

Instructions

Each student should submit the assignment in the form of hardcopy and the submission should contain the following

- Step by step solution of the problem with manual calculation
- Print out of the CODE with the outputs as per the question
- Plots without proper caption, axes title, etc., will not be considered.

Problem 1:

Consider a differential equation given below which governs the axial deformation behaviour of a structural member subjected to self-weight. The limit varies between 0 to 1.

$$-\frac{d}{dx} \left[A(x)E \frac{du}{dx} \right] = \rho A(x)g$$

Boundary Condition 1: $u(0)=0$ and $u(1)=0$

Boundary Condition 2: $u(0)=0$ and $AE \, du/dx=0$ at $x=1$

Assume that the cross-sectional area varies as $A(x) = 100(x^2-2x+1) \text{ mm}^2$;

$E=210 \text{ GPa}$; density = 7800 kg/m^3 ; $g=9.81 \text{ m/s}^2$;

Solve the problem **manually** using

- Two point Collocation
- Method of Least Squares with TWO TERM approximation
- Galerkin approach with TWO TERM approximation
- Ritz method with TWO TERM approximation
- Write a **computer code** and **show the convergence** by using a greater number of points/terms in the above methods.

Plot the results on a same graph for each approach and compare the converged results in each method on a same graph.

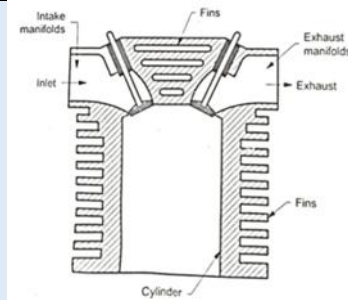
Problem 2:

Consider a differential equation given below which governs the heat transfer through fin in an IC engine, a schematic diagram is given. Compare the temperature distribution through the fin for the fin with uniform cross-section, linearly decreasing cross-section, and quadratically decreasing cross-section.

$$-\frac{d}{dx} \left[kA(x) \frac{du}{dx} \right] + hP(x)(T - T_{\infty}) = 0; \quad 0 < x < L \quad T(x=0) = 100^{\circ}C \quad \text{and} \quad \left[kA \frac{dT}{dx} \right]_{x=L} = 0$$

Assume

$$k = 300 \frac{W}{mC}; \quad h = 100 \frac{W}{m^2C}; \quad T_{\infty} = 20^{\circ}C; \quad L = 100cm$$



Uniform square cross section:

$$A(x) = 100mm^2$$

Linearly decreasing square cross-section:

$$A(x) = 100 \left[1 - \left(\frac{x}{L} \right) \right] mm^2$$

Parabolically Decreasing square cross-section:

$$A(x) = 100 \left[\left(\frac{x}{L} \right)^2 - 2 \left(\frac{x}{L} \right) + 1 \right] mm^2$$

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