---|---|---|
| **County 1** | (10, 0.2) | (15, 0.2) | (20, 0.2) | (25, 0.2) |
| **County 2** | (30, 0.2) | (45, 0.2) | (60, 0.2) | (75, 0.2) |
| **County 3** | (80, 0.2) | (95, 0.2) | (100, 0.2) | (105, 0.2) |
| **County 4** | (115, 0.2) | (120, 0.2) | (125, 0.2) | (130, 0.2) |
| **County 5** | (125, 0.2) | (130, 0.2) | (135, 0.2) | (140, 0.2) |

Table 2: Supply Realizations and Probabilities

| **Supply, Probability** | | | |
|---|---|---|---|
| (100, 0.25) | (160, 0.25) | (180, 0.25) | (200, 0.25) |

Using the small instance data, we analyze the expected cost for different equity constant values $\beta$. The expected costs against different $\beta$ are presented in Figure 1. Figure 1 exhibits the nature of equity. With $\beta = 0$ with perfect equity, the expected cost is higher and reduces as the model deviates from the perfect. After a certain threshold, the expected cost converges, meaning after a certain level of deviation, the optimal allocation policy is constant for all $\beta$ values, and the optimal cost is the same. Figure 2 shows that with smaller $\beta$ (with smaller supplies), the proportion of demand met among different counties is close to each other, whereas, with larger $\beta$ (with bigger supplies), the demand proportion met for different counties differs from each other, reflecting a higher deviation from equitable distribution among different counties.

Since the developed model reflects the characteristics of equitable distributions, we use the model for solving a case study in the next section.

### 4.2 FBCENC case study

We use the FBCENC food bank case study in Raleigh, NC, to distribute supplies to 10 agencies in Raleigh city, as shown in Figure 3. First, we use the model with only equity consideration. Then, we use the model with both equity and efficiency considerations.

The average demand for each of these agencies is presented in Table 3. For this case, considering a disaster event, we consider a demand surge for each agency. Different demand realizations for different levels of disaster are presented in the Appendix (Table 4). We discretized the demands

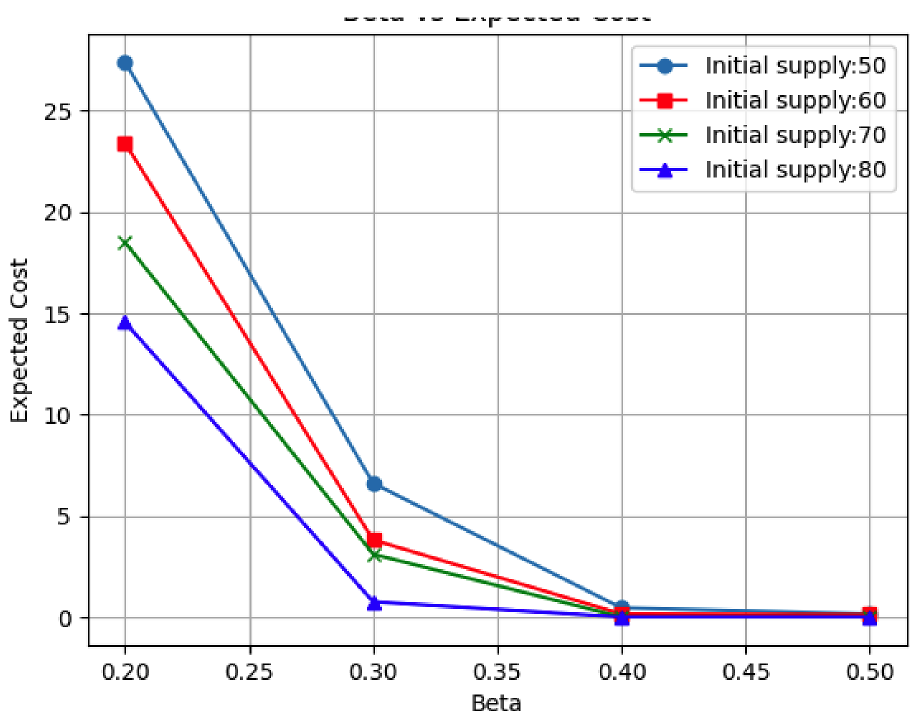



Figure 1: Expected costs with different level of equity

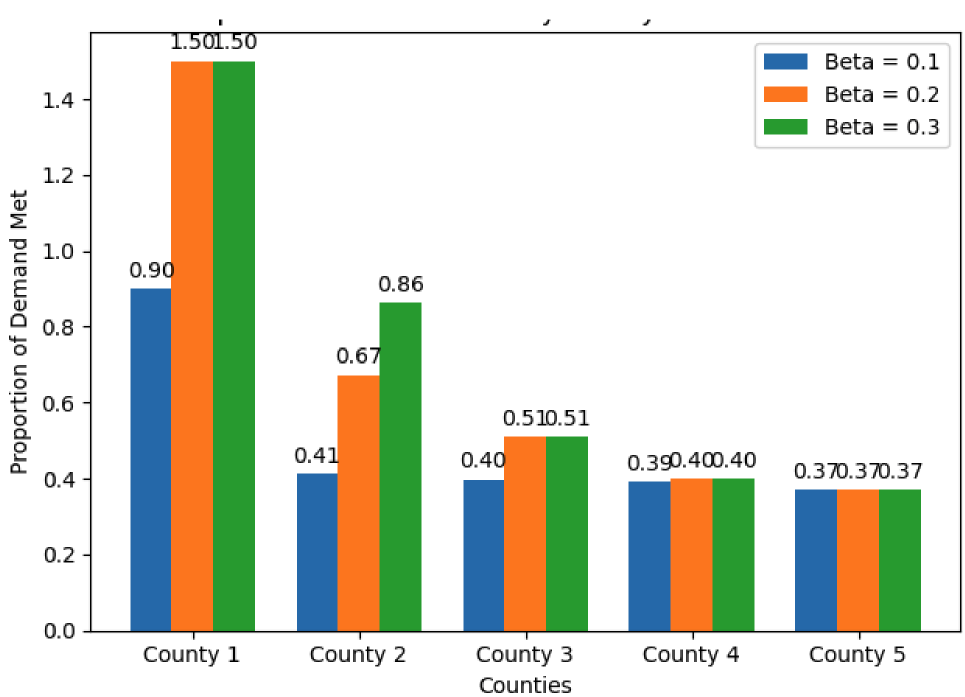



Figure 2: Proportion of demand met for counties

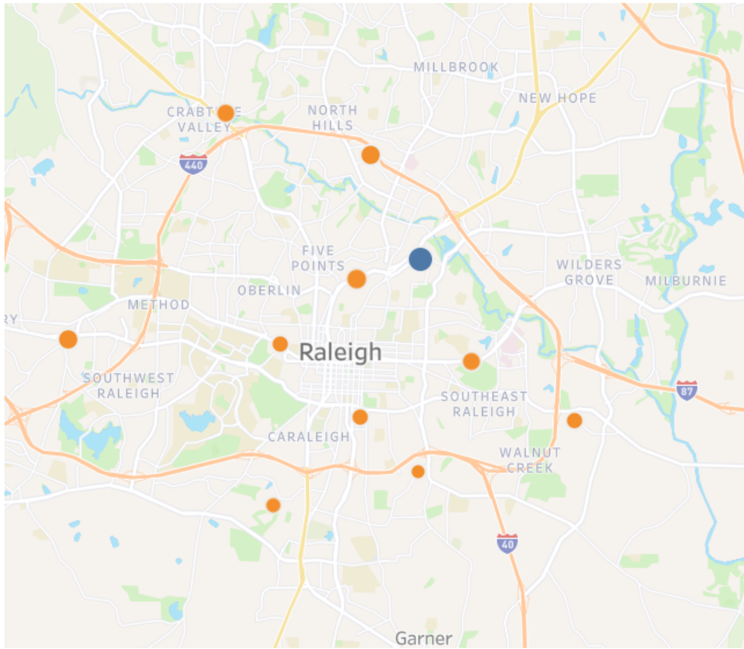
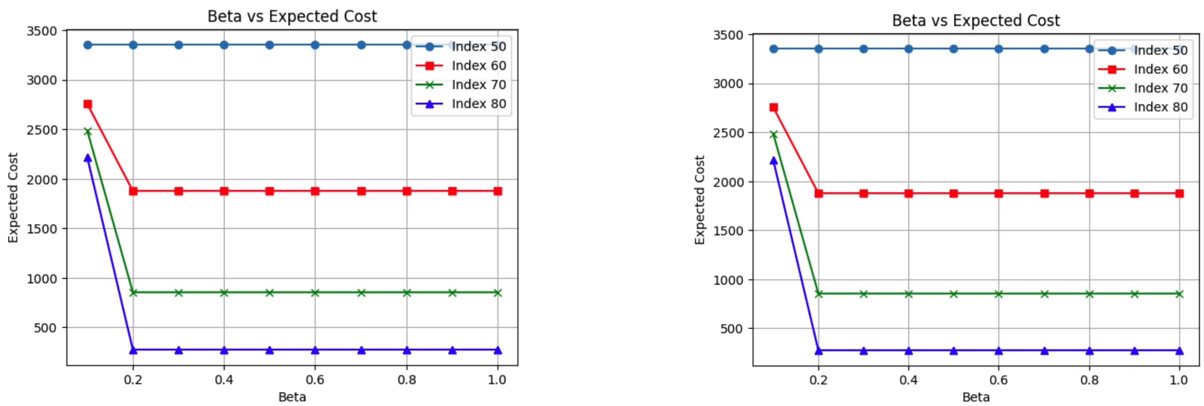



Figure 3: Foodbank and agency locations

to reduce the state space and took 1 for every 100 lbs of demand, similar to the previous instance. The supply realizations are also presented in the Appendix (Table 5). The distribution costs are approximated and presented in the Appendix (Table 6) for different distances from the food bank to agencies.

Using the data, we solved the case using Equations (1) (equity only) and (2) (equity and efficiency). The results are shown in Figure 4. From the figure, we see almost no change in the cost, as the scale of unmet demand costs is much higher, and it puts a higher priority on equity rather than efficiency. The figure also exhibits the exact nature of the amount of supply in hand. With a small supply (index 50 in the figure), the cost is higher than the larger initial supply. For all cases, the optimal expected costs converge after $\beta = 0.2$. Hence, we suggest the decision-makers allow a 20% equity deviation to ensure sufficient fairness among different agencies while the cost is also minimized. The optimal policy for 20% equity deviation given the donation realizations in the food bank is presented in Figure 5.




(a) Expected cost for equity only

(b) Expected cost for equity and efficiency

Figure 4: Optimal expected cost vs equity

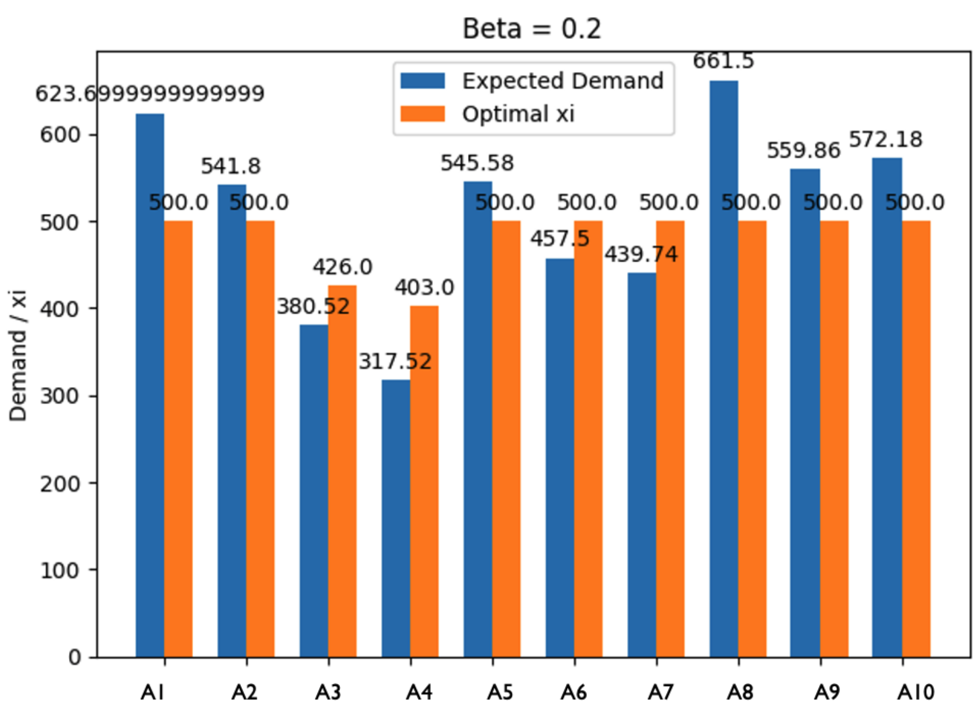



Figure 5: Proportion of demand met for counties

Table 3: Demand by Organization

| Code | Name | Demand(lbs) |
|---|---|---|
| A1 | Church Of God Of Prophecy | 49514 |
| A2 | Crabtree Valley Baptist Church | 43049 |
| A3 | Faith Missionary Baptist | 30252 |
| A4 | First Cosmopolitan MBC | 25218 |
| A5 | Iglesia De Dios Pentecostal | 43389 |
| A6 | N.C.A. Philip Randolph Inst. | 37527 |
| A7 | New Bethel Christian Church | 34966 |
| A8 | Urban Ministries of Wake County | 52545 |
| A9 | Victory Gospel Chapel | 37166 |
| A10 | NC Harm Reduction | 46942 |

# 5 Conclusion

In this project, we presented a dynamic programming approach for equitable allocation of donations to different demand regions. First, we formulated the recursive function with only equity considerations and then proposed another formulation with efficiency considerations. We used the equity-only formulation for a small instance and demonstrated that the costs are higher for achieving a higher equity level. Then, we used FBCENC case study and both formulations for the analysis. Since the distribution cost was far less than the unmet demand, the distribution cost did not affect the result much and prioritized equitable distribution. We also observed that the expected optimal cost converges at 20% equity deviation. Achieving slightly larger equity increases the cost significantly. So, for the case study, we suggest the optimal policy of allocation of distribution best results from 20% equity deviation.

Future works for this project include considering a multi-period distribution, as it was not considered in the current model. Additionally, the demand and supply distribution can be analyzed to create a more realistic presentation of their realizations. Furthermore, we can consider different distribution costs for different disaster scenarios.

# Appendix

## Small instance code

```
import numpy as np
import matplotlib.pyplot as plt
#Number of counties
NUM_COUNTIES = 5
#equity constant
start_beta = 0.00
end_beta = 1
step_beta = 0.1
BETAS = np.arange(start_beta, end_beta + step_beta, step_beta)

C_UI = np.array([5, 4, 3, 2, 1])  # Cost per unit of unmet demand for each county

# Additional supplies and their probabilities
A_realizations = np.array([100, 160, 180, 200])
A_probabilities = np.array([0.25, 0.25, 0.25, 0.25])

# Initial supply at hand in the foodbank
initial_supply = 50
MAX_SUPPLY = initial_supply + max(A_realizations)
# Demand realizations and probabilities for each of the five counties
D_realizations = [
    np.array([10, 15, 20, 25, 30]),
    np.array([30, 45, 60, 75, 80]),
    np.array([80, 95, 100, 105, 110]),
    np.array([115, 120, 125, 130, 135]),
    np.array([125, 130, 135, 140, 145])
]
D_probabilities = [
    np.array([0.2, 0.2, 0.2, 0.2, 0.2]),
    np.array([0.2, 0.2, 0.2, 0.2, 0.2]),
    np.array([0.2, 0.2, 0.2, 0.2, 0.2]),
    np.array([0.2, 0.2, 0.2, 0.2, 0.2]),
    np.array([0.2, 0.2, 0.2, 0.2, 0.2])
```

```
]
# Maximum realization for each county
max_values = [np.max(row) for row in D_realizations]


total_Demand = sum(max_values)

# DP table
dp = np.full((len(BETAS),NUM_COUNTIES + 1, (MAX_SUPPLY+1)), np.inf)
dp[:,NUM_COUNTIES , :] = 0  # Base case: no cost for the terminal condition
optimal_allocations = np.full((len(BETAS),NUM_COUNTIES, (MAX_SUPPLY+1)),-np.inf)

# DP recursion with equity only
for beta_index, beta in enumerate(BETAS):
    for i in reversed(range(NUM_COUNTIES)):
        for Si in range(MAX_SUPPLY+1):
            for xi in range(Si+1):
                if all(((abs((xi / (Si + a)) - (d / total_Demand ))<= beta)) \
                for a in A_realizations for d in D_realizations[i]):
                    cost = 0
                    for a_idx, a in enumerate(A_realizations):
                        for d_idx, d in enumerate(D_realizations[i]):
                            if Si + a - xi <= MAX_SUPPLY:
                                cost = cost + (C_UI[i] * max((d - xi),0) + \
                                dp[beta_index,i+1,Si + a - xi]) \
                                * D_probabilities[i][d_idx]*A_probabilities[a_idx]
                    if cost < dp[beta_index,i, Si]:
                        dp[beta_index,i, Si] = cost
                        optimal_allocations[beta_index,i, Si]= xi

#Minimum expected cost achieved for different equity constant values
#with a know initial supply at the food bank
for beta_index, beta in enumerate(BETAS):
    print(f"\nFor beta = {beta}:")
    print(f"Optimal Expected Cost: {dp[0, initial_supply, beta_index]}")
```

### FBCENC case study code

```
#Number of counties
NUM_COUNTIES = 10
#equity constant
start_beta = 0.00
end_beta = 1
step_beta = 0.1
BETAS = np.arange(start_beta, end_beta + step_beta, step_beta)
# Cost per unit of unmet demand for each count
C_UI = np.array([15, 15, 15, 15, 15,15, 15, 15, 15, 15])
y
# Cost per unit of unmet demand for each county
C_XI = np.array([3, 3, 3, 2, 1, 1.5, 2, .5, 1.5, 1])

# Additional supplies and their probabilities
A_realizations = np.array([2000, 2200, 2400, 2600])
A_probabilities = np.array([0.5, 0.3, 0.1, 0.1])

# Initial supply at hand in the foodbank
initial_supply = 500
MAX_SUPPLY = initial_supply + max(A_realizations)
# Demand realizations and probabilities for each of the five counties
D_realizations = [
    np.array([495, 495*1.2, 495*1.4, 495*1.6]),
    np.array([430, 430*1.2, 430*1.4, 430*1.6]),
    np.array([302, 302*1.2, 302*1.4, 302*1.6]),
    np.array([252, 252*1.2, 252*1.4, 252*1.6]),
    np.array([433, 433*1.2, 433*1.4, 433*1.6]),
    np.array([375, 375*1.2, 375*1.4, 375*1.4]),
    np.array([349, 349*1.2, 349*1.4, 349*1.6]),
    np.array([525, 525*1.2, 525*1.4, 525*1.6]),
    np.array([371, 371*1.2, 525*1.4, 525*1.6]),
    np.array([469, 469*1.2, 469*1.4, 469*1.4])
]
D_probabilities = [
    np.array([0.3, 0.3, 0.2, 0.2]),
```

```
]
max_values = [np.max(row) for row in D_realizations]
total_Demand = sum(max_values)
# DP recursion with equity and efficiency
for beta_index, beta in enumerate(BETAS):
    for i in reversed(range(NUM_COUNTIES)):
        for Si in range(MAX_SUPPLY+1):
            for xi in range(Si+1):
                if all(((abs((xi / (Si + a)) - (d / total_Demand ))<= beta))\
                for a in A_realizations for d in D_realizations[i]):
                    cost = 0
                    for a_idx, a in enumerate(A_realizations):
                        for d_idx, d in enumerate(D_realizations[i]):
                            if Si + a - xi <= MAX_SUPPLY:
                                cost = cost + (C_UI[i] * max((d - xi),0) + \
                                dp[beta_index,i+1,Si + a - xi]) \
                                * D_probabilities[0][d_idx]* A_probabilities[a_idx]
                    if cost < dp[beta_index,i, Si]:
                        dp[beta_index,i, Si] = cost
                        optimal_allocations[beta_index,i, Si]= xi
```

Table 4: Demand realizations for FBCENC case sttudy

| **Scenario** | **Demand(lbs)** |
|---|---|
| Normal | Average demand |
| Level 1 Disaster | 20% increase |
| Level 2 Disaster | 40% increase |
| Level 3 Disaster | 60% increase |

Table 5: Supply realizations for FBCENC case sttudy

| **Scenario** | **DemandProbability)** | |
|---|---|---|
| S1 | 2000 | .5 |
| S2 | 2200 | .3 |
| S3 | 2400 | .1 |
| S4 | 2600 | .1 |

Table 6: Distribution costs

| **Name** | **Distribution cost per unit** |
|---|---|
| Church Of God Of Prophecy | 3.0 |
| Crabtree Valley Baptist Church | 3.0 |
| Faith Missionary Baptist | 3.0 |
| First Cosmopolitan MBC | 2.0 |
| Iglesia De Dios Pentecostal | 1.0 |
| N.C.A. Philip Randolph Inst. | 1.5 |
| New Bethel Christian Church | 2.0 |
| Urban Ministries of Wake County | 0.5 |
| Victory Gospel Chapel | 1.5 |
| NC Harm Reduction | 1.0 |

## References