PROJECT REPORT

Image Processing and Noise Reduction

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Contents

Abstract

Images can be represented in the form of a 2D array of numbers called a matrix. This 2D array of numbers follows a special kind of mathematics called linear algebra. Singular Value Decomposition (SVD) operation is part of linear algebra that decomposes a matrix into 3 basic and simple-to-operate matrices. This method can be used to reduce the size and remove the noise from the Images. In this project, we will see that how we can remove the large noise from the images using the SVD. Later we will discuss the applications and plan that we are planning to implement in this project

1 Introduction

Image processing is a very important tool in the modern world of science. Extraction of information from the noisy signals has become a necessity. Not only this, storage and handling of data is very difficult. SVD or Singular Value Decomposition is a method derived from linear algebra that has become a powerful instrument for the multidisciplinary application of imaging.

In this project, we dive into the simple 2D-data structures that can be used for exploring and implementing SVD. We shall look into the theoretical framework of linear algebra that forms the foundation of SVD matrix decomposition and also see how it is implemented in practical problems.SVD decomposes an image into its constituent singular values arrays and offers a unique perspective that helps to perform transformation operations (stretching, elongation and rotation), denoising and compression to feature extraction and restoration.

The main objectives of this project include:

1. Understanding the basics of linear algebra (LA) and Singular Value Decomposition (SVD) method.
2. Relating LA and SVD to image processing.
3. Implementation of SVD for essential image compression and reconstruction.
4. Denoising an image by introducing some controlled noise (Gaussian noise) to the image for both greyscale and RGB (Red,Green and Blue).
5. Evaluation of the performance of SVD compression and denoising on a relative scale.
6. Discussion about the potential advance that can be made into the code for better performance.

2 Linear Algebra

2.1 What is Linear Algebra?

Linear Algebra is a branch of mathematics that deals with vectors (1D-array), matrices (2D arrays) and tensors (higher dimensional arrays). LA has lots of applications across different fields like Physics, Engineering, Computer Graphics, Signal Processing etc. In the context of Image image processing, every image can be represented as a single or multiple 2D arrays of numbers. Each number in the matrix corresponds to a colour which can be then used to reconstruct the image from the matrix.

Consider a point in the XY-plane, for example, $A(2,3)$ which can be represented in the form of an array as (23). We can translate this point A to a different point in space, say $A'(6,4)$ just by the multiplication of a 2D matrix $'S'$ which can be given by the relation:

$$\begin{bmatrix} -1 & -1 \\ -2 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -3 \\ -4 \end{bmatrix}$$

Here the matrix $((0,2),(2,0))$ is the S matrix also known as the translation matrix.

A similar transformation can be done for a line or 2D shape in the xy plane. For this to happen each point on the shape is multiplied by the transformation matrix 'S' for example for a circle of unit radius at the origin if multiplied by S = ((1,1),(1,-1)) will produce an ellipse.

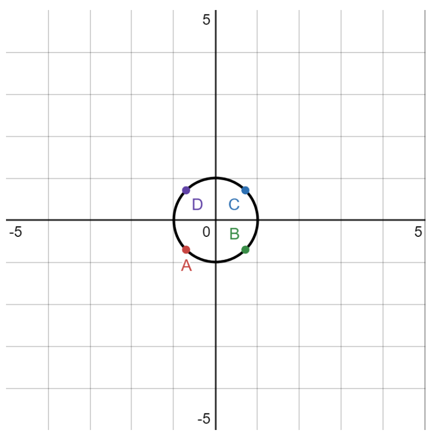

Figure 1: Circle of unit radius

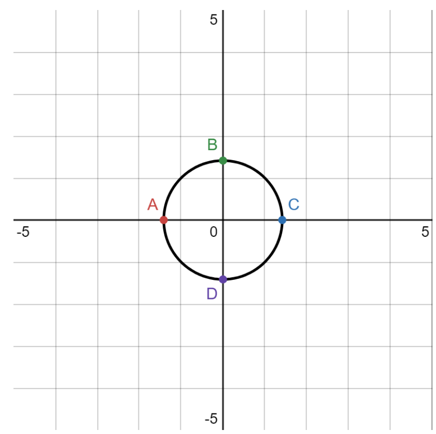

Figure 2: Circle of radius $\sqrt{2}$ units after transformation with a rotation of angle $\pi/4$ radians clockwise

Similarly changing the elements of the matrix would produce circles of different radii, rotations or even ellipses if the elements of the transformation matrix are not equal in magnitude.

2.2 SVD Matrix Decomposition

We can break down any matrix into its three guaranteed elements. A matrix can be factorised into three matrices with distinct characteristics using the SVD linear algebra process. Therefore for every matrix 'A' of dimension $m \times n$, there exists a U matrix of $m \times n$, an Σ matrix of $n \times n$ and a V matrix of $n \times n$.

$$A_{mn} = U_{mm} \Sigma_{mn} {V^T}_{nn} \tag{2}$$

Where S is a diagonal matrix that contains elements in decreasing order of $s_{11} \leq s_{22} \leq s_{33} \leq s_{44} \cdots \leq s_{nn}$.

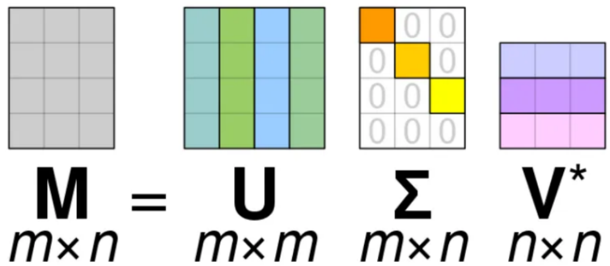

Figure 3: Singular Value Decomposition

Since the elements in the matrix 'S' are continuously decreasing we can approximate the smaller elements to be zero. For example, we can say that elements beyond r¡n are sufficiently small to be approximately zero, so now we can write the matrix 'A' as the product of three matrices with reduced dimensions:

$$A_{mn} = U_{mr}\Sigma_{rr}{V^T}_{rm} \tag{3}$$

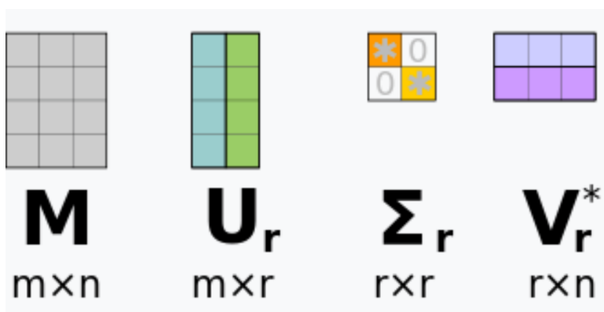

Figure 4: Reduced Singular Value Decomposition for $r < n$

3 SVD Image Processing

Any image can be thought of as a 2D or a collection of multiple 2D arrays. Hence it can be converted into a matrix and then decomposed into its SVD components. As of now, we will take an example image as shown in figure 5 and add some Gaussian noise to it given in 6

For verification purposes, I first converted the 'Original Image' to a greyscale image performed SVD and reconstructed the image by following the matrix summation formula given by the relation:

$$A = \sum_{i}^{n} s_{ii} U_{mi} (V_{ni})^T \tag{4}$$

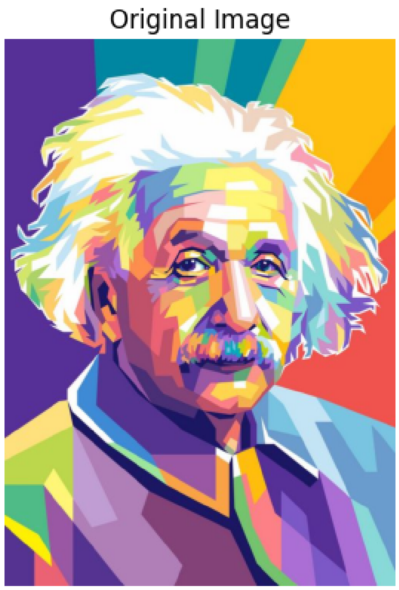

Figure 5: The Original Image

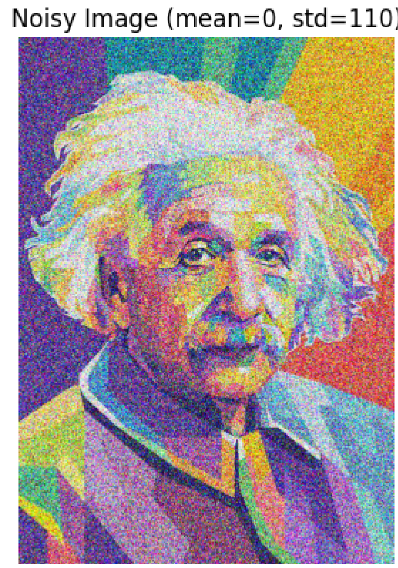

Figure 6: Adding Noise to the Original Image

3.1 Noise Reduction using SVD

Now we converted the 'noised image' to greyscale and performed an SVD to get $U_{514\times720}$, $\Sigma_{720\times720}$, V_{720} and a reconstruction was performed by using the Equation-4 for n = 50 to get an output as shown in the figure 9.

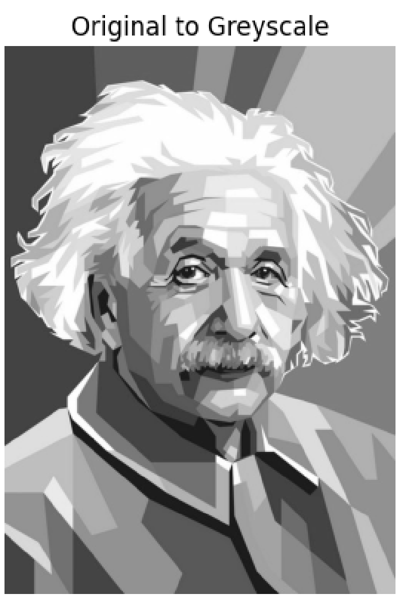

Figure 7: Original Image in Greyscale

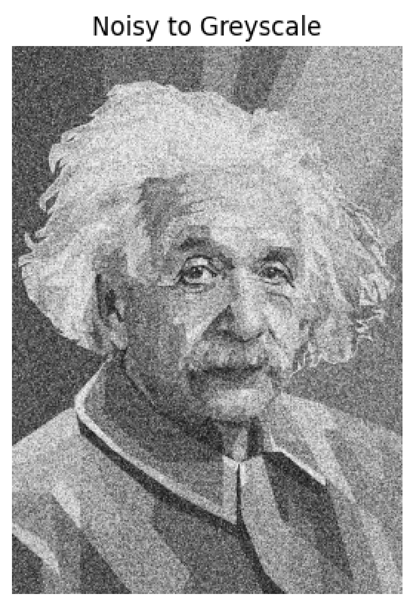

Figure 8: Noised Image in Greyscale

Similarly the same was done for the coloured noisy image with few additional steps. Firstly,

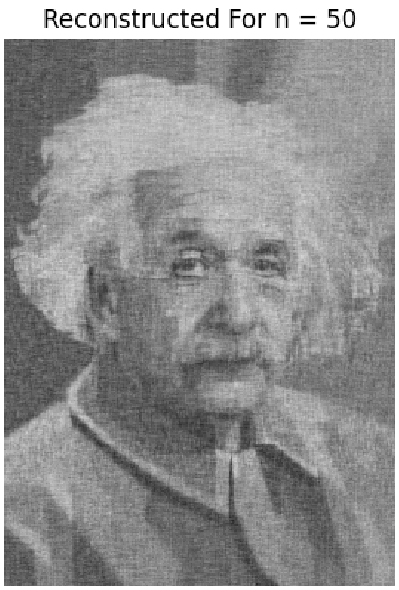

Figure 9: Reconstructed Image

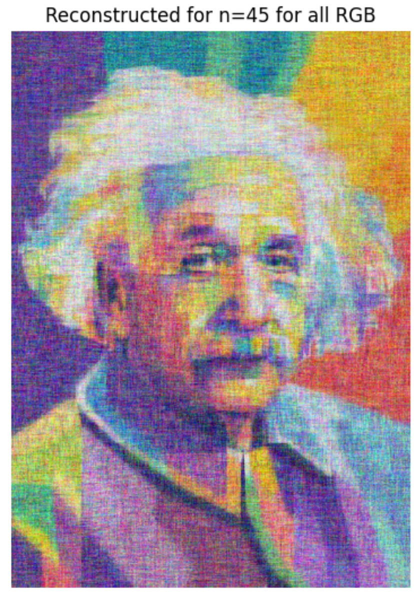

Figure 10: Reconstructed Coloured Image

the noisy image was segregated into its constituent Red, Blue and Green Components. Then SVD was performed for each of the segregated components and a reconstruction was performed with different numbers of 'n' for each colour. The final output image looked like **??**

4 Applications

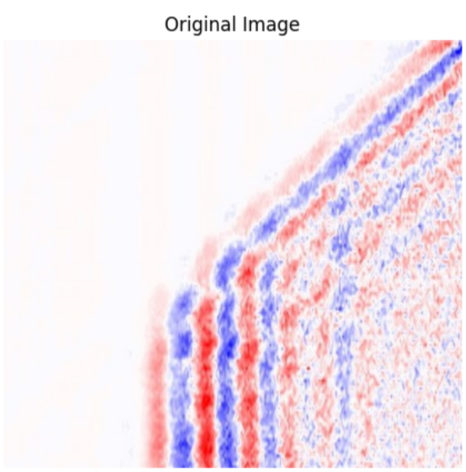

Figure 11: Original Temporal Signal

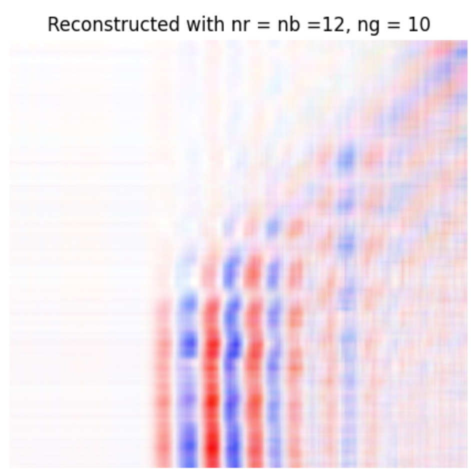

Figure 12: Reduced Noise

Noise reduction is important in the field of optics for both spectroscopy and microscopy. In the case of THz-pump THz-probe spectroscopy or 2D-Thz spectroscopy, it is important to filter out the noise from the temporal data to get the desired signal output. Figure 11 shows a typical temporal signature of the 2D-THz data. One additional step that was used here was multiplying the Σ matrix with the window(window) function to suppress the noise from the signal and the final image looks like Figure 12.

4.1 About the Code

The complete code for the SVD image filtering was done in 'Python' as well as 'MATLAB' both of them yielded similar results and both the Codes consist of the same steps which are as follows:

1. Open the Image(noised) in the desired platform.
2. Convert the image into the matrix or set of matrices.
3. Decompose all the matrices in V, U and Σ .
4. Merge the reconstructed matrices.
5. Choose the value of n for each reconstruction.
6. Convert the final matrix into the image.

5 Future Plan

The Future Plan for this project includes using and developing the same code using K-SVD which is a learning algorithm that is utilized for tasks such as feature extraction, dimensional reduction, denoising, and sparse representation learning. K-SVD can also be used to learn a dictionary of image patches, which can then be employed for tasks like image denoising or super-resolution. Also, I would like to pair it up with a suitable window function and smooth function.

References

[1] L. Cao, "Singular value decomposition applied to digital image processing," *Division of Computing Studies, Arizona State University Polytechnic Campus, Mesa, Arizona State University polytechnic Campus*, pp. 1–15, 2006.

[2] R. Hinno, "Simple svd algorithms," 2021. https://towardsdatascience.com/simple-svd-algorithms-13291ad2eef2.

[3] "Opencv24 python tutorials," 2016. https://opencv24-python-tutorials.readthedocs.io/en/latest/py_tutorials/py_imgproc/py_filtering/py_filtering.html.

[4] M. Lebrun and A. Leclaire, "An implementation and detailed analysis of the k-svd image denoising algorithm," *Image Processing On Line*, vol. 2, pp. 96–133, 2012.

[5] L. Serrano, "luisguiserrano/singular value decomposition," 2020. https://github.com/luisguiserrano/singular_value_decomposition.