PROBLEM 1

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.linalg import solve
from numpy.polynomial.legendre import leggauss

# ----------------------------
# Problem parameters
# ----------------------------
E = 210e9
rho = 7800
g = 9.81
x0, x1 = 0.0, 1.0
EPS = 1e-9

def A(x):
    return 100.0 * (x**2 - 2.0*x + 1.0) * 1e-6

def q(x):
    return rho * g * A(x)

def cumtrapz_custom(y, x):
    seg = 0.5 * (y[1:] + y[:-1]) * (x[1:] - x[:-1])
    return np.concatenate(([0.0], np.cumsum(seg)))

def gauss_points_weights(n):
    xs, ws = leggauss(n)
    xs = 0.5*(x1-x0)*xs + 0.5*(x1+x0)
```

```
    ws = 0.5*(x1-x0)*ws
    return xs, ws

# ----------------------------
# Exact solution & constants
# ----------------------------
def compute_reference_solution(npoints=10001):
    xs = np.linspace(x0, x1 - EPS, npoints)
    qvals = q(xs)
    Q = cumtrapz_custom(qvals, xs)
    denom = E * A(xs)

    # BC1 constants
    I1 = np.trapz((-Q) / denom, xs)
    I2 = np.trapz(1.0 / denom, xs)
    C1_BC1 = -I1 / I2
    C2_BC1 = 0.0

    uprime_BC1 = (-Q + C1_BC1) / denom
    u_BC1 = cumtrapz_custom(uprime_BC1, xs) + C2_BC1

    # BC2 constants
    C1_BC2 = Q[-1]
    C2_BC2 = 0.0

    uprime_BC2 = (-Q + C1_BC2) / denom
    u_BC2 = cumtrapz_custom(uprime_BC2, xs) + C2_BC2
```

```
    print(f"BC1: C1 = {C1_BC1:.6e}, C2 = {C2_BC1:.6e}")
    print(f"BC2: C1 = {C1_BC2:.6e}, C2 = {C2_BC2:.6e}")

    return (lambda x: np.interp(x, xs, u_BC1),
            lambda x: np.interp(x, xs, u_BC2))

# ----------------------------
# Basis functions
# ----------------------------
def basis_BC1(x, n):
    return np.array([ (x*(1.0-x)) * (x**(k-1)) for k in range(1, n+1) ])

def dbasis_BC1(x, n):
    return np.array([ k * x**(k-1) - (k+1) * x**k for k in range(1, n+1) ])

def basis_BC2(x, n):
    return np.array([ x**k for k in range(1, n+1) ])

def dbasis_BC2(x, n):
    return np.array([ k * x**(k-1) for k in range(1, n+1) ])

# ----------------------------
# Methods
# ----------------------------
def assemble_ritz(n, bc):
    phis = basis_BC1 if bc=='BC1' else basis_BC2
    dphis = dbasis_BC1 if bc=='BC1' else dbasis_BC2
    xs, ws = gauss_points_weights(200)
```

```
    phi_vals = phis(xs, n)
    dphi_vals = dphis(xs, n)
    K = np.zeros((n, n))
    f = np.zeros(n)
    for i in range(n):
        for j in range(n):
            K[i,j] = np.dot(ws, E * A(xs) * dphi_vals[i,:] * dphi_vals[j,:])
        f[i] = np.dot(ws, q(xs) * phi_vals[i,:])
    return solve(K, f), lambda x: phis(x, n)

def assemble_galerkin(n, bc):
    phis = basis_BC1 if bc=='BC1' else basis_BC2
    dphis = dbasis_BC1 if bc=='BC1' else dbasis_BC2
    xs, ws = gauss_points_weights(200)
    phi_vals = phis(xs, n)
    dphi_vals = dphis(xs, n)
    A_mat = np.zeros((n,n))
    b_vec = np.zeros(n)
    for j in range(n):
        for i in range(n):
            A_mat[j,i] = np.dot(ws, E * A(xs) * dphi_vals[j,:] * dphi_vals[i,:])
        b_vec[j] = - np.dot(ws, phi_vals[j,:] * q(xs))
    return solve(A_mat, b_vec), lambda x: phis(x, n)

def assemble_least_squares(n, bc):
    phis = basis_BC1 if bc=='BC1' else basis_BC2
    dphis = dbasis_BC1 if bc=='BC1' else dbasis_BC2
    xs, ws = gauss_points_weights(200)
```

```
    Aprime = (200.0*(xs - 1.0)) * 1e-6
    d2phis = np.gradient(dphis(xs, n), xs, axis=1)
    D_dphi = E * (Aprime * dphis(xs, n) + A(xs) * d2phis)
    M = np.zeros((n,n))
    rhs = np.zeros(n)
    for i in range(n):
        for j in range(n):
            M[i,j] = np.dot(ws, D_dphi[i,:] * D_dphi[j,:])
        rhs[i] = - np.dot(ws, q(xs) * D_dphi[i,:])
    return solve(M, rhs), lambda x: phis(x, n)

def assemble_collocation(n, bc):
    xs = np.linspace(x0 + EPS, x1 - EPS, n+2)[1:-1]
    if bc=='BC1':
        dphi_vals = dbasis_BC1(xs, n)
        phis = lambda xx: basis_BC1(xx, n)
    else:
        dphi_vals = dbasis_BC2(xs, n)
        phis = lambda xx: basis_BC2(xx, n)
    Aprime = (200.0*(xs - 1.0)) * 1e-6
    d2phi_vals = np.gradient(dphi_vals, xs, axis=1)
    D_dphi = E * (Aprime * dphi_vals + A(xs) * d2phi_vals)
    return solve(D_dphi.T, -q(xs)), phis

def eval_u(a, phis_func, xvals):
    return np.dot(a, phis_func(xvals))

# ----------------------------
```

```
# Main
# ----------------------------
u_ref_BC1, u_ref_BC2 = compute_reference_solution()
methods = ['collocation', 'least_squares', 'galerkin', 'ritz']
colors = ['tab:blue', 'tab:orange', 'tab:red', 'tab:purple']

# Plot n=8 BC1 solid, BC2 dashed
plt.figure(figsize=(8,5))
for m, c in zip(methods, colors):
    for bc in ['BC1', 'BC2']:
        if m=='collocation':
            a, phis = assemble_collocation(8, bc)
        elif m=='least_squares':
            a, phis = assemble_least_squares(8, bc)
        elif m=='galerkin':
            a, phis = assemble_galerkin(8, bc)
        else:
            a, phis = assemble_ritz(8, bc)
        style = '-' if bc=='BC1' else '--'
        plt.plot(np.linspace(0,1,200), eval_u(a, phis, np.linspace(0,1,200)),
                 style, color=c, label=f"{m} ({bc})" if bc=='BC1' else None)
plt.xlabel("x (dimensionless, 0..1)")
plt.ylabel("u(x) [m]")
plt.title("Comparison of methods (N=8) — solid: BC1, dashed: BC2")
plt.legend()
plt.grid(True)
plt.show()
```

```
# Two-term coefficients and expressions
print("\nTwo-term (N=2) coefficients and expressions for each method:")
for bc in ['BC1', 'BC2']:
    print(f"\n--- {bc} ---")
    for m in methods:
        if m=='collocation':
            a, phis = assemble_collocation(2, bc)
        elif m=='least_squares':
            a, phis = assemble_least_squares(2, bc)
        elif m=='galerkin':
            a, phis = assemble_galerkin(2, bc)
        else:
            a, phis = assemble_ritz(2, bc)
        print(f"\nMethod: {m}")
        print(f"a coefficients = {a}")
        terms = [f"{a[k]:.6e}*phi{k+1}(x)" for k in range(len(a))]
        print("u_approx(x) =", " + ".join(terms))

# BC1 & BC2 vs exact for n=2 and n=12
for n in [2, 12]:
    for bc, uref in [('BC1', u_ref_BC1), ('BC2', u_ref_BC2)]:
        plt.figure(figsize=(8,5))
        plt.plot(np.linspace(0,1,200), uref(np.linspace(0,1,200)),
                 'C0', label="Exact")
        for i, m in enumerate(methods):
            if m=='collocation':
                a, phis = assemble_collocation(n, bc)
            elif m=='least_squares':
```

```
            a, phis = assemble_least_squares(n, bc)
        elif m=='galerkin':
            a, phis = assemble_galerkin(n, bc)
        else:
            a, phis = assemble_ritz(n, bc)
        plt.plot(np.linspace(0,1,200),
                 eval_u(a, phis, np.linspace(0,1,200)),
                 color=colors[i], label=f"{m} (n={n})")
    plt.xlabel("x (m)")
    plt.ylabel("u(x) (m)")
    plt.title(f"{bc} - Methods vs Exact (n={n})")
    plt.legend()
    plt.grid(True)
    plt.show()
```

CODE

Two-term (N=2) coefficients and expressions for each method:

--- BC1 ---

- Method: collocation

c coefficients = [-1.21457142e-07 3.64371428e-07]

u_approx(x) = -1.214571e-07*phi1(x) + 3.643714e-07*phi2(x)

- Method: least_squares

c coefficients = [1.09561855e-07 4.82072327e-08]

u_approx(x) = 1.095619e-07*phi1(x) + 4.820723e-08*phi2(x)

- Method: galerkin

c coefficients = [-1.01214286e-07 -1.41700000e-07]

u_approx(x) = -1.012143e-07*phi1(x) + -1.417000e-07*phi2(x)

- Method: ritz

c coefficients = [1.01214286e-07 1.41700000e-07]

u_approx(x) = 1.012143e-07*phi1(x) + 1.417000e-07*phi2(x)

--- BC2 ---

- Method: collocation

c coefficients = [1.21457143e-07 -6.07285714e-08]

u_approx(x) = 1.214571e-07*phi1(x) + -6.072857e-08*phi2(x)

- Method: least_squares

c coefficients = [1.21457143e-07 -6.07285714e-08]

u_approx(x) = 1.214571e-07*phi1(x) + -6.072857e-08*phi2(x)

- Method: galerkin

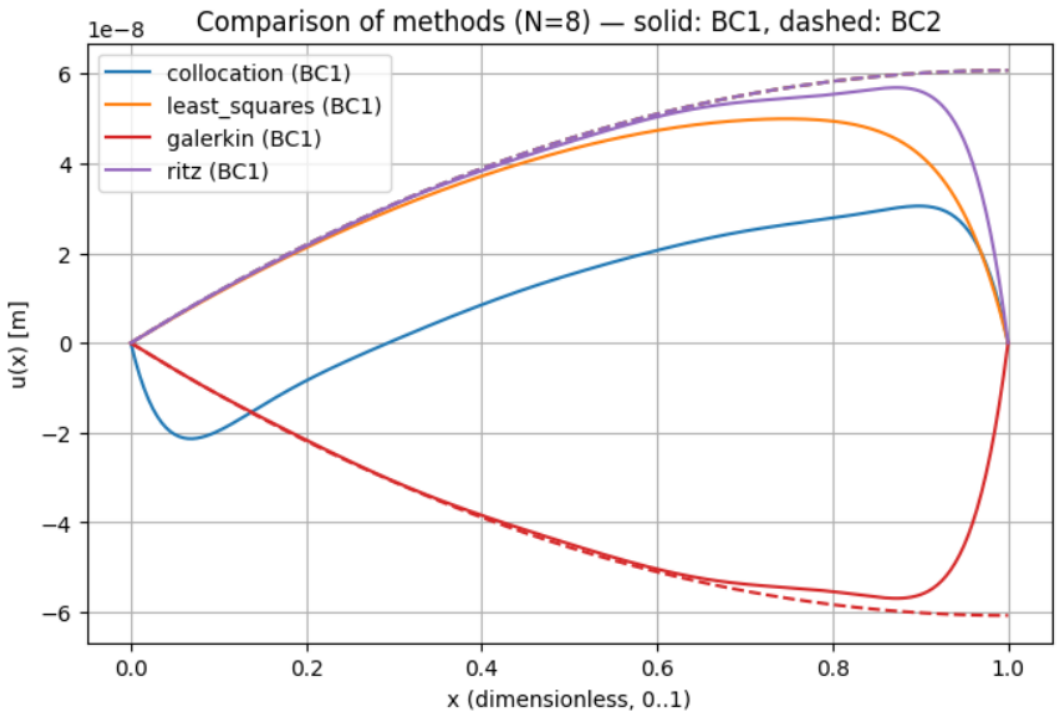
c coefficients = [-1.21457143e-07 6.07285714e-08]

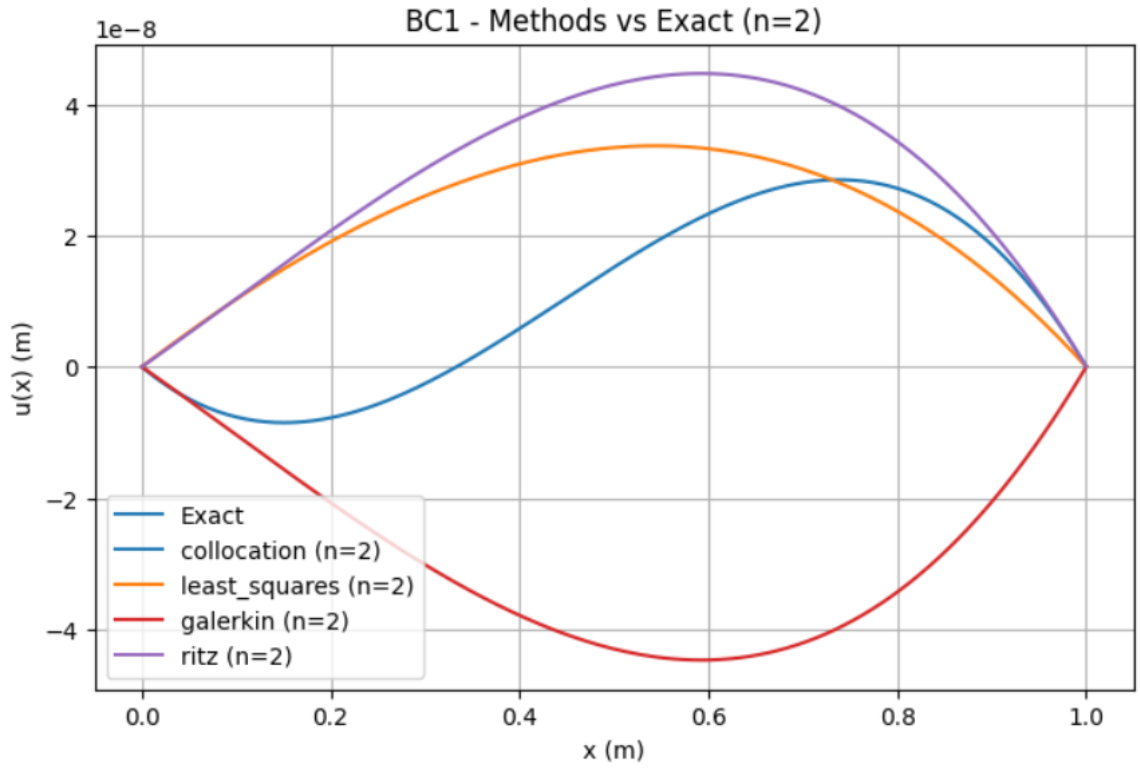
u_approx(x) = -1.214571e-07*phi1(x) + 6.072857e-08*phi2(x)

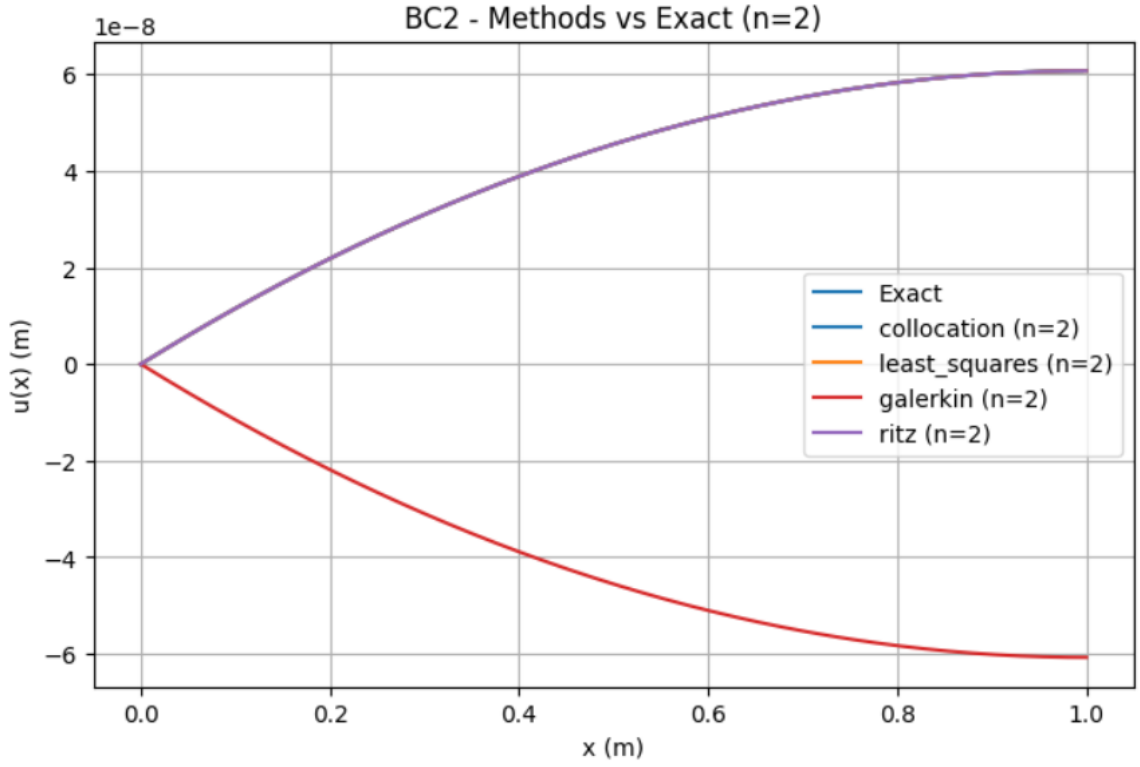
- Method: ritz

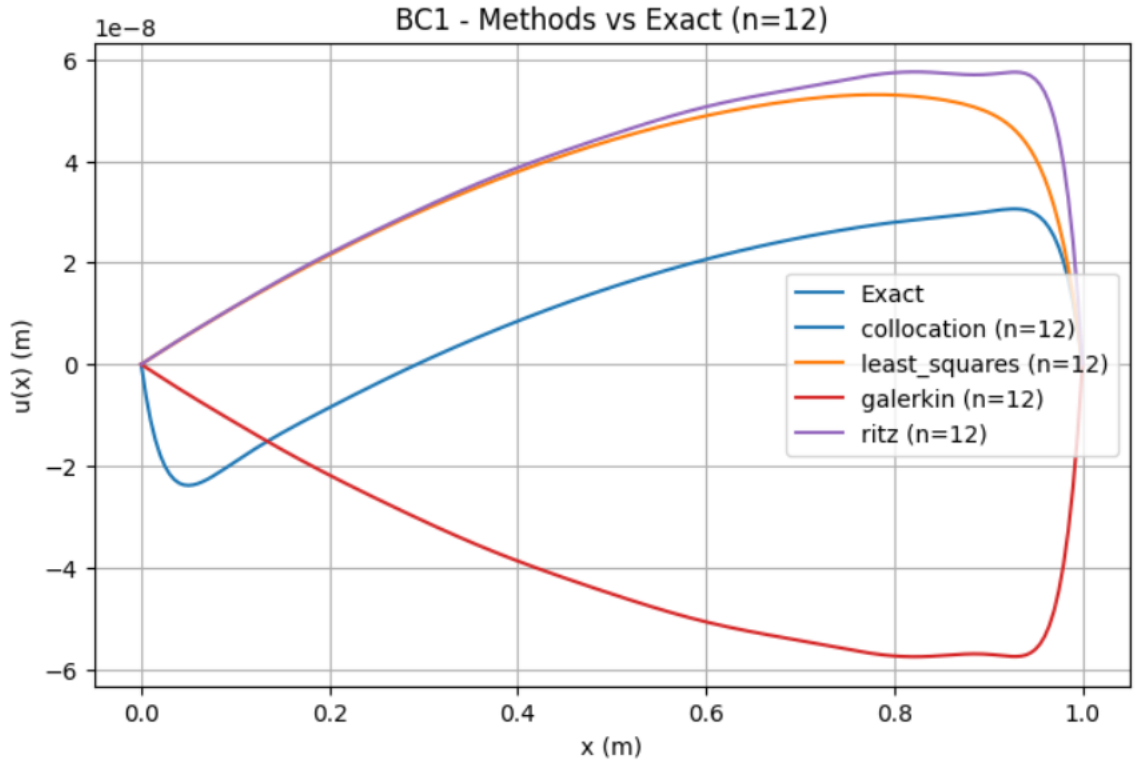
c coefficients = [1.21457143e-07 -6.07285714e-08]

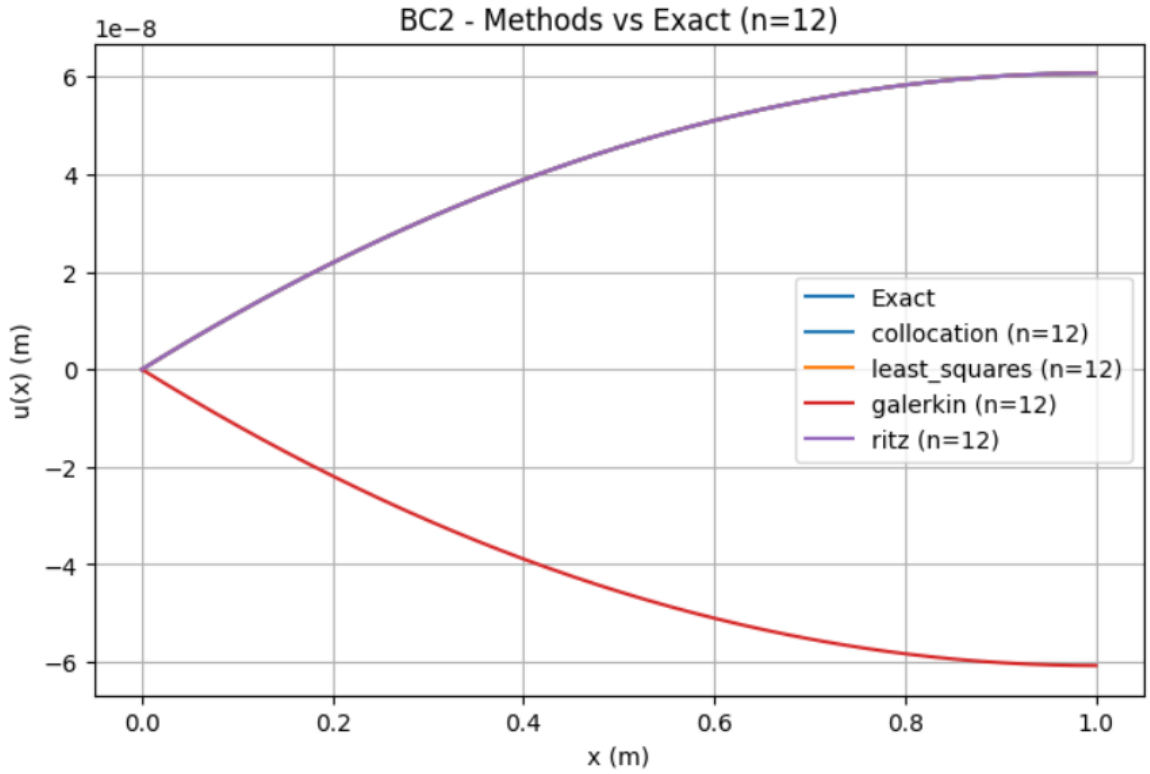
u_approx(x) = 1.214571e-07*phi1(x) + -6.072857e-08*phi2(x)




BC1 - Methods vs Exact (n=2)
1e−8
u(x) (m)
x (m)
Exact
collocation (n=2)
least_squares (n=2)
galerkin (n=2)
ritz (n=2)


BC2 - Methods vs Exact (n=2)
1e−8
u(x) (m)
x (m)
Exact
collocation (n=2)
least_squares (n=2)
galerkin (n=2)
ritz (n=2)





PROBLEM 2

import numpy as np

```
import matplotlib.pyplot as plt
from scipy.linalg import solve
from numpy.polynomial.legendre import leggauss

# ------------------------------
# Physical / problem parameters
# ------------------------------
k = 300.0          # W / (m C)
h = 100.0          # W / (m^2 C)
T_inf = 20.0       # C
T0_val = 100.0     # temperature at x=0 (C)
L = 1.0            # m (100 cm)

# ------------------------------
# Quadrature helper
# ------------------------------
def gauss_points_weights(nq, x0=0.0, x1=1.0):
    xs, ws = leggauss(nq)
    xs = 0.5*(x1-x0)*xs + 0.5*(x1+x0)
    ws = 0.5*(x1-x0)*ws
    return xs, ws

# ------------------------------
# Area definitions (m^2)
# ------------------------------
def A_uniform(x):
    return 100.0e-6 * np.ones_like(x)
```

```
def A_linear(x):
    return 100.0e-6 * (1.0 - x / L)

def A_parabolic(x):
    xi = x / L
    return 100.0e-6 * (xi**2 - 2.0*xi + 1.0)

# Perimeter for square cross-section: P = 4*sqrt(A)
def P_from_A(Avals):
    return 4.0 * np.sqrt(Avals)

# ------------------------------
# Reference solution: finite difference
# ------------------------------
def reference_fd(Afunc, Nx=800):
    # Nx intervals -> Nx+1 points
    x = np.linspace(0.0, L, Nx+1)
    dx = x[1] - x[0]
    Avals = Afunc(x)
    Pvals = P_from_A(Avals)

    # system arrays
    diag = np.zeros(Nx+1)
    low = np.zeros(Nx)    # subdiagonal (i,i-1)
    high = np.zeros(Nx)   # superdiagonal (i,i+1)
    rhs = np.zeros(Nx+1)

    # BC at x=0 (Dirichlet)
```

```
diag[0] = 1.0
rhs[0] = T0_val

# interior nodes i=1..Nx-1
for i in range(1, Nx):
    kA_w = k * 0.5 * (Avals[i] + Avals[i-1])
    kA_e = k * 0.5 * (Avals[i] + Avals[i+1])
    a_w = kA_w / dx**2
    a_e = kA_e / dx**2
    diag[i] = a_w + a_e + h * Pvals[i]
    low[i-1] = -a_w
    high[i] = -a_e
    rhs[i] = h * Pvals[i] * T_inf

# tip: insulated => T_N = T_{N-1} => T_N - T_{N-1} = 0
diag[Nx] = 1.0
low[Nx-1] = -1.0
rhs[Nx] = 0.0

# assemble matrix
M = np.zeros((Nx+1, Nx+1))
for i in range(Nx+1):
    M[i,i] = diag[i]
for i in range(Nx):
    M[i+1,i] = low[i]
    M[i,i+1] = high[i]

T = solve(M, rhs)
```

```
    return x, T


# ----------------------------
# Basis functions (phi_k vanish at x=0)
# phi_k(x) = (x/L)^(k+1)
# ----------------------------
def basis(x, n):
    xs = np.atleast_1d(x)
    # returns (n, len(xs))
    return np.vstack([ (xs/L)**(k+1) for k in range(n) ])

def dbasis(x, n):
    xs = np.atleast_1d(x)
    return np.vstack([ (k+1)/L * (xs/L)**k for k in range(n) ])

def eval_u_from_coeffs(a, x):
    phis = basis(x, len(a))
    return T0_val + np.dot(a, phis)

# ----------------------------
# Assemblers for the 4 methods
# ----------------------------
def assemble_ritz(n, Afunc, nq=160):
    xs, ws = gauss_points_weights(nq, 0.0, L)
    Avals = Afunc(xs)
    Pvals = P_from_A(Avals)
    phis = basis(xs, n)
    dphis = dbasis(xs, n)
```

```
    K = np.zeros((n,n))
    F = np.zeros(n)
    for i in range(n):
        for j in range(n):
            K[i,j] = np.dot(ws, k*Avals*dphis[i,:]*dphis[j,:] + h*Pvals*phis[i,:]*phis[j,:])
        F[i] = np.dot(ws, h*Pvals*(T_inf - T0_val)*phis[i,:])
    a = solve(K, F)
    return a

def assemble_galerkin(n, Afunc, nq=160):
    # identical to Ritz for this symmetric problem
    return assemble_ritz(n, Afunc, nq=nq)

def assemble_least_squares(n, Afunc, nq=220):
    xs, ws = gauss_points_weights(nq, 0.0, L)
    Avals = Afunc(xs)
    Pvals = P_from_A(Avals)
    phis = basis(xs, n)
    dphis = dbasis(xs, n)
    # D(phi) = -d/dx(k A dphi/dx) + h P phi
    kA_dphi = k * Avals * dphis      # shape (n, nq)
    d_kA_dphi = np.gradient(kA_dphi, xs, axis=1)  # approx derivative along x
    D_phi = -d_kA_dphi + h * Pvals * phis
    # R0 from T0 constant: R0 = h P (T0 - T_inf)
    R0 = h * Pvals * (T0_val - T_inf)
    M = np.zeros((n,n))
    rhs = np.zeros(n)
    for i in range(n):
```

```
        for j in range(n):
            M[i,j] = np.dot(ws, D_phi[i,:]*D_phi[j,:])
        rhs[i] = -np.dot(ws, D_phi[i,:]*R0)
    a = solve(M, rhs)
    return a

def assemble_collocation(n, Afunc):
    # choose n interior collocation points
    xs = np.linspace(0.0+1e-8, L-1e-8, n+2)[1:-1]   # length n
    Avals = Afunc(xs)
    Pvals = P_from_A(Avals)
    phis = basis(xs, n)   # (n, n)
    dphis = dbasis(xs, n)
    kA_dphi = k * Avals * dphis
    d_kA_dphi = np.gradient(kA_dphi, xs, axis=1)
    D_phi = -d_kA_dphi + h * Pvals * phis     # (n, n)
    R0 = h * Pvals * (T0_val - T_inf)
    # Solve D_phi.T * a = -R0  -> (n,n) * a = ...
    a = solve(D_phi.T, -R0)
    return a

# ----------------------------
# Conveniences
# ----------------------------
methods = {
    'ritz': assemble_ritz,
    'galerkin': assemble_galerkin,
    'least_squares': assemble_least_squares,
```

```
    'collocation': assemble_collocation
}

A_funcs = {
    'Uniform': A_uniform,
    'Linear': A_linear,
    'Parabolic': A_parabolic
}

x_plot = np.linspace(0.0, L, 400)

# ------------------------------
# Compute reference solutions for each area
# ------------------------------
ref_sols = {}
for name, Af in A_funcs.items():
    xr, Tr = reference_fd(Af, Nx=1000)   # fine FD
    ref_sols[name] = (xr, Tr)
    print(f"Computed FD reference for {name} (Nx=1000)")

# ------------------------------
# 1) Plot N=8 comparisons (all methods) for each area
# ------------------------------
N_plot = 8
for name, Af in A_funcs.items():
    plt.figure(figsize=(8,5))
    xr, Tr = ref_sols[name]
    plt.plot(xr, Tr, 'k-', lw=2, label='Reference (FD)')
```

```
    for mname, assembler in methods.items():
        a = assembler(N_plot, Af)
        Tapprox = eval_u_from_coeffs(a, x_plot)
        plt.plot(x_plot, Tapprox, label=f"{mname} (N={N_plot})")
    plt.title(f"[N=8] Methods comparison — {name} cross-section")
    plt.xlabel("x (m)")
    plt.ylabel("T (°C)")
    plt.legend()
    plt.grid(True)
    plt.show()

# ----------------------------
# 2) Two-term (N=2) coefficients printed for each area & method
# ----------------------------
print("\nTwo-term (N=2) coefficients (T(x) = T0 + a1*phi1 + a2*phi2):\n")
for name, Af in A_funcs.items():
    print(f"--- {name} cross-section ---")
    for mname, assembler in methods.items():
        a = assembler(2, Af)
        # format printing
        print(f"{mname:15s}: a = {np.round(a, 8)}")
    print()

# ----------------------------
# 3) For each area: plot N=2 and N=12 vs reference (separate figures)
# ----------------------------
for name, Af in A_funcs.items():
    xr, Tr = ref_sols[name]
```

```
# N = 2
plt.figure(figsize=(8,5))
plt.plot(xr, Tr, 'k-', lw=2, label='Reference (FD)')
for mname, assembler in methods.items():
    a = assembler(2, Af)
    Tapprox = eval_u_from_coeffs(a, x_plot)
    plt.plot(x_plot, Tapprox, label=f"{mname} (N=2)")
plt.title(f"{name} cross-section: N=2 comparison")
plt.xlabel("x (m)")
plt.ylabel("T (°C)")
plt.legend()
plt.grid(True)
plt.show()

# N = 12
plt.figure(figsize=(8,5))
plt.plot(xr, Tr, 'k-', lw=2, label='Reference (FD)')
for mname, assembler in methods.items():
    a = assembler(12, Af)
    Tapprox = eval_u_from_coeffs(a, x_plot)
    plt.plot(x_plot, Tapprox, label=f"{mname} (N=12)")
plt.title(f"{name} cross-section: N=12 comparison")
plt.xlabel("x (m)")
plt.ylabel("T (°C)")
plt.legend()
plt.grid(True)
plt.show()
```

```
# ----------------------------
# 4) Convergence: aggregated L2 error over three areas for N=1..12
# ----------------------------
Ns = list(range(1,13))
plt.figure(figsize=(10,6))
for mname, assembler in methods.items():
    err_list = []
    # Start collocation from N=2
    start_N = 2 if mname == 'collocation' else 1
    for N in range(start_N, 13):
        total_L2 = 0.0
        for name, Af in A_funcs.items():
            xr, Tr = ref_sols[name]
            a = assembler(N, Af)
            Tapprox_on_xr = eval_u_from_coeffs(a, xr)
            e = Tr - Tapprox_on_xr
            L2 = np.sqrt(np.trapz(e*e, xr))
            total_L2 += L2
        err_list.append(total_L2)
    # Adjust Ns for plotting collocation results
    plot_Ns = list(range(start_N, 13))
    plt.plot(plot_Ns, err_list, marker='o', label=mname)
plt.yscale('log')
plt.xlabel("Number of basis terms N")
plt.ylabel("Aggregated L2 error (sum over 3 cross-sections)")
plt.title("Convergence (aggregated L2) of methods")
plt.grid(True)
```

```
plt.legend()
plt.show()


# ----------------------------
# 5) Final: print summary of errors for N=2 and N=12 per area & method
# ----------------------------
print("\nSummary: L2 errors (per area & method) for N=2 and N=12\n")
for name, Af in A_funcs.items():
    xr, Tr = ref_sols[name]
    print(f"--- {name} ---")
    for N in [2, 12]:
        print(f" N = {N}:")
        for mname, assembler in methods.items():
            # Skip collocation for N=1 if it's still in the list
            if mname == 'collocation' and N < 2:
                continue
            a = assembler(N, Af)
            Tapprox_on_xr = eval_u_from_coeffs(a, xr)
            e = Tr - Tapprox_on_xr
            L2 = np.sqrt(np.trapz(e*e, xr))
            max_err = np.max(np.abs(e))
            print(f"   {mname:15s}: L2 = {L2:.6e}, max_abs = {max_err:.6e}")
    print()

print("Done.")
```

CODE

Two-term (N=2) coefficients (T(x) = T0 + a1*phi1 + a2*phi2):

- Uniform cross-section

ritz: c = [-291.02412021 229.86687755]

galerkin: c = [-291.02412021 229.86687755]

least_squares: c = [-311.45314622 263.16583673]

collocation: c = [-337.23653238 337.23653238]

- Linear cross-section

ritz: c = [-330.4099913   290.65882024]

galerkin: c = [-330.4099913   290.65882024]

least_squares: c = [-378.62207346 374.25426279]

collocation: c = [-347.61518273 351.58299575]

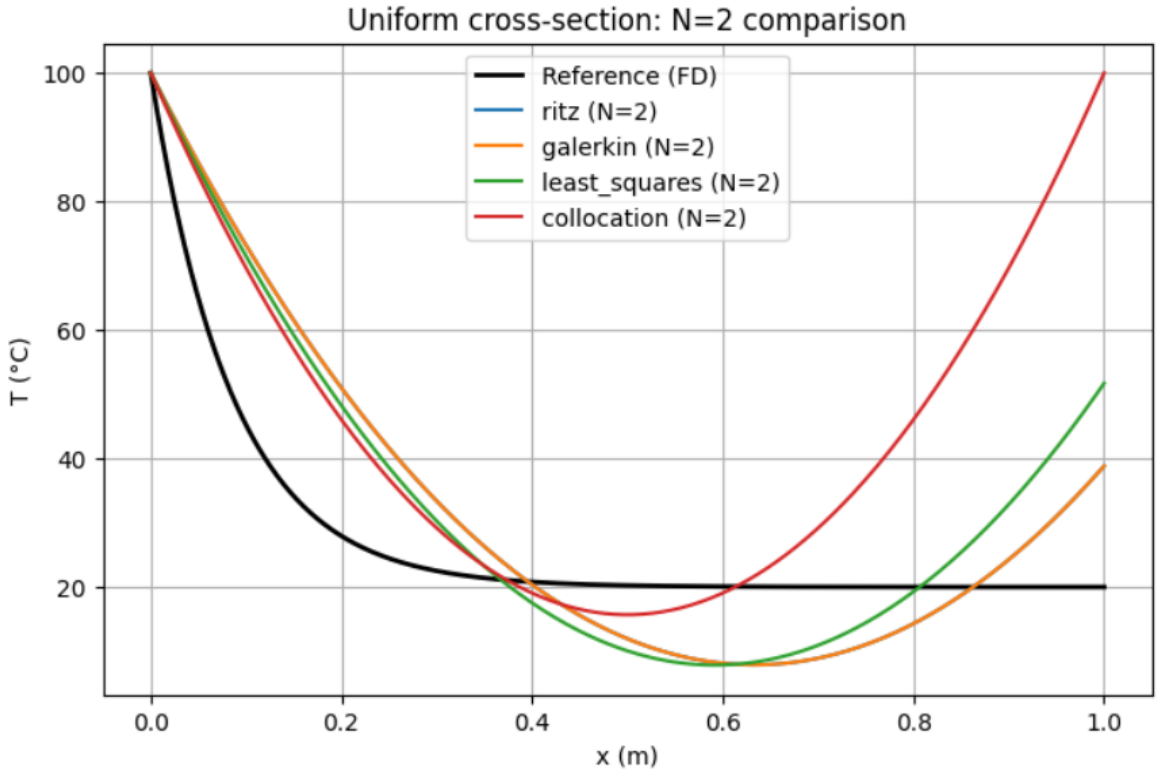
- Parabolic cross-section

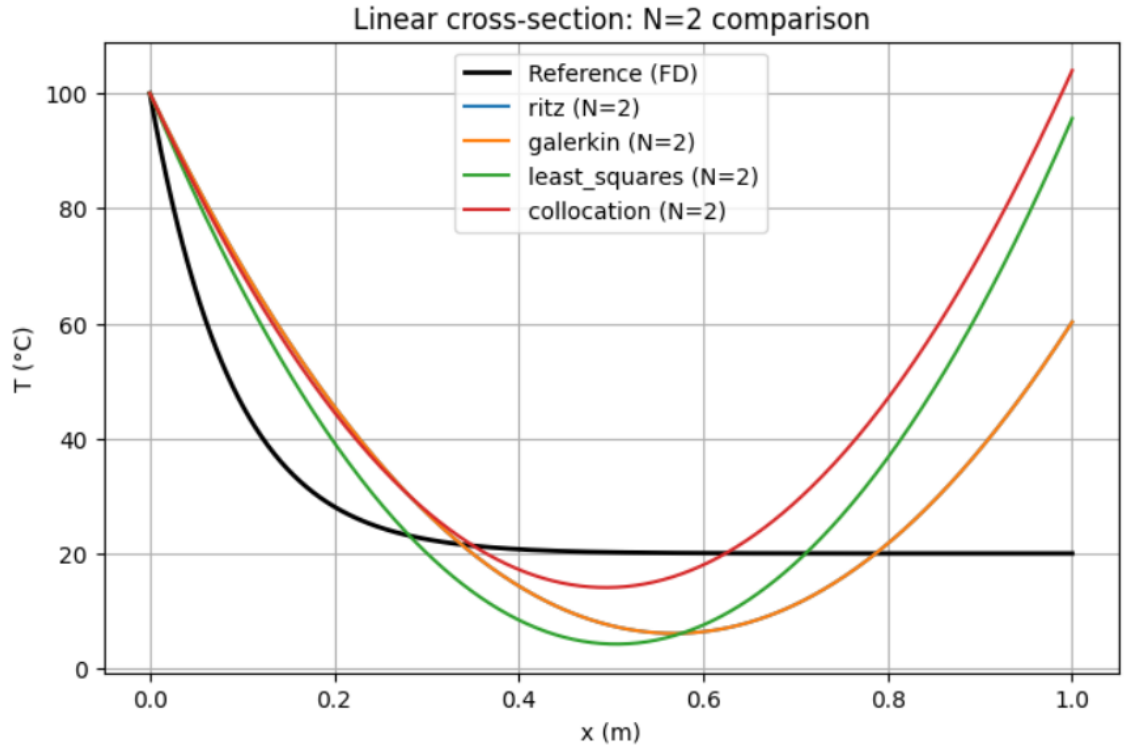
ritz: c = [-362.41465265 346.80828005]

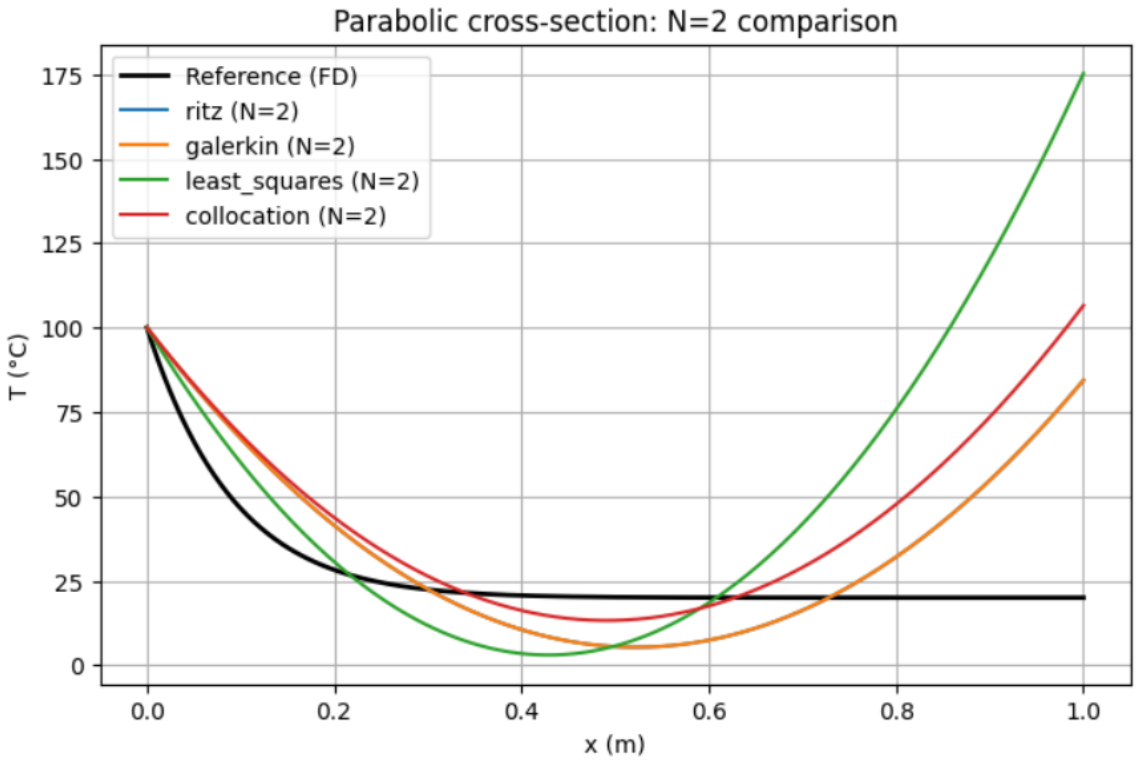
galerkin: c = [-362.41465265 346.80828005]

least_squares: c = [-452.47841719 527.71460554]

collocation: c = [-353.47037314 359.9999982]







L2 errors (per area & method) for N=2 and N=12

- Uniform

N = 2:

ritz: L2 = 1.409754e+01, max_abs = 2.837903e+01

galerkin: L2 = 1.409754e+01, max_abs = 2.837903e+01

least_squares: L2 = 1.451266e+01, max_abs = 3.171114e+01

collocation: L2 = 2.648003e+01, max_abs = 7.999844e+01

N = 12:

ritz: L2 = 1.585643e-04, max_abs = 3.317982e-04

galerkin: L2 = 1.585643e-04, max_abs = 3.317982e-04

least_squares: L2 = 1.421532e+01, max_abs = 6.830613e+01

collocation: L2 = 1.350953e+01, max_abs = 7.999845e+01

- Linear

N = 2:

ritz: L2 = 1.541824e+01, max_abs = 4.024867e+01

galerkin: L2 = 1.541824e+01, max_abs = 4.024867e+01

least_squares: L2 = 2.365457e+01, max_abs = 7.563203e+01

collocation: L2 = 2.726977e+01, max_abs = 8.396765e+01

N = 12:

ritz: L2 = 1.347436e-04, max_abs = 9.717044e-04

galerkin: L2 = 1.347436e-04, max_abs = 9.717044e-04

least_squares: L2 = 2.789194e-03, max_abs = 6.109117e-03

collocation : L2 = 1.674750e+01, max_abs = 1.182822e+02

- Parabolic

N = 2:

L2 = np.sqrt(np.trapz(e*e, xr))

ritz: L2 = 2.048396e+01, max_abs = 6.439363e+01

galerkin: L2 = 2.048396e+01, max_abs = 6.439363e+01

least_squares: L2 = 4.995653e+01, max_abs = 1.552362e+02

collocation: L2 = 2.784555e+01, max_abs = 8.652962e+01

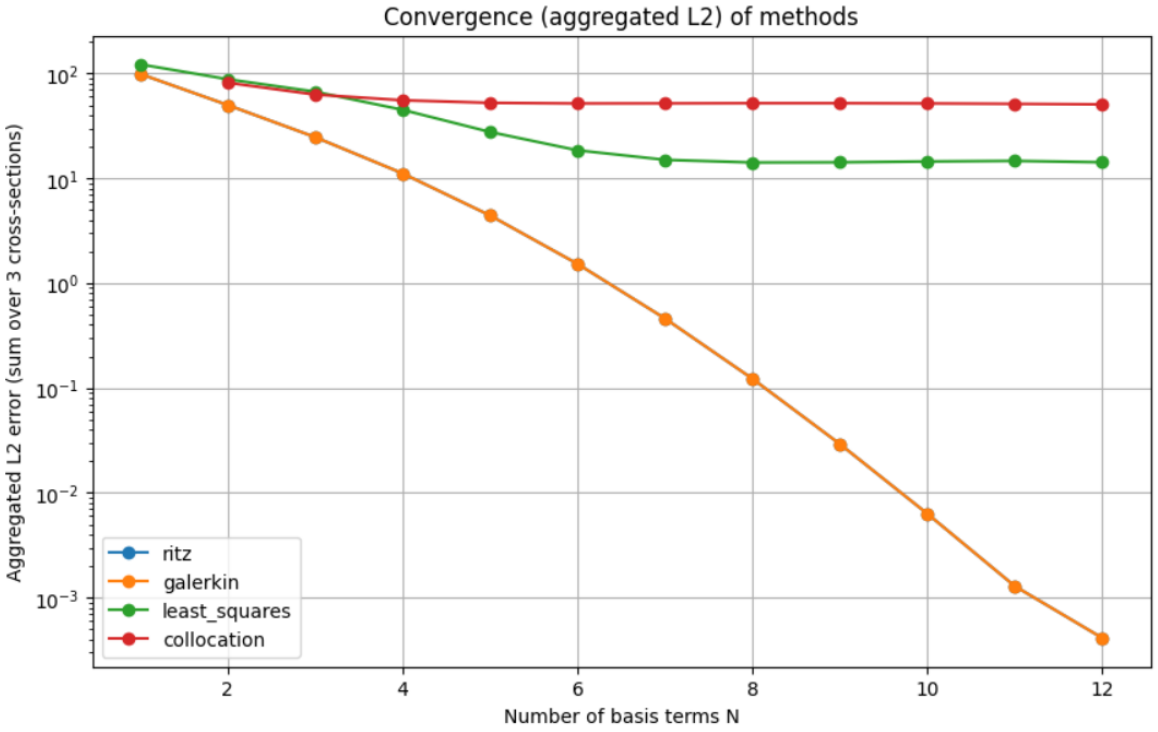
N = 12:

ritz: L2 = 1.192062e-04, max_abs = 1.302237e-03

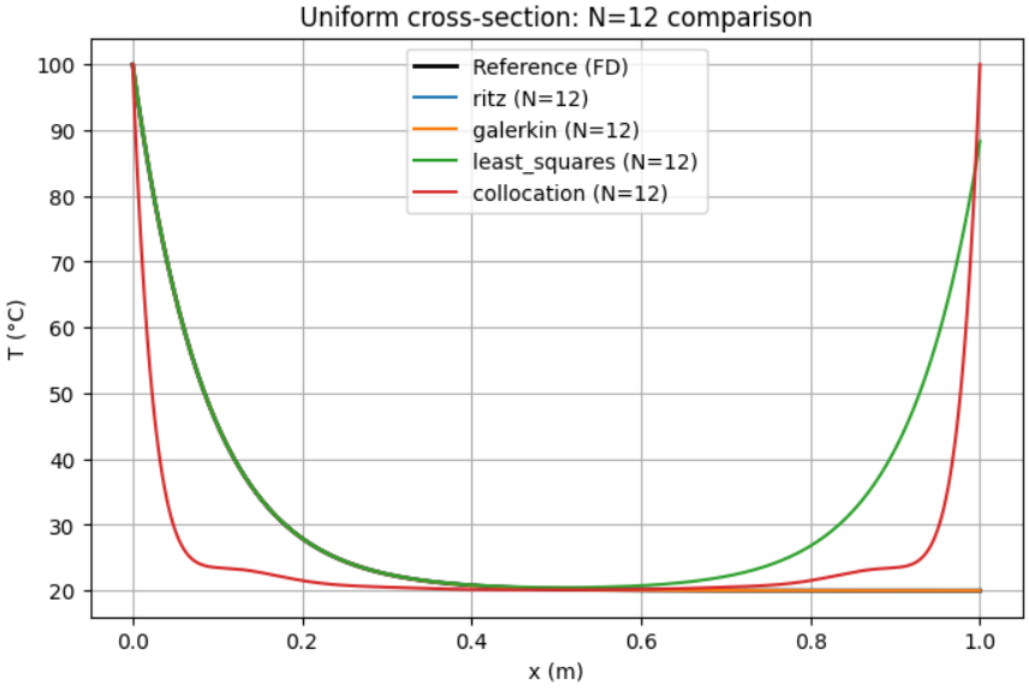
galerkin : L2 = 1.192062e-04, max_abs = 1.302237e-03

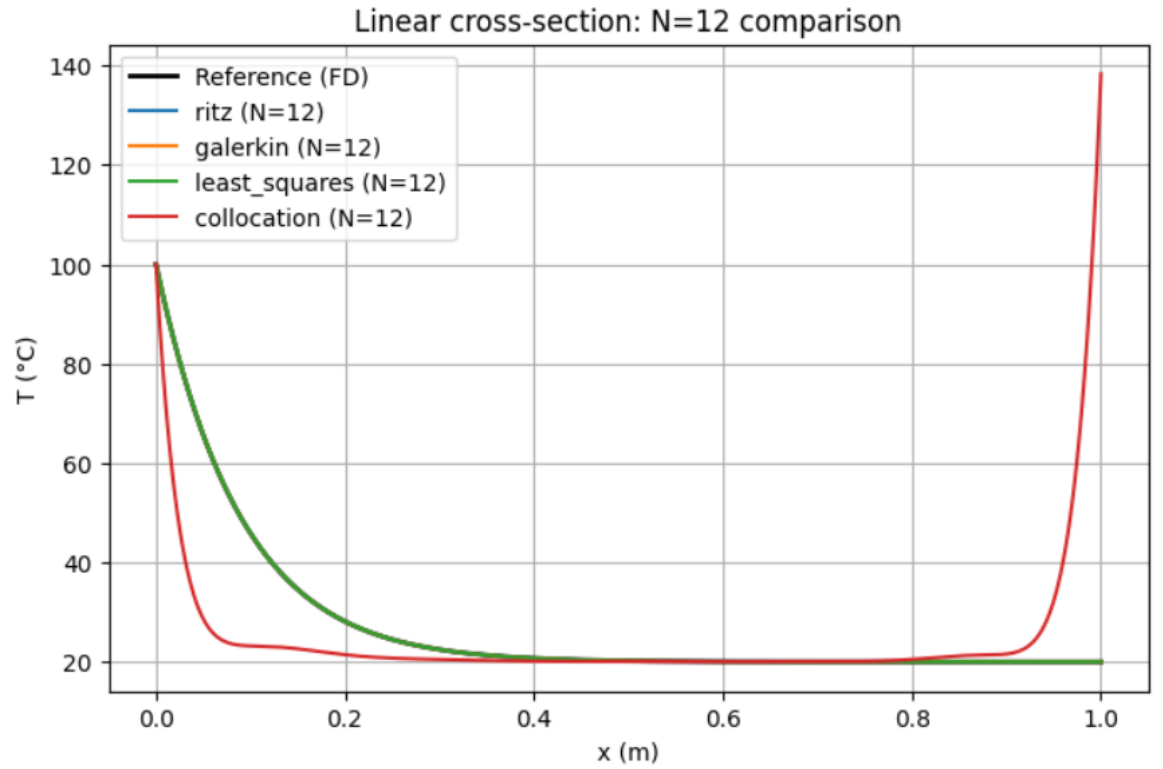
least_squares: L2 = 3.237296e-03, max_abs = 8.917016e-03

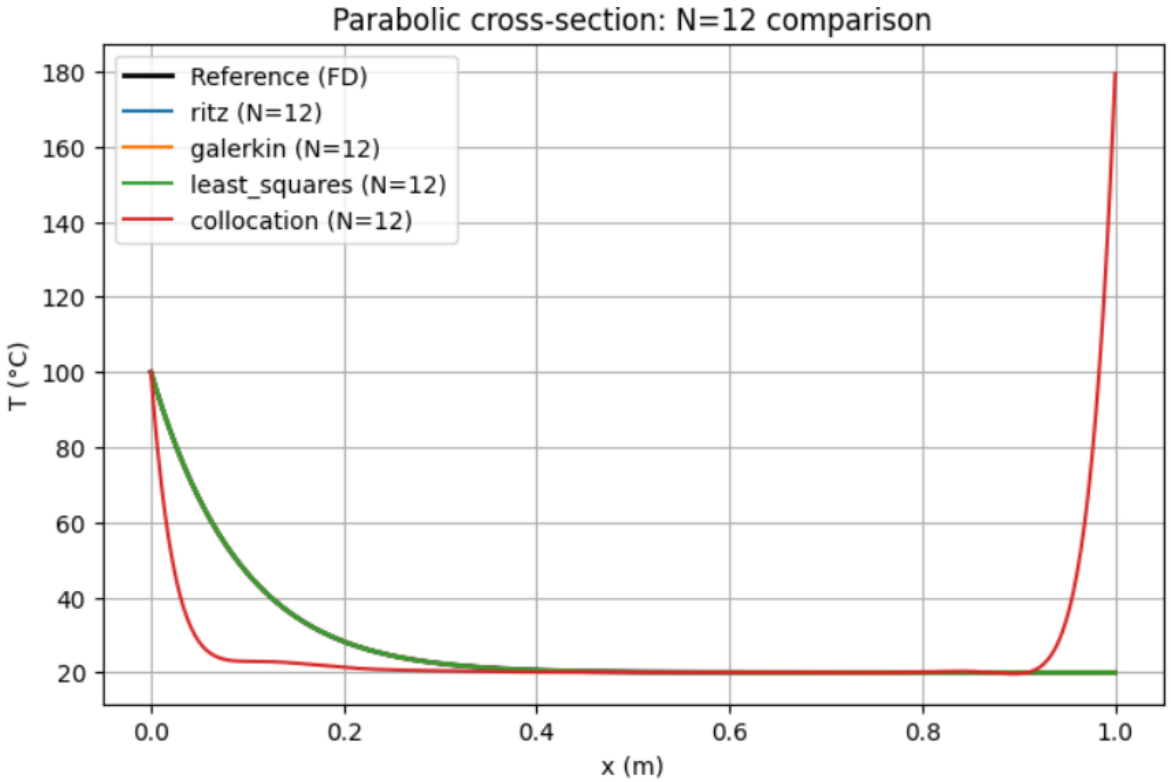
collocation: L2 = 2.049566e+01, max_abs = 1.594286e+02



Twelve-term (N=12) graphs:




Linear cross-section: N=12 comparison
Reference (FD)
ritz (N=12)
galerkin (N=12)
least_squares (N=12)
collocation (N=12)
T (°C)
x (m)
140
120
100
80
60
40
20
0.0
0.2
0.4
0.6
0.8
1.0


Parabolic cross-section: N=12 comparison
Reference (FD)
ritz (N=12)
galerkin (N=12)
least_squares (N=12)
collocation (N=12)
T (°C)
x (m)
180
160
140
120
100
80
60
40
20
0.0
0.2
0.4
0.6
0.8
1.0