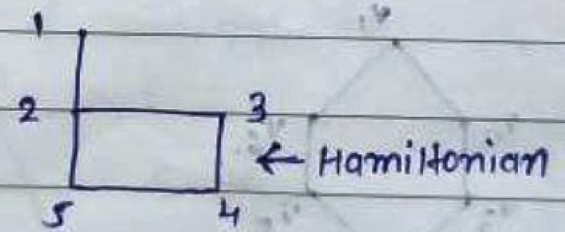
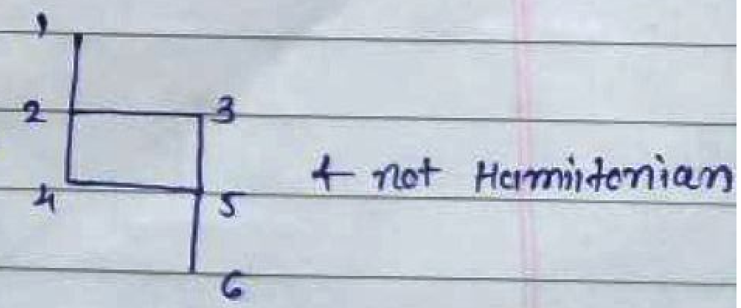
* <u>Hamiltonian path & circuit</u> :

→ In Hamiltonian Path or circuit, will pass through each of the vertex in a graph exactly once.

→ Graph which contains Hamiltonian circuit is called Hamiltonian graph.





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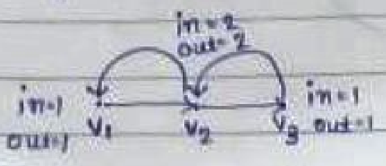
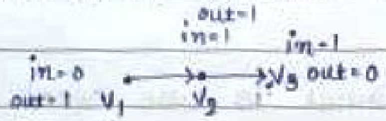
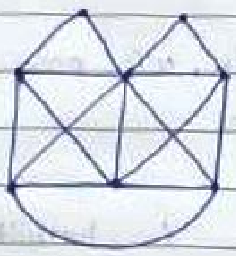
→ The directed graph contains euler circuit, if and only if it is connected and the incoming degree of each vertice is equal to its outgoing degree.



→ A directed graph contains euler Path if and only if it is Connected and indegree of each vertex is equals to outdegree except for 2 vertices where the diff. b/w indegree and outdegree is one.

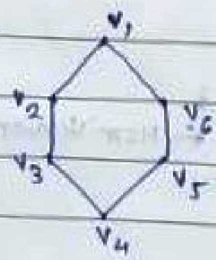


* Conditions for Hamiltonian Graph:



No Euler circuit
Euler Path

→ Let G is a graph with n vertices with the sum of degrees of each Pair of vertices is n-1 or more then there is a Hamiltonian path.

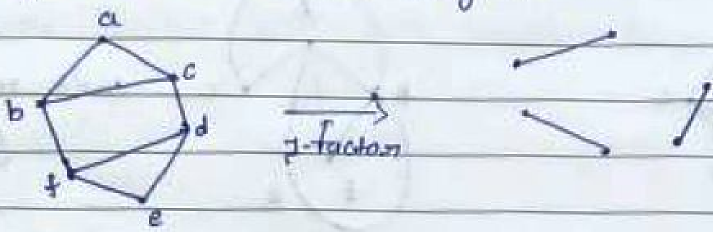


→ The above theorem is only a sufficient condition for the existance of Hamiltonian path.

→ Finding a Hamiltonian circuit and Path is a NP complete Problem.

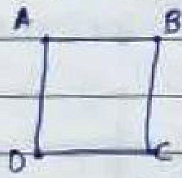
* Factors of the graph :

→ K-factor graph is a Spanning Subgraph of a original graph with the degree of each vertex being K

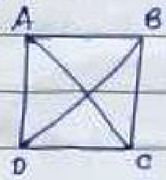


* ~~Standard~~ Planner Graph :

A graph is said to be planner if it can be drawn ~~in~~ on a Plane such that no two edges cross.



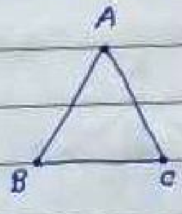
← planner



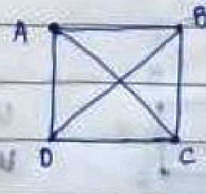
← not planner

* Kn (complete graph)

A complete graph of n vertices (Kn) is such a graph in which each pair of vertices are connected.

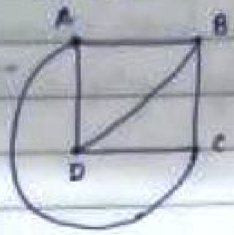


← K_3

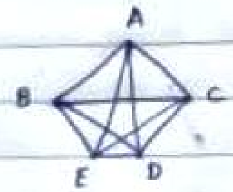


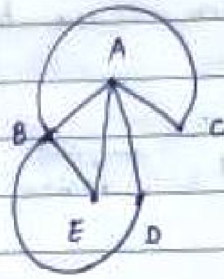
← K_4

K_4 planner graph:



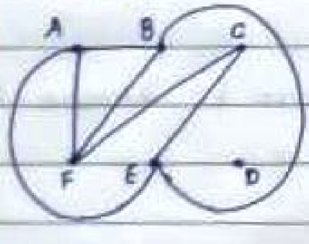
K_5





can't make planner graph

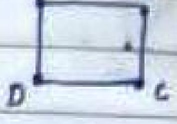
K_6



* Bipartite Graph:

→ It is a graph whose vertices can be divided into two independent set U and V such that every edge (u,v) either connects vertex from u to v or v to u.

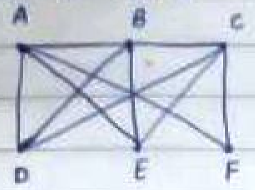
→ In other words $u \in U$ and $v \in V$.



$U = \{A, C\}$
$V = \{B, D\}$

* **Complete Bipartite Graph :**

→ In this graph every vertex of set U is connected with every vertex of set V



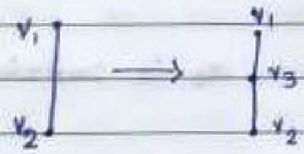
Simplest possible unplanner graph - $K_{3,3}$

* **Kuratwoski :**

K_5 and $K_{3,3}$ are reffered as Kuratwoski graph.

* **Homiomorphic Graph :**

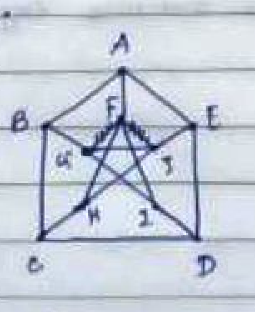
→ a homiomorphic graph of ∞ is a graph obtained by insertion or removal of a vertex of degree 2.



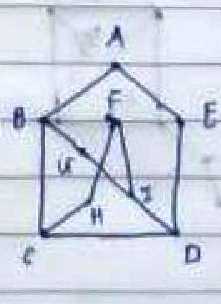
they are not Isomorphic graph.

* **Kuratwoski theorem :**

→ every non planner graph has a sub graph that is homiomorphic t K_5 or $K_{3,3}$.



Petersion Graph



Sub graph

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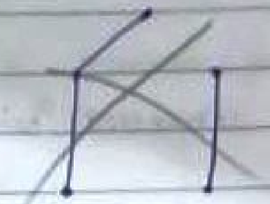
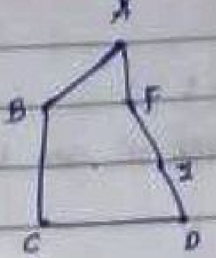
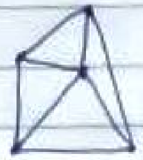


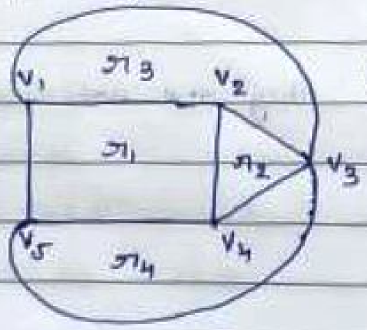




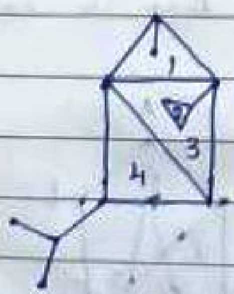
* Region of the graph :

→ Consider a planner graph, a region is an area of a plane that is bounded by edges and can not be further sub divided.

→ A Planner graph divides a plane into one or more regions in one of the regions is infinite.



r_5 is Infinite region



r_5 is infinite region.

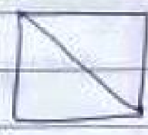
→ In a connected planner graph which has E edges and V vertices and r regions $v - e + r = 2$. (Euler's formula for planner graph)

* Proof of euler's formula :

→ basis of induction.
no of edges is one.

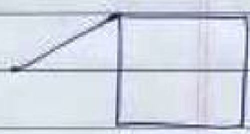
if $e = 1$ then $v = 2$, $r = 1$
$= 2 - 1 + 1$
$= 2$
basis is correct.

→ let's assume that the formula is true for $e = k$.
$v_k - e_k + r_k = 2$.



→ now, we need to prove $e = k+1$.
<u>Case-I</u> : so, $v_{k+1} - e_{k+1} + r_{k+1}$
$= v_k - (e_k + 1) + r_k + 1$
$= v_k - e_k + r_k$
$= 2.$

The formula holds when new edge is added.



<u>Case-II</u> : $v_{k+1} - e_{k+1} + r_{k+1}$
$= v_k + 1 - (e_k + 1) + r_k$
$= v_k - e_k + r_k$
$= 2.$

Tree

→ Tree is a connected acyclic graph.
→ Tree represents heirarchical relationship b/w the entities.

* Properties of the Tree :

→ There is a unique path b/w every two vertices in the tree
→ No. of vertices in the tree is one more than the number of edges. $(v = e+1)$
→ A Tree with two or more vertices has at least two leaves

<u>Proof</u> : Assume the no of vertices be n and leaves $= k$.

non-leaves $= n-k$

find out degree of vertices.

$$\sum_{i=1}^{n} d(v_i) = k(1) + (n-k)x$$

every non-leaf node has degree two at least.

$$\sum_{i=1}^{n} d(v_i) \geq k + 2(n-k)$$
$$\geq 2n - k.$$
$$2e \geq 2n - k$$
$$2(n-1) \geq 2n - k$$
$$2n - 2 \geq 2n - k$$
$$k \geq 2$$

* Characteristics of the tree :

→ Any graph in which there is a unique Path b/w every pair of vertices is a tree.
→ Connected graph with $e = v-1$ is a tree.

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Proof: let's assume there is a cycle in the graph.
→ If we remove the cycle from the graph then the number of edges will be $e = v - 2$. which will make the graph disconnected.
→ This is contradiction to a original statement. there is no cycle in the graph.

→ Graph with $e = v - 1$ that has no cycle is a Tree.

* Rooted Tree :

→ A directed graph is said to be directed tree if it becomes a tree where the directions are ignored.

→ rooted tree is a directed tree if there is exactly one vertex whose incoming degree is 0. and all other vertices directly or indirectly originate from the root.

→ All the nodes whose outdegree is not zero are internal nodes.

* Ordered Tree :

→ It is a rooted tree in which ~~it~~ edges originating from each branch node are numbered as 1, 2, 3 ... i.

→ An ordered tree in which every branch node has at most m children called m-ary tree.

→ An m-ary tree is called regular if every node has exactly m children.

* Path Length :

→ Path length of a vertex of rooted tree is number of edges ~~which~~ in ~~the~~ the path from root to the vertex.

→ Height of the tree is defined as max of Path length.

i = no of branch or internal nodes in a tree

t = no of leaf nodes in a tree

$i = t-1$

General formula for m-ary tree.

$(m-1)\, i = t-1$

→ 19 lamps to a single ele. outlet by extension board each board has 4 outlets.

$m = 4$

$t = 19$

$(4-1)\, i = 19-1$

$i = 6$

* Result related to path length.

I = sum of Path length of all branch nodes (Internal nodes)

E = sum of Path lengths of all leaf nodes

$E = I + 2i$

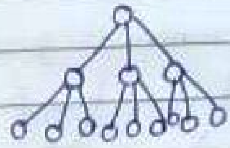
General formula for m-ary tree.

$E = (m-1) I + mi$

$I = 3$

$m = 3$

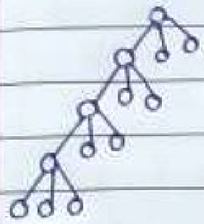
$i = 4$



$E = (3-1)3 + 3 \times 4$

$= 6 + 12$

$= 18$

→ 

$m = 3$

$I = 6$

$i = 4$

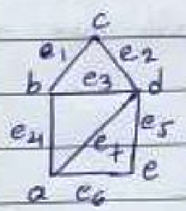
$E = (3-1) \times 6 + 3 \times 4$

$= 12 + 12$

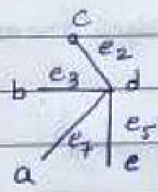
$= 24$

* Spanning Tree:

→ Spanning tree of connected graph is a spanning subgraph which is a tree.



→



Branches - $\{e_2, e_3, e_5, e_7\}$

Chords - $\{e_1, e_4, e_6\}$

→ Branch of a spanning tree is a set of edges which are present in a spanning tree.

→ The set of chords are also called complement of the spanning tree.

→ A connected graph always contains a spanning Tree.

→ In a connected graph with e edges and v vertices, there are v-1 branches in a spanning tree.

$b = v-1$

$c = e-(v-1) = e-v+1$

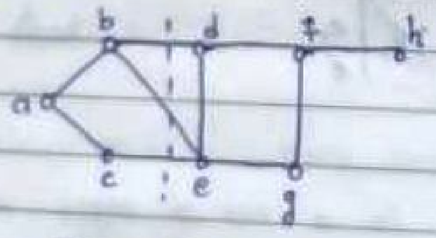
↑ no of chords

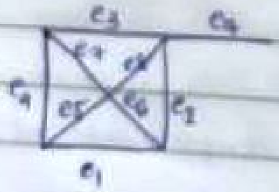
* Cut set :

→ cut set of a connected graph is a set of edges with the following Properties.

(i) removal of all edges from cut set will make the original graph disconnected

(ii) removal of some edges from cut set will not make original graph disconnected.





$g = \{e_1, e_3, e_7, e_5\}$

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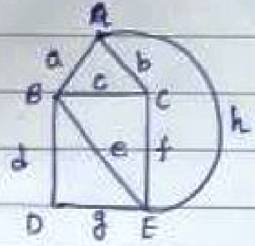
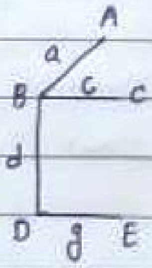
* Fundumental Circuit :



→ let T is spanning tree in a connected graph G where a cord is added to the spanning tree then exactly one circuit is formed. the circuit is called fundamental circuit.



chords - {b, e, f, h}

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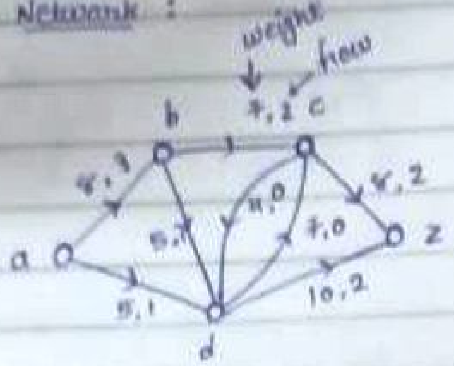
* Transport Network :



A weighted directed graph

(1) connected, no self loops

(2) Only one vertex, has no incoming edge (source)

(3) Only one vertex has no outgoing edge (sink)

(4) weight non-negative real no.

* Flow in the Network :

$\phi(i,j) \leftarrow$ flow

1. $\phi(i,j) \leq w(i,j)$
2. $\sum_{\text{all } i} (i,j) = \sum_{\text{all } k} (j,k)$

$$\phi_v = \sum_{\text{all } i} \underset{\substack{\uparrow \\ \text{source}}}{\phi(a,i)} = \sum_{\text{all } j} \underset{\substack{\uparrow \\ \text{sink}}}{(j,z)}$$

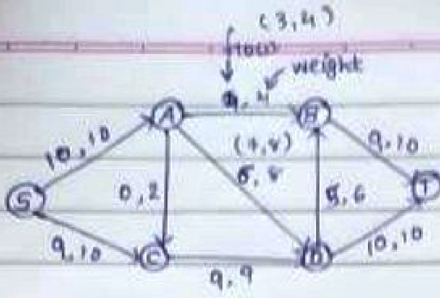
Cut :

$w(P, \bar{P})$

$P = \{a, d\}$

$\bar{P} = \{b, c, z\}$

$$\sum_{i \in P, j \in \bar{P}} w(i,j) = 8 + 7 + 10 = 25$$

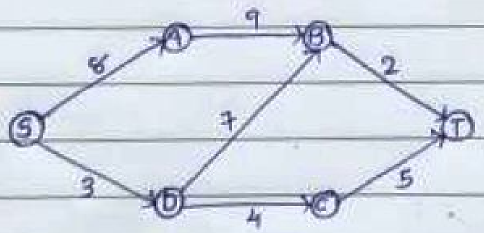


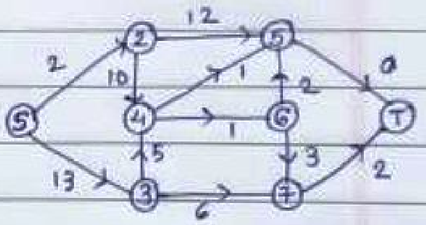
$W(P, \bar{P}) = P(S, C) \quad \bar{P}(A, B, D, T)$

$= 10 + 9 = 19$

Augmenting Path	Bottleneck capacity
S-A-B-T	4
S-A-D-T	6
S-C-D-T	4
S-C-D-B-T	5
	19

S-A-D-T	8
S-C-D-T	2
S-C-D-B-T	6
S-A-B-T	2
S-C-D-A-B-T	1
	19





Permutation:

⇒ r distinctly colored balls

n distinctly numbered boxes

$n \cdot (n-1) \cdot (n-2) \ldots (n-r+1)$

$\frac{n!}{(n-r)!} = P(n,r)$

⇒ 7 rooms

4 rooms : programmers

3 rooms : Terminals

$\underbrace{7 \times 6 \times 5 \times 4}_{\text{for programmers}} \times 1$

⇒ four digit numbers (Repeatation not allowed)

$9 \times 9 \times 8 \times 7$

⇒ 3 exam to schedule

5 days any no. of exams on a day.

$5 \times 5 \times 5$

⇒ n-digit quintary seq. with even no. of 1's.
(0, 1, 2, 3, 4)

$2, 3, 4 \rightarrow 3^n$

$0, 1, 2, 3, 4 \rightarrow 5^n$

$\frac{5^n - 3^n}{2} + 3^n$

⇒ Total number of distinct slips needed to Print all five digit numbers on slip of Papers with one number on each slip.

$\underline{10}\ \underline{10}\ \underline{10}\ \underline{10}\ \underline{10} = 10^5 - \frac{5^5 - 3\cdot 5^2}{2}$ slips needed

$64901 \rightarrow 10689 \mid 1, 0, 6, 8, 9 = \frac{5^5 - 3\cdot 5^2}{2}$

$16091 \rightarrow 16091$

⇒ r colored balls
n numbered boxes
q_1 - same color balls
q_2 - same color balls

$$\frac{P(n,r)}{q_1!\ q_2!}$$

⇒ 3 dash (–) 2 dots (·)

$$\frac{5!}{3!\ 2!}$$

⇒ r balls of same color
n numbered boxes

$$= \frac{n!}{(n-r)!\ r!} = C(n,r),\ {}^{n}C_{r},\ \binom{n}{r}$$

⇒ 11 MLA
committee of 5 members
$C(11,5)$
if A is already there $C(10,4)$
if we don't want A $C(10,5)$
if A or B should be there at least $C(9,4) + C(9,4) + C(9,3)$

Date: / /

* Discrete random Variable :

RV - discrete Sample space
- elements discontinouse

rolling 4 dice

rv - Sum of the outcome on faces on dice
$|S| = 6^4$

tossing 4 coins

rv - no of tails observed after tossing 4 coins
$|S| = 2^4$

$X = \{0, 1, 2, 3, 4\}$

* Probability distribution function (PDF)

If x is a rv with sample space S then a function denoted by f(x) or $P(X=x)$ and defined as $f(x) = P(X=x)$ + Probability for the rv $X=x$ is called the Probability function.

$P(X=0) = f(0) = \frac{^4C_0}{16}$

$P(X=1) = f(1) = \frac{^4C_1}{16}$

$P(X=2) = f(2) = \frac{^4C_2}{16}$

$f(3) = \frac{^4C_3}{16}$ $\quad f(4) = \frac{^4C_4}{16}$

* Find Pdf corresponding to the sum of numbers 2, 3, 4, 5 & 6 which appeared when rolling two dice once.

X = the sum on the faces of dice when rolled once

$X = \{2, \dots, 12\}$

$|S| = 36$

$P(X=2) = \frac{1}{36}$ // (1,1) $\quad$ $P(X=4) = \frac{3}{36} = \frac{1}{12}$ // (2,2), (1,3), (3,1)

$P(X=3) = \frac{1}{18}$ // (1,2) (2,1) $\quad$ $P(X=5) = \frac{4}{36} = \frac{1}{9}$ // (2,3) (3,2), (1,4), (4,1)

$P(X=6) = \frac{5}{36}$ // (1,5) (5,1) (2,4) (4,2) (3,3)

* Distribution Function : (CDF)

→ If X is a rv defined over the sample space S with Pdf (x) then a function given by

$$F(x) = P(X \le x) = \sum_{x_i \le x} f(x_i) \text{ is called cdf.}$$

* find cdf when $f(0) = f(1) = \frac{1}{3}$, when $f(2) = f(3) = \frac{1}{6}$

$X = \{0, 1, 2, 3\}$

$F(0) = \sum_{x \le 0} f(x) = f(0) = \frac{1}{3}$

$F(1) = \sum_{x \le 1} f(x) = f(0) + f(1) = \frac{2}{3}$

$F(2) = \sum_{x \le 2} f(x) = f(0) + f(1) + f(2) = \frac{1}{3} + \frac{1}{3} + \frac{1}{6} = \frac{5}{6}$

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$f(3) = \sum_{x \leq 3} f(x) = f(0) + f(1) + f(2) + f(3) = \frac{1}{3} + \frac{1}{3} + \frac{1}{6} + \frac{1}{6} = 1$

$$F_x(x) = \begin{cases} 0 & x \leq 0 \\ 1/3 & 0 \leq x < 1 \\ 2/3 & 1 \leq x < 2 \\ 5/6 & 2 \leq x < 3 \\ 1 & x \geq 3 \end{cases}$$

* $2P(X=1) = 3P(X=2) = P(X=3) = 5P(X=4)$

$X = \{1, 2, 3, 4\}$

$LCM = (2, 3, 1, 5) = 30$

$P(X=1) = 15K = 15/61$ $\quad$ $P(X=2) = 10K = 10/61$ $\quad$ $P(X=3) = 30K = 30/61$ $\quad$ $P(X=4) = 6K = 6/61$

$61K = 1$

$k = \frac{1}{61}$

$$F_x(x) = \begin{cases} 0 & x < 1 \\ 15/61 & 1 \leq x < 2 \\ 25/61 & 2 \leq x < 3 \\ \cancel{30/61}\ 55/61 & 3 \leq x < 4 \\ \cancel{6/61}\ 1 & x \geq 4 \end{cases}$$

* A coin is tossed twice let X be the no. of observed heads find the distribution f^n of X.

$X = \{0, 1,$

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* Suppose that a random variable we can take each with Probability half. find the distribution f^n.

* Conditional Probability :

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

* A coin was chosen and tossed. Prob. that the fair coin was chosen and

$P(\text{head}/\text{fair coin}) = \frac{1}{3}$

$P(\text{tail}/\text{fair coin}) = 1/3$

$P(\text{head}/\text{unfair coin}) = 1/12$

$P(\text{tail}/\text{unfair coin}) = 1/4$

$P(\text{head}) = \frac{1}{3} + \frac{1}{12} = \frac{5}{12}$

$P(\text{unfair}) = \frac{1}{12} + \frac{1}{4}$

$$P(\text{unfair}/\text{head}) = \frac{P(\text{unfair} \cap \text{head})}{P(\text{head})}$$

$$= \frac{1/12}{5/12}$$

$$= \frac{1}{5}$$

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* A : There is an ace
B : No two faces are same $\qquad P(B) = \frac{P(6,3)}{6^3}$

$$P(A|B) = \frac{P(A\cap B)}{P(B)}$$

$$P(A\cap B) = \frac{3\cdot 3\cdot P(5,2)}{6^3}$$

$$P(A|B) = \frac{3\cdot P(5,2)}{P(6,3)}$$

* Baye's Theorem :

-> If $A_1, A_2, \ldots, A_n$ are mutually disjoint events with $P(A_i) \neq 0$ then for arbitrary event E which is a subset of $\bigcup_{i=1}^{n} A_i$ such that $P(E) > 0$, we have

$$P(A_i/E) = \frac{P(A_i)\, P(E/A_i)}{\sum_i P(A_i)\, P(E/A_i)}$$

$$E \subset \bigcup_i A_i$$

$$E = E \cap \bigcup_i A_i$$

$$= \bigcup_i (E\cap A_i)$$

$$P(E) = P\left(\bigcup_i (E\cap A_i)\right)$$

$$= P\left(\sum_i (E\cap A_i)\right)$$

$$= \sum_i (P(E\cap A_i))$$

$$P(E) = \sum_i P(A_i)\, P(E/A_i)$$

$$P(A_i/E) = \frac{P(A_i \cap E)}{P(E)}$$

$$= \frac{P(A_i)\,P(E/A_i)}{\sum_i P(A_i)\,P(E/A_i)}$$

* A student knew only 60% of the questions in a test each with 5 answer. he simply guessed while answering the test. what is the Probability that he knew the ans to a question given that he answered it correctly.

A_1 : student knew the ans $\qquad P(A_1) = 0.6$

A_2 : Student guessed the ans $\qquad P(A_2) = 0.4$

E : Student answered it correctly

$P(E/A_1) = 1 \qquad P(E/A_2) = \frac{1}{5}$

$$P(A_1/E) = \frac{P(A_1)\cdot P(E/A_1)}{P(A_1)\,P(E/A_1) + P(A_2)\,P(E/A_2)}$$

$$= \frac{0.6}{0.6 + 0.4 \times 1/5}$$

$$= \frac{0.6}{0.68}$$

* Probabilities of x, y, z becoming managers is $4/9$, $2/9$, $1/3$ respectively. the Prob that the bonus Scheme will be introduced if x, y, z becomes managers are $3/10$, $1/2$, $4/5$ respectively. if the bonus is introduced what is the prob. that manager appo. was y ?

A_1 : x is a manager $\quad P(A_1) = 4/9$

A_2 : y is a manager $\quad P(A_2) = 2/9$

A_3 : z is a manager $\quad P(A_3) = 1/3$

E : bonus is introduced.

$P(E/A_1) = 3/10 \quad P(E/A_2) = 1/2 \quad P(E/A_3) = 4/5$

$$P(A_2/E) = \frac{P(A_2)\,P(E/A_2)}{P(A_1)P(E/A_1) + P(A_2)P(E/A_2) + P(A_3)P(E/A_3)}$$

$$= \frac{2/9 \times 1/2}{4/9 \times 3/10 + 2/9 \times 1/2 + 1/3 \times 4/5}$$

$$= \frac{1/9}{2/15 + 1/9 + 4/15}$$

$$= \frac{1/9}{2/5 + 1/9}$$

$$= \frac{1/9}{23/45}$$

$$= \frac{45}{23 \times 9} = \frac{5}{23}$$

* From a bag containing 3 white and 5 black balls. 4 balls are transfered into an empty bag. from this bag a ball is gone and it is found to be white. what is the Probability that out of 4 balls transfered 3 are white and 1 is black.

A_1 : 0 white 4 black $\quad {}^3C_0 \times {}^5C_4 / {}^8C_4$

A_2 : 1 white 3 black $\quad {}^3C_1 \times {}^5C_3 / {}^8C_4$

A_3 : 2 white 2 black $\quad {}^3C_2 \times {}^5C_2 / {}^8C_4$

A_4 : 3 white 1 black $\quad {}^3C_3 \times {}^5C_1 / {}^8C_4$

E :

$$P(A_4/E) = \frac{P(A_4)\ P(E/A_4)}{P(A_1)\ P(E/A_1) + P(A_2)\ P(E/A_2) + P(A_3)\ P(E/A_3) + P(A_4)\ P(E/A_4)}$$

$$= \frac{5/{}^{8}C_4}{}$$